Bouncing universes and dark energy

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Abstract: We consider the possibility of a universe whose dark energy (DE) is made out of a negative cosmological constant Λ and a dynamical component X, dubbed as "cosmon", which does not interact with Λ. Due to $\Lambda < 0$, the late-time accelerated expansion rate is caused by X. In this paper, the study of this framework has been performed by implementing a piece-wise function for the equation of state (EoS) parameter of the cosmon. This parametrization provides an analysis of the impact on the total DE behaviour when a transition from phantom-like to quintessence cosmon (or vice versa) takes place. It has been found that a stopping point in the Universe is displayed in the future when a transition of X from phantom-like to quintessence occurs. For such transition, the ratio of the DE density to matter density (ρ_D/ρ_m) is bounded, giving a possible solution to the cosmological coincidence problem. Furthermore, for a phantom-like cosmon at present, we can obtain values of the Hubble constant (H_0) compatible with the ones acquired experimentally for cosmic microwave background data [1] and local data [2].

I. INTRODUCTION

Since the late 1990s, the accepted cosmological model is the ΛCDM model. It is the simplest parametrization capable of explaining to an extend some properties of the cosmos, such as the existence and structure of the cosmic microwave background (CMB) and the late-time acceleration expansion rate. In this framework, gravity is described by Einstein's general relativity, space is considered to have zero curvature and the dark energy (DE) is made out of a positive cosmological constant Λ. This Λ is the source for the late-time acceleration of the Universe.

In the last decades, numerous experiments have been carried out to acquire precise data for cosmological parameters. Recent data shows some discrepancies with the concordance model, such as the value of the Hubble constant at present time: the H_0 tension. This tension is one of the main problems of current cosmology studies. The H_0 tension is produced due to the inconsistency of the value H_0 when determined from the anisotropies in CMB radiation [1] and from local data (high-z Type Ia supernovae) [2], reaching a discrepancy of $\sim 4\sigma$. As a result, the scientific community has theorized more sophisticated models capable of coping with the most recent data. Taking into account that the main problem of the ΛCDM model comes from considering the DE as simple as a constant Λ, these alternative models work with different parametrizations for the DE sector. Since the ultimate nature of the DE still remains unknown [3-5], some studies have proposed a dynamical dark energy (DDE) model [6], [7], referred usually as XCDM models. However, other studies have been performed by working on running vacuum models (RVMs) [8-11].

A promising cosmological model due to its generalization is the ΛXCDM model proposed by Grande et al. [12], [13]. In such model, the DE sector is a mixture of different compounds: a cosmological constant Λ and a dynamical fluid X. These Λ and X can interact between each other and the cosmological constant can be either positive or negative. However, in this paper we restrain ourselves to the scenario with a negative cosmological constant and without interactions between the DE composites (see [12] for an extensive and general work on ΛXCDM model). Hence, the introduction of X not only deals with the actual acceleration expansion rate but also has the function to make this model with $\Lambda < 0$ resemble the ΛCDM model for large values of the cosmological redshift z. This component X, dubbed as "cosmon", can be constant or dependent on time, i.e. dependent on z.

The determination of the ultimate nature of the cosmon is beyond the scope of this paper. However, its behaviour is crucial to the evolution of the Universe. For a cosmon with an equation of state (EoS) parameter within the phantom-like regime ($\omega_X < -1$), the acceleration of the Universe will keep on increasing. On the other hand, for a quintessence cosmon ($-1 < \omega_X < -1/3$), X exerts a force similar to gravity and is able to override the actual expansion. Therefore, this would result in a halt of the Universe, causing a bouncing universe. With the latter behaviour, we can shed some light into the cosmological coincidence problem (see Section III.B). In the ΛCDM model there is no explanation for having $\frac{\rho_\Lambda}{\rho_m} \sim 1$ at present (being ρ_Λ and ρ_m the energy density for Λ and matter-radiation, respectively), since ρ_Λ is constant through time and ρ_m diminishes with the expansion of the Universe. Hence, the ratio $\frac{\rho_\Lambda}{\rho_m}$ tends to infinite with time and having an actual value close to 1 is an utter coincidence. By having a bouncing universe, this ratio is bounded. Furthermore, by keeping the ratio to unity, the cosmological coincidence problem is no longer an issue.

The structure of this paper is as follows. In Section II we present our model, based on the ΛXCDM model from Grande et al. [12]. In Section III we perform an analysis for a bouncing universe within our model and compare it with an ever-expanding universe. Additionally, we discuss the behaviour of the framework with recent data.

Finally, in Section IV we sum up the previous discussions and present our conclusions.

II. ΛXCDM MODEL WITH A NEGATIVE COSMOLOGICAL CONSTANT

In order to study a bouncing universe with $\Lambda < 0$, we implement a dynamical fluid on the DE sector: the cosmon X. We consider the simple case in which X does not interact with Λ. In this framework, X will vary with time and its barotropic index ω_X will be described by a piece-wise function of the cosmological redshift (see II.A).

Thereby, the total DE is a mixture of the cosmological constant Λ and X, with an effective EoS parameter that can be expressed as

$$\omega_e(z) = \frac{p_D}{\rho_D} = \frac{\omega_X\rho_X - \rho_\Lambda}{\rho_X + \rho_\Lambda} = -1+(1+\omega_X)\frac{\rho_X(z)}{\rho_D(z)} \ , \quad (1)$$

where $\omega_\Lambda = -1$ and ω_X can be a phantom-like ($\omega_X < -1$) or a quintessence ($-1 < \omega_X < -1/3$) fluid.

With the addition of the cosmon, the energy density of DE (Ω_D) is expressed by

$$\Omega_D(z) = \Omega_\Lambda^0 + \Omega_X^0 f_X(z) \ , \quad (2)$$

where $\Omega_i \equiv \rho_i/\rho_c$ and $f_X(z) \equiv \rho_X(z)/\rho_X^0$ is given by the following expression [14]:

$$f_X(z) = \exp\left\{3\int_0^z dz' \frac{1+\omega_X(z')}{1+z'}\right\} \ . \quad (3)$$

Notice that the presence of the cosmon preserves the cosmic sum rule even though $\Omega_\Lambda^0 < 0$.

Finally, for a Friedmann-Lemaître-Robertson-Walker (FLRW) metric with no space curvature, the evolution of the Hubble parameter as a function of the cosmological redshift can be expressed as

$$H^2(z) = H_0^2\left[\Omega_m^0(1+z)^3 + \Omega_D(z)\right] \ , \quad (4)$$

where Ω_m^0 is the matter energy density. As we are working on the matter epoch, the radiation contribution is neglected.

A halt in the expansion of the Universe occurs for $H(z) = 0$. Since $\Omega_\Lambda^0 < 0$, only a quintessence cosmon is able to satisfy such condition.

A. Piece-wise model

Taking into account Eqs. (2) and (4), for a negative cosmological constant, a bouncing universe is only feasible for a quintessence cosmon. Therefore, we are going to work with a piece-wise function for ω_X where a transition from phantom-type to quintessence will be carried out at some redshift z, with

$$\omega_X(z) = \begin{cases} \omega_1, & z \le z_1 \\ \omega_2, & z_1 < z \le z_2 \\ -1, & z_2 < z \le z_3 \\ \omega_3, & z > z_3 \end{cases} \ , \quad (5)$$

where $\omega_3 < -1 < \omega_2 < \omega_1 < -1/3$ in order to ensure a phantom-like behaviour for $z > z_3$ which evolves into a quintessence behaviour for $z \le z_2$. We have set $z_1 = -0.5$, $z_2 = -0.2$ and $z_3 = -0.1$. Therefore, this transition takes place in the recent future. Furthermore, we have split in two the quintessence behaviour since some parameters (e.g. $r(z) = \rho_D/\rho_m$) present drastic changes for slight variations of ω_X.

For this piece-wise function of ω_X, the evolution of X follows

$$f_X(z) = \begin{cases} (1+z)^{\alpha_1}, & z \le z_1 \\ (1+z_1)^{\alpha_1-\alpha_2}(1+z)^{\alpha_2}, & z_1 < z \le z_2 \\ \beta, & z_2 < z \le z_3 \\ \beta(1+z_3)^{-\alpha_3}(1+z)^{\alpha_3}, & z > z_3 \end{cases} \ , \quad (6)$$

where $\alpha_i = 3(1+\omega_i)$ and $\beta = (1+z_1)^{\alpha_1-\alpha_2}(1+z_2)^{\alpha_2}$.

III. DATA COMPARISON AND DISCUSSION

Although a quintessence cosmon is needed in the future for a bouncing universe to take place, we have also studied the scenario where the cosmon behaves as a phantom fluid in the future. Hereinafter, we will refer as case I the scenario where the transition is from quintessence to phantom and case II as the opposite transition. Both scenarios have been studied since the behaviour of X and its transition are not relevant (at a background analysis level) as long as the overall DE sector behaves according to the experimental data. Therefore, both cases are possible within the model.

A. Effective EoS parameter

By multiplying and dividing the second term on the *r.h.s.* of Eq. (1) by ρ_c^0, the effective EoS parameter can be written as

$$\omega_e(z) = -1 + (1+\omega_X)\frac{\Omega_X(z)}{\Omega_D(z)} \ , \quad (7)$$

where the equation for the energy density of DE is stated in Eq. (2) and for the cosmon is

$$\Omega_X(z) = \Omega_X^0 f_X(z) \ . \quad (8)$$

FIG. 1 shows the evolution of $\omega_e(z)$ as a function of z for cases I and II. The experimental value of the EoS parameter at $z = 0$ obtained at Planck 2018 [1], $\omega_e = -1.03 \pm 0.03$, shows that both scenarios are able to reach a satisfactory value of ω_e at present time. Values of $\omega_e(z=0)$ closer to -1 can be obtained for $\omega_X \to -1$ (approaching from bellow in case I and from above in case II) and for a small negative value of Ω_Λ^0 .

Moreover, in case I a discontinuity of $\omega_e(z)$ can be spotted in the future. The source of such discontinuity is due to the change of sign of ω_e when the density value of ρ_D is zero. Hence, this singularity points out that the description of the EoS parameter is not correct at that point.

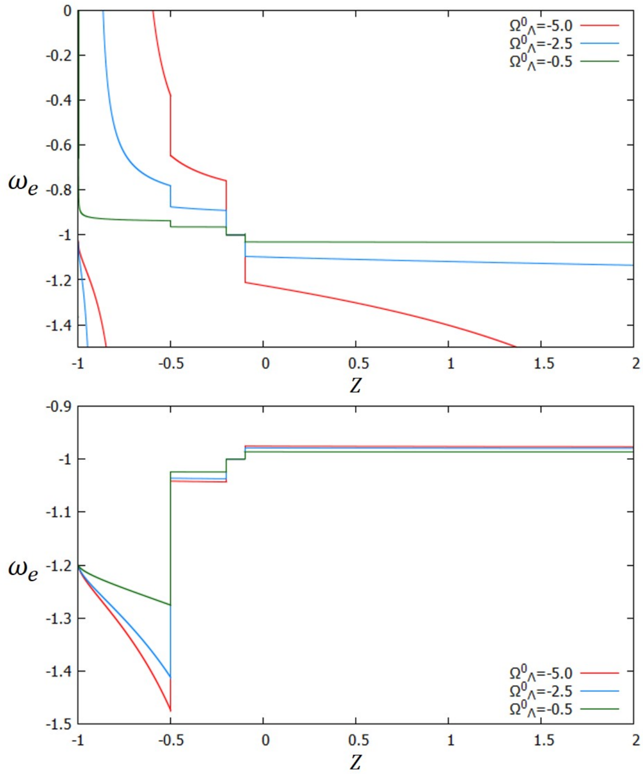

FIG. 1: Evolution of $\omega_e(z)$ for different values of Ω^0_Λ. In case I (top), we have $\omega_1 = -0.965$, $\omega_2 = -0.98$ and $\omega_3 = -1.0175$. In case II (bottom), we have $\omega_1 = -1.2$, $\omega_2 = -1.0175$ and $\omega_3 = -0.99$.

Even though FIG. 1 only shows the evolution of $\omega_e(z)$ up to $z = 2$, a study of the behaviour of the EoS parameter in the asymptotic past ($z \to \infty$) would manifest concordance with the ΛXCDM model (see Eq. (7.15) in [12]), i.e. $\omega_e = -1$ and $\omega_e = \omega_X$ for a phantom and quintessence cosmon in the asymptotic past, respectively.

B. Evolution of the ratio of the DE density to matter density

The ratio of the DE density to matter density for a flat universe is

$$r(z) \equiv \frac{\rho_D}{\rho_m} = \frac{\Omega_D}{\Omega_m} = \frac{\Omega_\Lambda + \Omega_X}{\Omega_m} = \frac{\Omega^0_\Lambda + \Omega^0_X f_X(z)}{\Omega^0_m (1+z)^{\alpha_m}} \quad , \quad (9)$$

where $\alpha_m = 3$ is the value for the matter epoch.

In order to have a bouncing universe, this ratio must be bounded. As stated in Section I, a bounded $r(z)$ could solve or at least alleviate the cosmological coincidence problem. The actual value of the ratio is $r_0 \sim 1$. Hence, a bounded ratio $r(z)$ with a maximum of the same order of magnitude (or one order of difference) would be compatible with the actual value. This fact implies that, unlike the ΛCDM model, having similar energy densities for the DE and matter at present time would no longer be a coincidence due to the bounded ratio $r(z)$.

As we can see from FIG. 2, only in case I can the ratio $r(z)/r_0$ be bounded. The behaviour of the ratio for a quintessence cosmon regime in the future varies significantly with small changes of ω_X. For larger values (less negative) of ω_X, the peak of the ratio remarkably diminishes. Therefore, the ratio would still be bounded even for cases with small negative values of Ω^0_Λ. In our plot, for $\Omega^0_\Lambda = -0.5$ the ratio is not bounded. However, it can be bounded for $-0.93 \le \omega_X < -1/3$, with a maximum in the range of $0.8 < r(z)/r_0 \le 5150$. In consequence, a universe where ω_X evolves into a quintessence regime with a value more detached from -1 can deal with the cosmological coincidence problem. Additionally, we can observe that the maximum of the ratio always takes place in the future.

On the other hand, for case II the ratio is never bounded. Besides, the presence of a phantom fluid cosmon in the future increases the expansion rate of the Universe and the ratio $r(z)$ will always tend to infinite. Note that for this case, the behaviour of the ratio is merely the same for different values of Ω^0_Λ.

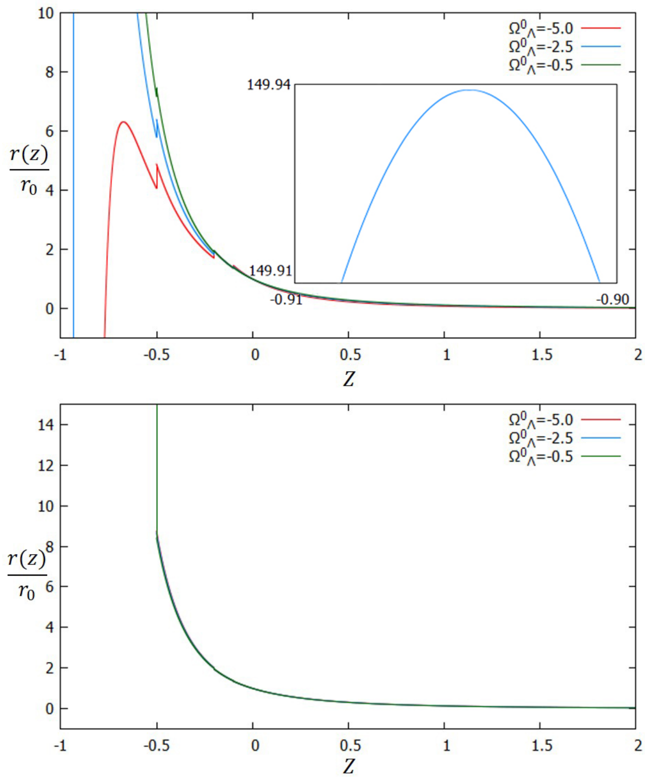

FIG. 2: Evolution of the ratio $r(z)/r_0$ for different values of Ω^0_Λ. In case I (top), we have $\omega_1 = -0.97$, $\omega_2 = -0.98$ and $\omega_3 = -1.0175$. An inset with the maximum of the ratio for $\Omega^0_\Lambda = -2.5$ is shown. In case II (bottom), we have $\omega_1 = -1.2$, $\omega_2 = -1.0175$ and $\omega_3 = -0.98$.

C. Evolution of the Hubble parameter

The evolution of the Hubble parameter H as a function of the cosmological redshift is

$$\frac{H^2(z)}{H_0^2} = \Omega_m^0(1+z)^3 + \Omega_\Lambda^0 + \Omega_X^0 f_X(z) \quad , \tag{10}$$

where $\Omega_m^0 = 0.3$, $\Omega_\Lambda^0 < 0$ and $\Omega_\Lambda^0 + \Omega_X^0 = 0.7$. The evolution of Eq. (10) has been plotted in FIG. 3.

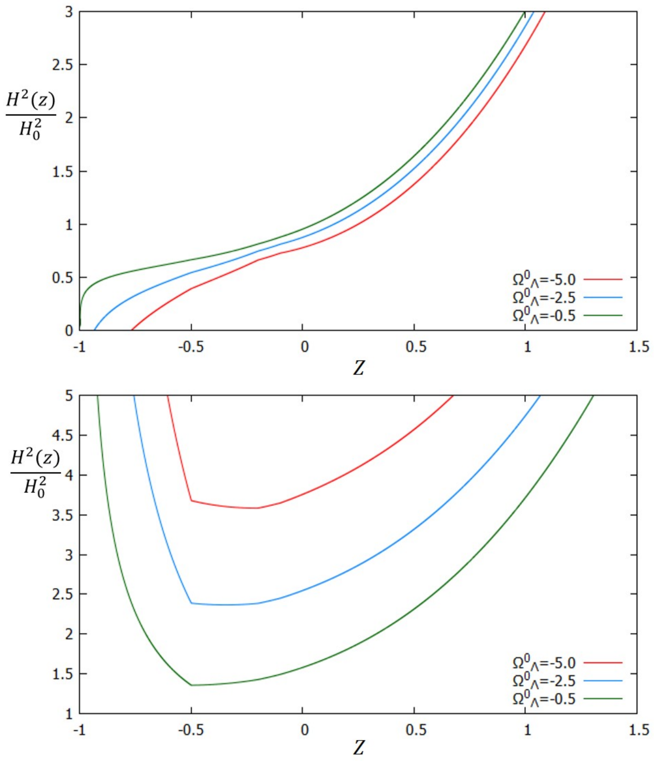

FIG. 3: Evolution of $H^2(z)/H_0^2$ for different values of Ω_Λ^0. In case I (top), we have $\omega_1 = -0.97$, $\omega_2 = -0.98$ and $\omega_3 = -1.0175$. In case II (bottom), we have $\omega_1 = -1.2$, $\omega_2 = -1.0175$ and $\omega_3 = -0.99$.

The results shown in FIG. 3 are in agreement with what has been previously discussed. For case I, H vanishes in the future due to the presence of a quintessence cosmon. In this scenario, the contribution of the cosmon in Eq. (10) is $\Omega_X^0(1+z)^{\alpha_X}$ for the future, with $0 < \alpha_X < 2$. Therefore, with the evolution of time, the contributions of Ω_m and Ω_X will diminish and the condition of $H(z) = 0$ will be achieved due to having $\Omega_\Lambda^0 < 0$. Note that for more negative values of Ω_Λ^0 and/or less negative values of ω_X, the stopping point will occur sooner. Nevertheless, the Universe will always stop in the future for this case; thus, a stopping point in the Universe bounds the ratio $r(z)$.

On the other hand, for case II the Universe will never stop its expansion. For this scenario the contribution of X in Eq. (10) is the same that for case I but with $\alpha_X < 0$. Hence, a phantom field X in the future displays an "explosion" of the energy density Ω_X and H will always be positive. Consequently, the Universe ends up in the Big Rip, where all bounded systems become unbounded and scattered [15].

The phantom behaviour of X need not compromise the unitarity of QFT since X is not necessarily assumed to be a fundamental field (e.g. a fundamental scalar field) but just an effective dynamical entity which may represent additional terms of the overall gravitational action beyond the usual Einstein term. If X would be a fundamental phantom field without a cosmological constant, then one has to cope with the usual consequences associated to it, in particular with the existence of a Big Rip scenario in the future [15]. But in this paper it is replaced by a bouncing behaviour, which is much more moderate.

D. The H_0 tension

The experimental value of the Hubble constant at present time, H_0, shows a considerate discrepancy between measurements from CMB data [1] and local data [2]. This inconsistency within the ΛCDM model manifests that a more complex DE sector is required instead of a simple cosmological constant.

We will work with the dimensionless Hubble parameter $h \equiv \frac{H_0}{100 \text{ km s}^{-1}\text{Mpc}^{-1}}$. Working with h provides the advantage that it can be described as a linear function of the EoS parameters of the DE sector [14] as

$$h = 0.673 + (\omega_X + 1)(0.93\omega_\Lambda - 0.33) \quad . \tag{11}$$

Therefore, using Eq. (11) with $\omega_\Lambda = -1$, we can obtain values in the range of $-0.167 < h < 0.673$ for a quintessence X and $h > 0.673$ for a phantom-like X. Taking into account that $h = (0.674 \pm 0.005)$ for CMB data [1] and $h = (0.74 \pm 0.01)$ for local data [2], a quintessence cosmon at $z = 0$ could achieve close values for the CMB data (for values of ω_X very close to -1). On the other hand, a phantom-like cosmon at present time can fit both experimental values.

IV. CONCLUSIONS

The starting point of this paper is the ΛXCDM model proposed by Grande et al. [12], where the DE sector is composed of two entities: the cosmological constant Λ and a generic fluid X. In our framework, the conditions of $\Lambda < 0$ and no interaction between Λ and X were imposed. By establishing $\Omega_\Lambda^0 < 0$, the cosmological constant does not rule over the acceleration of the Universe, being unable to produce the late-time accelerated expansion. Hence, the behaviour of the cosmon (i.e. quintessence or phantom behaviour) will be in charge of the expansion of the Universe.

Approaching this framework with a piece-wise function for ω_X allows us to study the overall behaviour when a transition of cosmon from phantom-type to quintessence (or vice versa) is displayed. However, a bouncing universe is only feasible for a quintessence X in the future. For this reason, even though a transition from quintessence to phantom for X is not forbidden in our model, it implies a never-ending expansion of the Universe and its further analysis is beyond the scope of this paper.

Hereby, the conclusions reached for a bouncing universe within this framework are:

- The transition of the behaviour of X (from phantom to quintessence) must be produced at present or in the near future. If the transition takes place in the upcoming future, X is for $z = 0$ a phantom fluid, enabling values of the EoS parameter for the DE compatible with the one obtained in Planck 2018 [1]. Moreover, a phantom-type cosmon at present alleviates the H_0 tension. On the other hand, if the transition takes place at present, a quintessence cosmon with $\omega_X \approx -1$ would be required at $z = 0$ to satisfy the experimental value of ω_e. However, it can only alleviate the H_0 tension for CMB data, being unable to reach high values of h. For this reason, we presume that the transition is more likely to happen in the near future.

- In the present context, the current expansion of the Universe will eventually stop at some point in the future. Therefore, the ratio of the DE density to matter density is bounded, giving a possible solution to the cosmological constant problem (CCP). The maximum ratio $r(z)$ varies significantly with ω_X, thus, less negative values of ω_X will produce a more satisfactory solution to the CCP.

- For a negative Λ and a quintessence X, the Hubble parameter will in due course vanish. This produces a stopping point in the Universe at some $z_c > 0$. At this point, the expansion stops and begins the further contraction of the Universe.

Taking into account the aforementioned points, we can conclude that the cosmological model discussed in this work may provide a fairly satisfactory description of the cosmological evolution of the Universe (at least at a background analysis level). Nevertheless, the ΛXCDM model proposed by Grande et al. [12] gives a more complex and general scenario, being capable of covering the framework of this paper. For this reason, the general ΛXCDM model stands as a promising cosmological model.

Acknowledgments

I want to thank my advisor Dr. Joan Solà for his help, guidance and support during this project. Being able to work with him has made me enjoy cosmology even more.

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