Minding the Gap between Schools and Universities*

Luiz Brotherhood†

Bruno R. Delalibera‡

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Abstract

During the 2000s decade, governments in South America reallocated per-student public expenditures from tertiary to basic education, potentially alleviating the economic gap between students in public schools and public universities. To investigate the welfare and macroeconomic effects of such types of policies, we build and calibrate a general equilibrium model using Brazilian data from the beginning of the 2000s. We find that the optimal utilitarian policy allocates per-student public expenditures equally across education stages, benefits almost the entirety of households, delivers significant welfare gains to the poorest families, and cuts back income inequality. We also use our framework to investigate economic differences between Brazil and the United States, and find that differences in the supply of vacancies in public universities is the education policy aspect that, alone, has the highest explanatory power over aggregate earnings and college attendance differences between the two countries.

Keywords: development, education, education stages, general equilibrium, optimal policy, public expenditures.

JEL Classification: I24, I25, I28, C68.

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†Universitat de Barcelona, Barcelona Economic Analysis Team, and EPGE Brazilian School of Economics and Finance.

Corresponding author. Universitat de Barcelona, Facultat d'Economia i Empresa. Avgda. Diagonal 696, 08034 Barcelona, Spain. Email: martinsbrotherhood@ub.edu.

‡EPGE Brazilian School of Economics and Finance.

1 Introduction

The public education system of many developing countries is characterized by a large gap between students' socioeconomic status in schools and universities. Public school students tend to have less favorable economic backgrounds than those who attend private schools (PISA, 2012), while public university students belong to wealthier families if compared to young adults with no college education.[1] Thus, public education expenditures affect different parts of the population depending on which stage they are allocated to.

At the beginning of the 2000s, governments in some developing countries, such as Brazil, Colombia, and Paraguay, allocated a high proportion of per-student education expenditures to public universities. This changed during that decade, when educational resources were redirected to students in basic education.[2] Figure 1 shows that, in 2002, for each dollar spent by the Brazilian government on a public university student, only 20 cents of dollars were allocated to a public school student. Ten years later, each dollar per university student corresponded to 83 cents per school student. Still, the Organisation for Economic Co-operation and Development (OECD) has recently recommended this project's intensification in some developing countries, with the objective of raising resource allocation efficiency and promoting equality (OECD, 2018a; OECD, 2018b; OECD, 2019).

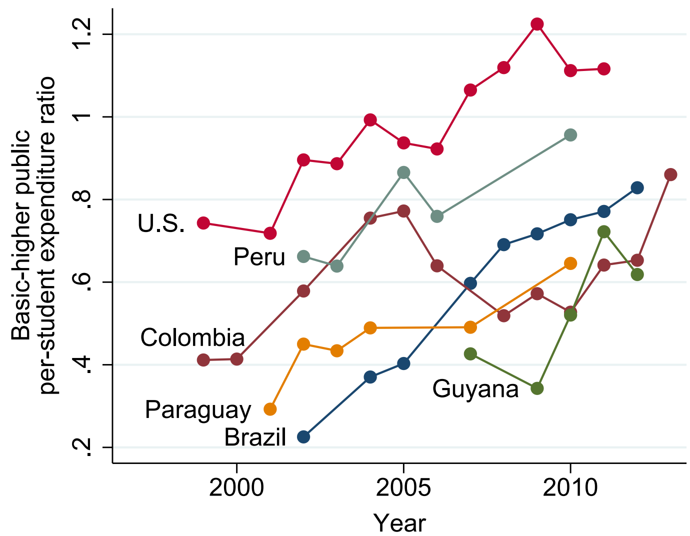
In this paper, we seek to answer the following research question: how should a government allocate public expenditures across schools and universities? To answer this question, we develop a general equilibrium model featuring heterogeneous agents, basic and higher education, public and private educational institutions, credit frictions, and complementarity between human capital inputs. We calibrate the model to Brazil and use it to simulate counterfactual experiments. We compute the optimal utilitarian policy that allocates per-student government expenditures across basic and higher education to maximize the average welfare across households. We also use our framework to understand how educational variables explain economic differences between Brazil and the United States.

In the model, each household is composed of a parent and a child. Parents make choices related to consumption, savings, and investments in their children's education.

[1] Studies show that there is inequality in access to higher education in countries in South Asia and sub-Saharan Africa (Ilie and Rose, 2016) and in developed countries (Hansen and Weisbrod, 1969; Radner and Miller, 1970; Peltzman, 1973; Jackson and Weathersby, 1975; Bishop, 1977; Oliver, 2000; Fernandez and Rogerson, 1995; Dynarski, 2000; Wälde, 2000; William, 2007; Koucky et al., 2009).

[2] We use the term "basic education" to refer to primary and secondary education, "higher education" to refer to tertiary education, "school" to refer to a basic education institution, and "university" or "college" to refer to a higher education institution.

Figure 1: The allocation of public expenditures per student between schools and universities for selected countries



Notes: Basic-higher public per-student expenditure ratio is the ratio between government expenditures per student in basic and higher education. Brazil's basic-higher expenditure ratio in 2002 was equal to 0.2, meaning that, for each dollar spent by the Brazilian government on a public university student, 20 cents of dollar were invested in a public school student. All countries shown in the figure reallocated per-student expenditures from higher to basic education during 2000-2013.
Data source: UNESCO (2017).

A child's human capital depends on her innate ability, basic and higher education expenditures, and parental education. Our human capital production function nests standard specifications, such as that in Cunha et al. (2010), but also allows for the existence of an optional educational stage.

There are credit frictions in the economy: households cannot take loans in the credit market, which generates the existence of poor parents with high ability children who are not able to borrow against their offspring's future labor earnings. In such an environment, public educational policies may be beneficial to financially constrained households.

Public and private schools differ in two aspects. First, each student in public school receives educational investments from the government through public expenditures.[3] Second, households' education expenditures made through public schools have a different marginal return in terms of generating human capital gains for students than

[3]Government spending per student does not vary across students within an education stage. For models with heterogeneous public expenditures across districts, see Cooper (1998) and Fernandez and Rogerson (1998). We also abstract from neighborhood and school sorting with peer-group effects. For this matter see Durlauf (1996), Fernandez and Rogerson (1996, 2001), and Caucutt (1997). Finally, for papers on the endogenous determination of public educational expenditures, including its political aspects, see Epple and Romano (1996), Glomm and Ravikumar (1998), Su (2006) and Rauh (2017).

investments made through private schools. Public and private colleges differ in the same aspects as schools, with one additional distinction: public universities have a limited number of seats and admissions are based on a noisy measure of the applicants' human capital, while vacancies in private universities are unlimited.[4] Due to this congestion effect in public universities, access to public education expenditures is unequal across families.

There are two main general equilibrium components in the model. First, since vacancies in public colleges are limited, a grade point cutoff is determined in equilibrium to make the mass of students entering public college consistent with seats supplied by the government. Second, government expenditure per student is determined endogenously in both educational stages, taking into account the mass of agents choosing public educational institutions and the fact that the government's budget must be balanced in equilibrium.

The model is calibrated to fit Brazilian data. Targeted statistics include the fraction of students in public and private schools and universities, households' educational expenditures, cross-sectional variability in labor earnings, private school and college wage premia. The model fits the data well, and we validate the calibration strategy by investigating the model's performance in fitting non-targeted statistics, such as percentiles of the wage distribution and educational choices by parental wage quartiles. We also replicate a quasi-experiment studied in Francis-Tan and Tannuri-Pianto (2018) as a validation exercise. Using a regression discontinuity approach, these authors find that, in Brazil, individuals admitted to an elite public university with scores close to the admissions cutoff have statistically significant higher wages than those who are barely rejected. We replicate this experiment in our model, running a discontinuity regression with simulated data, and obtain a coefficient estimate consistent with Francis-Tan and Tannuri-Pianto's (2018) findings.

We use the calibrated framework to look for the optimal utilitarian policy, which is given by the allocation of per-student expenditures across schools and universities that maximizes the average welfare across households. Since our objective is to compare the welfare of the current generation under different policies, our computations take into account the transitions from the benchmark equilibrium to the steady states associated with each policy that we simulate (Kambourov, 2009).

We find that the optimal utilitarian policy is the one in which the annual basic-higher per-student expenditure ratio is equal to 0.96. That is, for each dollar spent in

[4]Public universities are tuition-free in the model. In addition to Brazil, other countries with low-cost tuition in public universities are Argentina, Cuba, Czech Republic, Denmark, Estonia, Finland, Germany, Greece, Mexico, Norway, Poland, the Slovak Republic, Slovenia, Sweden, and Turkey.

the education of a public college student, the government allocates 96 cents to a public school student. Note in Figure 1 that this policy is similar to that followed by the United States during the middle of the 2000s.[5]

The optimal policy benefits almost all households (96%) and delivers considerable welfare gains for families in the bottom wage quartile: such households would be willing to pay on average 0.82% of their consumption in all periods and states of nature to implement the policy. This experiment has positive and small effects on aggregate earnings, and it cuts down income inequality by 2% in the long-run.

Our welfare analysis indicates that the allocation of public expenditures across schools and universities observed in Brazil in 2002 was an inferior policy. It also suggests that the strategy followed by some countries in South America during the 2000s, in which per-student public resources were reallocated from higher to basic education, were effective policies, and not only should have generated positive impacts in the 2000s, but also shall produce further gains in the future.

We re-calibrate the model to fit U.S. data and use it to understand aggregate earnings and college attendance differences between the United States and Brazil. According to the calibration results, public education institutions are relatively more efficient (with respect to private institutions) in the United States than in Brazil. In particular, we find that public universities in the U.S. model are overall more efficient than private universities, while the opposite happens in the model calibrated to Brazil. These facts show that the two models depict substantially different economies.

In 2000, the United States was approximately four times richer than Brazil and had a college attendance 2.3 times higher. We find that differences in the supply of vacancies in public universities is the education policy aspect that, alone, has the highest explanatory power over aggregate earnings and college attendance differences between the two countries. Increasing the number of vacancies by a factor of 6.5 in Brazil, to replicate the U.S. case, leads to 2% higher aggregate earnings and 53% higher college attendance in the long-run. This result on college attendance is not mechanical because applying to public university is a choice that entails costs in the model.

To the best of our knowledge, this is the first paper to study from a quantitative macroeconomic perspective the optimal allocation of public expenditures across basic and higher education in an environment where households choose between public and private education institutions. Therefore, we contribute to a large literature that studies education policies using structural models (Glomm and Ravikumar, 1992; Epple

[5]Given the differential patterns of income distribution in public education institutions across educational stages in the model, the optimal policy is characterized by wealthier families subsidizing low-income households, in line with the literature that studies mechanisms of income redistribution. See, for example, Glomm and Ravikumar (1992) and Benabou (2002).

and Romano, 1996; Epple and Romano, 1998; Glomm and Ravikumar, 1998; Glomm and Ravikumar, 2003; Restuccia and Urrutia, 2004; Erosa et al., 2010; Cunha, 2013; Manuelli and Seshadri, 2014; Blankenau and Youderian, 2015; Herrington, 2015; Cubas et al., 2016; Epple et al., 2017; Herskovic and Ramos, 2017; Abbott et al., 2019; Delalibera and Ferreira, 2019; Lee and Seshadri, 2019; Caucutt and Lochner, 2020).

Two papers that are close to ours are Caucutt and Lochner (2020) and Herskovic and Ramos (2017). Caucutt and Lochner (2020) study the role of several economic mechanisms in determining human capital investments in children at different ages. They use a rich dynastic household model and compute the optimal ratio of early to late government subsidies to education in the US. Our model is markedly different from theirs: we consider general equilibrium effects and endogenous choices among public and private education institutions, while they model aspects that we abstract from, such as unobserved costs of schooling (e.g., psychic costs). Another major difference is related to the classification of education stages: their "early education" stage ranges from early childhood to primary education, and "late education" includes secondary and tertiary education.

Herskovic and Ramos (2017) develop a model to study the economic implications of affirmative action in higher education in Brazil. They find that targeted quotas for public college admissions are a powerful policy to reduce intergenerational persistence of earnings and improve welfare and aggregate output. To answer our research question, we build on their model by adding new features, such as savings, dynamic complementarity across human capital stages, and a non-fixed government spending per student in public college.

The "hierarchical education" literature[6] is also particularly close to our paper (Lloyd-Ellis, 2000; Driskill and Horowitz, 2002; Su, 2004; Blankenau et al., 2007; Arcalean and Schiopu, 2010; Abington and Blankenau, 2013). These papers try to understand the macroeconomic consequences produced by different allocations of public expenditures across basic and advanced education stages ("advanced" generally meaning higher education). Most papers in this literature are theoretical, with Arcalean and Schiopu (2010) being the only quantitative paper that we are aware of. We contribute to this literature by building and calibrating a rich quantitative model that encompasses several relevant features not considered by previous papers, such as heterogeneous agents, credit constraints, and differences between public and private education institutions.

The rest of this paper is organized as follows. Section 2 describes the main model. Section 3 presents the calibration strategy. Section 4 discusses the optimal utilitarian

[6] This term was coined by Sarid (2016). See this reference for a literature review.

policy. Section 5 uses the framework to decompose economic differences between Brazil and the United States. Section 6 presents concluding comments.

2 A model of educational choices and public policy

The model is based on Restuccia and Urrutia (2004) and Herskovic and Ramos (2017).[7] It is composed of households, a representative final goods firm, and the government. Households are represented by overlapping generations, make decisions on consumption, savings, and education, and supply human capital to the final goods firm. The economy is small and households save using international credit markets. The final goods firm employs human capital and produces the consumption good. The government taxes families and provides educational services for free. There are two levels of education (basic and higher) and two types of educational institutions (private and public). Time is discrete and each period in the model is interpreted as an 18-year period.

Households An agent's economic life has four periods. In the first period, an agent is born as a young child (aged zero to 18 years) and in the next period she becomes an old child (18 to 36 years). In the third period of her life, an agent is a young parent (36 to 54 years), and in the fourth and final period she becomes an old parent (54 to 72 years). Each generation is a continuum of positive measure, and a family unit comprises one parent and one child.[8] At each point in time, there are two different family units: one with a young child and a young parent, and another with an old child and an old parent. There is no population growth.

A young household is characterized by the young parent's human capital, h, the young child's innate ability, π, and the household's asset wealth, a. When a child is born, her innate ability is drawn from a log-normal distribution:

$$\log(\pi) \sim N(0, \sigma_\pi^2). \tag{1}$$

All young children go to school and complete basic education.[9]

[7]Compared to these models, the main novelties in our framework comprise financial assets that allow households to transfer resources over time, and a richer human capital production function structure. Without the latter, it would not be possible to use the model to answer our research question. However, we do not adopt some features present in the other models, such as the possibility of dropping out of college (Restuccia and Urrutia, 2004) and a distinction between human capital and the public college exam score (Herskovic and Ramos, 2017).

[8]We discuss later the effect that this demographic structure hypothesis can have on our results.

[9]For simplicity's sake, in the model we posit that young children cannot work. In the 2002 Brazilian National Household Survey (PNAD), 6.15 percent of 6- to 17-year-old children work for 6 hours a day or more.

Similar to young households, an old household is characterized by the old parent's human capital and the household's asset wealth. An old child, however, is characterized by her acquired ability, $\hat{\pi}$, which is the human capital that she has in the stage immediately after completing her basic education but before going to college. An old parent may choose not to send her old child to college.

Decisions are made at the family level. Parents make consumption, savings, and educational investment choices, maximizing the expected discounted utility of its dynasty. The utility function is defined at the family level and depends on household consumption. A parent values her child's consumption in the future as if it were his own. Because of this, there is no bargaining problem between a parent and a child.[10,11]

Human capital An agent's human capital is her productivity in the labor market. It is determined by three types of variables: her innate ability, effective educational investments, and her parent's human capital. Effective educational investment combines governmental and household's education investments. Education investments represent monetary expenditures in educational goods and services, such as schools, books, etc. The transformation of an education investment into an effective educational investment depends on whether the household chooses a public or a private education institution.

Young households make decisions related to basic education. A young household chooses how much to invest in school, $e_y \geq 0$, and an education institution type, which can be public or private. Effective basic educational investment $\hat{e}_y$ is determined by[12]

$$\hat{e}_y = \begin{cases} e_y & \text{if agent attends a private school} \\ \alpha_y(g_y + e_y) & \text{if agent attends a public school,} \end{cases} \tag{2}$$

where $g_y \geq 0$ is the governmental expenditure per student in public school, and $\alpha_y > 0$ is a parameter that measures the productivity of educational investments made through public schools, relative to those made in private schools: if a student in public school receives a one-unit increment of educational investment, the increase in her effective education investment is equal to $100 \times \alpha_y$ percent of the increase that she would get if she were in a private school. Since α_y is a parameter, increasing governmental expenditures in public schools does not affect its productivity – it only means having a higher level

[10] Other papers that model families as dynasties include Becker and Barro (1988), Barro and Becker (1989), and Restuccia and Urrutia (2004).

[11] We do not need to define an age of death for agents. Since utility and consumption are at the family level, it is possible to interpret a young household as composed of one young child, one young parent, and one retired grandparent.

[12] Private education investments e_y may be different conditional on the type of education chosen (public or private).

of input in the effective education technology. A parent choosing to send her child to a public school may still decide to make positive educational expenditures, representing the fact that parents can complement governmental investments through purchasing additional books, private lessons, etc.

Old households make decisions related to college education. An old child will be in one of three situations: (i) having no college education, (ii) having a college degree from a private university or, (iii) having a college degree from a public university. Effective college investment $\hat{e}_o$ is given by

$$\hat{e}_o = \begin{cases} 0 & \text{if agent does not attend university} \\ e_o & \text{if agent attends a private university} \\ \alpha_o(g_o + e_o) & \text{if agent attends a public university,} \end{cases} \tag{3}$$

where in this case $e_o \geq 0$ represents the household's educational expenditure in college. Parameter $\alpha_o > 0$ captures the productivity difference between public and private colleges, and $g_o \geq 0$ is the public expenditure per student in public college.

A child's acquired ability, $\hat{\pi}$, is a function of her innate ability, π, her parent's human capital, h, and effective basic educational investment, $\hat{e}_y$, through a constant elasticity of substitution (CES) technology:

$$\hat{\pi} = \pi \left[\gamma_y h^{\phi_y} + (1 - \gamma_y) \hat{e}_y^{\phi_y} \right]^{1/\phi_y}, \tag{4}$$

where $0 \leq \gamma_y \leq 1$ is a parameter that measures the relative importance of her parent's human capital to her acquired ability formation, and $\phi_y \leq 1$ drives the elasticity of substitution between h and $\hat{e}_y$.[13]

The human capital of an individual, h', is determined by her acquired ability, $\hat{\pi}$, and the effective educational investment in her college education $\hat{e}_o$:

$$h' = \left[\gamma_o \hat{\pi}^{\phi_o} + (1 - \gamma_o) \left(\psi + \hat{e}_o \right)^{\phi_o} \right]^{\theta/\phi_o}, \tag{5}$$

where $0 \leq \gamma_o \leq 1$ and $\phi_o \leq 1$ have similar interpretations as in (4), $0 < \theta < 1$ measures the degree of decreasing returns to scale, and $\psi > 0$ drives the marginal return of higher education investment for an individual without college. Note that if

[13] Parental human capital as an input can be interpreted as a reduced form to capture transmission of moral values from parents to children, differences in parenting styles, and network effects (Doepke and Zilibotti, 2017). Based on Holter (2015) and Blankenau and Youderian (2015), one may also interpret that this mechanism captures differences in parental time investments. Some papers that explicitly model time allocations inside households are Erosa and Koreshkova (2007) and Del Boca et al. (2014).

$\psi = 0$, (5) is a standard CES production function. We assume that $\psi > 0$ because this specification makes an agent's human capital a strictly increasing function of her acquired ability, $\hat{\pi}$, even if she does not study in college ($\hat{e}_o = 0$) and inputs are complementary ($\phi_o < 0$). Parameter ψ measures the essentiality of higher education: if $\psi \to 0$, higher education is essential to human capital formation, since not attending an university would generate zero human capital regardless of the level of acquired ability. On the other hand, a higher ψ means a lower marginal human capital return to higher educational investments, all else constant.[14] The human capital production function specification in Cunha et al. (2010) is a special case of (5) with $\psi = 0$.

Young households We now present the recursive problem of households. The state variable of a young household is given by the parent's human capital h, the child's innate ability π, and household's asset wealth a. The parent inelastically supplies human capital services in the labor market, and her labor income is equal to wh, where w is the equilibrium wage rate. Asset earnings are given by ra, where r is the exogenous interest rate. The government proportionally taxes labor and asset earnings at rate τ and makes lump-sum transfers T to all households. The young parent decides how to allocate her income between consumption, c_y, basic education expenditures, e_y, and the household's asset level taken to the next period, a'_y, and chooses whether to send her child to a public or private school. Conditional on choosing education type $s_y \in \{\text{public}, \text{private}\}$, the problem of a young parent is given by

$$
\begin{gathered}
V_y^{s_y}(h, \pi, a) = \max_{c_y, e_y, a'_y \geq 0} u(c_y) + \beta V_o(h, \hat{\pi}, a'_y) \\
\text{subject to} \\
c_y + e_y + a'_y = (1-\tau)wh + (1 + r(1-\tau))a + T, \\
\text{(2) and (4).}
\end{gathered} \tag{6}
$$

The function $V_y(\cdot)$ is the value function of the young parent and $V_o(\cdot)$ is the value function of the old parent. Since assets cannot be negative, households cannot borrow in the credit market. The child's acquired ability in the next period, $\hat{\pi}$, depends on her innate ability, π, her parent's human capital, h, and decisions on educational investment, e_y, and type, s_y, according to (2) and (4).

The value function of a young household is the maximum between the values of

[14] Another interesting interpretation for ψ is in the case with perfect complementarity. If $\phi_o \to -\infty$, (5) becomes a Leontief production function with decreasing returns to scale, $\min\{\gamma_o \hat{\pi}, (1-\gamma_o)(\psi + \hat{e}_o)\}^{\theta}$. In this case, ψ sets a threshold such that if an agent's acquired ability is higher than this cutoff, the agent necessarily needs to enter university and start investing in college education if she desires to obtain a higher human capital level.

sending the child to public and private schools:

$$V_y(h, \pi, a) = \max \left\{ V_y^{\text{public}}(h, \pi, a), V_y^{\text{private}}(h, \pi, a) \right\}. \tag{7}$$

Old households An old parent makes discrete choices with respect to higher education and chooses how to allocate her family income between consumption, savings, and investment in college. Specifically, an old parent chooses whether or not to have her child apply to college, and, therefore, her value function is defined as

$$V_o(h, \hat{\pi}, a) = \max \left\{ V_o^{\text{no college}}(h, \hat{\pi}, a), V_o^{\text{apply}}(h, \hat{\pi}, a) \right\}, \tag{8}$$

where $V_o^{\text{no college}}(\cdot)$ is the value of not applying and having no college education, and $V_o^{\text{apply}}(\cdot)$ is the expected value of applying to college.

If a child does not apply to college, then she immediately joins the labor market, inelastically supplying her human capital services. Thus, under no college education, the value of not applying is

$$\begin{gathered} V_o^{\text{no college}}(h, \hat{\pi}, a) = \max_{c_o, a_o' \geq 0} u(c_o) + \beta \mathbb{E}_{\pi'} \left[V_y(h', \pi', a_o') \right] \\ \text{subject to} \\ c_o + a_o' = (1-\tau)[\kappa \bar{\xi} w h + \underline{\xi} w h'] + (1 + r(1-\tau))a + T, \\ (3) \text{ and } (5). \end{gathered} \tag{9}$$

The budget constraint includes post-tax labor income from all family members combined. An old parent with human capital h has labor earnings equal to $\kappa \bar{\xi} w h$ and an old child with human capital h' has labor income equal to $\underline{\xi} w h'$. $\bar{\xi} > 1$ and $\underline{\xi} < 1$ are life-cycle earnings parameters that measure, respectively, how much more productive old parents are than young parents, and how much less productive old children than young parents. Parameter κ represents the fraction of the time period that old parents work. The continuation value of the old parent is the expected value function of her own child as a young parent in the next period.[15] This expectation is over the innate ability distribution of the old parent's grandchild, which follows an independent and identically distributed log-normal distribution as in (1). In the case of having no college education, the old child's human capital h' is determined by her innate ability $\hat{\pi}$ and

[15] Intergenerational altruism could be formulated in different ways, as presented by Glomm and Ravikumar (1992). One option is for parents to value their children's allocations as in Kohlberg (1976). Another is to directly value the income they give to their children (human capital and bequests) (Banerjee and Newman, 1991). We choose the formulation where the current generation values their descendants' utilities, as in Loury (1981).

effective educational investment in higher education equal to zero, according to (3) and (5).

To enter either public or private college, children must apply. We assume that all college applicants are automatically admitted to private colleges.[16] The same is not true for a public college. The outcome of an application to public college is random. The probability of admission depends on applicants' acquired ability, and college applicants with higher acquired abilities are likelier to be admitted. The probability of admission is given by $q(\hat{\pi})$, which is an equilibrium object that we discuss later in this section, along with a description of the college admissions market. Taking the probability function as given, the expected value of applying to college is a combination of the value if admitted, $V_o^{\text{admitted}}(\cdot)$, and the value if not admitted, $V_o^{\text{not admitted}}(\cdot)$:

$$V_o^{\text{apply}}(h, \hat{\pi}, a) = q(\hat{\pi}) V_o^{\text{admitted}}(h, \hat{\pi}, a) + [1 - q(\hat{\pi})] V_o^{\text{not admitted}}(h, \hat{\pi}, a). \tag{10}$$

If an old child's public college application is successful, she may go to either a private or public college:

$$V_o^{\text{admitted}}(h, \hat{\pi}, a) = \max \left\{ V_o^{\text{public}}(h, \hat{\pi}, a), V_o^{\text{private}}(h, \hat{\pi}, a) \right\}. \tag{11}$$

The value of going to a college of type $s_o \in \{\text{public}, \text{private}\}$ is

$$\begin{gathered} V_o^{s_o}(h, \hat{\pi}, a) = \max_{c_o, e_o, a'_o \geq 0} u(c_o) + \beta \mathbb{E}_{\pi'} \left[V_y(h', \pi', a'_o) \right] \\ \text{subject to} \\ c_o + e_o + a'_o = (1 - \tau) \left[\kappa \bar{\xi} w h + (1 - \eta_1 - \eta_2) \underline{\xi} w h' \right] + (1 + r(1 - \tau)) a + T. \\ \text{(3) and (5).} \end{gathered} \tag{12}$$

Parameter η_1 represents time spent applying to college, and η_2 captures time spent studying in college. Both η_1 and η_2 represent opportunity costs associated with a college education, since the old child can only work a fraction $1 - \eta_1 - \eta_2$ of the period. The old child's human capital h' is determined by her innate ability $\hat{\pi}$, educational investment e_o, and school type s_o, according to (3) and (5).[17]

Conditional on not being admitted to public college, an old parent chooses between

[16] As Herskovic and Ramos (2017) describe, according to 2008 Brazilian data, there are nearly seven times more students applying to public college than public college seats available. However, for a private college, the ratio of applicants to private college seats is about 1.3. Moreover, only 5 percent of public college seats are unfilled, while 50 percent of private college seats go unfilled.

[17] It is possible to include mandatory tuition fees in public colleges by adding a fixed cost payment if the household chooses to attend a public university. In this case, the measure of public college students will be positive if this fixed cost is lower than g_o, or if $\alpha_o > 1$.

enrolling her child in private college or not:

$$V_o^{\text{not admitted}}(h, \hat{\pi}, a) = \max\left\{V_o^{\text{private}}(h, \hat{\pi}, a), V_o^{\text{no college}^*}(h, \hat{\pi}, a)\right\}. \tag{13}$$

The problem of an old household whose child is not admitted to public college and whose parent decides not to send her child to private college is

$$\begin{gathered} V_o^{\text{no college}^*}(h, \hat{\pi}, a) = \max_{c_o, a_o' \geq 0} u(c_o) + \beta \mathbb{E}_{\pi'}\left[V_y(h', \pi', a_o')\right] \\ \text{subject to} \\ c_o + a_o' = (1-\tau)\left[\kappa \bar{\xi} w h + (1-\eta_1)\underline{\xi} w h'\right] + (1 + r(1-\tau))a + T. \\ (3) \text{ and } (5). \end{gathered} \tag{14}$$

Note that this problem is the same as that of an old household that decides not to apply to college at all, with the only difference that, in (14), the old child spends a fraction of the time period, η_1, away from the labor market with the objective of applying to public college.

Public college admissions We model public college admissions as a competitive market in which applicants compete for college seats. As a result of an old child's application, the college observes a noisy signal of her ability. This signal is interpreted as her grade on the admissions exam, and it is determined by

$$\log(\hat{\pi}^{\text{observed}}) = \log(\hat{\pi}) + \varepsilon, \qquad \varepsilon \sim N(0, \sigma_\varepsilon^2). \tag{15}$$

Parameter σ_ε^2 captures the noisiness of admissions in public college. The measure of seats available at the public college is given by exogenous parameter μ. Given the number of college seats available, the public college admits the applicants with the highest observed scores until all seats are filled or until there is no more demand for seats. There is a grade point cutoff, π^*, such that every applicant with $\hat{\pi}^{\text{observed}} \geq \pi^*$ is admitted. The cutoff π^* is the equilibrium variable that clears the public admissions market. Denote by ν_o the mass of old children who are both admitted and choose to study in public college. The condition for the admissions market clearing is:

$$\begin{aligned} \mu &= \nu_o \quad \text{if} \quad \pi^* > 0, \\ \mu &\geq \nu_o \quad \text{if} \quad \pi^* = 0. \end{aligned} \tag{16}$$

That is, if the grade point cutoff is strictly positive, then all seats must be filled. However, it is also possible that the grade point cutoff is zero and there are empty slots in equilibrium.

Firm The representative final goods firm takes the wage rate w as given and has a constant returns to scale production technology that only depends on labor. The problem of the firm is

$$\max_{H \geq 0} \mathcal{A}H - wH, \tag{17}$$

where $\mathcal{A}$ is total factor productivity (TFP) and H is labor. In equilibrium, we must have $\mathcal{A} = w$. Let H^* denote the optimal choice of the firm which, in equilibrium, is determined by the total human capital supplied by households.

Government The government offers public education and finances itself by taxing households. For each student in public school and college, the government spends g_y and g_o, respectively. Denote by ν_y and ν_o the equilibrium mass of households that study in public school and public college. Total public expenditures in education are given by

$$G = g_y \nu_y + g_o \nu_o. \tag{18}$$

Aggregate earnings are given by total labor earnings and total asset earnings:[18]

$$Y = wH + rA, \tag{19}$$

where aggregate wealth A and aggregate human capital H are determined by households' choices. The condition for the government's balanced budget requires expenditures to be equal to revenues:

$$G + T = \tau Y. \tag{20}$$

Public spending per student for college education, g_o, is an equilibrium variable, while the ratio of public spending per student in the two educational stages, g_y/g_o, the tax rate, τ, and the proportion of aggregate earnings spent on education by the government, G/Y, are parameters.[19]

Equilibrium A stationary recursive competitive equilibrium (or steady state) in this economy is given by value and policy functions, aggregate labor demand H^*, wage rate

[18] If one interprets that the model describes a small economy in which national assets A are held overseas, then aggregate earnings Y are equal to the gross national product (GNP). Other papers that use general equilibrium models with exogenous interest rates are Santos and Weiss (2016) and Rauh (2017).

[19] We include lump-sum transfers, T, in the model in order to calibrate a value for τ that is consistent with the data (which is higher than G/Y). This is important because the tax rate captures marginal incentives that households face when making savings and education decisions.

w, public spending per student in public college g_o, grade point cutoff π^*, lump-sum transfers T, and distributions of young and old households across state variables, such that:

1. Value and policy functions solve households' functional equations.
2. Labor demand solves the final good firm's problem.
3. The labor market clears: the firm's optimal labor demand is equal to the aggregate human capital supplied by households.
4. The mass of public college students is less than or equal to the mass of public college vacancies, with equality if $\pi^* > 0$.
5. The government's budget is balanced.
6. Lump-sum transfers received by households are consistent with (20).
7. The distributions of young and old households are constant over time and consistent with households' optimal decisions and the exogenous distribution of innate abilities. That is, denoting the aggregate distributions (density functions) of young and old households by $\mu_y(\cdot)$ and $\mu_o(\cdot)$, respectively,

$$\begin{aligned}\mu_y(h,\pi,a) = f(\pi)\int_{\tilde{h}}\int_{\hat{\pi}}\int_{\tilde{a}} \mu_o(\tilde{h},\hat{\pi},\tilde{a})\mathbb{I}\left(f_o^1(\tilde{h},\hat{\pi},\tilde{a}) = a\right) \\ \Big\{\mathbb{I}^{\text{apply}}(\tilde{h},\hat{\pi},\tilde{a})\,\Big[q(\hat{\pi})\mathbb{I}\left(f_o^2(\tilde{h},\hat{\pi},\tilde{a}) = h\right) + \\ [1-q(\hat{\pi})]\,\mathbb{I}\left(f_o^3(\tilde{h},\hat{\pi},\tilde{a}) = h\right)\Big] + \\ \Big[1-\mathbb{I}^{\text{apply}}(\tilde{h},\hat{\pi},\tilde{a})\Big]\,\mathbb{I}\left(f_o^4(\tilde{h},\hat{\pi},\tilde{a}) = h\right)\Big\}\,\mathrm{d}\tilde{a}\mathrm{d}\hat{\pi}\mathrm{d}\tilde{h},\end{aligned} \tag{21}$$

$$\mu_o(h,\hat{\pi},a) = \int_{\tilde{h}}\int_{\pi}\int_{\tilde{a}} \mu_y(\tilde{h},\pi,\tilde{a})\mathbb{I}\left\{f_y^1(\tilde{h},\pi,\tilde{a}) = a\right\}\mathbb{I}\left\{f_y^2(\tilde{h},\pi,\tilde{a}) = \hat{\pi}\right\}\mathrm{d}\tilde{a}\mathrm{d}\pi\mathrm{d}\tilde{h}, \tag{22}$$

where $f(\cdot)$ is the log-normal density function (omitting its dependence on variance parameter σ_π^2); $\mathbb{I}(\cdot)$ is the indicator function; $\mathbb{I}^{\text{apply}}(h,\hat{\pi},a)$ indicates if the optimal choice of the old household $(h,\hat{\pi},a)$ is to apply to higher education; $f_y^1(\cdot)$ and $f_o^1(\cdot)$ are asset policy functions of young and old households, respectively; $f_y^2(h,\pi,a)$ is the acquired ability that a young child in household (h,π,a) has when becoming an old child; and $f_o^2(h,\hat{\pi},a)$, $f_o^3(h,\hat{\pi},a)$, and $f_o^4(h,\hat{\pi},a)$, respectively, are functions that yield the human capital that an old child in household $(h,\hat{\pi},a)$ has if admitted to public college, if not admitted to public college, and if not applying to higher education. Functions $f_y^1(\cdot)$, $f_y^2(\cdot)$, $f_o^1(\cdot)$, $f_o^2(\cdot)$, $f_o^3(\cdot)$, and $f_o^4(\cdot)$ are determined by households' optimal choices.[20,21]

3 Calibration and benchmark equilibrium

We calibrate the model using 2002 Brazilian data.[22] The model has 23 parameters, which we separate into two groups. The first group is composed of 13 parameters, whose values are directly obtained from the data or borrowed from the literature. Values for the remaining ten parameters in the second group are obtained through a minimum distance procedure, in which parameter values are chosen so that the simulated model reproduces selected statistics related to educational choices and the labor market in Brazil.

Table 1 displays values for parameters in the first set. First, we normalize TFP to one. We assume that agents have constant relative risk aversion (CRRA) preferences, $u(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$, with a risk aversion of σ. We calibrate this parameter at 1.5, which is a reasonable parameter value considering several estimates from the literature (Restuccia and Urrutia, 2004). The discount factor β is calibrated at 0.947 per year, following Berriel and Zilberman (2018). To calibrate r, we use an annual interest rate of 3.87% before taxes (Berriel and Zilberman, 2018), implying that $r = (1.0387)^{18} - 1$.

To estimate κ, the fraction of the model period in which old parents work in the labor market, we use the 2002 Brazilian National Household Survey (PNAD) cross-section data and find that 61 years is the lowest age value at which the majority of individuals do not work. Therefore, we posit that old parents stop working when they reach 61 years, implying that those agents work for 7/18 of their last period.[23] We use 2002 PNAD data to estimate life-cycle parameters $\underline{\xi}$ and $\overline{\xi}$ following a strategy similar to Restuccia

[20] When computing the benchmark steady state, we verify that using several different guesses for the initial distribution of households across states leads to the same stationary equilibrium (see Appendix C for more information on the numerical algorithm to compute a steady state). See Galor and Zeira (1993) for a discussion of steady state multiplicity in heterogeneous agents models with human capital choices.

[21] We abstract from economic growth in the sense that, in a steady state, aggregate variables are constant over time. However, in the next sections we compute transitions between steady states. Thus, our analysis is also able to describe situations in which aggregate variables change over time.

[22] An advantage of using 2002 data to calibrate the model is that three important educational programs were either implemented or expanded after this year in Brazil, and we do not include those programs in the model. First, a government program that provides scholarships for low-income individuals to study in private universities (ProUni) was implemented in 2004. Second, a program in which the government provides loans to students of private universities (Fundo de Financiamento ao Estudante do Ensino Superior (FIES)) was expanded after 2002. FIES reached 8.3% and 40% of students in private universities in 2002 and 2015, respectively (Ministério da Fazenda, 2017). Third, affirmative action programs were implemented in several public universities starting from 2002.

[23] This average retirement age for Brazil is in line with Queiroz (2006).

Table 1: Externally calibrated parameters

Parameter	Description	Value
$\mathcal{A}$	Total factor productivity	1
σ	CRRA utility	1.5
β	Time discounting	0.947^{18}
r	Interest rate (annual)	0.0387
κ	Old parent work period	7/18
$\overline{\xi}$	Life-cycle earnings (old parent)	1.28
$\underline{\xi}$	Life-cycle earnings (old child)	0.7
η_1	College duration	4/18
η_2	Time spent applying to college	1/18
μ	Vacancies in public college	0.029
τ	Tax rate	0.175
G/Y	Public expend. in educ. over GDP	0.035
g_y/g_o	Basic-higher expenditure ratio	$0.10 \times (12/4)$

Notes: This table shows parameters whose values are directly found in datasets or in other papers in the literature.

Data sources: Row 1: normalization. Row 2: literature. Rows 3-4: Berriel and Zilberman (2018). Row 5-7 and 10: 2002 PNAD. Rows 8-9: Herskovic and Ramos (2017). Row 11: IBGE. Row 12: UNESCO (2017). Row 13: INEP (2018).

and Urrutia's (2004): we compare mean earnings of college-educated individuals in the age intervals of old children, young parents, and old parents. We find that the mean earnings of college-educated old children (old parents) are equal to 70% (128%) of those of young parents.

We follow Herskovic and Ramos (2017) and suppose that students take one year to apply for higher education exams and that college lasts for four years, implying that $\eta_1 = 1/18$ and $\eta_2 = 4/18$. According to 2002 PNAD, 11% of 18- to 35-year-old individuals were either college students or had completed college. Out of all 18- to 35-year-old college students, 25% were enrolled in a public college. Thus, the measure of public seats is calibrated at $\mu = 0.029$, which is 25% of 0.11.[24]

We calibrate τ at 17.5%, which was the government's spending as a proportion of GDP in 2002, according to National Accounts data. In this same year, public expenditures in education (primary, secondary, and tertiary) as a proportion of GDP were equal to 3.5% in Brazil (UNESCO, 2017).[25] We use this value for G/Y. In yearly terms, the ratio between public spending per student in basic education and college

[24]Footnote 16 justifies using the mass of students enrolled in public college as the measure of public seats.

[25]UNESCO's data on public educational expenditures include research and development investments.

was 0.1 in 2002 (INEP, 2018).[26,27] To calibrate g_y/g_o, we multiply this number by 12/4 to account for the fact that the duration of basic education is 12 years, while students take 4 years to complete an undergraduate course in college.

Values for the remaining parameters in the model are not directly observable in the data or cannot be borrowed from the literature. Therefore, we use a minimum distance procedure and choose parameter values such that the model produces statistics related to educational choices and the labor market that are close to the ones observed in the data.

We calibrate ten parameters. Eight of these are related to the human capital function: the relative productivities of public education institutions, α_y and α_o; the relative importance of inputs, γ_y (h vs. $\hat{e}_y$) and γ_o ($\hat{\pi}$ vs. $\hat{e}_o$); the elasticity of substitution drivers, ϕ_y and ϕ_o; the marginal return to college educational investment driver, ψ; and a return to scale parameter, θ. We also calibrate innate ability dispersion, σ_π, and public college admissions process noisiness, σ_ε.

We select ten statistics to be fitted by the model, which can be split into three groups. The first group is composed of moments related to households' discrete choices: the proportion of students in private school, and the mass of old children with no university education.[28] The second group contains statistics related to households' continuous choices. We target the total size of educational expenditures (as a proportion of aggregate earnings), and its coefficient of variation across households. We also make the model reproduce how households' educational expenditures are split between basic and higher education and, for each educational level, how expenditures are divided between private and public institutions. The third group contains moments related to the labor market: the standard deviation of log labor earnings, the college wage premium,[29] and the private school wage premium among university-educated workers.[30]

[26] INEP's (Ministry of Education) data on governmental education investments is composed of active staff payments, social charges, capital investments, and research and development. It does not include retirement costs, scholarships, student financing and interest expenses, amortizations and debt charges of the educational area.

[27] Cross-country g_y/g_o data shown in Figure 1 comes from UNESCO (2017). We decide to use INEP data to feed g_y/g_o into the model because INEP constructs this variable exclusively for Brazil and, therefore, it is superior in terms of considering particularities of this country.

[28] In the regions of the parameter space where all public university vacancies are filled in equilibrium, which is what happens in the calibrated steady state, the proportion of old children in private universities is a deterministic function of the mass of old children without college education. This is why we do not count the mass of private university students as a targeted moment in the calibration strategy.

[29] The college wage premium is the mean labor earnings of university-educated workers divided by the mean labor earnings of workers who do not have higher education.

[30] We thank Priscilla Bacalhau for providing us with this statistic. The private school wage premium among university-educated workers was calculated through a dataset constructed by merging the Brazilian employer-employee matched dataset (RAIS) with a questionnaire that tertiary students

Table 2: Internally calibrated parameters

Parameter	Description	Value
σ_π	Std. dev. of innate abilities	1.5566
σ_ε	Std. dev. of shock that determines university exam scores	0.1005
α_y	Relative productivity of educ. expend. in public schools	0.6774
α_o	Relative productivity of educ. expend. in public universities	0.6157
γ_y	Relative importance of parental human capital (w.r.t. basic educ.)	0.3803
γ_o	Relative importance of acquired ability (w.r.t. higher educ.)	0.9820
ϕ_y	Substitution parameter (parental human capital vs. basic educ.)	-1.8957
ϕ_o	Substitution parameter (acquired ability vs. higher educ.)	-1.0268
ψ	Importance of higher education to human capital	0.0005
θ	Concavity of human capital production function	0.4816

Notes: The values for the parameters shown in this table were obtained so that the model produces statistics close to those in the data. Table 3 shows the targeted statistics and the model's fit performance.

Table 2 shows the parameter values obtained when using the minimum distance procedure, and Table 3 displays targeted statistics in the data and in the model. To motivate our choice of the targeted moments, we next discuss the relationship between parameters and statistics in the model. These arguments are based on a numerical exercise where we simulate the model for different values of a given parameter, while fixing the remaining parameters at the calibrated values. Figure A1 displays the relationship between pairs of parameters and moments obtained through this exercise.

Parameter σ_π is the essential source of heterogeneity in the model, and therefore is positively associated with the standard deviation of (log) labor earnings. The model generates a dispersion in labor earnings close to that in the data. Parameter σ_ε affects the number of students who apply to a public university. If there is no uncertainty in the admissions process, $\sigma_\varepsilon = 0$, then only students with high acquired ability apply and are admitted. As σ_ε increases, more students apply. Those who are not admitted may choose to attend private universities, increasing higher education expenditures. The model replicates how total educational expenditures made by households are split between schools and universities.

The relative productivity of public schools, α_y, is positively correlated with the mass of public school students in the model. We find that 9.3% of students in the model

answer when taking ENADE, a mandatory exam for university students. The ENADE questionnaire includes a question on the type of secondary school (public or private) attended by a student. Using the student's national id code (CPF), this information can be merged with RAIS, allowing the computation of the mean hourly wage of formal workers conditional on having taken the ENADE exam and having attended a public or private secondary school. This dataset uses the 2004-2006 ENADE questionnaires and the 2013-2015 RAIS. See da Silveira (2018) for more details.

Table 3: Targeted moments in the data and model

Moments	Data	Model
Household discrete choices		
Students in private schools	0.1236	0.0937
Old children with no higher educ.	0.8924	0.9069
Old children in private universities	0.0789	0.0643
Household continuous choices		
Educ. expenditure over GDP	0.0255	0.0492
Educ. expenditure (higher/basic)	0.3077	0.3423
Basic educ. expenditure (private/public)	8.0512	11.4535
Higher educ. expenditure (private/public)	3.0831	3.4380
Coef. of var. of educ. expenditures	2.1002	2.4652
Labor market statistics		
Standard deviation of (log) labor earnings	0.9906	0.8206
College wage premium	3.2851	4.4545
Private school wage premium	1.3877	1.2199

Notes: This table shows the statistics that were targeted in the calibration strategy. The values for the parameters shown in Table 2 were chosen so that the model generates the statistics as close as possible to those in the data.

Data sources: Rows 1-3 and 9-10: 2002 PNAD. Rows 4-8: 2002-2003 POF. Row 11: see footnote 30.

choose private education, while the proportion in the data is 12%. The calibration yields $\alpha_y = 0.67$, meaning that human capital marginal returns from education spending in public schools are 67% that of private schools, all else constant. Herskovic and Ramos (2017) find $\alpha_y = 0.5$ using a similar model. In the case of higher education, the relative productivity of public universities, α_o, is positively associated with the private school wage premium among university-educated workers. If α_o increases, public universities become more attractive, raising the incentives for educational expenditures in basic education. Since private school students have higher marginal human capital returns from investments, the economy's adjustment to an increase in α_o will lead to a higher private school wage premium among university-educated workers. This statistic is equal to 1.38 in the data, and 1.21 in the model.

The parameter γ_y measures the importance of parental input to a child's human capital formation. In the benchmark equilibrium, a high proportion of young children have their education financed purely by governmental investments, which are relatively low. In a world where parental human capital is more important, children depend less on basic educational investments and, therefore, will have higher acquired abilities, increasing the proportion of students who choose to attend universities. In the benchmark equilibrium, 90% of old children do not have higher education, while the corresponding number in the data is 89%. The calibration yields $\gamma_y = 0.38$. Blankenau and Youderian (2015) use a similar framework to ours and obtain a 0.5 estimate for a similar parameter. The parameter γ_o measures the relative importance of acquired ability with respect to higher education expenditures. A higher γ_o increases the marginal return of basic education investments, leading to a higher dispersion of educational spending across households. The model does a good job of fitting the coefficient of variation of educational expenditures.

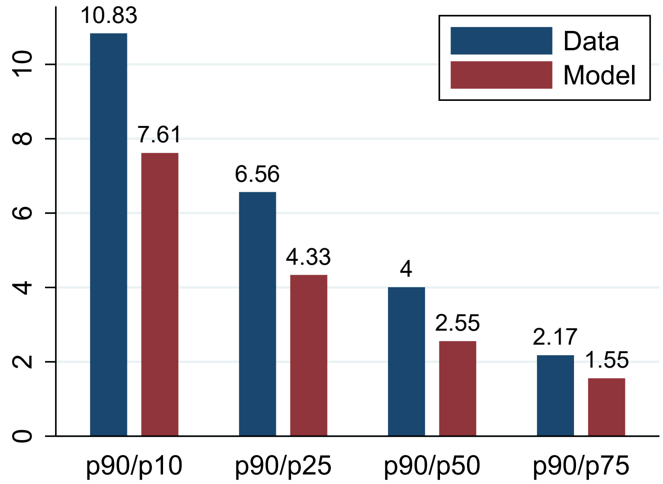
Since inputs are complementary to human capital formation, parental human capital may be a constraint to a child's development through educational expenditures. With higher substitutability between h and $\hat{e}_y$, the human capital gain of a child is less constrained by the parent's human capital, so that the marginal return of educational investments is higher. This relationship is stronger in private schools due to its superior productivity, relative to that of public schools. Therefore, ϕ_y is positively associated with the proportion of total basic education expenditures made through private institutions. A similar argument shows that a higher ϕ_o implies in a larger portion of higher education expenditures made through private universities. We find that $\phi_y = -1.89$ and $\phi_o = -1.02$, implying in a higher complementarity between inputs than that found in other papers (Cunha et al. (2010) obtain estimates that range from -0.78 to -0.18). A potential reason for this difference is that our human capital production function has a new specification due to the parameter ψ, and we also work with a different classification of educational stages.

The parameter ψ affects the college wage premium. The higher ψ is, the less essential is higher education and, therefore, only more selected students will attend universities, increasing the college wage premium. Finally, θ regulates the overall concavity of the human capital production function. The higher θ is, the higher are the marginal returns to educational investments and, therefore, the higher is the proportion of aggregate earnings spent in education by households.

Non-targeted statistics In this section, we discuss the model's performance in fitting statistics that were not targeted in the calibration process.

In the calibration procedure, we target the standard deviation of labor earnings across workers, but we can also investigate if the model does a good job in terms of fitting other moments of the wage distribution, such as the ratios between the 90^{th} percentile and percentiles 10, 25, 50, and 75. These statistics are shown in Figure 2.

Figure 2: Labor earnings distribution percentile ratios



Notes: This figure shows the ratio between percentiles of the distribution of labor earnings in the data and in the model. For example, the ratio between percentiles 90 and 10 of the labor earnings distribution is equal to 10.83 in the data, whereas the model generates a ratio equal to 7.61. These statistics were not targeted in the calibration strategy.

Data source: 2002 PNAD.

Table 4 shows discrete choices related to basic education by parents' labor earnings quartiles. The model does a good job of fitting the choices of the first two wage quartiles, and overestimates public schooling choice for the two top income quartiles. The maximum absolute error is in the top quartile, and is equal to 13%.

Table 5 shows the pattern of higher education choices by parents' labor earnings quartiles. The model replicates the statistics for the first three wage quartiles, but it slightly underestimates the measure of private university students in the top wage

Table 4: Basic education choices by parents' labor earnings quartiles

	Data		Model	
	Private	Public	Private	Public
1	1.54	98.46	1.62	98.38
2	4.73	95.27	2.02	97.98
3	11.97	88.03	2.60	97.40
4	47.85	52.15	34.36	65.64

Notes: This table shows the distribution of children across public and private schools conditional on parental labor earnings quartiles in the data and in the model. For example, 1.54% of households in the bottom wage quartile send their children to private schools in the data, whereas 1.62% of such type of households in the model do the same.

Data source: 2002 PNAD.

Table 5: Higher education choices by parents' labor earnings quartiles

	Data			Model		
	No univ.	Private	Public	No univ.	Private	Public
1	98.52	0.98	0.5	95.97	3.61	0.42
2	95.98	2.81	1.21	95.53	3.99	0.48
3	90.83	6.62	2.55	91.91	6.00	2.10
4	68.69	23.93	7.38	77.63	13.01	9.37

Notes: This table shows the discrete choices related to higher education conditional on parental labor earnings quartiles in the data and in the model. The interpretation of these numbers are similar to that in the previous table.

Data source: 2002 PNAD.

quartile. The model performs well in terms of fitting the distribution of parents' wages among public university students.

In the calibration procedure, we target the overall variance of educational investments made by households. Here we evaluate if the model also fits the variance of education expenditures conditional on the two educational stages. We find that the fit is good: the coefficient of variation in basic education is 2.43 in the data and 1.78 in the model, while for higher education, it is 0.91 in the data and 0.78 in the model. This is summarized in the first two rows of Table 6.

Our last validation exercise is to replicate a quasi-experiment exploited by Francis-Tan and Tannuri-Pianto (2018), who estimate the local average treatment effect on wages of being admitted to an elite public university in Brazil. These authors investigate the impact of 2004–2005 admissions results for the University of Brasilia on applicants' labor earnings eight years after the selection exams. Using a regression discontinuity approach, they find that students who are admitted with a score close to the admissions cutoff earn 12% higher wages than those who are rejected close to the threshold. Since

Table 6: Other non-targeted moments

	Data	Model
Coef. of variation of hh. educ. expenditures (basic)	2.43	1.78
Coef. of variation of hh. educ. expenditures (higher)	0.91	0.78
Francis-Tan and Tannuri-Pianto (2018) RD replication (%)	12.64	7.76

Notes: This table shows the model's performance in terms of fitting some statistics that were not used as targets in the calibration strategy. In the third row, we run a regression discontinuity (RD) using simulated data from the model. We use exam scores as the running variable and old children's log labor earnings as the outcome variable, and compare the discontinuity coefficient estimate with that obtained by Francis-Tan and Tannuri-Pianto (2018) using Brazilian microdata.

Data sources: Rows 1-2: 2002-2003 POF.

the University of Brasilia is an elite university, we can take the authors' estimate as an upper bound for the public university wage premium among applicants close to the exam cutoff score in the model.

We replicate the experiment in the model by simulating exam scores for applicants. We select a subsample of applicants using the same rule as in Francis-Tan and Tannuri-Pianto (2018)[31] and run a regression discontinuity using the simulated data. We obtain a discontinuity coefficient equal to 0.0747, implying that barely-admitted agents earn 7.76% (= $\exp[0.0747] - 1$) higher wages than barely-rejected applicants. This value is inside a plausible range, suggesting that the magnitude of the public college wage premium is disciplined in the model.

Francis-Tan and Tannuri-Pianto (2018) find that applicants who barely pass the exam do not display a statistically-significant higher probability of having completed higher education than applicants who are barely rejected. This is consistent with our model since all agents who apply but are not admitted decide to study in private universities. Also, student state migration was relatively small at that time (Machado and Szerman, 2016), suggesting that the probability of a non-admitted applicant in Francis-Tan and Tannuri-Pianto's (2018) sample deciding to study in a public university other than the University of Brasilia should be small.

4 Optimal utilitarian policy

In this section, we use the calibrated framework to investigate the effects of educational policies on welfare and aggregate variables, such as aggregate earnings and income inequality. The relationship between welfare and the allocation of government resources across public schools and public universities is characterized by a trade-off. On one hand, devoting resources to public schools may generate significant welfare gains because credit frictions lead to the existence of credit-constrained parents with high-ability children that would benefit significantly from additional educational investments.

On the other hand, public universities are characterized by the selection of high-ability students. Allocating too few resources to public universities may lead to unexplored human capital potential. In such a scenario, since educational stages are complementary, part of the investments in basic education for low-income public college students are lost.

To simulate a given allocation of government per-student expenditures across the two educational stages in the model, we choose a value for the basic-higher public

[31] Treated agents consist of all admitted students, while agents in the control group are those with scores greater than or equal to the 90$^{\text{th}}$ score percentile among rejected applicants.

expenditure per student ratio parameter (g_y/g_o). Increasing this ratio means reallocating government spending (per student) from universities to schools, keeping fixed the proportion total earnings spent on education by the government.

We start by asking the following question: how should an utilitarian social planner, i.e. a planner who takes into account the average discounted lifetime utility of agents, allocate a fixed educational budget across basic and higher education? To answer this question, we simulate the model for different allocations of per-student expenditures across public schools and public universities and evaluate their effects on aggregate welfare.

In this environment, comparing the well-being of households in two different steady states is not an appropriate way of executing welfare analysis because it could take a long time for the economy to transit from the benchmark stationary equilibrium to the new steady state. In such a case, comparing households in the two steady states would be equivalent to comparing two different generations very apart from each other across time.

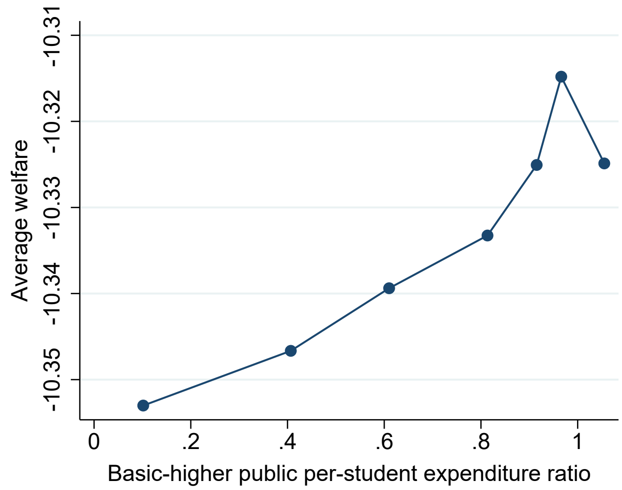
Thus, to appropriately make welfare analysis in our model, we first compute the transition from the benchmark equilibrium to the new steady state associated with a new policy. We follow Kambourov (2009) and assume that households don't expect the policy change, which is completely and immediately implemented in the first period of the transition. A household in the first period of the transition knows the economy's full path toward the new steady state. This family's expected discounted value in the first period of the transition summarizes how this policy affects its welfare, which takes into account the utilities of all future generations of its dynasty.[32]

To look for the optimal utilitarian policy, we simulate transitions from the benchmark equilibrium to steady states associated with different values for g_y/g_o. For each simulation, we compute the average discounted lifetime utility across agents in the first period of the transition. Figure 3 plots this average welfare for different values of g_y/g_o and shows that the optimal utilitarian policy is the one in which the annual basic-higher per-student expenditure ratio is equal to 0.96. That is, for each dollar spent in the education of a public college student, the government allocates 96 cents to a public school student. This allocation of per-student expenditures is similar to that followed by the United States in the middle of the 2000s.[33]

[32]In Appendix B we formally define a transition between two steady states and describe the computational algorithm to compute transitions.

[33]The simulations show that there is no reallocation of expenditures across education stages that makes all households better-off. A Pareto-improving policy could potentially be obtained through a reallocation policy coupled with transfers that depend on households' characteristics (compensating families that are worse-off using resources from benefited households). Looking for a reallocation policy that benefits all households is an interesting venue of research. However, since implementing

Figure 3: Looking for the optimal utilitarian policy



Notes: This figure shows the average discounted lifetime utilities across all households for different allocations of government per-student expenditures across education stages, g_y/g_o. These welfare values are measured in the first period of the transition from the benchmark equilibrium to the steady states associated with each g_y/g_o. The optimal utilitarian policy maximizes average welfare by making annual g_y/g_o equal to 0.96.

To have a better grasp on the welfare effects of the optimal policy, we study two additional welfare measures. The first is the mass of households who are better off with the new policy. The second welfare measure is the willingness to pay, which can be understood as the answer to the following question: "how much an agent would be willing to pay to be in the situation with the policy?". Formally, it is computed as the maximum decrease in consumption that a household would be willing to lose in all periods and contingencies of the equilibrium associated with the policy, such that its lifetime utility equals that of the initial steady state (Chatterjee et al., 2007). To compute both welfare measures, we again compare the situations of households in the benchmark equilibrium with that in the first period of the transition to the steady state associated with the new policy.

Table 7 shows the two welfare measures associated to the optimal utilitarian policy. We find that almost all of the households are better off with in the new situation. The mass of households that are worse-off, 3.5%, is lower than the measure of households that apply to the public university in the benchmark equilibrium, 4.6%, showing that even some of the agents who would apply to public university if there was no policy change are better off if the government reallocates resources from universities to schools.

The mass of households choosing public schools increases from 90% in the bench-

the optimal utilitarian policy is already close to a Pareto-improvement, we choose not to follow this path.

Table 7: Welfare measures associated to the optimal utilitarian policy

Welfare measures	%
Better-off households	96.47
Income quartile 1	100
Income quartile 2	100
Income quartile 3	99.08
Income quartile 4	86.80
Willingness to pay	0.49
Income quartile 1	0.82
Income quartile 2	0.66
Income quartile 3	0.49
Income quartile 4	0.01

Notes: This table shows two welfare measures associated with the optimal utilitarian policy. It shows that 96.47% of the households are better-off with the new allocation of public expenditures. Households in the first income quartile would be willing to pay 0.82% of their consumption in all periods and states of nature to implement the optimal utilitarian policy.

mark equilibrium to almost 92% in the first period of the transition to the new steady state. Since 96% of the households are better-off, this shows that the new policy benefits even households that send their children to private schools in the new situation. Although such a family does not immediately profit from higher public basic educational investments, it takes into account that they may be needed in the future, in case a sequence of bad shocks related to the innate ability of their descendants take place. That is, public schools work as an insurance in this model.

Households in the first income quartile would be willing to pay on average 0.82% of their consumption in all periods and contingencies to implement the policy, which is a considerable amount of resources. The average willingness to pay for the policy across all families is equal to 0.49% of consumption in all periods.[34]

Table 8 shows the long-run effects produced by the optimal utilitarian policy. In the new steady state, g_y increases by 9%, and g_o decreases by 88%. This vast difference between the proportional changes in both variables is due to the existence of a large number of public school students and a small number of public university students.

[34] Scholz and Seshadri (2009) find that, since poorer households have more children, they face stronger credit constraints than wealthier families. This suggests that, in a model with non-uniform demographic structure, the optimal policy would be more focused on basic education than the one we find. Other similar papers that work with a uniform demographic structure are Restuccia and Urrutia (2004), Lee and Seshadri (2019), and Caucutt and Lochner (2020).

Table 8: Long-run effects of the optimal utilitarian policy

	Benchmark	Optimal
g_y/g_o (annual)	0.1017	0.9661
g_y (%Δ)		9.0541
g_o (%Δ)		-88.5206
Macroeconomic variables		
Aggregate earnings (%Δ)		0.0639
Human capital (%Δ)		0.2530
Asset holdings (%Δ)		-4.1895
Consumption (%Δ)		0.2494
Income Gini coefficient	0.3594	0.3521
Income p90/p10	5.5139	5.3682

Notes: This table shows the long-run effects of the optimal utilitarian policy in some variables, such as government's per-student spending in public schools, g_y, public colleges, g_o, aggregate earnings, and income inequality.

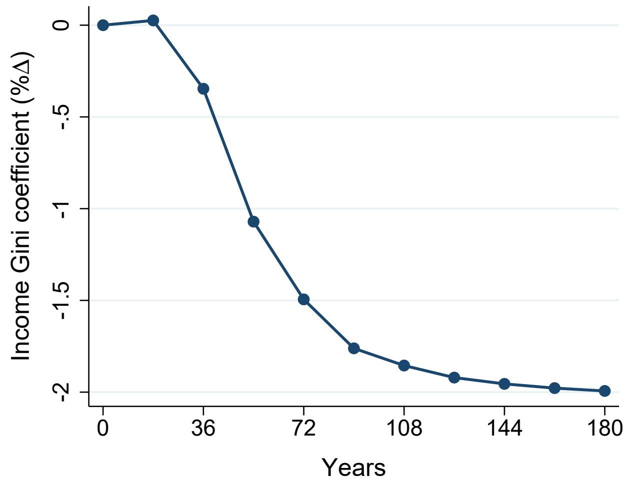
If credit frictions are intense in this economy, there will be a significant fraction of poor households underinvesting in their offspring. In this case, reallocating governmental expenditures from public universities to schools may have a positive impact on aggregate human capital, even after accounting for the fact that investment return might be small due to public schools' relative lower productivity.

On the other hand, asset holdings and public schools compete as insurance mechanisms in this environment. As in standard economic models, households accumulate assets in good times to maintain a smooth consumption. Public schools work similarly, providing a minimum human capital level to children in families that receive a sequence of adverse shocks over time. Since in the steady state associated with the optimal policy households encounter an environment where the insurance provided by public schools is higher, they may choose to cut back on asset holdings and consume part of it.[35] Consistent with this, we find that aggregate human capital increases by 0.25% in the long-run, whereas aggregate asset holdings decrease by 4%. These two opposing forces lead to a small but positive long-run effect of aggregate earnings of 0.06%.[36]

[35] Adding income shocks to the model would make credit constraints more binding for poor families. Furthermore, income shocks reduce the correlation between the decisions of parents and the outcomes of their children, potentially affecting rich households in a highly negative way. These facts suggest that including labor income uncertainty to the model would lead the optimal policy to be even more tilted towards basic education. See Restuccia and Urrutia (2004) for a robustness exercise including post-college idiosyncratic permanent earnings shocks in a similar model.

[36] In the final good's production function (17), a college graduate with a given human capital level is perfectly substitutable by a non-graduate with the same human capital. A different modeling choice

Figure 4: Income inequality during the transition



Notes: This figure shows the fall of the income Gini coefficient during the transition from the benchmark equilibrium to the steady state associated with the optimal utilitarian policy. Since there are credit frictions and asset accumulation in the model, the transition dynamics take several years to be complete.

The new policy generates less economic inequality, decreasing the income Gini coefficient by 2%, and the ratio between percentiles 90 and 10 of the income distribution by 2.6%.[37] Figure 4 shows the behavior of the income Gini coefficient during the economy's transition to the new steady state. In the first period, inequality barely moves because public school students who benefit from the new policy still have not entered the labor market. However, in the second period of the transition, the Gini coefficient begins to change because the first cohort affected by the policy starts to work.

The fourth period of the transition (after 72 years) is the first period in which all agents who are in the labor market are such that, when they were young children, the

would be to use a CES production function that depends on the measures of college graduates and non-graduates. Adopting this modeling choice would have an ambiguous effect on the optimal policy result. On one hand, increasing g_y and decreasing g_o in our model may lead to a lower demand for higher education in the long-run due to a lower government spending in universities. Using a CES production function for the final good's firm could alleviate this effect because the wage rate of college graduates would increase as a response to a lower supply of college graduates. This would mitigate the welfare losses of rich households and, therefore, the optimal utilitarian policy would be associated with a higher g_y/g_o than the one that we find. On the other hand, switching public resources from higher to basic education may lead to a higher demand for universities because of the complementarity of education stages. In this case, using a CES production function for the final good could lead to an optimal policy where g_y/g_o is lower than the one that we find.

[37] There are other education policy aspects that may be more important to explain income distribution than the allocation of public expenditures across basic and higher education. For example, Tamura (2001) finds that teacher quality is an important element that explains the convergence of income per capita across U.S. states over 1882-1990. Similarly, Turner et al. (2013) show that unobserved variables (such as the efficiency through which inputs are used, including education inputs) explain most part of the convergence of output per worker across U.S. states from 1840 to 2000.

new policy was already in place. However, this does not mean that the transition to the new steady state is over in the fourth period. In fact, after 72 years, the change in the income Gini coefficient over time is still not close to coming to an end, with almost one fourth of the transition still to happen in the next 72 years. In this environment, the transition dynamics take several years to be complete because there exist credit frictions and asset accumulation, so that it takes a long time for the economy to fully adjust to the new conditions.[38,39]

4.1 Inspecting the mechanisms

Next, we investigate the quantitative importance of some mechanisms present in the model in shaping the long-run effects of the optimal utilitarian policy by running three counterfactual exercises. First, we study the macroeconomic impacts of agents' reactions to the policy. Second, we evaluate the importance of general equilibrium effects. Third, we investigate how the parent-child human capital transmission affects the results in the simulations. In each exercise, we compare the benchmark equilibrium with a counterfactual steady state associated with the optimal policy, but also featuring some modification that allows us to clearly investigate a given mechanism in shaping the new policy's effects.

To investigate the importance of behavior, we simulate a counterfactual steady state by iterating the aggregate distribution assuming that households make the same choices as in the benchmark equilibrium (i.e., we fix the policy function), while governmental per-student expenditures are those associated with the optimal utilitarian policy. The results are presented in the second column of Table 9, where numbers represent percentage changes from the benchmark equilibrium to the fixed-choices counterfactual simulation. This exercise leads to a fall in aggregate earnings, aggregate human capital, total asset holdings, and to an sizable decline in income inequality.

The simulations show that, although the average human capital of the three first income quartile increases, the human capital of wealthy households falls by a considerably larger amount. Since educational stages are complementary, part of the sizable school expenditures that households in the top income quartile make are lost because

[38]The algorithm that we use to compute transitions is such that the distance between the distributions of households across states in the last period of the simulated transition and in the terminal steady state is smaller than a given tolerance level. Therefore, in our computations, the economy reaches the steady state (up to a numerical tolerance) in a finite number of periods.

[39]If public and private educational institutions have the same productivities, $\alpha_y = \alpha_o = 1$, the macroeconomic effects of increasing annual g_y/g_o from 0.1 to 0.96 are stronger: aggregate earnings increase by 0.5% (eight times more), total human capital increases by 0.7% (2.8 times more), and assets fall by -2.9% (-4.1% with the calibrated parameters). The impact on income inequality (-1.7%) is smaller than in the baseline exercise (-2.02%).

Table 9: Inspecting mechanisms in the model

(%Δ)	Complete exercise	Fixed choices	No general equilibrium effects	No parent-child human capital transmission
Aggregate earnings	0.0639	-0.5589	0.1644	-0.1710
Human capital	0.2530	-0.1950	0.3328	-0.0081
Asset holdings	-4.1895	-8.7426	-3.6250	-4.7183
Income Gini coefficient	-2.0286	-6.8562	-2.0366	-2.3719

Notes: This table shows the results of exercises that shed light on how some mechanisms shape the optimal policy's long-run effects. The numbers represent percentage changes from the benchmark equilibrium to a given counterfactual simulation (corresponding to each column). The first column shows the long-run effects of the optimal utilitarian policy, replicating some values in Table 8. In the second column, the counterfactual simulation consists in implementing the optimal utilitarian policy but keeping household choices fixed as in the benchmark equilibrium. In the third column, we use a simple back-of-the-envelope rule to update general equilibrium variables (g_o, π^*, and T) when simulating the optimal policy. In the last column, we turn off the direct transmission of human capital from parent to child ($\gamma_y = 0$) and recalculate the general equilibria before and after the policy change.

they are not backed up anymore by public expenditures in public universities, leading to lower aggregate human capital.

If the government reallocates expenditures from higher to basic education, families that send their children to public schools crowd out government expenditures by decreasing their private educational spending. The third row of Table 9 shows that, if we fix choices as in the benchmark equilibrium, households with children in public schools move to states where they have relatively high levels of human capital (because they do not crowd out public expenditures) and, therefore, compensate by holding fewer assets. The negative effects on human capital and assets imply in a fall of aggregate earnings in this scenario.

Crowing out effects also explain the policy's impact on income inequality. In the initial steady state, government basic education per-student spending, g_y, is low, so that even relatively poor households must make strictly positive educational investments to assure that their offspring end up with modest human capital levels. Thus, simulating the new policy while keeping choices fixed as in the benchmark equilibrium leads to a higher fall in labor earnings inequality. Similarly, poor households have more assets in the benchmark equilibrium than in the new steady state due to the insurance effect. These two facts together lead to a stronger decline in income inequality if we keep choices fixed as in the benchmark equilibrium. That is, agents reacting to the new policy end up mitigating the fall of inequality.

Next, we investigate the importance of general equilibrium effects. An essential general equilibrium variable in the model is the government spending per student in higher education. When we compare the benchmark equilibrium with the optimal policy's steady state, two ambiguous forces determine general equilibrium effects with respect to the government's balanced budget condition. First, when public resources are reallocated from tertiary to basic education, the demand for public schools increases, while that of public universities decreases. In fact, the number of public school students increases from 90% to 92%, and the number of public university students falls from 2.87% to 0.02% after the implementation of the optimal policy. A higher number of individuals in public schools implies that, in equilibrium, the government basic education expenditure per student must be lower than if the mass of public school students does not change.

Second, since we find that earnings are slightly higher in the new steady state, governmental revenues are also larger because they are a constant proportion of the aggregate earnings. This implies that in general equilibrium, governmental expenditures must be higher than if we ignore the fact that public revenues increase.

To study these general equilibrium forces, we simulate the previous educational policy using a simple rule to update equilibrium variables. First, we increase annual g_y/g_o from 0.1 to 0.96 as before, but to obtain the values of g_y and g_o, we assume that all remaining variables that show up in equation (18) (i.e., the mass of students in public educational institutions and governmental revenues) are kept constant at their values before the policy change. This simple rule represents a naive back-of-the-envelope calculation that ignores the endogeneity of important equilibrium variables. Second, we keep the public university exam score cutoff and lump-sum transfers constant.

The third column of Table 9 shows that this simple general equilibrium updating rule would lead us to conclude that the policy's impact on aggregate earnings is equal to 0.16%, which overestimates the real effect by a factor of 2.5. Since we would escalate the policy's macroeconomic effects, we would also overestimate the impact on income inequality. This result shows that taking into account the change in the number of students in public schools and public universities is quantitatively more important than considering the endogeneity of public revenues as forces that compose the general equilibrium effects in the baseline experiment.

In the model, parental human capital is an input of a child's human capital. This may be an important mechanism in terms of boosting the effects of education policies because it generates a dynamic mechanical force associated with educational investments: if the government makes a higher education investment in a given child, then after that child becomes a parent (with greater human capital), the return of investing in this parent's child (the second-generation child) will be higher than otherwise it would have been. We can observe the importance of this mechanism by turning it off. To do so, we set the parameter that governs this force, γ_y, equal to zero, and recalculate the general equilibria before and after the policy.

Comparing these two counterfactual steady states, we find that ignoring the parent-child human capital transmission mechanism would lead us to conclude that the reallocation policy leads to a large fall in the human capital of the wealthier households, resulting in a lower aggregate human capital level. The fall in aggregate assets are also larger than in the baseline counterfactual, leading to lower aggregate earnings in general equilibrium.

This same mechanism has a distinct impact on economic inequality. All else equal, children of parents with high human capital have a higher human capital return from educational investment than poor children. Therefore, increasing public expenditures in schools may lead to higher inequality among benefited families, and possibly among all households. In fact, shutting down this mechanism increases the educational policy's absolute impact on inequality from -2.03% to -2.37%. The fourth column of Table 9 summarizes the results of this exercise.

5 Comparing Brazil and the United States

The economies of Brazil and the United States are different in several aspects. In particular, the U.S. was approximately four times richer than Brazil (in terms of per capita GDP) and had a college attendance 2.3 times higher in 2000. Next, we study how our framework can replicate such differences. In Section 5.1 we re-calibrate the model to fit data on the United States. In Section 5.2 we use the calibrated model to understand aggregate earnings and college attendance differences between the two countries.

5.1 Re-calibrating the model to the United States

We start by dividing the model parameters in three sets. The first set is composed of parameters that have the same values for both the United States and Brazil, which includes utility parameters (risk aversion σ, and time discount β), innate ability variance (σ_π), and human capital production function parameters ($\gamma_y, \gamma_o, \phi_y, \phi_o, \psi, \theta$). We also assume that the time cost to apply to higher education is one year (η_1), the same as that in the benchmark calibration, and that college education takes four years (η_2) as in Restuccia and Urrutia (2004). The U.S. Bureau of Labor Statistics (1998) reports that the average retirement age in the U.S. in 1995 was approximately equal to 62 for

Table 10: Externally calibrated parameters (U.S. economy)

Parameter	Description	Value
r	Interest rate (annual)	0.027
$\overline{\xi}$	Life-cycle earnings (old parent)	1.12
$\underline{\xi}$	Life-cycle earnings (old child)	1
μ	Vacancies in public college	0.189
τ	Tax rate	0.145
G/Y	Public expend. in educ. over GDP	0.0389
g_y/g_o	Basic-higher expenditure ratio	$0.977 \times (12/4)$

Notes: This table shows parameters that are externally calibrated using data for the U.S., and whose values are different from the parameters that are externally calibrated to fit the Brazilian data.

Data sources: Row 1: U.S. Department of the Treasury (2020) and Federal Reserve Bank of Minneapolis (2020). Rows 2-3: Restuccia and Urrutia (2004). Row 4: NCES (2018) and OECD (2020). Rows 5-6: World Bank (2020). Row 7: UNESCO (2017).

men and women. Since this number is similar to that for Brazil (61), we use the same value for κ in the two economies.

The second set of parameters is composed by those whose values can be directly found in datasets or other papers, and are shown in Table 10. The average 1-year nominal interest rate on US' treasury bonds was 6.1% in 2000 (U.S. Department of the Treasury, 2020), whereas the inflation rate for that year according to the Consumer Price Index was 3.4%, yielding a real annual interest rate of $6.1\% - 3.4\% = 2.7\%$. We use the same values for the life-cycle labor earnings parameters, $\underline{\xi}$ and $\overline{\xi}$, as those used by Restuccia and Urrutia (2004). The proportion of people aged 25 or over who had completed college education in the U.S. in 2000 was equal to 23.6% (NCES, 2018), while 80% of university students attended public colleges that year (OECD, 2020), leading to a mass of 18.9% vacancies with respect to the measure of old children in the model. According to the World Bank (2020), US' general government final consumption expenditure was 14.5% of GDP in 2000, and the American government spent 3.89% of the GDP for educational purposes that year. Therefore, we set $\tau = 0.145$ and $G/Y = 0.0389$. We use UNESCO (2017) data and set the annual g_y/g_o equal to US' mean basic-higher expenditure ratio over the years 1999-2011.

The third set of parameters is composed by the total factor productivity, $\mathcal{A}$, the relative productivity of public institutions, α_y and α_o, and the noisiness of the admission process to public college, σ_ε. To determine the values of these parameters, we select four statistics for the model to match: US' per capita GDP (purchasing power parity) with respect to Brazil (aggregate earnings in the model),[40] the measure of students in

[40] Using GNP data instead of GDP would lead to a similar statistic: the ratio of per capita GNP

Table 11: Internally calibrated parameters (U.S. economy)

Parameter	Description	Value
$\mathcal{A}$	Total factor productivity (TFP)	1.79
α_y	Relative productivity of educ. expend. in public schools	0.70
α_o	Relative productivity of educ. expend. in public universities	1.13
σ_ε	Std. dev. of shock that determines univ. exam scores	0.21

Notes: This table shows the parameter values that are internally calibrated so that the model fits U.S. data.

public schools, the mass of individuals without college education, and the measure of students in public universities.

Tables 11 shows the values for the internally calibrated parameters and Table 12 shows the targeted statistics in the data and in the model. For the model to replicate the per capita GDPs of U.S. with respect to Brazil, TFP in the U.S. economy needs to be 1.79.[41] We find that the relative productivity of expenditures made through public schools (with respect to private schools) in the U.S., 70%, is slightly higher than that in the calibrated Brazilian economy, 67%. Differently from the model fitted to Brazil, public universities in the U.S. model have a higher productivity than that of private colleges. Finally, the calibrated noisiness of the admission process is higher in the American economy.

The model replicates the distribution of students across public and private educational institutions in basic and higher education observed in the data, and predicts that the aggregate earnings in the U.S. economy are 3.5 times that of the Brazilian benchmark equilibrium, underestimating the data by 12%.

Table 12: Targeted moments in the data and model (U.S. economy)

Moments	Data	Model
Per capita earnings (w.r.t. to Brazil)	4.0101	3.5055
Students in public schools	0.8900	0.9144
Old children with no higher educ.	0.7440	0.6996
Old children in public universities	0.1890	0.1899

Notes: This table shows the statistics that were targeted in the calibration strategy for the US. *Data sources:* NCES (2018), OECD (2020), and World Bank (2020).

(purchasing power parity) between U.S. and Brazil was 4.1755 in 2000.

[41]The total factor productivity in the U.S. model does not need to be 3.5 to generate aggregate earnings 3.5 higher than in the Brazilian economy. This happens mainly because the effect of TFP on aggregate earnings is boosted by endogenous investments in human capital by households, in line with several studies (Erosa et al., 2010; Manuelli and Seshadri, 2014; Cubas et al., 2016).

Although public universities in the U.S. are not tuition-free as in Brazil, the calibration to the U.S. delivers statistics that are consistent with the data in this dimension. We find that in the equilibrium calibrated to the U.S. economy, only 7% of the public college students make education expenditures that are lower than the average (across students) total tuition paid by students enrolled in 4-year U.S. institutions in 2000-01.[42] Besides, the proportion of public college students who choose not to make positive education spending is insignificant (0.25%).

5.2 Understanding differences between Brazil and the United States

To have a better grasp on some differences between Brazil and the United States, we conduct several exercises. In each exercise, we follow the steps: (i) start with the model calibrated to Brazil, (ii) change the value of one parameter of interest to correspond to that of the U.S., and (iii) evaluate how aggregate earnings and college attendance vary in general equilibrium. This exercise corresponds to evaluating how Brazil would be different if it had a given aspect of the U.S. economy (such as the number of openings in public universities).

Public universities in the United States have six times more students than those in Brazil. A priori, increasing the number of public university vacancies in Brazil has an ambiguous effect on aggregate earnings. On one hand, expanding access leads to a reduced government spending per student. It may also imply in a lower mean acquired ability among admitted students because there will be less selection due to a lower exam score cutoff in general equilibrium. These facts potentially lead to diminished marginal human capital returns to education spending among public college students and, therefore, to lower aggregate earnings gains from public expenditures.

On the other hand, in the environment with few vacancies, low-income families with high-ability children may decide not to apply because of the opportunity costs of doing so. This may lead to a situation in which some old children who do not apply to public universities have higher acquired abilities than applicants who are admitted. More vacancies imply in a higher probability of admission, and the entry of such credit-constrained applicants may result in a higher acquired ability among university students. Thus, expanding access to public universities potentially generates higher aggregate earnings, even though government expenditures per student decrease. This effect becomes stronger if we consider the fact that, due to dynamic complementarity

[42]We use U.S. data on average total tuition, fees, room and board rates charged for full-time undergraduate students in 4-year degree-granting public institutions from NCES (2019).

and a higher probability of admission, families will increase schooling investments in basic education.

We start the first exercise by simulating the Brazilian economy with the same number of vacancies in public higher education as that in the U.S., which corresponds to a large increase in the measure of openings in the benchmark economy from 2.9% to 18.9%. The third column of Table 13 shows that this policy leads to 2.06% higher aggregate earnings in the long-run. At the same time, public expenditures per student in public universities decrease by 20%.

The fourth column computes an approximate elasticity measure related to this exercise by dividing the percentage effect on aggregate earnings by the percentage increase in μ: doubling the number of vacancies leads to a 0.37% rise in aggregate earnings. We show later that other parameters are associated with much higher elasticities with respect to aggregate earnings.

Replicating the number of U.S. vacancies in Brazil leads to an increase in the measure of public college students by 53%. Interestingly, this policy change is not enough to close the gap of public college attendance between the two countries. This happens because part of the TFP differences are related to disparities in the overall quality of education institutions, which are lower in Brazil. Our framework does not distinguish how a country's TFP is split between overall education quality and other factors that contribute to aggregate productivity (such as a country's degree of property rights enforcement). Differences between the overall qualities of education institutions in Brazil and the U.S. would potentially explain a large part of the human capital discrepancies across the two countries.

In the second exercise, we assume that the Brazilian government allocates the same proportion of aggregate earnings to public education as in the United States. The corresponding increase of G/Y leads to 1.5% higher aggregate earnings. In elasticity terms, G/Y has a stronger impact than that of μ: a one percent increase in the proportion of aggregate earnings spent on education by the government leads to a 0.13% increase in earnings and to 0.24% higher college attendance.

We saw in Section 4 that increasing the basic-higher expenditure ratio per student, g_y/g_o, has no effect on aggregate earnings and, therefore, has no explanation power over earnings differences between Brazil and the U.S.[43] Still, reallocating expenditures to public schools leads to higher demand for private universities, increasing college attendance by 1.57% in the long-run.

The next two exercises are related to the relative productivities of public educational

[43]Remember that the optimal expenditure ratio per student in Brazil is 0.96, which is almost the same as that calibrated to the U.S. (0.98).

Table 13: Replicating U.S. education characteristics in Brazil

	Parameter value		Aggregate earnings		College attendance	
	Brazil	U.S.	%Δ	Elast. (×100)	%Δ	Elast. (×100)
μ	0.029	0.189	2.06	0.37	53.11	9.53
G/Y	0.035	0.039	1.51	13.53	2.70	24.25
g_y/g_o	0.10	0.98	0.07	0.01	1.57	0.18
α_y	0.68	0.70	1.04	31.03	1.01	30.12
α_o	0.62	1.13	0.90	1.09	1.49	1.79
σ_ε	0.10	0.21	0.10	0.09	0.28	0.25

Notes: This table shows the results of several exercises to understand differences between the two countries. In each exercise we simulate the Brazilian economy assuming that a given parameter (corresponding to each row) has the same value as that of the U.S. economy. The first two columns show the values of the parameters for Brazil and the US. The third column shows the long-run aggregate earnings percentage change (with respect to the benchmark Brazilian economy) from replicating a given U.S. parameter value. The fourth column shows the percentage change in aggregate earnings divided by the percentage change in a given parameter (multiplied by 100), which gives an approximate elasticity measure. Columns 5-6 are related to college attendance and have similar interpretations than those of columns 3-4.

institutions, α_y and α_o. Our calibration suggests that the relative productivity of expenditures made through public schools in the U.S. (0.7 the productivity of private schools) are slightly higher than in Brazil (0.68). Despite this difference being small, aggregate earnings in Brazil would be 1% higher if it had U.S.' relative productivity of public schools. In comparison to the other parameters in Table 13, α_y has the greatest impact on earnings and college attendance in terms of elasticity: a one percent increase in α_y leads to 0.31% higher aggregate earnings and to 0.30% higher college attendance.

Public universities in the U.S. are more productive than private universities (1.13), while the opposite happens in Brazil (0.62). Table 13 shows that, if Brazilian public universities had the relative quality of public colleges in the U.S., aggregate earnings would be 1.25% higher. Note that the aggregate earnings gain driven by this large increase in α_o is lower than the one generated by the relatively lower increase in α_y from 0.67 to 0.7.

Our last experiment shows that differences between the noisiness of the admission process to public universities in Brazil and the U.S. have small explanatory power over the aggregate earnings gap and the college attendance gap.

Overall, these simulations suggest that the education aspect that, alone, has the highest explanatory power over aggregate earnings and college attendance differences between the two countries is the measure of vacancies offered by the government in public universities. The proportion of aggregate income taxed by the government to be spent on educational purposes also plays a relatively big role in explaining such differences.

However, note that μ and G/Y differ by a great extent across the two countries. Our results show that the relative productivity of public schools is the education parameter that would deliver the highest macroeconomic gains from small (marginal) positive improvements, suggesting that policies that increase the productivity of expenditures made through public schools[44] are important to boost productive efficiency in this economy.

6 Conclusion

In this paper, we study the allocation of public educational expenditures between basic and higher education from a quantitative macroeconomic perspective. Our setting is useful to understand the welfare and macroeconomic effects of educational policies adopted in South America during the 2000s decade. We also use our framework to understand how educational variables explain economic differences between Brazil and the United States.

The model is calibrated using Brazilian 2002 data, and it performs well in terms of fitting targeted and non-targeted statistics, including a quasi-natural regression discontinuity coefficient estimate of public university exam scores on labor earnings (Francis-Tan and Tannuri-Pianto, 2018). The calibrated framework reproduces several characteristics of the economic and educational environment of some developing countries, such as a highly dispersed distribution of income, private schools attended by wealthier students, and inequality of access into public universities, resulting in a significant amount of public resources being allocated to high-income individuals.

We find that the optimal utilitarian policy that maximizes average welfare across households allocates per-student public expenditures equally across basic and higher education. This policy benefits almost all families and delivers significant welfare gains to the poorest households, as measured by a willingness to pay measure. The macroeconomic effect on aggregate earnings is small but positive, and income inequality falls by 2% in the long-run, although these effects take several years to occur.

Similarly to Brazil, countries such as Colombia, Paraguay, and Peru have high income inequality, sizable economic disparity between students in public and private education institutions, and were allocating a relatively high proportion of per-student

[44]Policies such as establishing proper incentives for teachers in basic education, or improving the allocation of public expenditures across different categories (e.g., teachers' wages, infrastructure, parental programs, etc.).

expenditures to higher education in the beginning of the 2000s. Our results suggest that the strategies adopted by these countries during that decade, when public resources were reallocated from universities to schools, were accurate policies that should have generated significant welfare gains during the last years.

We inspect the role of some mechanisms included in the model in shaping the optimal policy's macroeconomic effects. First, we find that crowding out effects are quantitatively important. Second, properly accounting for general equilibrium effects prevents us from overestimating the policy's impact on aggregate earnings. Third, the parent-child human capital direct transmission mechanism weakens the policy's impact on income inequality.

We re-calibrate the model to fit U.S. data on GDP per capita and education choices. We find that public educational institutions in the United States are more efficient (with respect to private institutions) than in Brazil. Simulations suggest that the education aspect that, alone, has the highest explanatory power over aggregate earnings and college attendance differences between the two countries is the number of vacancies offered by the government in public universities. We also find that increasing the productivity of public schools in Brazil delivers high marginal impacts on aggregate earnings.

The Organisation for Economic Co-operation and Development has recently recommended some developing countries to shift their educational policies' focus from tertiary to pre-tertiary education (OECD, 2018a; OECD, 2018b; OECD, 2019). In consonance with this, a clear policy recommendation that emerges in our paper is that it is important for policy makers to mind the gap between schools and universities.

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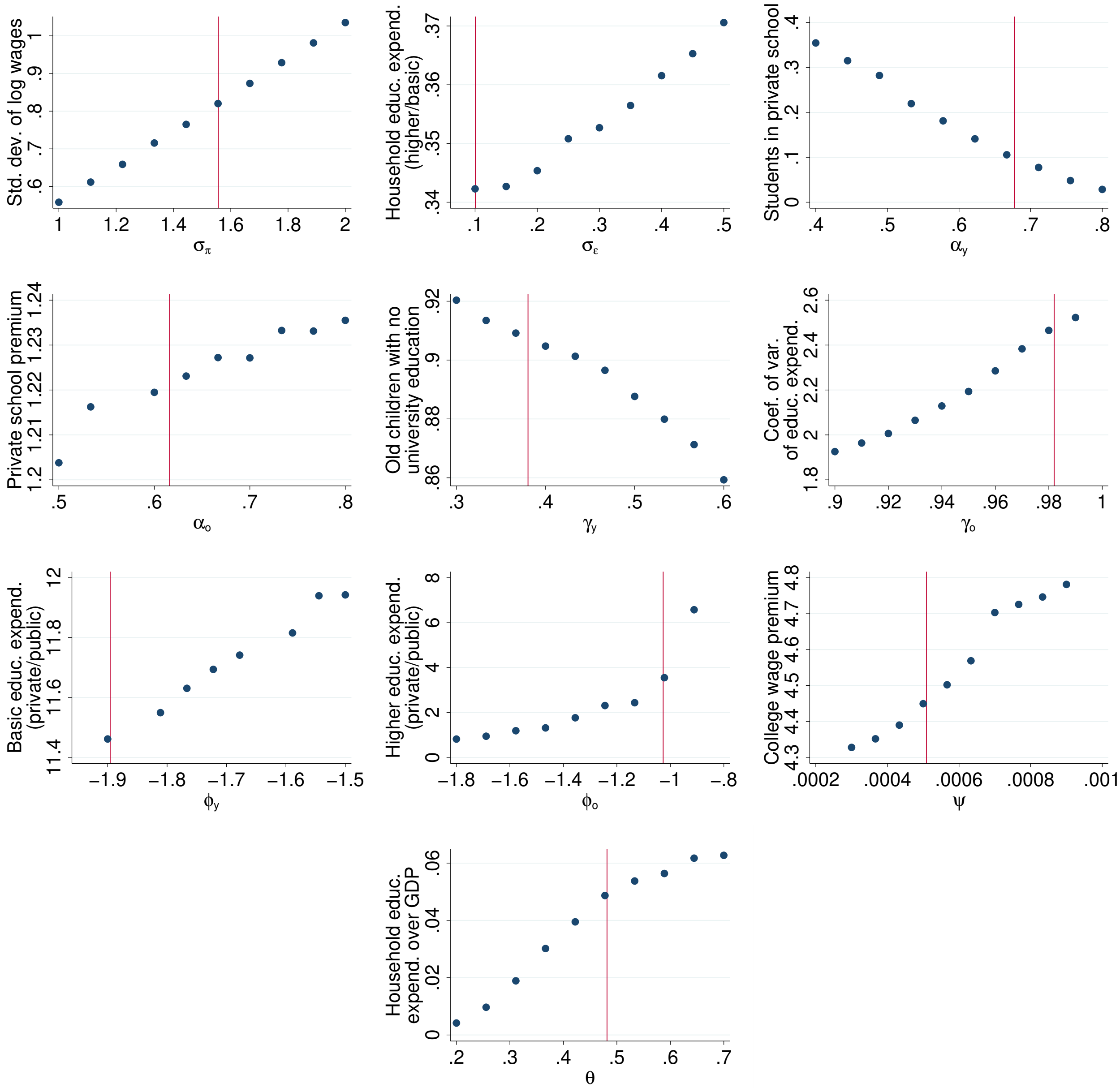
Appendix

A Additional figures

Figure A1: Relationship between parameters and targeted moments



Notes: Each plot shows how a targeted moment in the vertical axis is affected by a parameter in the horizontal axis in general equilibrium. The vertical line shows the calibrated parameter value. To construct a plot, we run the model for different values of a given parameter, fixing the remaining parameters at calibrated values (Table 2).

B Transition between steady states

Next, we define transition between two steady states. It is helpful to think that the initial steady state is the benchmark economy, calibrated to fit the data, and the final steady state is the stationary equilibrium associated with a counterfactual policy change, such as reallocating government expenditures from higher to basic education.

The transition between two steady states is given by paths of variables (value and policy functions, public spending per student in public college g_o, grade point cutoff π^*, lump-sum transfers T, and distributions of young and old households across state variables) over time such that:

1. Taking as given the paths of g_o, π^*, and T over time, value and policy functions solve households' problems.

2. The government's budget is balanced in all periods.

3. The public college entry condition holds in all periods.

4. Lump-sum transfers that households receive are consistent with those used to evaluate the government's budget balance condition in all periods.

5. The path of the distributions of households across states is determined by the choices of agents and the exogenous stochastic distributions. The distribution of households in the first period of the transition is the same as that in the initial steady state.

6. Over time, all equilibrium variables converge to those associated with the final steady state.

Using the benchmark stationary distribution of households across states as the distribution of households in the first period of transition means that agents do not expect the policy change before it is implemented. In the first period of the transition, the new policy is implemented, and households make choices after knowing about the new situation. Households take as given the paths of equilibrium variables g_o, π^*, and T. Since these variables change over time, the recursive problem of the household is non-stationary.

We compute the transition between steady states to make welfare analyzes. Our objective is to compare the well-being of households living in the benchmark steady state with the welfare of the same households in case a new policy is implemented. This is done by comparing the welfare of families living in the benchmark equilibrium with that of families living in the first period of the transition to the new steady state.

Numerical algorithm We use the following algorithm to compute transitions, which is based on Kambourov (2009):

1. Compute the initial and final steady states.
2. Assume the transition takes N periods.
3. Guess the path of equilibrium variables along the transition, $\Theta = \{g_o(t), \pi^*(t), T(t)\}_{t=1}^N$.
4. Given Θ, compute the value and policy functions going from the last transition period to the first one (backwards).
5. Given the value and policy functions along the transition, iterate the aggregate distribution from the first transition period to the last (going forward).
6. Evaluate market clearing conditions (government's budget balance, public college entry condition, and lump-sum transfers consistency) in all periods. If there is no market clearing in all periods, update Θ and go back to step 4; if it holds for all periods, go to next step.
7. Evaluate whether the aggregate distribution in the last transition period is sufficiently close to that in the final steady state. If it is not, increase N and go back to step 3; if aggregate distributions are sufficiently close to each other, the computation is complete.

C Numerical issues: benchmark equilibrium computation and calibration

Calibration and computation of the benchmark equilibrium requires three steps: an inner loop, where value function and aggregate distribution are iterated until convergence; a middle loop, where general equilibrium variables are obtained; and an outside loop, where parameters are chosen so that the model fits targeted statistics in the data.

We discretize the state space using grids for h, π, $\hat{\pi}$, and a using 15 points for π and 45 for the remaining variables. In the benchmark equilibrium, the maximum mass across young and old household types is 0.73% and 0.87%, respectively, suggesting that grid sizes are appropriate. Restuccia and Urrutia (2004) use 15 points for π and 60 for h and $\hat{\pi}$.[45] Given benchmark general equilibrium variables, iterating the value function and the aggregate distribution using 15 points for π and 60 for the remaining variables produces a similar simulation: public deficit, excess demand for public college, and T's consistency condition hold with precision (equal to 0.1%, 2%, and 0.1%, respectively).

Since the household's problem includes discrete choices, the value function is not differentiable due to it having kinks. Because of this, we use the grid search method to solve the household's problem. The educational expenditure log-spaced grid has 100 points, and the asset choice grid is the same as the state space asset grid. Running the model using a grid with 500 points for educational expenditures does not significantly change the simulation: the 99$^{\text{th}}$ percentile difference between educational policies obtained with the small and large grids is 19% (in modulus) regardless whether we weight all points equally or use the stationary aggregate distribution associated with the larger grid. Moreover, given benchmark equilibrium variables, iterating the value function and aggregate distributions using a larger educational expenditure grid produces a similar simulation: public deficit, excess demand in public college, and T's consistency condition hold with precision (all smaller than 0.6%).

We use Fortran 90 to simulate the model. Finding a general equilibrium takes approximately 30 minutes using 20 Xeon processors. Calibrating the model takes more than one thousand general equilibrium computations.

[45] See https://www.economics.utoronto.ca/diegor/research/RU2_AER_detailed_algorithm.pdf.