Quantum computation via global operations on a chain of fermions

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Abstract: A quantum computation model via global operations is presented. We first review how to get local control in a chain of qubits by using only global unitary operations. Next we discuss how to translate these ideas to fermions through an appropriate mapping. Finally, some experimental aspects are briefly discussed.

I. INTRODUCTION

In quantum mechanics, two classes of elementary particles are distinguished: bosons and fermions. Bosons are particles with integer spin and they obey Bose-Einstein statistics. They have no restriction on the occupation number of a single-particle state. On the other hand, fermions are particles with half-integer spin and they obey Fermi-Dirac statistics. Only one fermion can occupy a single-particle state. That constraint is known as Pauli's exclusion principle. Despite these differences, both particle classes have been proven to be suitable for performing quantum computation [1], [2].

When trying to build a quantum computer, the symmetry of the system used for implementation can play a major role as it reduces the complexity. For that reason, in this report we will consider a 1D chain of particles where only global operations can be performed. However, the conventional quantum computational model assumes we can perform single-qubit and neighbouring two-qubits operators. Therefore, we will need to get local control from our global operations. We will first discuss the computational model considering our particles as bosons and then we will provide a mapping from bosons to fermions.

Developing this quantum computation model via global operations is very interesting when we consider experimental limitations since the possibility to perform some local operations on a physical system of qubits is still quite recent and they are technologically more demanding than global ones. If we consider fermions instead of bosons, then performing these local operations represents even a bigger challenge.

Even though having access to local operations still represents a technological challenge, having access to single-particle measurements is more feasible [3], [4]. For that reason, in this project we don't consider the need to perform local measurements via global operations.

A computational model with those properties in the case of a chain of qubits is already developed in [1]. Being able to map this model for fermions can be very useful as it provides an alternative for its physical building. Some experimental advances that have recently been made in the manipulation of fermions can support its implementation.

This report is organised as follows. In section II, we review the quantum computation model based on global operations acting on a chain of qubits proposed in [1]. In section III, we provide a mapping to translate what is shown in section II into fermions. In section IV, some experimental aspects are discussed. Finally, section V contains our conclusions.

II. LOCAL CONTROL AND UNIVERSAL GATE SET

In this section we will show how to get local control on an open chain of qubits only using global operations. This idea, which may seem counterintuitive, is made possible because of the existence of boundaries in our chain. After achieving local control we will get a universal gate set. A gate set is universal if any unitary operation can be approximated to arbitrary accuracy using the gates included in the set. It is proven that single-qubit gates and one neighbouring two-qubit gate (such as CNOT gate) are enough to form a universal gate set [5].

We start by defining the following global transition gate:

$$T = \left(\prod_{i=1}^{N-1} \Lambda(\sigma^z)_{i,i+1}\right)\left(\bigotimes_{i=1}^{N} H_i\right), \tag{1}$$

where N is the number of qubits in our chain, H_i is the one-qubit Hadamard gate acting on qubit i, σ^z is the Pauli Z-matrix and $\Lambda(U)_{i,i+1}$ is the controlled-U operation where i is the control qubit and $i+1$ is the target qubit. Pauli matrices are defined as

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

The Hadamard gate maps every state of the basis ($|0\rangle$ and $|1\rangle$) to a state where the measurement has the same probability to result in 0 or 1. Its matrix form is

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{3}$$

A controlled-U gate is a gate that acts on two qubits: the control qubit and the target qubit. Its action consists in

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performing U to the target qubit if and only if the control qubit is $|1\rangle$. We can represent this operation as

$$\Lambda(U) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & U_{00} & U_{01} \\ 0 & 0 & U_{10} & U_{11} \end{pmatrix} \quad \text{with} \quad U = \begin{pmatrix} U_{00} & U_{01} \\ U_{10} & U_{11} \end{pmatrix}. \tag{4}$$

We also define a global unitary transformation of the form

$$U_A(\alpha) = \bigotimes_{i=1}^{N} \exp\left(i\frac{\alpha}{2}A_i\right), \tag{5}$$

where A is any 2×2 hermitian matrix.

An interesting property of the transition function is that:

$$T^{N+1} = R, \tag{6}$$

where R is the reflection operator that sends the qubit chain state onto its mirror image. That property is a direct consequence of the system having boundaries.

In order to understand better how the presence of edges in our system has equation (6) as a consequence we will study how a local Pauli observable evolves under conjugation by T until we get its reflection image. An example of such an evolution for the 8-qubits case is given in Table I. During the evolution, three different phases can be distinguished: expansion (when the number of local Pauli observables increases after a T conjugation), transmision (when the number of local Pauli observables remains constant after conjugating by T but they act on different qubits) and contraction (when the number of local Pauli observables decreases after conjugating by T). The duration of each phase depends on the initial position of the local observable as it is related to their distance to the boundaries. The operator starts in an expansion phase until it reaches one of the boundaries. After that, the operators enter a transmission phase. Once again, this phase continues until one of the operators reaches the other boundary. When that happens, the operators enter the contraction phase until they become the mirror image of the original local operator.

We further define a Y-pulse as:

$$Y := U_{\sigma^y}(\pi) = \bigotimes_{i=1}^{N} \sigma_i^y. \tag{7}$$

If we conjugate a local Pauli observable by Y at any time, it can pick up a sign factor as $\sigma^y\sigma^\alpha\sigma^y = -\sigma^\alpha$ with $\alpha \in \{x, z\}$. Depending on the phase in which the observable is when we conjugate by Y the sign will be: $+1$ during transmission (as we have an even number of Pauli operators) and -1 during expansion and contraction (as we have an odd number of Pauli operators). That is true when our initial operator acts only on one qubit. The fact that the durations of expansion, transmission and contraction phase depend on the initial position of the observable implies that the sign it gets when we conjugate by Y after a certain number of conjugations by T also depends on its initial position. That is really important as it has the following consequence: if we perform a global operation $U_A(-\alpha/2)$, we conjugate it by T a certain number of times and then we conjugate by Y, some local rotations will pick a negative sign while others won't. The operators that picked up the negative sign will then perform a rotation of $\alpha/2$ instead of $-\alpha/2$ on their corresponding qubits. If we now perform another global operation $U_A(\alpha/2)$, the local rotations which picked up a positive sign will get cancelled while the ones which picked up the negative sign will add another $\alpha/2$ rotation. We see it is therefore possible to perform a rotation of α on certain qubits. That provides a local control even though we only performed global operations. Being able to rotate exactly the qubits we want is just a matter of conjugating by Y after the correct number of T conjugations. Therefore, spatial control and temporal control in our system are related.

Time	Observable	Y conjugation sign
0	σ_6^z	-
1	$T\sigma_6^z T^{-1} = \sigma_5^z\sigma_6^x\sigma_7^z$	-
2	$T^2\sigma_6^z T^{-2} = \sigma_4^z\sigma_5^x\sigma_6^z\sigma_7^x\sigma_8^z$	-
3	$T^3\sigma_6^z T^{-3} = \sigma_3^z\sigma_4^x\sigma_5^z\sigma_6^x\sigma_7^z\sigma_8^x$	+
4	$T^4\sigma_6^z T^{-4} = \sigma_2^z\sigma_3^x\sigma_4^z\sigma_5^x\sigma_6^z\sigma_7^x$	+
5	$T^5\sigma_6^z T^{-5} = \sigma_1^z\sigma_2^x\sigma_3^z\sigma_4^x\sigma_5^z\sigma_6^x$	+
6	$T^6\sigma_6^z T^{-6} = \sigma_1^x\sigma_2^z\sigma_3^x\sigma_4^z\sigma_5^x$	-
7	$T^7\sigma_6^z T^{-7} = \sigma_2^x\sigma_3^z\sigma_4^x$	-
8	$T^8\sigma_6^z T^{-8} = \sigma_3^x$	-
9	$T^9\sigma_6^z T^{-9} = \sigma_3^z$	-

TABLE I: Example of the evolution of the local Pauli observable σ_6^Z (Pauli gate Z applied on the sixth qubit) in a chain of 8 qubits under conjugation by T. The sign we get under Y conjugation of the observable is also indicated. This sign is useful in order to understand how we can get spatial control from our temporal control. It may be interesting to notice that equation (6) is satisfied as $R\sigma_6^Z R = \sigma_3^Z$

.

As we mentioned before, the duration of expansion phase, transmission phase and contraction phase depends on the distance between the original local observable and the edges of our system. However, because of the symmetry of our system, a local observable acting on qubit i has the boundaries of the system at the same distance as a local observable acting on its mirror qubit $\bar{i} = N+1-i$. This means that the duration of every phase will be the same for local operators at sites i and $\bar{i}$. This reflection symmetry implies that every operation will be performed simultaneously on the left side and as a mirror image on the right side of our chain.

We have already provided intuition on why spatial and temporal control are related in our system. We now state that we can perform a Z-rotation on qubits i and $\bar{i}$ as

follows:

$$e^{i\alpha\sigma_i^z}e^{i\alpha\sigma_{\bar{i}}^z} = \left(T^{N+1-i}YTYT^{i-1}\right)U_{\sigma^z}(-\alpha/2) \left(T^{N+1-i}YTYT^{i-1}\right)^{-1}U_{\sigma^z}(\alpha/2). \tag{8}$$

A formal proof for (8) is provided in [1] but we have found it more insightful to show an example for a Z-rotation of qubits $i = 3$ and $\bar{i} = 6$ in a chain of $N = 8$ qubits. We start by showing the evolution of $e^{-i\frac{\alpha}{2}\sigma^z{}_3}$ as we perform the conjugations in (8):

$$\begin{aligned}
&T^2e^{-i\frac{\alpha}{2}\sigma_3^z}T^{-2} = \exp(-i\frac{\alpha}{2}\sigma_1^z\sigma_2^x\sigma_3^z\sigma_4^x\sigma_5^z)\\
&YT^2e^{-i\frac{\alpha}{2}\sigma_3^z}T^{-2}Y = \exp(i\frac{\alpha}{2}\sigma_1^z\sigma_2^x\sigma_3^z\sigma_4^x\sigma_5^z)\\
&TYT^2e^{-i\frac{\alpha}{2}\sigma_3^z}T^{-2}YT^{-1} = \exp(i\frac{\alpha}{2}\sigma_1^x\sigma_2^z\sigma_3^x\sigma_4^z\sigma_5^x\sigma_6^z)\\
&YTYT^2e^{-i\frac{\alpha}{2}\sigma_3^z}T^{-2}YT^{-1}Y = \exp(i\frac{\alpha}{2}\sigma_1^x\sigma_2^z\sigma_3^x\sigma_4^z\sigma_5^x\sigma_6^z)\\
&T^6YTYT^2e^{-i\frac{\alpha}{2}\sigma_3^z}T^{-2}YT^{-1}YT^{-6} = \exp(i\frac{\alpha}{2}\sigma_6^z)\\
&T^6YTYT^2e^{-i\frac{\alpha}{2}\sigma_3^z}T^{-2}YT^{-1}YT^{-6}e^{i\frac{\alpha}{2}\sigma_6^z} = \exp(i\alpha\sigma_6^z).
\end{aligned} \tag{9}$$

Similarly,

$$T^6YTYT^2e^{-i\frac{\alpha}{2}\sigma_6^z}T^{-2}YT^{-1}YT^{-6}e^{i\frac{\alpha}{2}\sigma_3^z} = \exp(i\alpha\sigma_3^z). \tag{10}$$

We can notice that the reason why we can reverse the original rotation angle $-\alpha/2$ is because we apply one Y-pulse just before expansion phase ends and one Y-pulse just after transmission phase begins. As the duration of every phase depends on the initial position of the observable and only observables acting on i and $\bar{i}$ have the same phase duration, we will get $T^6YTYT^2e^{-i\frac{\alpha}{2}\sigma_j^z}T^{-2}YT^{-1}YT^{-6}e^{i\frac{\alpha}{2}\sigma_j^z} = \mathbb{1}\ \forall j \neq 3, 6$. We will only show an example of this behaviour for qubit 2 as we can get the remaining operators evolution in an analogous way:

$$\begin{aligned}
&T^2e^{-i\frac{\alpha}{2}\sigma_2^z}T^{-2} = \exp(-i\frac{\alpha}{2}\sigma_1^x\sigma_2^z\sigma_3^x\sigma_4^z)\\
&YT^2e^{-i\frac{\alpha}{2}\sigma_2^z}T^{-2}Y = \exp(-i\frac{\alpha}{2}\sigma_1^x\sigma_2^z\sigma_3^x\sigma_4^z)\\
&TYT^2e^{-i\frac{\alpha}{2}\sigma_2^z}T^{-2}YT^{-1} = \exp(-i\frac{\alpha}{2}\sigma_2^x\sigma_3^z\sigma_4^x\sigma_5^z)\\
&YTYT^2e^{-i\frac{\alpha}{2}\sigma_2^z}T^{-2}YT^{-1}Y = \exp(-i\frac{\alpha}{2}\sigma_2^x\sigma_3^z\sigma_4^x\sigma_5^z)\\
&T^6YTYT^2e^{-i\frac{\alpha}{2}\sigma_2^z}T^{-2}YT^{-1}YT^{-6} = \exp(-i\frac{\alpha}{2}\sigma_7^z)\\
&T^6YTYT^2e^{-i\frac{\alpha}{2}\sigma_2^z}T^{-2}YT^{-1}YT^{-6}e^{i\frac{\alpha}{2}\sigma_7^z} = \mathbb{1}.
\end{aligned} \tag{11}$$

Hence, we have shown that (8) actually works for this particular case with $i = 3$ and $N = 8$. Further discrimination of qubits i and $\bar{i}$ is not possible. For that reason, and as we may be interested in modifying only qubit i, we will double the size of the chain by adding its mirror image on its right. By doing that we still modify i and $\bar{i}$ but now $\bar{i}$ is just a copy of i.

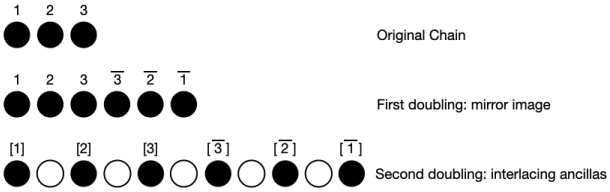

FIG. 1: Illustration of the two doublings the chain goes through. "•" represents a logical qubit and "○" represents an ancilla in the state $|0\rangle$. The numbers above the qubits are their position in the chain with $\bar{i} = N + 1 - i$ and $[i] = 2i - 1$.

We can now get the X-rotation gate by using $T^{-1}\sigma_i^zT = \sigma_i^x$:

$$e^{i\alpha\sigma_i^x}e^{i\alpha\sigma_{\bar{i}}^x} = \left(T^{N-i}YTYT^{i}\right)U_{\sigma^x}(-\alpha/2) \left(T^{N-i}YTYT^{i}\right)^{-1}U_{\sigma^x}(\alpha/2). \tag{12}$$

Equations (8) and (12) tell us how to implement single-qubit gates. We now turn to two-qubits gates. For that purpose we introduce the operator $K := T^{-1}\sigma_i^xT = \sigma_i^z\bigotimes_{j|\Gamma_{ij}=1}\sigma_j^x$ where Γ_{ij} is the adjacency matrix of the interaction graph. In our case, we only consider nearest-neighbour interactions and a non-directed graph so $\Gamma_{ij} = \delta_{i,j+1} + \delta_{i,j-1}$. Once the gate K has been defined, we can conjugate (12) by T^{-1} and get

$$e^{i\alpha K_i}e^{i\alpha K_{\bar{i}}} = \left(T^{N-1-i}YTYT^{i+1}\right)T^{-1}U_{\sigma^x}(-\alpha/2)T \left(T^{N-1-i}YTYT^{i+1}\right)^{-1}T^{-1}U_{\sigma^x}(\alpha/2)T. \tag{13}$$

In order to get a universal gate set we need a two-qubit gate that induces entanglement. However, gate $e^{i\alpha K_i}$ induces entanglement in three qubits. We can solve that by adding ancillas in the state $|0\rangle$ in between qubits. It can be useful to distinguish between physical position and logical position. We define logical position by $[j] = 2j-1$ and it corresponds to the physical position of the qubit in the chain before performing this second doubling. An illustration of how we double the chain length is shown in Fig. 1.

When we double the chain the second time we get $e^{i\alpha K_i}$ to induce entanglement in only two logical qubits as it will effectively work as:

$$e^{i\alpha K_{2j}}|\psi\rangle \equiv e^{i\alpha\sigma^x_{[j]}\sigma^x_{[j+1]}}|\psi\rangle. \tag{14}$$

Gates from (8), (12) and (13) can be converted to the standard universal gate set $\{H, \sigma^{z\,1/4}, \Lambda(X)\}$.

III. ENCODING QUBITS IN FERMIONS

We now transpose our findings of the previous section to fermions [2]. We consider m sites which can be either empty or occupied by a spinless fermionic particle. We will work in a Foch space where the basis states are

$|n_0 n_1 \dots n_{m-1}\rangle$ and n_i is the occupation number of the site i. As we are working with spinless fermions, n_i will be either 0 or 1 and the sites will be called "local fermionic modes" (LFMs). On our basis states, the creation $a_i^\dagger$ and annihilation a_i operators act as follows:

$$a_i^\dagger|n_0 \dots n_i \dots\rangle = \begin{cases} 0 & \text{if } n_i = 1, \\ (-1)^{\sum_{j=0}^{i-1} n_j}|n_0 \dots 1 \dots\rangle & \text{if } n_i = 0, \end{cases} \tag{15}$$

$$a_i|n_0 \dots n_i \dots\rangle = \begin{cases} (-1)^{\sum_{j=0}^{i-1} n_j}|n_0 \dots 0 \dots\rangle & \text{if } n_i = 1, \\ 0 & \text{if } n_i = 0. \end{cases} \tag{16}$$

This behaviour can be easily proven by taking into account the anticommutation rules of fermionic creation and annihilation operators ($\{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} = 0$, $\{a_i, a_j^\dagger\} = \delta_{ij}$).

In order to see how parity-preserving operators X act on fermions, we consider the operator $a_j^\dagger a_k$. Its action on fermions is

$$a_j^\dagger a_k|\dots n_j \dots n_k \dots\rangle = \begin{cases} (-1)^{\sum_{s=j+1}^{k-1} n_s}|\dots 1_j, \dots, 0_k, \dots\rangle & \text{if } n_j = 0 \text{ and } n_k = 1 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

We introduce the following notation: $X[i,j]$ represents a parity-preserving qubit operator acting on qubits i, j and $X\{i,j\}$ represents a parity-preserving fermion operator acting on qubits i, j. We notice that, in general, $X[j,k] \neq X\{j,k\}$. However, $X[j] = X\{j\}$ and $X[j, j+1] = X\{j, j+1\}$. This allows to represent fermionic gates in terms of qubit gates and vice versa by performing a certain number of permutations to place the target qubits or fermions together.

Now that we know how to represent fermionic gates in terms of qubits gates, we can express the universal gate set $\{e^{i\alpha\sigma_i^z}e^{i\alpha\sigma_{\bar{i}}^z}, e^{i\alpha\sigma_i^x}e^{i\alpha\sigma_{\bar{i}}^x}, e^{i\alpha K_i}e^{i\alpha K_{\bar{i}}}\}$ which acts on qubits in a way that it performs the same operations on fermions.

First of all we notice that the gates that form the universal gate set can be expressed with gates that act on either one qubit or on two nearest-neighbour qubits. One may be tempted to claim that, taking into account this fact and the identities $X[j] = X\{j\}$ and $X[j, j+1] = X\{j, j+1\}$, no modification is required on the universal gate set. However, it's important to notice that these identities are true only for parity-preserving operators and some of the global operations we have considered don't preserve parity.

To solve this problem we define the following mapping from qubits to fermions:

$$J: \quad |n_0 n_1 \dots\rangle_Q \longmapsto |n_0 n_0 n_1 n_1 \dots\rangle_F \tag{18}$$

where $|\dots\rangle_Q$ represents a qubit state and $|\dots\rangle_F$ represents a fermionic state.

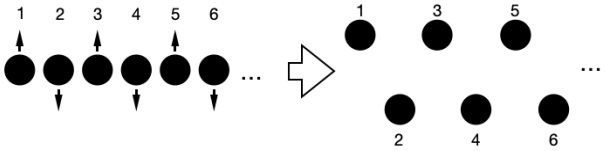

FIG. 2: Illustration of how we can split the fermion register in two chains by moving fermions located at odd positions upwards and fermions located at even positions downwards."•" represents a fermion and the numbers above or under the fermions correspond to their original physical position.

With a non parity-preserving operator U which acts on one qubit we can associate a two-fermions operator $\widetilde{U}$ as follows:

$$\widetilde{U} = \sum_{i,j}\langle i|U|j\rangle|ii\rangle\langle jj| + \sum_{i\neq j}|ij\rangle\langle ij| \tag{19}$$

Those operators $\widetilde{U}$ actually preserve parity in the fermion register and are related to the original U by the indentity $U = J^{-1}\widetilde{U}J$ where J^{-1} is the pseudo-inverse of J and it is defined as

$$J^{-1}: \quad |n_0 n_{0'} n_1 n_{1'} \dots\rangle_F \longmapsto \prod_i \delta_{n_i, n_{i'}}|n_0 n_1 \dots\rangle_Q. \tag{20}$$

As the only two-qubit operator we need in our model is $\Lambda(\sigma^z)$ and it is already parity-preserving we don't need to associate any four-fermions operator. Then, if (1) acts on qubits, we can define T_F acting on fermions as

$$T_F = \left(\prod_{i=1}^{M/2-1}\Lambda(\sigma^z)_{2i-1,2i+1}\right)\left(\bigotimes_{i=1}^{M/2}\widetilde{H}_{2i-1,2i}\right), \tag{21}$$

where M is the number of LFMs in the register. This expression for T_F has two drawbacks: every $\Lambda(\sigma^z)$ acts on two fermions that are not nearest-neighbours and $\prod_{i=1}^{M/2-1}\Lambda(\sigma^z)_{2i-1,2i+1}$ is not a global operation. In order to solve those problems we suppose we are able to split our fermion register in two equal chains by being able to move fermions located at odd positions upwards and fermions located at even positions downwards (Fig. 2). Then we can perform a controlled-Z gate on every fermion from one of our chains and put our chains back together. We discuss the feasibility of that operation in section IV.

This mapping therefore provides a powerful and easy way to implement the universal gate set on a chain of fermions instead of qubits at a constant overhead.

IV. EXPERIMENTAL ASPECTS

In this section we will discuss some experimental aspects that support the possibility of using this model with fermions instead of qubits.

Nowadays, a lot of research in fermionic manipulation is being made. Most of the research in this area is focused on fermionic quantum simulation of different models such as the Hubbard model [6]. However, we can take advantage of their fermionic manipulation advances to implement our computational model.

Our computational scheme is based on a 1D chain of fermions. This chain can be easily built by trapping fermions in a 1D optical lattice [7]. An optical lattice is a structure which can simulate the lattices from Solid State Physics thanks to the interference pattern of an external electric field originated from an optical dipole trap. It provides a good way to simulate a solid state system without impurities and with a great control of the system Hamiltonian. It relies on the interaction between the external electric field and the induced dipole moment in the atom. In our case, we can get an optical lattice simulating our chain from two counter-propagating coherent laser beams. That will generate a standing wave with a wave length of $\lambda_L/2$ where λ_L is the wave length of one of the laser beams. The potential generated can be either attractive (we will have our atoms located at intensity maximums) or repulsive (atoms will be placed at intensity minimums). We can get this control by taking into account the laser beam frequency and an atomic resonance frequency. If the laser beam frequency is lower than the atomic resonance frequency, the potential will be attractive while in the opposite case the potential will be repulsive. This control of our system provided by the lasers manipulation is enough to move fermions located at odd positions upwards and fermions located at even positions downwards. Hence, the operation we considered at section III in order to perform $\prod_{i=1}^{N/2-1} \Lambda(\sigma^z)_{2i-1,2i+1}$ as a global operation should actually be feasible.

One important thing we claimed in the introduction was that it is technologically possible to perform single-qubit measurements. That has recently been made possible for fermions. Currently, it is possible to image single fermions trapped in the sites of an optical lattice [3],[4]. This ability to perform single-site images of atoms was developed before in the case of bosons. That is because in the case of fermions usually alkaline atoms are used and their low mass and small hyperfine-structure splitting complicates the standard cooling techniques used to increase the fluorescence of the atoms. In fact, the fluorescence rate of fermionic atoms is more than a factor of ten lower than the one obtained for bosons. However it is large enough to unambiguously determine the presence or absence of an atom in a site.

Even though single-site imaging provides a great option as a measurement tool for our computational model, it has not enough resolution to determine the number of atoms that are on every site, it only distinguishes between the case where there is none and the case where there is at least one. To solve that issue we will take advantage of the Pauli's principle of exclusion which imposes that, in order to have two fermions at the same site, their spin must be different. We can then perform an optical pumping with a certain laser beam which is resonant with the internal transition we want our atoms to go through in order to have all of them at the same internal state [8]. This technique changes the internal state of every atom in the lattice with 95% fidelity. We can remove the atoms that remain in another internal state by applying a resonant light pulse. This way of changing the internal state of an atom by using a magnetic field also represents a good option for performing global operations in our system.

V. CONCLUSIONS

We developed a mapping that allows us to implement a computational model based on global operations in a chain of fermions. The possibility to develop this model in the case of fermions offers an alternative for the already developed model for bosons. This provides a good way to have a better understanding of fermionic systems and represents another example to show that both bosons and fermions are suitable for quantum computation.

We have also provided some examples of recent research that has developed useful tools to recreate this model with fermions.

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