Hadron production in statistical model with vorticity

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Abstract: Relativistic heavy ion collisions and the produced particle spectra supply essential information about the early universe and the states of nuclear matter. Using the statistical production model within the framework of some approximations, this study finds the chemical freeze-out conditions of temperature and baryochemical potential that origin a certain production of particles in such collisions. Then, the effect on the parameters of the model of a vortical component in the flow is verified. The inclusion of this magnitude indeed affects the parameters of this model, but in order for these changes to be substantial, thermal vorticity is required to take non physical values.

I. INTRODUCTION

Relativistic heavy ion collisions provide a way of reproducing the conditions of the early universe and set the environment for the obtaining of quark-gluon plasma (QGP) [1]. Surprisingly, the statistical production model has shown satisfactory results when approaching this kind of systems with a number of particles much smaller than the Avogadro number [2], correctly reproducing the detected particle spectra in heavy ion collisions while treating $10^3 - 10^4$ particles. This has allowed a statistical treatment of the phenomena originated in these collisions within the grandcanonical ensemble, defining its partition function as dependent on the temperature of the system and the chemical potential [3]. Despite the simplicity of this method, the obtention of essential properties of the hot dense subatomic matter has been possible. Since the usage of T has proved effective, local equilibrium exists and therefore one can use an Equation of State. Consequently, this allows to talk about QGP, a state of matter of which quarks and gluons are the main degrees of freedom, according to experimental evidence.

Thanks to the options offered by the statistical model (and others), it has been possible to stablish the strongly interacting matter phase diagram [4], which sums up the properties of subatomic matter under certain temperature and baryochemical potential conditions and distinguishes between two radically different regions: the hadronic world and the QGP. The current state of this diagram is shown in FIG. 1.

Heavy ion collision experiments such as the AGS, the SPS and the RHIC provide the detected hadron yields at their respective energies. In these experiments, when a very energetic collision happens, QGP is formed [5]. Then, by expanding, the fireball cools down following the yelow lines found in the diagram until the first order phase transition is reached. That is, when the hadronization of the deconfined quarks takes place, but inelastic collisions are still predominant. However, as the system reduces its temperature, inelastic collisions cease and the system achieves chemical freeze-out conditions, which can be obtained from the analysis of the particle production, since the yield at this point remains unchanged.

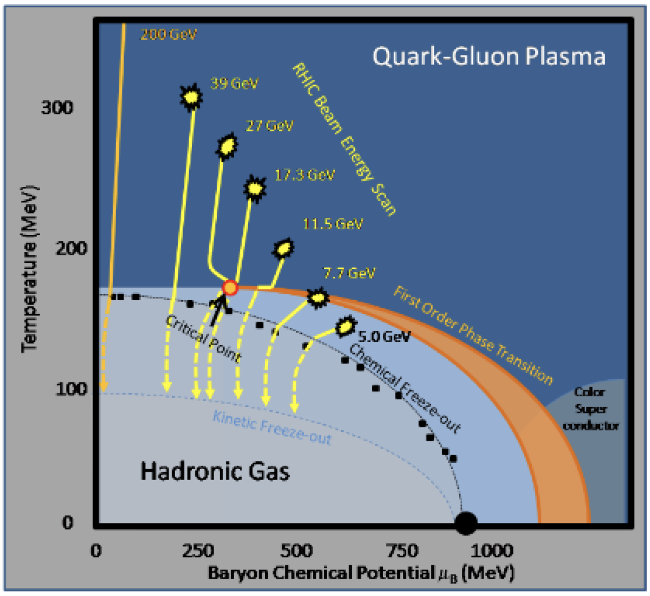

FIG. 1: Phase diagram of the strongly interacting matter in the (T, μ_B) plane [4]. Besides from the known phases (hadron gas and QGP, which was firstly found at the SPS@CERN experiment in 2000 [5]), some theoretically suggested ones are represented (such as the color super-conductor phase). Also the first order transition line between QGP and the hadronic world is denoted

Afterwards, elastic interactions persist until the system reaches kinetic freeze-out at a temperature which can be computed from the momentum spectra of the detected particles. Therefore, by finding the parameters that give rise to a given particle spectrum, the chemical freeze-out conditions are found and placed into the chart.

On the other hand, experimental observations have shown that for non-central collisions, there is a certain polarization of the detected hadrons [6–9]. This can be explained by taking into account the angular momentum conservation, which entails a speed gradient, and therefore a vortical component:

$$\vec{\omega} = \vec{\nabla} \times \vec{v} \tag{1}$$

According to recently published results [10], vorticity plays a role in the particle production in relativistic heavy ion collisions, enhancing it following a quadratic dependence. Additionally, its significance is intensified for particles with greater spin. Thus, the number of pions produced would result unaltered, due to their null spin, whereas other particles like Ω^- and Λ^* baryons would be affected in a very notable way. All in all, the accentuation of the yield of a particle "i" of spin j_i is given by:

$$N_i(\omega) = N_i(\omega = 0)\left[1 + \frac{j_i(1+j_i)}{6}\left(\frac{\omega}{T}\right)^2\right] \quad (2)$$

By using the aforementioned model, and considering RHIC energies, the aim is to find the temperature and baryochemical potential that best fit production ratios, so as to study its validity within the framework of some approximations which will be commented later on. Moreover, by contrasting the results of the simulation with and without vorticity, the main goal of this work is to numerically check whether a significant discrepancy in temperature and baryochemical potential arises when introducing this magnitude, as is postulated in [10]: "Since finite ω enhances the hadron yield on average, the existence of the strong local vorticity would result in a reduction of the coalescence/chemical freeze-out temperatures." If such were to be the case, considering vorticity would be a key factor when correctly reproducing freeze out and determining the strongly interacting matter phase diagram. Instead, if this effect is negligible, fittings of the statistical model up-to-date could be considered acceptable, and one could conclude that there is no need to remake these fits including vorticity.

II. FITTING PROCEDURE

A. Model description

As stated above, the statistical production model will be used in order to fit the experimental data of yielded particles in the Relativistic Heavy Ion Collider (RHIC) ($\sqrt{s_{NN}}$ = 200 GeV). Although the number of constituents is small, data analysis suggests that the grand canonical partition function can be used [2], and it has to be derived with respect to the chemical potential to compute the number of constituents [3]:

$$N_i = -T\frac{\partial lnZ_i}{\partial \mu} \quad (3)$$

The result, however, depends on the volume of the fireball, which is an unknown parameter. In order to avoid this problem, ratios have to be calculated instead.

Taking spin degeneracy into account, the considered hadrons will follow a familiar distribution (Bose-Einstein for bosons, Fermi-Dirac for fermions) [3]:

$$N_i = \frac{g_i V}{2\pi^2}\int_0^\infty \frac{p^2 dp}{exp[(E_i - \mu_i)/T] \pm 1} \quad (4)$$

Choosing the denominator sign according to the same criteria. Thus, each particle "i" of spin j_i will have degeneracy $g_i = 2j_i + 1$, energy $E_i = \sqrt{p^2 + m_i^2}$, and chemical potential $\mu_i = \mu_B B_i + \mu_S S_i + \mu_Q I_{3_i} + \mu_C C_i$, where B_i is its baryon number, S_i its strangeness, C_i its charm and I_{3_i} the third component of its isospin [2]. Note also that each quantum number carries an associated chemical potential, which ensures the compliance of the correspondant conservation law in average. These are:

1. Baryon number: $V\sum_i n_i B_i = N_B$ (5)
2. Strangeness: $V\sum_i n_i S_i = 0$ (6)
3. Isospin (electric charge): $V\sum_i n_i I_{3_i} = I_{3_i}^T$ (7)
4. Charm: $V\sum_i n_i C_i = 0$ (8)

This model, nevertheless, is far too complicated for the purposes of this study. In order to simplify it some constraints have to be established.

Firstly, the range of momenta can be narrowed so as the integral is more easily computable. Since no temperatures greater than 300 MeV will be considered, the integration can be safely stopped at $p^{max} = 4000$ MeV

Furthermore, only T and μ_B are tipically taken as free parameters of the fit, while μ_S and μ_Q are calculated with the equations (6) and (7). However, that would require the computation of the whole production of particles, and since a "toy" model will be used, the benefits to the aim of this study do not compensate the complications, so both parameters will be included as free parameters. This approximation is justified, for strangeness and electric charge potentials are significantly smaller than then baryochemical potential, and their variations are not so important. Finally, given that no charmed particles will appear in the fittings, its associated term can be neglected. The total chemical potential will then be:

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_Q Q_i \quad (9)$$

On the other hand, the decay of unstable particles is neglected as well [2]. The consideration of this phenomenon would affect the yield and consequently the ratios, but that is of little interest for the purposes of this simplified study.

Thus, the parameters of the fit will be the temperature (T), the baryochemical potential (μ_B), the strangeness potential (μ_S), the electric charge potential (associated to the third component of the isospin) (μ_Q) and the vorticity (ω), in the most general case.

Ratios	**Experimental values**	**Fit ($\omega/T = 0$)**	**Fit with free vorticity**	**Fit ($\omega/T = 0.1$)**
π^-/π^+	0.984 ± 0.061	0.959	0.946	0.959
K^-/K^+	0.933 ± 0.061	0.920	0.890	0.920
$\bar{p}/p$	0.731 ± 0.073	0.711	0.686	0.711
$\bar{\Lambda}/\Lambda$	0.759 ± 0.015	0.773	0.770	0.772
$\bar{\Xi}/\Xi$	0.808 ± 0.021	0.808	0.823	0.808
$\bar{\Omega}/\Omega$	1.01 ± 0.08	0.877	0.924	0.878
K^+/π^+	0.171 ± 0.011	0.226	0.157	0.226
K^-/π^-	0.162 ± 0.011	0.216	0.147	0.217
$\bar{p}/\pi^-$	0.047 ± 0.003	0.034	0.034	0.034
Λ/π^-	0.041 ± 0.005	0.012	0.010	0.012
Ξ/π^-	0.0078 ± 0.0010	0.0048	0.0030	0.0048
Ω/π^-	0.00095 ± 0.00010	0.00099	0.00132	0.00099

TABLE I: Ratios obtained after completing the MINUIT minimization of the chisquared function for different vorticities, contrasted with the experimental values and errors from [11, 12].

Parameters	**Fit ($\omega/T = 0$)**	**Fit with free vorticity**	**Fit ($\omega/T = 0.1$)**	**Ref [2]**
T (MeV)	140 ± 1	114 ± 4	140 ± 1	155 ± 2
μ_B (MeV)	20 ± 3	19 ± 2	20 ± 3	26 ± 5
μ_S (MeV)	2.4 ± 1.7	4.1 ± 1.6	2.4 ± 1.8	–
μ_Q (MeV)	5.6 ± 3.0	2.6 ± 2.5	5.6 ± 3.0	–
ω/T	–	4.5 ± 0.9	0.1	–
χ^2	113	99	113	1.5

TABLE II: Resulting parameters of the fitting after the MINUIT minimization of the chisquared function, compared to those obtained from [2], where a much more sofisticated model is used.

B. Parameter determination

Having calculated each ratio as $R_k = N_i/N_j$, the next step is to find T, μ_S μ_S and μ_Q so that the experimental results are optimally adjusted.

In order to evaluate the discrepancy between the computed and the experimentally found ratios, the chisquared mathematical function is ideal, and it is defined as:

$$\chi^2 = \sum_k^{12} \frac{(R_k^{exp} - R_k^{model})^2}{\sigma_k^2} \tag{10}$$

The aim is then to minimize it while obtaining the parameters that do so, and the MINUIT fortran library will take care of that task. By receiving a subroutine as an input, which in this case will be the chi-squared function, MINUIT will look for its minima and will provide the fitting parameters at convergence and their pertinent errors. The 12 experimental ratios considered will be taken from [11, 12], which correspond to $Au + Au$ collisions at the RHIC at $\sqrt{s_{NN}} = 200$ GeV.

C. Vorticity implementation

As commented in the introduction, vorticity (ω) has an impact in the hadronic production in relativistic heavy ion collisions, according to equation (2) The MINUIT minimization will be redone taking this effect into account by considering the so called thermal vorticity ω/T as a free parameter.

III. CALCULATIONS

The statistical production model has been used to fit the experimental data. The results of the obtained ratios are shown in *Table I*, while the parameters of the fit are shown in *Table II.* Each table contains a column displaying the outcomes for different values of vorticity:

1. No vorticity ($\omega/T = 0$).
2. Vorticity included as a free parameter of the minimization, i.e., without constraints.
3. Vorticity taking a typical value found in simulations ($\omega/T = 0.1$ [13, 14]).

The results for each condition will be commented and explained hereunder.

A. Fit without vorticity

The obtained temperature and baryochemical potential are 140 ± 1 MeV and 20 ± 3 MeV respectively. The temperature substantially differs from the [2] fit, it being significantly lower. This disagreement could be due to having disregarded the decay of unstable particles. The baryochemical potential fits into the experimental error and can be considered an adequate outcome, and is considerably bigger than strangeness and electric charge potential.

The ratios, on the other hand, suit the experimental errors in half of the cases, while the others are rather far from the error range. The consequence is a rather big chisquared value: $\chi^2 = 113$. However, the quality of the fit is not really important for the purposes of this simplified study, since the only real interest in minimizing this function is to find the coordinates (T, μ_B) of the minimum, and later to check whether or not its position is sensitive to the presence of vorticity

B. Fit considering vorticity

If vorticity is included as a parameter, temperature radically lowers, which is the expected results by the authors in [10]. Nonetheless, the obtained value for the vorticity is not physical and will never be found in relativistic heavy ion collisions at energies within reach [13, 14]. That is, $\omega/T = 4.5 \pm 0.9$, whereas its typical simulation values lay in the interval $[0, 0.5]$. Thus, one must check whether this difference is of such magnitude when fixing the parameter at a true physical value.

Concerning the ratios, the lower value of the chisquared function ($\chi^2 = 99$) indicates a better quality of the fit, even though it is of little importance.

C. Fit with a fixed vorticity

By fixing vorticity at a typical quantity ($\omega/T = 0.1$) one can see whether or not the effect is substantial (under normal conditions), thus determining the importance of taking it into account in chemical freeze out modeling. If the effect is negligible, there will be no need to remake the fittings with this new parameter.

That being said, the tables clearly show an insignificant difference. Besides from a slight discrepancy in some of the ratios, all the results are practically equal to the ones obtained without vorticity. The quality of the fit is also the same, and one can consider that low vorticity effects are inconsequential.

Yet, there must be a transition from a meaningless to a significant influence for higher vorticity values. In order to get a deeper insight on the change of temperature and baryochemical potential, this effect has been studied over the whole range of values of ω, up to 4.5 MeV, since for the future experiments FAIR (GSI, Germany) and NICA (Dubna, Russia) there are predictions with $\omega/T \geq 1$ for some peripheral cells [9].

D. Parameter dependence on vorticity

The graphical representations of the variation of T and μ_B as a function of vorticity for $\omega \in [0, 4.5]$ are shown in FIG. 2 and 3.

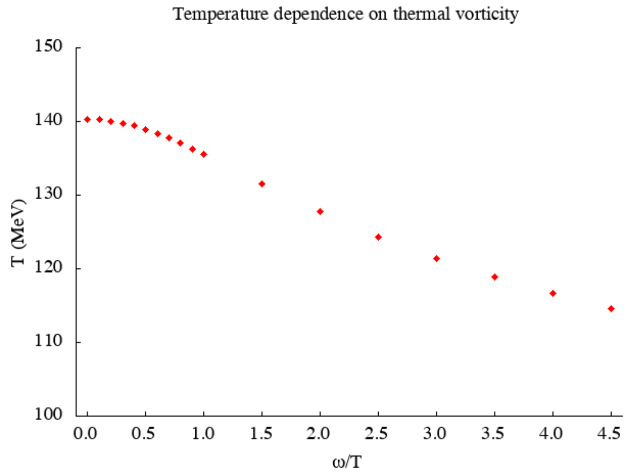

FIG. 2: Representation of T as a function of ω/T.

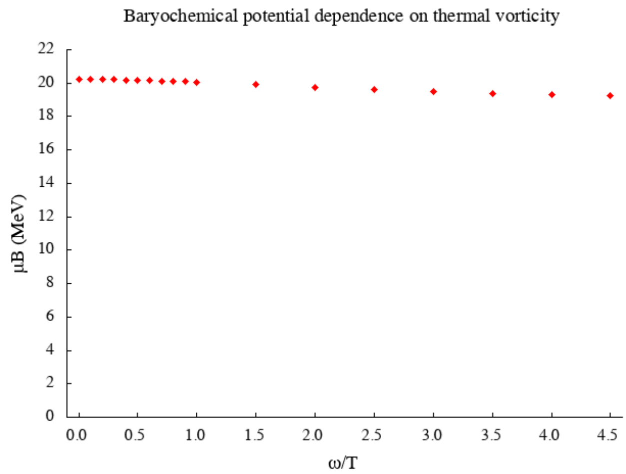

FIG. 3: Representation of μ_B as a function of ω/T.

One can easily see that there is a tendency to lower values of T and μ_B as thermal vorticity grows, significantly more relevant for temperature which clearly shows a considerable drop for the top ω/T. On the other hand, baryochemical potential is very slighly reduced.

All in all, effects are very small in the range of physical values, whereas for higher vorticities a substantial difference exists.

IV. CONCLUSIONS

The statistical production model has proved to effectively reproduce the experimental hadron production ratios. This "toy" model, despite finding inaccurate minima, has provided a qualitatively correct result. Moreover, the obtained μ_S and μ_Q are much lower than μ_B, in agreement with the anticipated outcome.

Also, the origin of the significant discrepancy between the outcomes of this simplified model and the ones obtained using a more sophisticated approach is clear [2]. The first main effect is due to disregarding the decay of unstable particles, which entails a difference between the number of hadrons that are produced when chemical freeze-out takes place and the number of hadrons detected. On the other hand, in order to attain a better result one should compute the strangeness and charge potentials independently, instead of considering them free parameters.

This simplified model has proved suitable for the inclusion of vorticity, as is detailed in [10], and it has been possible to test and verify the modification of particle yields in relativistic heavy ion collisions.

The inclusion of a free vorticity has thoroughly changed the temperature conditions for chemical freeze-out which is the effect anticipated by the authors in [10]. Nevertheless, it led to a non physical value.

On the other hand, the incorporation of a physically realistic vorticity has left the ratios unchanged, giving rise to identical freeze-out temperature and baryochemical potential. Thus, voricity is meaningless in the determination of the chemical freeze out conditions, and is therefore a parameter that can be neglected. However, it is absolutely crucial to explain other phenomena found in relativistic heavy ion collision, such as the observed baryon polarization.

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