

Competition between different groundwater uses under water scarcity

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Article published in *Water Resources and Economics* (2021)
DOI: 10.1016/j.wre.2020.100173

Abstract

We study groundwater management under a regime shock affecting water availability, using a dynamic common-property resource game. The different players correspond to different groundwater uses (irrigation or urban water supply), enabling us to consider competition between economic sectors for the stock with limited availability. The players have different water demand functions and, under certain circumstances depending on the shock, different discount rates. The effects of asymmetries in both demand and discount rates are analyzed, comparing cooperative and non-cooperative solutions. A numerical analysis for the particular case of the Western La Mancha aquifer in Spain is conducted to analyze the degree of inefficiency of non-cooperative solutions with respect to cooperative solutions in terms of welfare. We show that a higher asymmetry in discount rates reduces the inefficiency of non-cooperative solutions. The opposite result is obtained when considering the asymmetry in demand.

Keywords: Groundwater resource; Subgame Perfect Equilibria; Asymmetric players; Exogenous shock; Dynamic game

JEL: C72, Q25

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1 Introduction

Many studies have analyzed and compared cooperative versus non-cooperative solutions for common groundwater resources used for irrigation (e.g., Gisser and Sanchez [1980], Negri [1989], Provencher and Burt [1993], Rubio and Casino [2001]), given that groundwater is mainly used for irrigation in the majority of aquifers. These studies indicate that non-cooperative solutions are inefficient in terms of stock when compared to cooperative (or Pareto efficient) solutions. However, Gisser and Sanchez [1980] showed analytically that these solutions get closer when the capacity of the aquifer is very large, giving rise to the so-called Gisser-Sánchez effect (GSE). In the calculation of the non-cooperative (or competitive) solution, these authors considered that farmers behave myopically, that is, farmers make decisions over a short period of time without considering the future consequences of their actions at each moment. Subsequent studies (Negri [1989], Provencher and Burt [1993], Rubio and Casino [2001] and de Frutos Cachorro et al. [2019]) extended the work of Gisser and Sánchez by using game theory, confirming the GSE after taking into account the dynamic and strategic interactions among farmers. However, previous studies also pointed out that cooperation could not be justified if the difference in welfare between the solutions is relatively low. To estimate the degree of inefficiency of non-cooperative solutions in terms of welfare, numerical applications have been performed using real cases.

When there is a structural lack of water or a regime shock affecting water availability in an aquifer, competition tends to increase not only among the farmers, but also among the different types of uses due to the limited availability of the stock (i.e., stock externality). For example, as explained in De Stefano et al. [2015], the domestic water supply company uses groundwater mostly only during drought periods in the city of Madrid. Furthermore, recent studies have warned that due to the growing concerns about environmental issues, competition between uses (Kahil et al. [2016]) and/or the conflict between economic and environmental objectives (Pereau [2018]) could increase, especially under water scarcity. However, there are only a few studies in the literature on cooperative solutions where the users are heterogeneous (or asymmetric) and on the consequences of shocks on these solutions.

Concerning the optimal extraction behavior under water scarcity, most agro-economic studies treat it as a short-term problem (i.e., agronomic drought), where water scarcity is modeled as a decrease in potential crop yields (e.g., Reynaud [2009], Graveline and Mérel [2014], De Frutos Cachorro et al. [2017]). In this paper, we consider water scarcity as a long-term problem affecting the water availability of the resource (hydrological drought). We

model a shock on the aquifer dynamics that decreases the recharge rate and, subsequently, water availability, with this decrease maintained over time. Effects of similar shocks have already been studied in Tsur and Zemel [2014] and de Frutos Cachorro et al. [2014] for centralized management, as well as in de Frutos Cachorro et al. [2019] considering dynamic and strategic interactions among several farmers.

Focusing on the few economic studies on common resources that combine the above mentioned research lines, there are those that have assumed symmetric players (e.g., de Frutos Cachorro et al. [2019]) and those that have considered asymmetric users (e.g., Karp and Tsur [2011]). However, most of the studies on groundwater resources that have taken into account asymmetric players, such as Roseta-Palma and Brasão [2004] (asymmetric water demands and costs), Erdlenbruch et al. [2008] (asymmetric opportunity costs) and Saleh et al. [2011] (revenue parameters and quantity stock access), have not considered the possibility of shock occurrences in their modeling. Karp and Tsur [2011] addressed a relatively similar methodological problem to ours, that is, a consumption problem, with heterogeneous agents facing a shock in the system. They considered an intergenerational model in which each generation is represented by one agent and has different time preferences, giving rise to a problem of non-constant discounting over time. When we depart from the standard assumption of a common (unique) and constant discount rate of time preferences, a problem of time-inconsistency¹ arises, as had been already observed by Strotz [1955]. As a result, standard dynamic optimization techniques fail to characterize time-consistent optimal policies (in the sense that solutions calculated at a given time t are no longer optimal at a different time t' , for $t < t'$), and modified dynamic programming equations are required (see, for instance, Karp [2007] or De-Paz et al. [2013]). Moreover, Karp and Tsur [2011] introduced an uncertain and catastrophic shock that depended on the stock of greenhouse gasses and affected the utility function, comparing the Markov perfect equilibrium with the case of constant preferences over time.

In this paper, we study how the occurrence of an exogenous shock that decreases water availability influences competition between groundwater users showing asymmetries in demand and time preferences. Our contribution with respect to previous studies can be summarized as follows. First of all, in contrast to Karp and Tsur [2011], we consider the players to be simultaneous users of a groundwater resource and not the different generations that exploit a common resource over time. Moreover, as explained previously, the players

¹Collective temporal decisions when agents have different rates of time preferences - as is the case in our cooperative setting - lead to time-inconsistent aggregate time preferences, giving rise to efficiency problems (Jackson and Yariv [2015]).

face a different type of shock compared to that in Karp and Tsur [2011], which consists in an abrupt decay in the recharge rate of the aquifer at a given date. Second, in our work, the different players correspond to two types of groundwater uses, namely irrigation and drinking water (or domestic use), which have different water demands, as in Roseta-Palma and Brasão [2004]. As is usual in most economic studies, if we ignore the shock effect, the different players making the same use of groundwater (such as farmers) apply the same time preferences. However, we consider that the different users can also show differences in time preferences in some situations. In particular, we assume that farmers using water for irrigation react in advance to the future period of water scarcity by increasing their valuation of their current use of water (i.e., their discount rate) until the shock occurs. Indeed, since urban water use is usually prioritized over agricultural use in periods of drought (Molle and Berkoff [2006]), it is realistic to assume that farmers may become more impatient to extract water (leading to a higher discount rate) before the shock arrives. An application to the Western La Mancha aquifer is performed to analyze the effect of different types of asymmetries between the users (demand and discount rates asymmetries) on extraction behavior. We compare cooperative and non-cooperative solutions to estimate the inefficiency of non-cooperative solutions with respect to cooperative solutions in terms of stock and welfare. We find that the consideration of asymmetric uses in groundwater exploitation does not necessarily increase the inefficiency of non-cooperative behavior. This will depend on the type of asymmetry considered.

The paper is organized as follows. In Section 2, we present the theoretical game, while in Section 3 we solve the model for non-cooperation and (time-consistent) cooperation. In Section 4 we describe the results from the numerical analysis of the Western La Mancha aquifer, while Section 5 provides the conclusion.

2 The model

In this work, we adapt the game of exploitation of a common groundwater resource described in Rubio and Casino [2001] and de Frutos Cachorro et al. [2019] for two type of uses, namely irrigation and urban water supply. To simplify the modeling, the first use is characterized by a representative farmer, while the second use corresponds to a private urban water supplier operating over the entire water cycle (extraction, water treatment, storage and distribution). Concerning the private participation in the provision of urban water services, this has been an increasing movement since the the privatization wave of public monopolies and public services from the 1980s (see, e.g., González-Gómez et al.

[2014] for an analysis of this trend in Spain). In particular, in the Spanish case, while urban water supply and sanitation services are under municipal jurisdiction, municipalities can decide on how to provide these water services, and choose between a public, private or joint management model. As reported in Dige et al. [2017], a 40% of the population is provided by the private urban industry, although regardless of the type of management, tariffs must be approved by the public administration. In the case of the European Union, the EU Water Framework Directive (Directive 2000/60/EC) establishes that water-pricing policies must provide the adequate incentives to use water resources efficiently, under the principle of recovery of cost for water services, including the environmental cost. These services include abstraction, impoundment, storage, treatment and distribution of surface water or groundwater, as well as waste-water collection and treatment facilities. In the following, we describe the elements of the model.

2.1 Revenue function

For the group of farmers, we consider a representative agent with a linear demand for irrigation given by $g_f = a - bp_f^w$ ($a > 0$ and $b > 0$), where g_f represents the water extraction rate and p_f^w the price of water. Under the assumption that the farmer is a price taker in the output markets, the price of water² will equal the value of marginal product of water. We also assume that the agricultural production function exhibits constant returns to scale, and that production factors other than water and land are optimized conditioned to the water extraction rate. Under these assumptions, the representative farmer's revenue function can be obtained by integrating the inverse of the derived demand for water

$$\int p_f^w(g_f) dg_f = \int \frac{a - g_f}{b} dg_f = \frac{a}{b}g_f - \frac{1}{2b}g_f^2.$$

For the the urban water supplier, we follow a similar model construction than that of farmers, and consider a private firm with a concession contract that manages the entire water cycle, from abstraction to distribution. The water supplier has a linear demand for urban water provision given by $g_u = a_u - b_u p_u^w$ ($a_u > 0$ and $b_u > 0$) where g_u represents the water extraction rate and p_u^w the price of water as input. While urban water is provided on an exclusivity basis, final consumer prices are approved by a public entity³, so we consider

²In the case that water was provided free of charge, an alternative approach is to consider that this price represents the shadow price associated to a given maximum availability of water, i.e., how much the farmer would pay to relax that constraint (see, e.g., Tsur et al. [2004]).

³For illustrative purpose, see for instance Picazo-Tadeo et al. [2020] for a description of the administrative process of urban water pricing in Spain.

our water supplier as a price taker in its output market, and the price of water as input will equal the value of marginal product of water. Moreover, approved final water consumer prices are assumed to guarantee positive profits to the water supplier when providing the service. We also assume that the supplier production function exhibits constant returns to scale, and that production factors other than water and water infrastructures required for the service provision are optimized conditioned to the water extraction rate. From this, the water supplier firm's revenue function can be obtained by integrating the inverse of the derived demand for water

$$\int p_u^w(g_u) dg_u = \int \frac{a_u - g_u}{b_u} dg_u = \frac{a_u}{b_u} g_u - \frac{1}{2b_u} g_u^2 .$$

While input water demands for both irrigation and urban uses are independent, to simplify the analysis when introducing asymmetries, we write $a_u = \theta a$ and $b_u = \frac{\theta b}{k}$, i.e.

$$g_u = \theta \left(a - \frac{b}{k} p_u^w \right), \quad \text{with } 0 < \theta < 1, \quad \text{and } k \geq 1 . \quad (1)$$

In the equation above, θ is linked to the proportion between both water demands, and at their maximum it fixes the exact ratio. For example, according to different Spanish reports (e.g., Hernández-Mora et al. [2007]) and recent press releases (e.g., Greenpeace [2019]), just around 17% – 19% of groundwater pumped from aquifers is devoted to urban uses in most of places in Spain. This helps us to choose the value of the parameter θ . Parameter k describes the ratio between the reservation prices of urban users and farmers, and modifies the price elasticity of the demand function of urban use with respect to that of irrigation use. Price elasticities for both urban and agricultural water have been extensively studied in the literature at the theoretical and empirical level, and are highly dependent on specific circumstances as climate conditions, type of uses, diversity of crops, etc. In the Spanish case, and in fact for most of the EU-28 countries, residential water demand is price inelastic, as reported in Reynaud [2015], while water demand for agricultural use depends on the particular region. In Berbel Vecino et al. [2005], the authors report an elastic demand for agricultural use in the Duero basin case, in the northern of Spain, and a two-segment elasticity demand (inelastic for low prices and elastic for high prices) in the Guadalquivir basin case, in the southern of Spain. Since our aim is to explore the effects of introducing an asymmetry between the two uses that are here considered (i.e. urban supply and irrigation), we will assume without loss of generality that, concerning price elasticity of demand, the groundwater demand function for urban use is comparatively less elastic than that for irrigation use. The proposed values $\theta < 1$ and $k \geq 1$ recover all

the above mentioned hypotheses. Considering the demand for urban water described in equation (1), the revenue of the water supplier firm becomes

$$k \left(\frac{a}{b} g_u - \frac{1}{2b\theta} g_u^2 \right).$$

2.2 Cost function and aquifer dynamics

In our model, the cost supported by groundwater users and the description of the aquifer dynamics follow the standard assumptions in the literature. The marginal cost of extraction of user i is a linear function that depends on the stock of the aquifer G . The total costs of extraction of user $i \in \{f, u\}$ are therefore

$$\bar{C}_i = (z_i - cG)g_i, \quad z, c > 0,$$

where c is the slope of the marginal pumping cost function and z_i is all the remaining costs per unit. Since different users will have different needs (for example, in terms of water quality), the values of z_f and z_u can be different.

The dynamics of the aquifer is given by $\dot{G} = -(1 - \gamma) \sum_i g_i + r$, where r is the recharge rate and γ the return flow coefficient⁴, $\gamma \in [0, 1)$. As in de Frutos Cachorro et al. [2014, 2019], the system is disturbed by an exogenous and deterministic shock that leads to a sudden decrease on water availability from t_a on. We model this effect as a sudden reduction in the recharge rate, r , at time t_a , which is previously known by all the users. Thus, at time t_a , the recharge rate switches from $r = r_1$ to $r = r_2$, with $r_1 > r_2$. As a result, the dynamics of the water resource becomes

$$\dot{G} = \begin{cases} -(1 - \gamma) \sum_i g_i + r_1 & \text{if } t \leq t_a \\ -(1 - \gamma) \sum_i g_i + r_2 & \text{if } t > t_a, \end{cases} \quad (2)$$

with $r_1 > r_2$.

For many economic applications, assuming a deterministic value for t_a is actually less realistic than assuming a random value. However, in our model, this can be justified, for example, in case of policy implementation when the date of an exceptional extraction of groundwater sustained over time (equivalent to a sustained recharge rate reduction) is announced in advance. This exceptional extraction of groundwater from t_a onwards can be due to the construction of a reservoir, a transfer to another river basin or a particular

⁴We assume a unique return flow coefficient for the different uses in order to simplify the model. Potential implications of different values of γ are briefly discussed in Section 4.4.

need for other uses in case of drought. Moreover, the choice of a deterministic setting has important advantages for our analysis. Since we aim to study the effects of the different asymmetries of the players (in demand and discount rates) in the short term (before the time the event occurs) and in the long term (later on), this analysis is clearer if t_a is deterministic. From a mathematical point of view, the problem can also be solved for a random t_a following an exponential distribution.

2.3 Discount rates

In our model, we depart from the standard assumption of a unique and constant discount rate for both players. Moreover, the players know in advance that there will be a regime shift in the future, so they can anticipate the future effects of that change. More specifically, during water scarcity, the regulator could act by prioritizing urban uses over agricultural uses through quota implementation or/and by increasing water prices⁵. This risk should be taken into account by farmers. Based on these considerations, it seems natural to assume that until the shock event at moment t_a , farmers will assign a higher value to present profits (they exhibit more impatience) than urban users. Therefore, we analyze the case in which the discount rates of farmers and urban users, ρ_f and ρ_u , are given by

$$\rho_f = \begin{cases} \bar{\rho}_f & \text{for } t \leq t_a \\ \rho & \text{for } t > t_a \end{cases}$$

and

$$\rho_u = \begin{cases} \bar{\rho}_u & \text{for } t \leq t_a \\ \rho & \text{for } t > t_a \end{cases}$$

with $\bar{\rho}_f \geq \bar{\rho}_u = \rho$. In the numerical application in Section 4.4, we briefly analyze the situation in which discount rates remain unchanged under non-cooperation.

2.4 Problem statement

The problem of user $i \in \{f, u\}$ is to maximize his individual welfare, defined as the present value of his future profits. If ρ_i is the discount rate of user i , we must solve

$$\max_{g_i(\cdot)} \int_0^{\infty} F_i(G, g_i) e^{-\rho_i t} dt, \quad (3)$$

⁵We acknowledge that priority rules are not explicitly modeled in this paper and the introduction of them could be an interesting extension to the current work, as explained in Section 5.

where

$$F_f(G, g_f) = \frac{a}{b}g_f - \frac{1}{2b}g_f^2 - (z - cG)g_f, \quad (4)$$

$$F_u(G, g_u) = k \left(\frac{a}{b}g_u - \frac{1}{2b\theta}g_u^2 \right) - (z - cG)g_u, \quad (5)$$

subject to (2), with $G(0) = G_0$ given, and

$$g_i \geq 0, \quad G \geq 0, \quad i = f, u. \quad (6)$$

3 Model resolution

Our objective is to compare the results obtained under cooperation and non-cooperation in terms of extraction levels, stock and welfare. Both types of players, farmers and urban users, are assumed to be able to observe the level of the resource, i.e., the water table level, during the whole planning horizon. In this context, we calculate the corresponding subgame perfect non-cooperative and cooperative solutions. Furthermore, we briefly consider the non-cooperative case under myopic behavior and under open-loop information structure, with the objective to compare the level of inefficiency in the resource stock to that under cooperation. Detailed calculations are presented in the Appendix.

3.1 Subgame perfect non-cooperative Nash equilibrium

Under the Markovian information structure, farmers and urban users observe the level of the resource (i.e., the water table level) during the planning horizon, for example, by using groundwater monitoring instruments, and make their extraction decisions accordingly. To calculate the subgame perfect non-cooperative Nash equilibria (SPNE), we solve the problem described in Section 2.4 by applying the standard dynamic programming techniques that are described in detail in e.g. Dockner et al. [2000] and Haurie et al. [2012]. Since the dynamics changes at time t_a , we solve the problem, first, for the time period (t_a, ∞) , incorporating this result later on to find the solution in the interval $[0, t_a]$.

3.1.1 Solution for $t > t_a$

In the first step, for $t \in (t_a, \infty)$, the dynamic programming equation to solve for each user $i \in \{f, u\}$ is

$$\rho V_i^{NC+}(G) = \max_{\{g_i\}} \left\{ F_i(G, g_i) + (V_i^{NC+}(G))'(r_2 - (1 - \gamma)(g_i + \phi_j^{NC+}(G))) \right\}, \quad (7)$$

for $i, j \in \{f, u\}$, $j \neq i$. In (7), $\phi_j^{NC+}(G)$ denotes the strategy of player j . We focus our attention on stationary linear (affine) strategies in this linear-quadratic differential game, so that $\phi_j^{NC+}(G) = \alpha_j^{NC+}G + \beta_j^{NC+}$, in which case $V_j^{NC+}(G) = A_j^{NC+}G^2 + B_j^{NC+}G + C_j^{NC+}$. In Appendix A, Lemmas 1 and 2 present the equations to be verified by all these coefficients.

It remains to characterize the equilibria. If we integrate the differential equation describing the evolution of the stock of the resource for $t > t_a$ with the initial condition $G(t_a^+) = G_{t_a}$, we obtain

$$G(t) = (G_{t_a} - G_\infty^{NC})e^{-(1-\gamma)(\alpha_f^{NC+} + \alpha_u^{NC+})(t-t_a)} + G_\infty^{NC},$$

where G_∞^{NC} represents the steady state. The steady state is the solution to $\dot{G}(t) = r_2 - (1-\gamma)(\alpha_f^{NC+} + \alpha_u^{NC+})G_\infty^{NC} - (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+}) = 0$, i.e.

$$G_\infty^{NC} = \frac{r_2 - (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+})}{(1-\gamma)(\alpha_f^{NC+} + \alpha_u^{NC+})}.$$

Since we are interested in solutions converging to a steady state, we must impose the condition $\alpha_f^{NC+} + \alpha_u^{NC+} > 0$. In addition, since we are looking for interior solutions and $G(t) \geq 0$ for all t , we assume that the resource is not exhausted in finite time, therefore $r_2 \geq (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+})$. Hence, we look for stationary linear solutions satisfying

Condition A: $\alpha_f^{NC+} + \alpha_u^{NC+} > 0$, $r_2 \geq (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+})$.

In Lemma 2, to calculate α_f^{NC+} and α_u^{NC+} , we have to solve a system of two nonlinear equations. If players are symmetric ($k = \theta = 1$), in the symmetric equilibrium the equation system can be simplified to a second degree equation and the convergence condition typically selects one solution corresponding to a unique SPNE. In the asymmetric case, the situation is more complicated, since the equation system for α_f^{NC+} and α_u^{NC+} simplifies to a unique fourth degree equation, that can have up to four roots. Although the convergence condition to a steady state typically removes one of these roots, these linear quadratic differential games can have multiple equilibria, as illustrated e.g. in Theorem 8.10 in Engwerda [2005]. However, in our game, if Condition A is imposed, there exists, at most, one equilibrium. If r_2 is too small ($r_2 < (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+})$) for the unique solution to equations (A.3)-(A.6) (see Appendix A) satisfying Condition A, then there are no interior equilibria.

Proposition 1 *In Problem (3)-(6) for $t \geq t_a$, there exists, at most, one stationary linear subgame perfect non-cooperative equilibrium. If it exists, it is given by the unique solution to equations (A.3)-(A.6) satisfying Condition A.*

Proof: See Appendix A. □

Remark 1 *In Section 4.2, to analyze the sources of the stock inefficiency, we will compare the values of the steady state for the SPNE with that for the other non-cooperative solutions: the Nash equilibrium under open-loop information structure and the myopic solution. We refer to Appendix B for their calculation.*

3.1.2 Solution for $t \leq t_a$

In the second step, for $t \in [0, t_a]$, the dynamic programming equation to solve for each user is

$$\bar{\rho}_i V_i^{NC-}(G, t) - \frac{\partial V_i^{NC-}(G, t)}{\partial t} = \max_{\{g_i\}} \left\{ F_i(G, g_i) + \frac{\partial V_i^{NC-}(G, t)}{\partial G} (r_1 - (1 - \gamma)(g_i - \phi_j^{NC-}(G, t))) \right\},$$

for $i, j \in \{f, u\}$, $j \neq i$, with the initial condition $G(0) = G_0$, where $\phi_j^{NC-}(G, t)$ denotes the strategy followed by player j . In addition, we have the final conditions

$$V_i^{NC-}(G(t_a), t_a) = V_i^{NC+}(G(t_a^+)), \quad \text{for } i \in \{u, f\}.$$

As a result, the quadratic value functions $V_i^{NC-}(G, t) = A_i^{NC-}(t)G^2 + B_i^{NC-}(t)G + C_i^{NC-}(t)$ associated to the linear (affine) strategies $g_j^{NC-} = \phi_j^{NC-}(G, t) = \alpha_j^{NC-}(t)G + \beta_j^{NC-}(t)$, $i, j \in \{f, u\}$, satisfy $A_i^{NC-}(t_a) = A_i^{NC+}$, $B_i^{NC-}(t_a) = B_i^{NC+}$ and $C_i^{NC-}(t_a) = C_i^{NC+}$. For the calculation of $A_i^{NC-}(t)$, $B_i^{NC-}(t)$ and $C_i^{NC-}(t)$, and the corresponding values of functions $\alpha_j^{NC-}(t)$ and $\beta_j^{NC-}(t)$ describing the pumping levels for $t < t_a$, we use a numerical approximation method.

3.2 Subgame perfect cooperative solution

Next, we compute the cooperative solution as the sum of discounted individual payoffs (see equations (4) and (5)). As in the previous case, we will solve the problem in two steps. In the first step, for $t > t_a$, both types of players discount the future at the same rate. Hence, we can apply standard techniques for the problem starting at t_a with the initial condition $G(t_a) = G_{t_a}$. On the contrary, for $t \leq t_a$, since the discount rates are different during this period, the solution provided by standard optimal control theory becomes time-inconsistent. In the search for time-consistent solutions, we compute the subgame perfect cooperative equilibrium (SPCE) proposed in De-Paz et al. [2013] (we refer also to Ekeland et al. [2013]).

3.2.1 Solution for $t > t_a$

In the first step, in the time interval (t_a, ∞) , since $\rho_f = \rho_u = \rho$, we have to solve a standard optimal control problem. The dynamic programming equation is

$$\rho V_f^{C+}(G) + \rho V_u^{C+}(G) = \max_{\{g_f, g_u\}} \left\{ F_f(G, g_f) + F_u(G, g_u) + \left((V_f^{C+}(G))' + V_u^{C+}(G)' \right) (r_2 - (1 - \gamma)(g_f + g_u)) \right\}, \quad (8)$$

where $V_i^{C+}(G) = A_i^{C+}G^2 + B_i^{C+}G + C_i^{C+}$, $i \in \{u, f\}$. This is a standard linear-quadratic optimal control problem, whose solution is presented in Appendix C. If we analyze the steady state, from equations (C.5), (C.6) and (C.7) in Appendix C, it becomes clear that

$$\frac{\partial G_\infty^C}{\partial r_2} > 0, \quad \frac{\partial g_{i,\infty}^C}{\partial r_2} > 0.$$

As a result, when the value of the recharge rate after the occurrence of the shock, r_2 , decreases (respectively increases), both the level of the stock and the pumping rates of both users at the steady state decrease (respectively increase) for the SPCE, in agreement with the symmetric case (de Frutos Cachorro et al. [2019]). Moreover, from (2), it becomes clear that at the steady state, the sum of the pumping rates coincides for all the solution concepts and is given by

$$g_{f,\infty} + g_{u,\infty} = \frac{r_2}{1 - \gamma}.$$

Concerning the "demand asymmetry effect" on the stock at the steady state, from equation (C.5) in Appendix C we obtain

$$\frac{\partial G_\infty^C}{\partial k} = \frac{\theta}{(k^2 + \theta^2)bc} \left[\frac{r_2}{1 - \gamma} - a(1 + \theta) \right].$$

Therefore, for $a(1 + \theta) > \frac{r_2}{1 - \gamma}$, i.e. when a (the demand parameter) is clearly higher than $\frac{r_2}{1 - \gamma}$ (as is the case of the Western La Mancha aquifer analyzed in Section 4), the stock of the resource at the steady state under cooperation decreases the higher the demand asymmetry. On the contrary, if the recharge rate is very high in comparison to the demand parameter, the introduction of asymmetric demands can be profitable in terms of the stock of the resource in the long run.

3.2.2 Solution for $t \leq t_a$

In the second step, for $t \in [0, t_a]$, since the discount rates in this period are different, joint preferences become time-inconsistent. Subgame perfect cooperative decision rules can be

obtained by solving

$$g_i^{C-} = \phi_i^{C-}(G, t) = \arg \max_{\{g_i\}} \left\{ F_f(G, g_f) + F_u(G, g_u) + \left(\frac{\partial V_f^{C-}(G, t)}{\partial G} + \frac{\partial V_u^{C-}(G, t)}{\partial G} \right) (r_1 - (1 - \gamma)(g_f + g_u)) \right\}, \quad i \in \{f, u\}, \quad (9)$$

together with

$$V_f^{C-}(G, t) = \int_t^{t_a} F_f(G(s), \phi_f^{C-}(G(s), s)) e^{-\bar{\rho}_f(s-t)} ds + e^{-\bar{\rho}_f(t_a-t)} V_f^{C+}(G_{t_a^+}),$$

$$V_u^{C-}(G, t) = \int_t^{t_a} F_u(G(s), \phi_u^{C-}(G(s), s)) e^{-\bar{\rho}_u(s-t)} ds + e^{-\bar{\rho}_u(t_a-t)} V_u^{C+}(G_{t_a^+}).$$

Alternatively, we can write

$$\bar{\rho}_f V_f^{C-}(G, t) + \bar{\rho}_u V_u^{C-}(G, t) - \frac{\partial V_f^{C-}(G, t)}{\partial t} - \frac{\partial V_u^{C-}(G, t)}{\partial t} = F_f(G, \phi_f^{C-}(G, t)) + F_u(G, \phi_u^{C-}(G, t)) + \left(\frac{\partial V_f^{C-}(G, t)}{\partial G} + \frac{\partial V_u^{C-}(G, t)}{\partial G} \right) (r_1 - (1 - \gamma)(\phi_f^{C-}(G, t) + \phi_u^{C-}(G, t))).$$

For more details, we refer to De-Paz et al. [2013] (equations (22-24) and (32-33)) and Ekeland et al. [2013] (Theorems 3-4 and equation (5.19)).

For this linear-quadratic problem, linear decision rules exist and are obtained by taking

$$V_i^{C-}(G) = A_i^{C-}(t)G^2 + B_i^{C-}(t)G + C_i^{C-}(t), \quad i \in \{f, u\}.$$

As with the SPNE, the solution is calculated by using a numerical approach.

4 Numerical application

In this section, we apply the theoretical model described in Section 2 to data from the Western La Mancha (WLM) aquifer, using the parameter values reported by Esteban and Albiac [2011], Esteban and Dinar [2016] and de Frutos Cachorro et al. [2019], which are listed in Table 1.

The WLM aquifer is located in the southern part of central Spain in a semi-arid region where dry periods are frequent. In the last decades of the 20th century, the WLM aquifer suffered a critical decrease in water table levels due to the development of an intensive

Parameters	Description	Units	Value
b	Water demand slope	(Million Cubic Meters/Year) ² Euros ⁻¹	0.097
a	Water demand intercept	Million Cubic Meters/Year	4400.73
z_f	Pumping costs intercept farmer	Euros/Million Cubic Meters	266 000
z_u	Pumping costs intercept urban	Euros/Million Cubic Meters	266 000
c	Pumping costs slope	Euros/(Million Cubic Meters) ²	3.162
G_0	Stock level (in volume)	Million Cubic Meters	80960
γ	Return flow coefficient	<i>unitless</i>	0.2
θ	Proportion demand uses	<i>unitless</i>	0.16
r_1	Natural recharge before shock	Million Cubic Meters/Year	360
ρ	Farmer discount rate after shock	Year ⁻¹	0.05
ρ_u	Urban discount rate	Year ⁻¹	0.05
r_2	Natural recharge after shock	Million Cubic Meters/Year	$r_2 \in [300, 360]$
k	water demand parameter	(Million Cubic Meters/Year) ⁻² Euros	$k \in [1, 2]$
$\bar{\rho}_f$	Farmer discount rate before shock	Year ⁻¹	$\bar{\rho}_f \in [0.05, 0.1]$

Table 1: Values of parameters of the Western La Mancha aquifer.

irrigated agriculture coupled with inefficient management regimes. Despite the implementation of efficient policy measures such as water restrictions, the problem of water scarcity is still present and is expected to become severe in a future climate change scenario due to the increases in the magnitude and intensity of drought periods. In the WLM aquifer, around 80-95% of the groundwater is used for irrigation depending on the year and the specific agricultural area (see e.g. the extraction regimes of 2017 in Unión CLM [2019]),

with the rest mainly used for domestic or urban use. Therefore, we can assume that the proportion between the groundwater demand for irrigation use and that for domestic use (see Section 2.1) corresponds approximately to $\theta = \frac{1}{6}$. The demand parameters of both users also differ in the presence of k , which takes into account the fact that urban water use is typically more inelastic to changes in prices than agricultural use ($k \in [1, 2]$ in numerical simulations). Moreover, in periods of scarcity, groundwater is usually prioritized for domestic use over irrigation. As explained in Section 2.3, we assume that the impatience and, subsequently, the discount rate of the farmer ρ_f may increase before the shock occurs ($\bar{\rho}_f \in [0.05, 0.1]$ in numerical simulations). The urban discount rate before and after the shock and the farmer discount rate after the shock are therefore set to 0.05. Finally, we simulate a shock in the aquifer dynamics from a given date that decreases the aquifer recharge rate ($r_2 \in [300, 360]$ in numerical simulations).

In what follows, numerical simulations are shown, in which the extraction behavior and stock implications are compared for the different solutions and asymmetries, before the shock occurrence and in the long run. To disentangle the effects of different asymmetries, we first analyze simulated results for symmetric costs and return flow coefficients between the users. Additional simulations are described in Section 4.4 demonstrating the robustness of our numerical results when considering additional asymmetries. We also present a welfare analysis to validate the numerical results in terms of stock in Section 4.3. The results are illustrated in Tables 2, 3, 4 and 5, and in Figure 1.

4.1 Analysis of extraction behavior before the shock occurrence

To analyze the numerical results before the shock occurrence, total extractions⁶ between $t = 0$ and the date of the shock occurrence t_a are compared. The results are presented in Table 2 for the cases without shock and with a high-intensity shock ($r_2 = 300$) that takes place at $t_a = 20$, as well as for two different levels for each type of asymmetry⁷.

4.1.1 Demand effect

We first analyze the effect of the different levels of asymmetry in the demand function of the urban user, which is represented by the parameter k (compare, for example, columns

⁶This is obtained by integrating the rate of extraction over a given period.

⁷Additional simulations for different levels of shocks and asymmetries have been computed, are consistent with numerical results and are available upon request from the authors.

Column	2	3	4	5	6	7
	$k = 1$			$k = 2$		
	No shock	Big shock ($t_a = 20$)		No shock	Big shock ($t_a = 20$)	
	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.1$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.1$ $\bar{\rho}_u = 0.05$
A. Farmer Cooperative	10 259	10 593	12 766	5 220	5 571	7 456
B. Urban Cooperative	1 710	1 765	2 128	7 770	7 799	7 956
C. Farmer Non-cooperative	12 874	13 016	14 139	11 543	11 749	12 475
D. Urban Non-cooperative	5 124	5 117	4 612	8 816	8 803	8 592
E. Total Cooperative	11 968	12 359	14 894	12 990	13 070	15 412
F. Total Non-cooperative	17 999	18 133	18 752	20 359	20553	21 067

Table 2: Value of total extractions between $t = 0$ and the date of occurrence of the shock t_a (i.e. $[0, t_a]$) for different values of k and $\bar{\rho}_f$ in the cases without shock and with a high-intensity shock ($r_1 - r_2 = 60$) that takes place at $t_a = 20$ years.

3 and 6). In the baseline case ($k = 1$), the non-cooperative farmer presents the highest extraction (in absolute numbers), while the cooperative urban water user exhibits the lowest extraction. Individual extractions increase in presence of the shock (compare, for example, columns 2 and 3) except in the non-cooperative urban case. These results are expected in a situation in which the farmer shows, on average, a higher water demand and is in competition with the urban water user. When asymmetry between the uses increases, the total extractions increase for the urban water user, but decrease for the farmer. This is a consequence of the effect of the value of k on the benefit function of the urban water user, with higher values of k increasing the profits.

We note that non-cooperative solutions are influenced more by a change in k than cooperative solutions (compare rows E and F of the table). For example, in the case with shock and where $k = 2$ (see column 6), total extractions in the non-cooperative case are around 13% higher than for $k = 1$ (see column 3), with this increase only around 8% in the cooperative case.

4.1.2 Discount effect

In the analysis of the effect of a change in time preferences of the groundwater users (represented by the parameter $\bar{\rho}_f$), the demands are fixed to the case in which $k = 1$ (see columns 3 and 4) to isolate the discount effect.

We note that total extractions increase in all cases when considering a higher discount rate for the farmer except in the case of the urban non-cooperative solutions. Logically, the farmer in the cooperative and non-cooperative cases exhibits a more aggressive extraction behavior due to the higher impatience (higher discount rate). However, while in the cooperative case the same extraction behavior is observed for the urban water user, in the non-cooperative case we observe the opposite tendency. The presence of competition seems then to slow down the effect of heterogeneous discounting on the intensity of extractions.

This is confirmed when comparing total extractions for the cooperative and non-cooperative cases (see rows E and F of the Table 2). The discount effect is much higher in the cooperative case, that is, in the absence of competition, which appears counterintuitive. However, both types of users exhibit the same cooperative behavior leading to higher extractions. For example, in the case in which the different users exhibit different discount rates and $\bar{\rho}_f = 0.1$ (see column 4), total extractions in the cooperative case are around 20% higher than in the case with equal discount rates (see column 3), with this increase only around 3% in the non-cooperative case.

4.1.3 Discount and demand effects

To analyze the impact of asymmetries in both demand and discount rates on the simulated results, it is necessary to compare columns 3 and 7 in Table 2.

In fact, regarding the extraction behavior of the different individual users (see rows A-D), total extractions decrease for the farmer, but increase for the urban water user. Similar results are obtained when analyzing the demand effect.

If we now compare the sum of total extractions of both users for the cooperative and non-cooperative cases (see rows E-F), the effects of both demand and discount rate mainly influence the cooperative case. Indeed, there is an increase in total extractions of around 25% in the cooperative case and only of around 16% in the non-cooperative case (see column 7) with respect to the baseline case (see column 3). Similar results are obtained when analyzing the discount effect.

Numerical simulations show that the impact of the discount effect on extraction behavior is higher at the group level (i.e., cooperative or non-cooperative action) than at

the individual level (i.e., farmer or urban user). In Section 4.3, we will analyze if this interesting result is also true in terms of welfare.

4.2 Long-term stock analysis

Table 3 presents the results from the analysis of the influence of the shock intensity, $r_1 - r_2$ (or equivalently a decrease in r_2 with respect to r_1), and asymmetric demand, i.e., k , on the steady-state stock (see also Figure 1). The discount effect is not analyzed as it does not affect the steady state, as shown in the top-side of Figure 1.

Column		2	3	4	5	6	7
		No shock			Big shock		
		$r_1 = r_2 = 360$			$r_1 = 360, r_2 = 300$		
		$k = 1$	$k = 1.5$	$k = 2$	$k = 1$	$k = 1.5$	$k = 2$
A.	Cooperative	78 187	77 530	77 176	76 777	76 109	75 750
B.	Open-loop	75 353	73 671	72 432	74 415	72 656	71 360
C.	Non-cooperative	73 446	72 107	71 115	72 826	71 364	70 280
D.	Myopic	70 987	70 330	69 976	70 777	70 109	69 750
E.	Cost externality (A)-(B)	2 834	3 859	4 744	2 362	3 453	4 390
F.	Strategic externality (B)-(C)	1 907	1 564	1 317	1 589	1 292	1 080
G.	Inefficiency non-cooperative (A)-(C)	4 741	5 423	6 061	3 951	4 745	5 470
H.	Inefficiency myopic (A)-(D)	7 200	7 200	7 200	6 000	6 000	6 000

Table 3: Results of steady-state stock (in Mm^3) for the different cooperative and non-cooperative solutions and differences between solutions.

4.2.1 Influence of the shock intensity

When the recharge rate upon the shock (r_2) decreases (respectively increases), the level of the stock at the steady state decreases (respectively increases) for the cooperative and non-cooperative solutions. This has been shown theoretically in Section 3.2 for the cooperative case and is now also confirmed for the subgame perfect non-cooperative case (see row C, for example, columns 3 and 6 in Table 3). Moreover, subgame perfect non-cooperative solutions are always inefficient with respect to cooperative solutions in terms of stock, with

this inefficiency decreasing with lower values of r_2 (see row G, e.g., columns 3 and 6, and Figure 1). Therefore, our numerical results are consistent with the findings in the literature regarding this type of shock involving symmetric players (de Frutos Cachorro et al. [2019]).

It is important to consider other types of non-cooperative solutions (see analytical solutions in Appendix B) that are described in the literature to better understand the sources of possible inefficiencies. As shown in Table 3, the open-loop solution (respectively the myopic solution) is less (respectively more) inefficient than the subgame perfect non-cooperative solutions in terms of stock. These results are also in line with previous literature (e.g., Gisser and Sanchez [1980], Rubio and Casino [2001]).

4.2.2 Influence of the demand asymmetry

Regarding the impact of asymmetric players, when the value of k increases (see, for example, columns 2, 3 and 4), steady-state stocks decrease, but the difference between the subgame perfect cooperative and non-cooperative stocks increases (see row G in Table 3 and bottom-side of Figure 1). This means that the inefficiency of non-cooperative solutions in terms of stock increases with higher asymmetry in demand between the users. The calculation of the open-loop solution helps us to interpret this result. Indeed, this increase in inefficiency is mainly due to the pumping cost externality (see row E), which captures the fact that the extractions made by one user lower the water table level, resulting in an increase in pumping costs for the other users, and not to the strategic (or stock) externality (see row F), which represents competition arising from the limited stock. Finally, the inefficiency of myopic solutions (see row H of the table) is not influenced by the asymmetry in demand, demonstrating the importance of considering dynamic interactions between the users in the modeling.

4.3 Welfare analysis

To study the efficiency of different extraction behaviors per type of user and solution, individual total welfare⁸ and total welfare per group over the whole planning horizon are calculated. The simulated results are shown in Table 4.

Concerning group (or joint) welfare (see row G of the table), the inefficiency of non-cooperative solutions with respect to cooperative solutions seems to increase with higher levels of k (see, for example, columns 3 and 6). However, additional simulations show that the inefficiency reaches a minimum at around $k = 1.2$ due to the additional asymmetry

⁸Individual welfare is defined as the present value of future individual profits (see equation (3))

Column	2	3	4	5	6	7
	$k = 1$			$k = 2$		
	No shock	Big shock ($t_a = 20$)		No shock	Big shock ($t_a = 20$)	
	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.1$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.05$ $\bar{\rho}_u = 0.05$	$\bar{\rho}_f = 0.1$ $\bar{\rho}_u = 0.05$
A. Farmer Cooperative	267 258	257 069	173 667	123 604	121 665	112 362
B. Urban Cooperative	44 543	42 845	41 409	380 818	370 783	345 748
C. Farmer Non-cooperative	198 145	193 383	149 180	155 522	153 849	128 896
D. Urban Non-cooperative	54 855	52 712	45 594	280 582	274 188	262 513
E. Profitability Farmer	69 112	63 686	24 487	-31 918	-32 183	-16 533
F. Profitability urban	-10 312	-9 867	- 4 185	100 236	96 597	83 235
G. Inefficiency non-cooperation (in %)	58 800 -	53 819 (-8.5 %)	20 302 (-65.5 %)	68 318 (16.2%)	64 414 (9.5%)	66 702 (13.4 %)

Table 4: Welfare (value function) analysis (in thousand euros) corresponding to Table 2.

in parameter θ in the demand function⁹. By contrast, the inefficiency of non-cooperative solutions decreases when only the discount effect is considered (see, for example, columns 3 and 4).

As for individual welfare (see rows A-D), it is interesting to see that for a higher asymmetry in demand, it is not worth it for the farmer to cooperate (see row E, columns 5 and 6), while this is not observed for the baseline case (i.e., $k = 1$). The opposite result is obtained for the urban water user (see row F, columns 2 and 3). When considering the discount effect (see for example, columns 3 and 4), the same tendencies are observed for each type of use (see rows E and F).

Thus, we confirm the results obtained when analyzing extraction behavior. The demand effect mainly influences individual welfare, producing situations in which it is not always profitable for the farmer to cooperate. However, the asymmetry in discount rates mostly

⁹Indeed, $\theta = \frac{1}{6}$ in our numerical example. However, when $\theta = 1$ and the conditions of Proposition 1 are fulfilled, the inefficiency of non-cooperative solutions compared with cooperative solutions is monotonic with respect to k (in $k \in [1, 2]$) and increases with greater asymmetry in demand.

affects total group welfare, leading to a lower inefficiency of non-cooperative solutions with respect to cooperative solutions.

4.4 Additional simulations

Here, we present additional simulations for a fixed level of shock ($r_1 - r_2 = 60$, see Table 5). In the case of different uses, it could be more realistic to consider different units of fixed costs (represented by z) and different return flow coefficients (represented by γ). To check the robustness of our numerical results, we assume a slightly higher fixed cost per unit and a lower return flow coefficient¹⁰ for the urban water user. Our analysis confirms that the main results regarding inefficiency in terms of stock and welfare are maintained.

	Demand effect			Discount effect		
	$z_u > z_f$	$\gamma_f > \gamma_u$	No adaptation	$z_u > z_f$	$\gamma_f > \gamma_u$	No adaptation
Stock inefficiency	↗	↗	↗	No effect	No effect	↗
Welfare inefficiency	↘↗	↘↗	↘↗	↘	↘	↗

Table 5: Sensitivity analysis with respect to cost and return flow parameters and the non-adaptation case. Symbol ↘↗ refers to non-monotonicity.

Numerical simulations are also performed for what we call the non-adaptation case to the shock, in the sense that we assume a unique discount rate for the cooperative case, and constant (but different) discount rates over the whole planning horizon for the non-cooperative case. The main results are maintained for the demand effect, but the tendencies change for the discount effect. This indicates the importance of considering heterogeneous time preferences when studying possible inefficiencies, especially in the case of shocks.

¹⁰Due to the absence of data and in order to obtain stationary and positive solutions, we assume that z_2 is 1% higher than z_1 and $\gamma_2 = 0$.

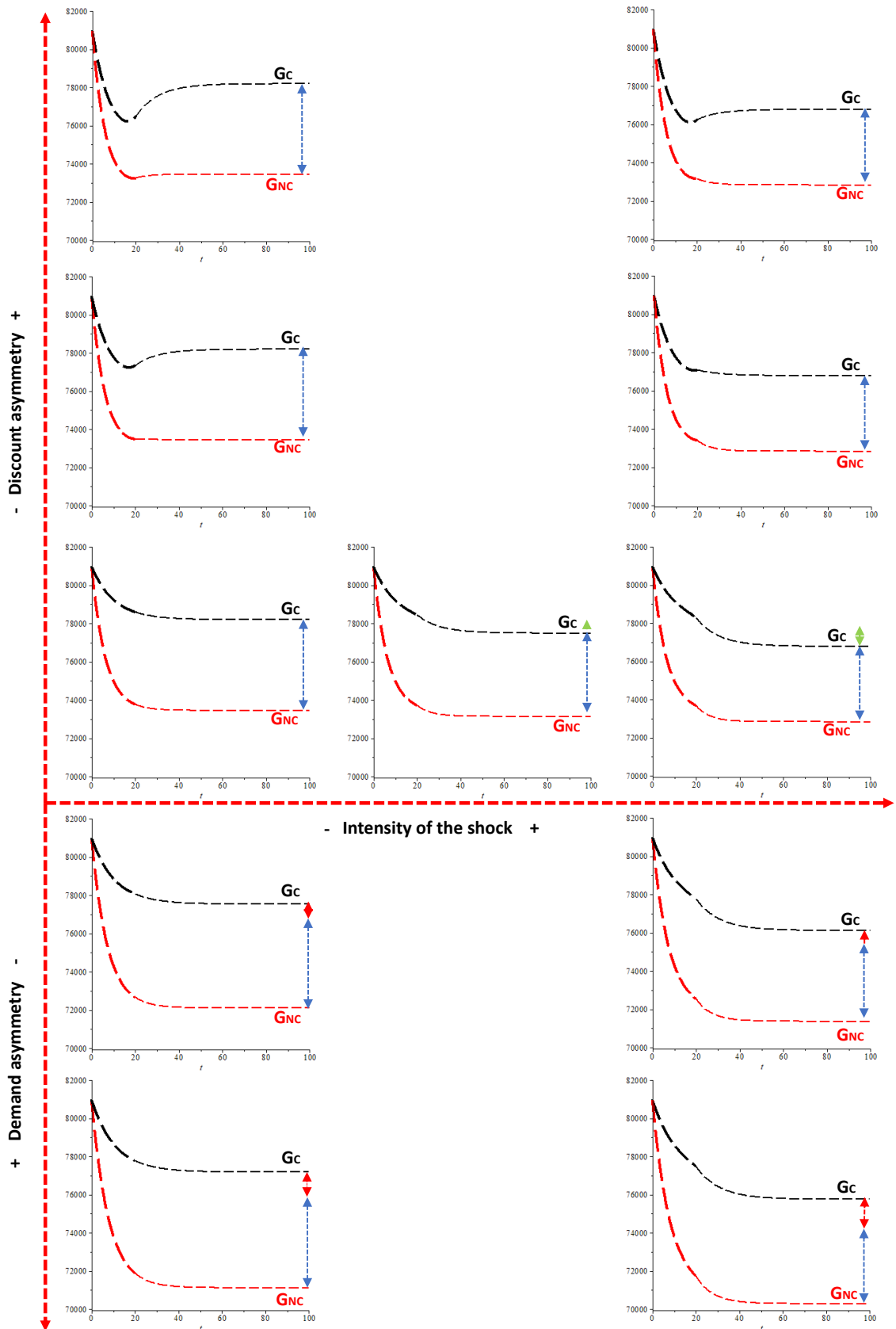


Figure 1: Simulations of the evolution of the stock (in Mm^3) for the subgame perfect cooperative (G_C , in black) and non-cooperative (G_{NC} , in red) cases, and inefficiency of non-cooperation (bidirectional vertical arrows) for the different model scenarios.

5 Conclusions and discussion

In this paper, we study how different asymmetries in demand and time preferences between users affect groundwater exploitation, when facing a future shock in water availability. In particular, we compute cooperative and different types of non-cooperative solutions, namely open-loop, subgame perfect and myopic non-cooperative solutions.

First of all, we provide steady-state analytical solutions for cooperative and non-cooperative cases under the open-loop information structure and myopic behavior. We show that not only the steady-state stock levels, but also the differences between the stock solutions decrease with higher levels of the shock, as reported in de Frutos Cachorro et al. [2019] for the symmetric case.

To compare the subgame perfect cooperative and non-cooperative solutions and calculate the difference between the solutions, which reflects the whole inefficiency of private exploitation (i.e., pumping costs and strategic externalities), we apply the model to the real case of the Western La Mancha (WLM) aquifer. The WLM aquifer has suffered from several droughts in the last decades and is characterized mainly by two highly heterogeneous groundwater uses: urban water use (mainly drinking water) and irrigation. This makes it a good case for studying our theoretical setting.

Regarding steady-state numerical results, simulated stock solutions confirm that the inefficiency of subgame perfect non-cooperative solutions also decreases with greater shocks, but increases with a higher asymmetry in demand. The latter result is mainly due to the increase in pumping costs for individual users when the stock level is lowered by the other users.

When analyzing extraction behavior before the shock occurrence, we find that while the asymmetry in the users' demand affects individual strategic behavior more than group behavior (i.e., cooperation or not) before the shock, the opposite result is obtained when considering the asymmetry in discount rates. This result is maintained when analyzing total welfare over the whole planning horizon. When the demand effect is considered, cooperation might not always be the most profitable solution for the farmer. However, group inefficiency (in terms of welfare) due to non-cooperative behavior is reduced when considering only the discount effect.

Important policy implications could be derived from our numerical results. Indeed, Spanish studies such as Maestu et al. [2007] have highlighted the need to obtain more information about different economic variables to promote a more efficient use of water resources, e.g., "estimation on price-demand elasticities of water use are needed in order

to predict the impact of current water price policies on demand" and on adequate price levels. This is reflected in our work when studying the impact of the demand effect on welfare inefficiency for different types of uses. More specifically, we observe that the more inelastic the demand curve of the urban water user, the less incentive the farmer has to cooperate, and the higher the inefficiency of non-cooperation between uses. For the case of irrigation water, empirical findings in Tsur et al. [2004] support the view that water pricing should be designed primarily to increase the efficiency of water use. However, for the case of drinking water, since it is a necessity good, pricing policies could not be so effective as for irrigation water, and could thus be boosted by actions based on water saving such as improvement in urban water networks to reduce water losses, awareness campaigns, home flow reducers, among others. Furthermore, under conditions of water scarcity, regulators (e.g. water agencies) could act by prioritizing urban uses over agricultural uses through quota implementation or/and by increasing water prices. In this context, it seems reasonable to assume that farmers will exhibit a higher impatience rate than urban users when internalizing this risk. We obtain that while the consideration of this risk (or, equivalently, of heterogeneous time preferences in the irrigation and urban sectors) before the period of water scarcity could decrease the inefficiency of non-cooperative solutions, and therefore reduce the incentive to cooperate in the long term, it also increases the overexploitation of the aquifer during this period. In fact, not only the user with the higher impatience rate will increase total extractions, as expected, but also total extractions by the other user are affected, increasing in the cooperative case and decreasing in the noncooperative case because of considering strategic effects. Therefore, measures aimed at reducing this perceived higher risk will not only influence the decisions of the farmer but also of the urban user, improving their efficiency in terms of water use. Some of these measures could be to facilitate the transition to less risky cropping patterns, investments in improved irrigation technologies, use of treated urban wastewater for irrigation, or specific insurances for irrigated crops with drought risk, among other possible long-term and short-term strategies.

This study opens up the possibility of further investigations. First, it would be interesting to consider priority rules in case of water scarcity for agricultural and urban uses by introducing, for example, a minimum level of groundwater extraction for drinking water. We could also introduce a stochastic shock in terms of the date or the intensity of the shock, which might be a more realistic case from a practical point of view. Moreover, annual and seasonal water demands (per type of use) often depend on weather conditions as well as social and economic activities. Changes on the proportion between annual water

demands could be introduced by considering different values of the parameter θ . Related to seasonal needs, demand for urban user is needed throughout the year, while farmers need water only during the growing season. To consider this in the modeling, it would be necessary a short-term decision model, including different decision periods within the year and estimates of different demand price functions for each of these periods. Finally, we could apply our theoretical model to other aquifers with different climatic characteristics and types of users.

Acknowledgments

The authors acknowledge financial support from the Project 'Dynamic Analysis of Environmental Policies and Dynamic Games. Time-Consistency and Sustainability of Economic Growth and Environmental Agreements', funded by the Spanish State Research Agency, with reference ECO2017-82227-P (AEI).

APPENDIX

A Subgame Perfect Non-cooperative Equilibrium

Lemma 1 *In Problem (3)-(6) if, for $t > t_a$, stationary linear SPNE of farmers and urban users are given by $g_f^{NC+} = \phi_f^{NC+}(G) = \alpha_f^{NC+}G + \beta_f^{NC+}$ and $g_u^{NC+} = \phi_u^{NC+}(G) = \alpha_u^{NC+}G + \beta_u^{NC+}$, respectively, then the corresponding value functions are $V_i^{NC+}(G) = A_i^{NC+}G^2 + B_i^{NC+}G + C_i^{NC+}$, for $i = u, f$, where the coefficients A_i^{NC+} , B_i^{NC+} and C_i^{NC+} are given by*

$$\begin{aligned}
 A_f^{NC+} &= \frac{-\frac{1}{2b}(\alpha_f^{NC+})^2 + c\alpha_f^{NC+}}{\rho + 2(1-\gamma)(\alpha_f^{NC+} + \alpha_u^{NC+})}, \\
 B_f^{NC+} &= \frac{-\frac{1}{b}\alpha_f^{NC+}\beta_f^{NC+} + (\frac{a}{b} - z_f)\alpha_f^{NC+} + c\beta_f^{NC+} + 2(r_2 - (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+}))A_f^{NC+}}{\rho + (1-\gamma)(\alpha_f^{NC+} + \alpha_u^{NC+})}, \\
 C_f^{NC+} &= \frac{-\frac{1}{2b}(\beta_f^{NC+})^2 + (\frac{a}{b} - z_f)\beta_f^{NC+} + (r_2 - (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+}))B_f^{NC+}}{\rho}, \\
 A_u^{NC+} &= \frac{-\frac{k}{2b\theta}(\alpha_u^{NC+})^2 + c\alpha_u^{NC+}}{\rho + 2(1-\gamma)(\alpha_f^{NC+} + \alpha_u^{NC+})}, \\
 B_u^{NC+} &= \frac{-\frac{k}{b\theta}\alpha_u^{NC+}\beta_u^{NC+} + (\frac{ka}{b} - z_u)\alpha_u^{NC+} + c\beta_u^{NC+} + 2(r_2 - (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+}))A_u^{NC+}}{\rho + (1-\gamma)(\alpha_f^{NC+} + \alpha_u^{NC+})}, \\
 C_u^{NC+} &= \frac{-\frac{k}{2b\theta}(\beta_u^{NC+})^2 + (\frac{ka}{b} - z_u)\beta_u^{NC+} + (r_2 - (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+}))B_u^{NC+}}{\rho}.
 \end{aligned}$$

Proof: From the first order conditions for a maximum in the right hand term of equation (7) we easily obtain

$$\begin{aligned}
 \frac{1}{b}\phi_f^{NC+} &= \left[c - 2A_f^{NC+}(1-\gamma) \right] G + \frac{a}{b} - z_f - (1-\gamma)B_f^{NC+}, \\
 \frac{k}{b\theta}\phi_u^{NC+} &= \left[c - 2A_u^{NC+}(1-\gamma) \right] G + \frac{ka}{b} - z_u - (1-\gamma)B_u^{NC+}.
 \end{aligned}$$

Therefore,

$$\alpha_f^{NC+} = b \left[c - 2(1-\gamma)A_f^{NC+} \right], \quad \alpha_u^{NC+} = \frac{b\theta}{k} \left[c - 2(1-\gamma)A_u^{NC+} \right], \quad (\text{A.1})$$

$$\beta_f^{NC+} = a - bz_f - b(1-\gamma)B_f^{NC+} \quad \text{and} \quad \beta_u^{NC+} = a\theta - \frac{b\theta}{k}z_u - \frac{b\theta}{k}(1-\gamma)B_u^{NC+}. \quad (\text{A.2})$$

The result follows by substituting the above expressions in equation (7) and identifying terms in G^2 , G and independent terms. \square

Next, we describe the nonlinear equations to be satisfied by α_f^{NC+} , β_f^{NC+} , α_u^{NC+} and β_u^{NC+} .

Lemma 2 *In Problem (3)-(6) for $t > t_a$, if there are SPNE of the form $g_f^{NC+} = \phi_f^{NC+}(G) = \alpha_f^{NC+}G + \beta_f^{NC+}$ and $g_u^{NC+} = \phi_u^{NC+}(G) = \alpha_u^{NC+}G + \beta_u^{NC+}$, then coefficients α_f^{NC+} and α_u^{NC+} solve the nonlinear equation system*

$$\frac{\rho}{2(1-\gamma)} \left(c - \frac{1}{b} \alpha_f^{NC+} \right) = -\frac{1}{2b} (\alpha_f^{NC+})^2 + c \alpha_f^{NC+} - \left(c - \frac{1}{b} \alpha_f^{NC+} \right) (\alpha_f^{NC+} + \alpha_u^{NC+}), \quad (\text{A.3})$$

$$\frac{\rho}{2(1-\gamma)} \left(c - \frac{k}{b\theta} \alpha_u^{NC+} \right) = -\frac{k}{2b\theta} (\alpha_u^{NC+})^2 + c \alpha_u^{NC+} - \left(c - \frac{k}{b\theta} \alpha_u^{NC+} \right) (\alpha_f^{NC+} + \alpha_u^{NC+}), \quad (\text{A.4})$$

whereas, given α_f^{NC+} and α_u^{NC+} , β_f^{NC+} and β_u^{NC+} satisfy the linear equation system

$$\frac{1}{b} \left(\frac{\rho}{1-\gamma} + \alpha_f^{NC+} + \alpha_u^{NC+} \right) \beta_f^{NC+} + \left(\frac{\alpha_f^{NC+}}{b} - c \right) \beta_u^{NC+} = \quad (\text{A.5})$$

$$\begin{aligned} & \left(\frac{a}{b} - z_f \right) \left(\alpha_u^{NC+} + \frac{\rho}{1-\gamma} \right) + \frac{r_2}{1-\gamma} \left(\frac{\alpha_f^{NC+}}{b} - c \right), \\ & \left(\frac{k\alpha_u^{NC+}}{b\theta} - c \right) \beta_f^{NC+} + \frac{k}{b\theta} \left(\frac{\rho}{1-\gamma} + \alpha_f^{NC+} + \alpha_u^{NC+} \right) \beta_u^{NC+} = \quad (\text{A.6}) \\ & \left(\frac{ka}{b} - z_u \right) \left(\alpha_f^{NC+} + \frac{\rho}{1-\gamma} \right) + \frac{r_2}{1-\gamma} \left(\frac{k\alpha_u^{NC+}}{b\theta} - c \right). \end{aligned}$$

Proof: From equations (A.1) and (A.2) we obtain

$$A_f^{NC+} = \frac{1}{2(1-\gamma)} \left(c - \frac{\alpha_f^{NC+}}{b} \right), \quad A_u^{NC+} = \frac{1}{2(1-\gamma)} \left(c - \frac{k\alpha_u^{NC+}}{\theta b} \right),$$

$$B_f^{NC+} = \frac{1}{b(1-\gamma)} \left(a - bz_f - \beta_f^{NC+} \right) \quad \text{and} \quad B_u^{NC+} = \frac{k}{b\theta(1-\gamma)} \left(a\theta - \frac{b\theta}{k} z_u - \beta_u^{NC+} \right).$$

The result follows by substituting in the expressions of A_f^{NC+} , A_u^{NC+} , B_f^{NC+} and B_u^{NC+} in Lemma 1 and rearranging terms. \square

Proof of Proposition 1: From Lemma 2, we know that the SPNE satisfy conditions (A.3) and (A.4). In fact, SPNE are the solutions to equations (A.3)-(A.6) satisfying Condition A.

First we prove that there exists a unique solution to (A.3)-(A.4) such that $\alpha_f^{NC+} + \alpha_u^{NC+} > 0$. Note that, if we isolate α_u^{NC+} in (A.3) and we substitute its value in (A.4), we obtain a fourth degree polynomial. So the question is how many roots of this polynomial meet condition $\alpha_f^{NC+} + \alpha_u^{NC+} > 0$. By introducing the new variables $x = \alpha_f^{NC+} - bc$ and $y = \alpha_u^{NC+} - \frac{\theta}{k}bc$, equations (A.3)-(A.4) can be written as

$$\begin{aligned} x^2 + 2xy + \left[2bc \left(1 + \frac{\theta}{k} \right) + \frac{\rho}{1-\gamma} \right] x + b^2c^2 &= 0, \\ y^2 + 2xy + \left[2bc \left(1 + \frac{\theta}{k} \right) + \frac{\rho}{1-\gamma} \right] y + \frac{\theta^2}{k^2}b^2c^2 &= 0. \end{aligned}$$

Condition $\alpha_f^{NC+} + \alpha_u^{NC+} > 0$ becomes $x + y + (1 + \frac{\theta}{k})bc > 0$.

Next, let us define

$$z = x + y + \left(1 + \frac{\theta}{k} \right) bc + \frac{\rho}{2(1-\gamma)}. \quad (\text{A.7})$$

By substituting in the above equations, we have to solve the constrained system of algebraic equations

$$x^2 - 2zx - b^2c^2 = 0, \quad y^2 - 2zy - \frac{\theta^2}{k^2}b^2c^2 = 0, \quad \text{with } z > \frac{\rho}{2(1-\gamma)},$$

hence

$$x = z \pm \sqrt{z^2 + b^2c^2}, \quad y = z \pm \sqrt{z^2 + \frac{\theta^2}{k^2}b^2c^2}, \quad \text{with } z > \frac{\rho}{2(1-\gamma)}.$$

By denoting $t_1, t_2 \in \{-1, 1\}$, from the above equations we obtain

$$x + y = 2z + t_1\sqrt{z^2 + b^2c^2} + t_2\sqrt{z^2 + \frac{\theta^2}{k^2}b^2c^2}$$

and, using equation (A.7) we can write

$$z + t_1\sqrt{z^2 + b^2c^2} + t_2\sqrt{z^2 + \frac{\theta^2}{k^2}b^2c^2} = - \left(1 + \frac{\theta}{k} \right) bc - \frac{\rho}{2(1-\gamma)}. \quad (\text{A.8})$$

Since $z > \frac{\rho}{2(1-\gamma)} > 0$, it is clear that equation (A.8) has no solution for $t_1 = t_2 = 1$. In addition, condition $\frac{\theta}{k} < 1$ (recall that $0 < \theta < 1$ and $k \geq 1$) implies that equation (A.8) has not also solution for $t_1 = 1, t_2 = -1$. For $t_1 = -1, t_2 = 1$, we can write (A.8) as

$$f_1(z) = - \left(1 + \frac{\theta}{k} \right) bc - \frac{\rho}{2(1-\gamma)}, \quad \text{where } f_1(z) = z - \sqrt{z^2 + b^2c^2} + \sqrt{z^2 + \frac{\theta^2}{k^2}b^2c^2}.$$

Next, note that $f_1'(z) > 0$ for $z > 0$ (and, in particular, for $z > \frac{\rho}{2(1-\gamma)} > 0$), so that $f_1(z)$ is strictly increasing. But $f_1(0) = \left(\frac{\theta}{k} - 1\right) bc > -\left(1 + \frac{\theta}{k}\right) bc - \frac{\rho}{2(1-\gamma)}$, so that equation (A.8) has no solution for $t_1 = -1$ and $t_2 = 1$ verifying condition $z > \frac{\rho}{2(1-\gamma)} > 0$.

It remains to check if has solution for $t_1 = t_2 = -1$. In that case, (A.8) becomes

$$f_2(z) = -\left(1 + \frac{\theta}{k}\right) bc - \frac{\rho}{2(1-\gamma)}, \quad \text{where} \quad f_2(z) = z - \sqrt{z^2 + b^2 c^2} - \sqrt{z^2 + \frac{\theta^2}{k^2} b^2 c^2}.$$

Elementary calculations show that $f_2''(z) < 0$, hence $f_2(z)$ is strictly concave. Moreover, $\lim_{z \rightarrow \infty} f_2(z) = -\infty$. In addition, since

$$f_2\left(\frac{\rho}{2(1-\gamma)}\right) + \left(1 + \frac{\theta}{k}\right) bc + \frac{\rho}{2(1-\gamma)} = \left[\frac{\rho}{2(1-\gamma)} + bc - \sqrt{\left(\frac{\rho}{2(1-\gamma)}\right)^2 + b^2 c^2} \right] + \left[\frac{\rho}{2(1-\gamma)} + \frac{\theta}{k} bc - \sqrt{\left(\frac{\rho}{2(1-\gamma)}\right)^2 + \frac{\theta^2}{k^2} b^2 c^2} \right] > 0,$$

therefore $f_2\left(\frac{\rho}{2(1-\gamma)}\right) > -\left(1 + \frac{\theta}{k}\right) bc - \frac{\rho}{2(1-\gamma)}$. As a consequence, there is a unique $z^* > \frac{\rho}{2(1-\gamma)}$ verifying condition $f_2(z^*) = -\left(1 + \frac{\theta}{k}\right) bc - \frac{\rho}{2(1-\gamma)}$. For this value of z^* ,

$$\alpha_f^{NC+} = z^* - \sqrt{(z^*)^2 + b^2 c^2} + bc, \quad \alpha_u^{NC+} = z^* - \sqrt{(z^*)^2 + \frac{\theta^2}{k^2} b^2 c^2} + \frac{\theta}{k} bc$$

is the unique candidate to a SPNE verifying condition $\alpha_f^{NC+} + \alpha_u^{NC+} > 0$. For these α_f^{NC+} and α_u^{NC+} , next we solve the linear equation system (A.5)-(A.6). If a solution exists satisfying $r_2 \geq (1-\gamma)(\beta_f^{NC+} + \beta_u^{NC+})$, then a unique SPNE exists. On the contrary, if the recharge rate r_2 is small enough so that such inequality is not met, no interior solutions exist (the groundwater well will be depleted in finite time). \square

B Other noncooperative solutions. Steady state

First, we compute the solution under myopic behavior. By solving $\frac{\partial F_f}{\partial g_f} = \frac{\partial F_u}{\partial g_u} = 0$, we obtain $g_f = a - b(z_f - cG)$, $g_u = \theta a - \frac{b\theta}{k}(z_u - cG)$. By substituting in (2) for $r = r_2$, the steady state ($\dot{G} = 0$) is given by

$$G_\infty^{MY} = \frac{k}{bc(k+\theta)} \left[r_2 - (1-\gamma)(1+\theta)a + (1-\gamma) \left(z_f + \frac{\theta}{k} z_u \right) \right].$$

Next, let us calculate the open-loop Nash equilibrium. We center our attention in the analysis of the steady state. The corresponding Hamiltonian functions are given by

$$H_i = F_i(G, g_i) + \lambda_i (r_2 - (1 - \gamma)(g_f + g_u)) ,$$

for $i \in \{f, u\}$, with $F_i(G, g_i)$ given by equations (4-5). Functions $G(t)$ and $\lambda_i(t)$ (the adjoint variables) are continuous. By applying the Pontryaguin maximum principle and assuming the existence of interior solutions, we can easily solve $\frac{\partial H_f}{\partial g_f} = \frac{\partial H_u}{\partial g_u} = 0$, together with $\dot{\lambda}_f = \rho\lambda_f - \frac{\partial H_f}{\partial G}$, $\dot{\lambda}_u = \rho\lambda_u - \frac{\partial H_u}{\partial G}$ and $\dot{G} = r_2 - (1 - \gamma)(g_f + g_u)$. In the steady state, $\dot{G} = \dot{\lambda}_f = \dot{\lambda}_u = 0$. After some algebra, we easily obtain

$$G_\infty^{OL} = \frac{k\rho^2 + (\theta + k)\rho bc(1 - \gamma) + \theta b^2 c^2 (1 - \gamma)^2}{\rho bc(1 - \gamma)[(\theta + k)\rho + 2\theta bc(1 - \gamma)]} r_2 - \frac{(a - bz_f)(k\rho + \theta bc(1 - \gamma)) + \theta(ka - bz_u)(\rho + bc(1 - \gamma))}{bc((\theta + k)\rho + 2\theta bc(1 - \gamma))} .$$

C Subgame Perfect Cooperative Solution

Solution for $t > t_a$

From the maximization of (8), with $V_f^{C+}(G) = A_f^{C+}G^2 + B_f^{C+}G + C_f^{C+}$, $V_u^{C+}(G) = A_u^{C+}G^2 + B_u^{C+}G + C_u^{C+}$, we easily obtain

$$\phi_f^{C+}(G) = a - b[z_f + (1 - \gamma)(B_f^{C+} + B_u^{C+})] + b[c - 2(1 - \gamma)(A_f^{C+} + A_u^{C+})]G , \quad (C.1)$$

$$\phi_u^{C+}(G) = a\theta - b\frac{\theta}{k} [z_u + (1 - \gamma)(B_f^{C+} + B_u^{C+})] + b\frac{\theta}{k} [c - 2(1 - \gamma)(A_f^{C+} + A_u^{C+})] G . \quad (C.2)$$

For the calculation of coefficients A_f^{C+} , A_u^{C+} , B_f^{C+} , B_u^{C+} , C_f^{C+} and C_u^{C+} , we numerically solve the system of six equations with six unknown variables obtained when we identify the second degree polynomials

$$\rho \left(A_i^{C+}G^2 + B_i^{C+}G + C_i^{C+} \right) = F_i(G, \phi_i^{C+}(G)) +$$

$$\left[2A_i^{C+}G + B_i^{C+} \right] \left[r_2 - (1 - \gamma) \left(\phi_f^{C+}(G) + \phi_u^{C+}(G) \right) \right] ,$$

for $i = f, u$, with $\phi_i^{C+}(G)$ given by (C.1)-(C.2), together with the transversality condition guaranteeing the convergence to an interior steady state.

For the calculation of the steady state, it is easier to apply the Pontryaguin maximum principle. The Hamiltonian function is

$$H = F_f(G, g_f^{C+}) + F_u(G, g_u^{C+}) + \lambda[r_2 - (1 - \gamma)(g_f^{C+} + g_u^{C+})] .$$

By assuming that the optimal solution is interior, from the first order optimality conditions we obtain

$$g_f^{C+} = a - bz_f + bcG - b(1 - \gamma)\lambda \quad \text{and} \quad g_u^{C+} = \theta a - \frac{b\theta}{k}z_u + \frac{b\theta}{k}cG - \frac{b\theta}{k}(1 - \gamma)\lambda . \quad (\text{C.3})$$

In addition $\dot{\lambda} = \rho\lambda - (\partial H/\partial G) = \rho\lambda - c(g_f^{C+} + g_u^{C+})$. Hence, from the state equation (2) we obtain

$$\begin{aligned} \dot{G} &= r_2 - (1 - \gamma) \left[(1 + \theta)a - \left(z_f + \frac{\theta}{k}z_u \right) b + \left(1 + \frac{\theta}{k} \right) bcG - b(1 - \gamma) \left(\lambda + \frac{\theta}{k}\lambda \right) \right] , \\ \dot{\lambda} &= \rho\lambda - c \left[(1 + \theta)a - \left(z_f + \frac{\theta}{k}z_u \right) b \right] - \left(1 + \frac{\theta}{k} \right) bc^2G + bc(1 - \gamma) \left(\lambda + \frac{\theta}{k}\lambda \right) . \end{aligned} \quad (\text{C.4})$$

The solution to the linear differential equation system above converging to a steady state is

$$\begin{aligned} G^{C+}(t) &= e^{\mu(t-t_a)} \left(G_{t_a}^{C+} - G_{\infty}^{C+} \right) + G_{\infty}^{C+} , \\ \lambda^{C+}(t) &= e^{\mu(t-t_a)} \left(\lambda_{t_a}^{C+} - \lambda_{\infty}^{C+} \right) + \lambda_{\infty}^{C+} , \end{aligned}$$

with $\mu = \frac{1}{2} \left[\rho - \sqrt{\rho^2 + 4\rho(1 - \gamma)bc \left(1 + \frac{\theta}{k} \right)} \right]$, where $G_{\infty}^C, \lambda_{\infty}^C$ represent the steady state solutions, i.e. when $\dot{G} = \dot{\lambda} = 0$. From (C.4), we easily derive $\lambda_{\infty}^C = r_2c/(\rho(1 - \gamma))$ and

$$G_{\infty}^C = \left[\frac{1}{\rho} + \frac{1}{bc(1 - \gamma) \left(1 + \frac{\theta}{k} \right)} \right] r_2 - \frac{a(1 + \theta)}{bc \left(1 + \frac{\theta}{k} \right)} + \frac{z_f + \frac{\theta}{k}z_u}{c \left(1 + \frac{\theta}{k} \right)} . \quad (\text{C.5})$$

By substituting in (C.3), the corresponding extraction rates at the steady state are

$$g_{f,\infty}^C = \frac{k}{(1 - \gamma)(k + \theta)} r_2 - \frac{a\theta}{\theta + k}(k - 1) + \frac{b\theta}{\theta + k}(z_u - z_f) , \quad (\text{C.6})$$

$$g_{u,\infty}^C = \frac{\theta}{(1 - \gamma)(k + \theta)} r_2 + \frac{a\theta}{\theta + k}(k - 1) - \frac{b\theta}{\theta + k}(z_u - z_f) . \quad (\text{C.7})$$

Solution for $t \in [0, t_a]$

From the maximization of (9), with $V_f^{C-}(G, t) = A_f^{C-}(t)G^2 + B_f^{C-}(t)G + C_f^{C-}(t)$, $V_u^{C-}(G, t) = A_u^{C-}(t)G^2 + B_u^{C-}(t)G + C_u^{C-}(t)$, we obtain

$$g_f^{C^-}(G, t) = a - b[z + (1 - \gamma)(B_f^{C^-}(t) + B_u^{C^-}(t))] + b[c - 2(1 - \gamma)(A_f^{C^-}(t) + A_u^{C^-}(t))]G ,$$

$$g_u^{C^-}(G, t) = a\theta - \frac{b\theta}{k}[z + (1 - \gamma)(B_f^{C^-}(t) + B_u^{C^-}(t))] + \frac{b\theta}{k}[c - 2(1 - \gamma)(A_f^{C^-}(t) + A_u^{C^-}(t))]G .$$

Finally, we substitute the above expressions in the associated dynamic programming equations. We solve the corresponding system of differential equations by using a numerical approach.

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