# Casimir forces between real materials

Author: Lior Tetro
*Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.*

Advisor: Agustin Perez Madrid

**Abstract:** In this project, the Casimir force between two object will be studied in light of the Lifshitz theory. The general expression as well as the small and large separation distance limits are discussed in detail. The analysis is complemented with numerical computations of the force as a function of the separation for Au and SiC at different temperatures. We numerically verify the analytical limits and compare the original result by Casimir with the force arising between real materials at non-zero temperature.

## I. INTRODUCTION

The discovery by Casimir that two electrically-neutral objects can attract to each at small separations plays an important role in several branches of physics. The celebrated result states that for two perfect conductors with planar surfaces at zero temperature, the attraction force per unit area is given by [1]

$$P_{\text{Casimir}} = \frac{\hbar c \pi^2}{240 d^4}, \tag{1}$$

where $\hbar$ is the reduced Planck constant, $c$ is the speed of light and $d$ is the thickness of the vacuum gap between the objects. A generalization of this phenomenon was developed by Lifshitz [2] using the theory of fluctuating electromagnetic fields developed by Rytov [3]. The idea behind this formulation is to incorporate a random current to Maxwell equations that arise from the fluctuations occurring inside the materials. We can understand this current as analogous to the random force introduced in the Langevin equation for the Brownian motion. As shown by Lifshitz [2, 4], the use of the Maxwell equations with a stochastic term gives the possibility to take into account the particularities of different materials, and the inclusion of the fluctuation-dissipation theorem (FDT) in the deduction leads to a temperature dependence of the force. Thus, Lifshitz's theory not only includes purely quantum fluctuations of the electromagnetic field at zero temperature, but also the contribution of thermal fluctuations.

In this project, the general expression for the force firstly derived by Lifshitz will be obtained following the scattering-matrix approach [5–9]. Once the force is obtained, we will study in detail the small and large separation distance limits and analyze how these limits are related to the temperature. The original result by Casimir is obtained in the proper limit. Finally, we will compute numerically the force for two materials in concrete configurations: we consider gold (Au) as an example of a metal and silicon carbide (SiC) as an example of a polar dielectric. These numerical results will be compared with the theoretical limits.

## II. ELECTROMAGNETIC FIELD

The considered system is composed of two plane-parallel slabs, denoted by $j = 1, 2$, which are supposed to be infinite in the $x$ and $y$ directions and which are located along the $z$-axis. The slabs are centered at $z_j$ and are separated by a distance $d$. We formally assume that the bodies have a thickness $\delta$, but below we will take $\delta \to \infty$ considering them to be optically opaque. As shown in Fig. 1, the slabs define three vacuum regions that we name as $\gamma = 0, 1, 2$. Furthermore, the full system is assumed in thermal equilibrium with a bath of thermal radiation at temperature $T$.

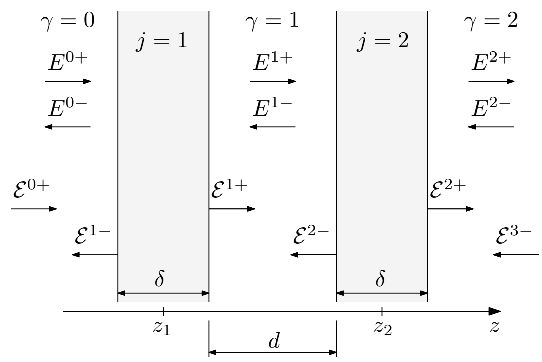


FIG. 1: Our system in the $y$-$z$ plane. The sketch shows the total field modes $E^{\gamma\phi}$ in each region and the modes $\mathcal{E}^{j\eta}$ coming out of the different sources.

The radiation emitted by the bodies and the environmental radiation of the thermal bath constitute sources for the electromagnetic field in the different vacuum regions. The total electric field at a point $\vec{R} = (x, y, z)$ in region $\gamma$ at time $t$ can be expressed as the following Fourier expansion $\vec{E}^{\gamma}(\vec{R}, t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \vec{E}^{\gamma}(\vec{R}, \omega)$, where $\omega$ is the frequency and we require that $\vec{E}^{\gamma}(\vec{R}, \omega) = \vec{E}^{\gamma *}(\vec{R}, -\omega)$ in order for $\vec{E}^{\gamma}(\vec{R}, t)$ to be real, where the asterisk denotes complex conjugate. In turn, the monochromatic field can be expanded in terms of the wave vector parallel to the surfaces $\vec{k} = (k_x, k_y)$, the direction of propagation $\phi$ and the polarization $p = \text{TE}, \text{TM}$, where $\phi = +$ indicates propagation to the right and $\phi = -$ to the left. Hence, taking into account all the previous expansions, we obtain the complete expressions for the field as

$$\vec{E}^{\gamma}(\vec{R},t) = \int_0^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \sum_{\phi,p} \int \frac{d^2\vec{k}}{(2\pi)^2} e^{i\vec{K}^{\phi}\cdot\vec{R}} \times \hat{\epsilon}^{\phi}(\omega,\vec{k},p) E^{\gamma\phi}(\omega,\vec{k},p) \quad + \quad \text{c.c.}, \tag{2}$$

where $E^{\gamma\phi}(\omega,\vec{k},p)$ is a field mode in this decomposition, $\vec{K}^{\phi} = (\vec{k},\phi k_z)$ is the total wave vector, $k_z = \sqrt{\omega^2/c^2 - k^2}$ is the normal component for which $k = |\vec{k}|$, $\hat{\epsilon}^{\phi}$ are the polarization vectors [8], and c.c. indicates complex conjugate. The magnetic field $\vec{B}^{\gamma}(\vec{R},t)$ in any vacuum region is obtained from the Maxwell equation $\nabla \times \vec{E}^{\gamma}(\vec{R},t) = -\partial_t \vec{B}^{\gamma}(\vec{R},t)$ and expression (2), and can be written in a plane-wave decomposition as

$$\vec{B}^{\gamma}(\vec{R},t) = \frac{1}{c}\int_0^{\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \sum_{\phi,p} \int \frac{d^2\vec{k}}{(2\pi)^2} e^{i\vec{K}^{\phi}\cdot\vec{R}} \times \hat{\beta}^{\phi}(\omega,\vec{k},p) E^{\gamma\phi}(\omega,\vec{k},p) \quad + \quad \text{c.c.}, \tag{3}$$

where $\hat{\beta}^{\phi}$ are directly related to the polarization vectors [8]. We remark that both the electric and magnetic fields are expanded in terms of the same modes $E^{\gamma\phi}(\omega,\vec{k},p)$. Furthermore, the total field in region $\gamma$ is a superposition of the source fields present in that region. In other words, the modes of the total field $E^{\gamma\phi}(\omega,\vec{k},p)$ are a linear combination of the source field modes that we now denote as $\mathcal{E}^{j\eta}(\omega,\vec{k},p)$, where the index $j = 0,1,2,3$ labels the corresponding source and $\eta = +,-$ indicates the direction which, in general, can be different from $\phi$. Here $j = 1,2$ identifies radiation emitted by the bodies, while $j = 0$ and $j = 3$ correspond to the contribution of the thermal bath arriving to the system from the left and from right, see Fig. 1. The linear relation between the fields can then be written as $E^{\gamma\phi} = \sum_{j,\eta} L^{\gamma\phi}_{j\eta}\mathcal{E}^{j\eta}$, where $L^{\gamma\phi}_{j\eta}$ are the coefficients solving the scattering problem and which depend on the reflection coefficients of the sources [9].

## III. CALCULATION OF THE FORCE

In order to describe the Casimir-Lifshitz forces, we have to consider the Maxwell stress tensor in any region $\gamma$ of the system, that in cartesian components is [8]

$$T^{\gamma}_{ij}(\vec{R},t) = \epsilon_0[E^{\gamma}_i(\vec{R},t)E^{\gamma}_j(\vec{R},t) + c^2 B^{\gamma}_i(\vec{R},t)B^{\gamma}_j(\vec{R},t)] - \frac{\epsilon_0}{2}\delta_{ij}[|\vec{E}^{\gamma}(\vec{R},t)|^2 + c^2|\vec{B}^{\gamma}(\vec{R},t)|^2] \tag{4}$$

with $i,j = x,y,z$ and where $\epsilon_0$ is the vacuum permittivity. The momentum flux in region $\gamma$ is equal to the symmetrized statistical average defined by

$$P_{\gamma} \equiv \langle T^{\gamma}_{zz}(\vec{R},t)\rangle. \tag{5}$$

The expression of the stress tensor involves products of the electric and magnetic field components. These components are written in terms of the field modes, so to compute the stress tensor we need to solve the averaged products of these field modes. To proceed, we introduce the correlation functions $C^{\gamma\phi\phi'} = C^{\gamma\phi\phi'}(\omega,k,p)$ which are defined as

$$\langle E^{\gamma\phi}(\omega,\vec{k},p)E^{\gamma\phi'*}(\omega',\vec{k}',p')\rangle = (2\pi)^3\delta(\omega-\omega')\delta(\vec{k}-\vec{k}')\delta_{pp'}C^{\gamma\phi\phi'}. \tag{6}$$

Here we have set the fields to be stationary and taken into account the planar geometry of the problem. The correlation functions are obtained by means of the fluctuation-dissipation theorem (FDT) for the total field outside a single body (assuming the fields emitted from different bodies are uncorrelated), which can be formulated as [10]

$$\langle E^{(tot)\phi}_i(\omega,\vec{R})E^{(tot)\phi'*}_i(\omega',\vec{R}')\rangle = \frac{2h\omega^2}{\epsilon_0 c^2}\delta(\omega-\omega')\mathcal{N}(w,T)\text{Im}G_{ij}(\vec{R},\vec{R}',\omega), \tag{7}$$

where $G_{ij}(\vec{R},\vec{R}',\omega)$ is the Green function of the system and

$$\mathcal{N}(\omega,T) = \frac{1}{2}\coth\left(\frac{\hbar\omega}{2k_B T}\right) = n(\omega,T) + \frac{1}{2}, \tag{8}$$

$n(\omega,T)$ being the thermal photon distribution. It is important to note the term 1/2 in this expression, which leads to zero-point energy fluctuations persisting even as $T \to 0$. This contribution is essential to understand the limit at which we will find the Casimir formula.

With the above results and taking into account the coefficients solving the scattering problem, we can work out an explicit expression for the momentum flux in any region $\gamma$ as defined by (5). In region $\gamma = 0$, we get

$$P_0 = -\frac{\hbar}{3c^3\pi^2}\int_0^{\infty} d\omega\omega^3\Big[\frac{1}{e^{\hbar\omega/k_B T}-1} + \frac{1}{2}\Big], \tag{9}$$

which is simply the blackbody radiation pressure including the contribution of the zero-point energy fluctuations, $k_B$ being the Boltzmann constant. This flux is thus a diverging quantity. Although this solution might seem unphysical, we are able to normalize this result when taking into account the momentum flux in region $\gamma = 1$ and look at the net force acting on body 1 given by $P \equiv P_1 - P_0$. From (5), (7) and the scattering coefficients, this net force reads [9]

$$P = -\frac{\hbar}{\pi^2}\text{Re}\int_0^{\infty} d\omega\Big[\frac{1}{e^{\hbar\omega/k_B T}-1} + \frac{1}{2}\Big] \times \int_0^{\infty} dkk \sum_p \frac{k_z r^1_p r^2_p e^{i2k_z d}}{1 - r^1_p r^2_p e^{i2k_z d}}, \tag{10}$$

where $r^j_p = r^j_p(\omega,k)$ are the vacuum-medium Fresnel reflection coefficients that depend on $\epsilon_j = \epsilon_j(\omega)$, the dielectric permittivity of the body $j$. Although expression (10) still contains the contribution of the zero-point fluctuations, the net force $P$ is finite. In addition, we can express this force as a sum over the poles in the imaginary axis of the frequencies instead of the integral over the real positive line. If we introduce the bosonic Matsubara frequencies $\zeta_n = 2\pi n k_B T/\hbar$ and the variables $k = \zeta_n \sqrt{q^2-1}/c$ and $s_j = \sqrt{\epsilon_j - 1 + q^2}$, we obtain the same expression as [2]

$$\begin{aligned} P = &\frac{k_B T}{\pi c^3} {\sum_{n=0}^{\infty}}' \zeta_n^3 \int_1^\infty dq q^2 \\ &\times \Big\{ \Big[ \frac{(s_1+q)(s_2+q)}{(s_1-q)(s_2-q)} \exp\big(2q\zeta_n d/c\big) - 1 \Big]^{-1} \\ &+ \Big[ \frac{(s_1+q\epsilon_1)(s_2+q\epsilon_2)}{(s_1-q\epsilon_1)(s_2-q\epsilon_2)} \exp\big(2q\zeta_n d/c\big) - 1 \Big]^{-1} \Big\}, \end{aligned} \tag{11}$$

where the prime indicates that the pole at $n = 0$ has an additional factor of $1/2$ that comes from the quarter-turn contribution in the complex plane integration. The integral is performed with a contour in the positive-positive quadrant of the complex plane.

### A. The limit of small separations

In order to obatain approximate expressions for the force (11), we consider in this section a situation such that $k_B T d/c\hbar \ll 1$. The quantity $k_B T d/\hbar c$ is a dimensionless parameter that relates quantum effects to thermal effects, and the considered limit here is equivalent to take $T \to 0$. Since $\hbar c/k_B T \approx 10\,\mu$m at room temperature, neglecting thermal effects is a good approximation even for separation distances up to about $1\,\mu$m. Furthermore, we first assume that the distance $d$ is small as compared to the absorption wavelengths $\lambda_0$ of the slabs, $d \ll \lambda_0$. At room temperature, and for separations above 1 nm, these conditions are met for metals, since $\lambda_0$ typically lies in the visible or ultraviolet. For polar dielectrics, this regime can be achieved at very small separations (below 1 nm) because $\lambda_0$ lies in the infrared. Under these conditions, the dominant terms of the sum (11) satisfy $n \sim c\hbar/k_B T d$ (which appears in the exponential through $\zeta_n$), therefore meaning a large $n$ limit. With this, we can convert the sum over $n$ into an integral with $\mathrm{d}n = \hbar \mathrm{d}\zeta/2\pi k_B T$, so

$$\begin{aligned} P = &\frac{\hbar}{2\pi^2 c^3} \int_0^\infty \mathrm{d}\zeta \zeta^3 \int_1^\infty \mathrm{d}q q^2 \\ &\times \Big\{ \Big[ \frac{(s_1+q)(s_2+q)}{(s_1-q)(s_2-q)} e^{2q\zeta d/c} - 1 \Big]^{-1} \\ &+ \Big[ \frac{(s_1+q\epsilon_1)(s_2+q\epsilon_2)}{(s_1-q\epsilon_1)(s_2-q\epsilon_2)} e^{2q\zeta d/c} - 1 \Big]^{-1} \Big\}. \end{aligned} \tag{12}$$

This expression can be further simplified by noting that due to the increasing exponential term, the dominant contribution takes place when $q\zeta d/c \sim 1$. This requires $q \gg 1$, and remembering the definition of $s_j$, we can set $s_1 \approx s_2 \approx q$. Finally, we change to a new integration variable $x = 2q\zeta d/c$ instead of $q$ and obtain

$$P = \frac{\hbar}{16\pi^2 d^3} \int_0^\infty \int_0^\infty x^2 \Big[ \frac{(1+\epsilon_1)(1+\epsilon_2)}{(1-\epsilon_1)(1-\epsilon_2)} e^x - 1 \Big]^{-1} \mathrm{d}\zeta \mathrm{d}x. \tag{13}$$

The previous equation gives the force for small separations in the quasi-static limit where retardation effects are negligible. We highlight that the force is inversely proportional to $d^3$ in this regime.

Next we consider a limit in which the separations are large as compared to the absorption wavelength, $d \gg \lambda_0$ (intermediate distances), but the condition $k_B T d/c\hbar \ll 1$ is still fulfilled (quantum effects dominate). At room temperature, these conditions can be reached for both metals and dielectrics at separations above 1 nm. We will use the same variable as in the previous limit, although now $x = 2q\zeta d/c$ is the new integration variable instead of $\zeta$. We then obtain

$$\begin{aligned} P = &\frac{\hbar c}{32\pi^2 d^4} \int_0^\infty \int_1^\infty \frac{x^3}{q^2} \Big\{ \Big[ \frac{(s_1+q)(s_2+q)}{(s_1-q)(s_2-q)} e^x - 1 \Big]^{-1} \\ &+ \Big[ \frac{(s_1+q\epsilon_1)(s_2+q\epsilon_2)}{(s_1-q\epsilon_1)(s_2-q\epsilon_2)} e^x - 1 \Big]^{-1} \Big\} \mathrm{d}q \mathrm{d}x. \end{aligned} \tag{14}$$

If we take a closer look to the integrand, we see a factor of the form $x^3(\alpha e^x - 1)^{-1}$. The dominant contribution to the integral then occurs for $x \sim 1$. In addition, the permittivity is a function of the frequency that becomes $\epsilon_j = \epsilon_j(i\frac{xc}{2qd})$ when written as a function of $x$. Since $q \geq 1$ (lower limit of the integral) and we assume large separations as compared to $\lambda_0$, the argument of the permittivity is almost always zero. Therefore, we can make the approximation $\epsilon_j(i\zeta) \approx \epsilon_j(0)$. If we further consider metals for which $\epsilon_j(i\zeta) \to \infty$ as $\zeta \to 0$, the force (14) reduces to

$$P = \frac{\hbar c}{16\pi^2 d^4} \int_0^\infty \int_1^\infty \frac{x^3\, \mathrm{d}q \mathrm{d}x}{q^2(e^x - 1)} = \frac{\hbar c \pi^2}{240 d^4}, \tag{15}$$

which is the original formula obtained by Casimir [1]. We observe that this force does not depend on material properties nor any other parameter except for the distance between the slabs with a dependence $d^{-4}$.

### B. The limit of large separations

In contrast to the previous section, we now consider a situation for distances so large that $k_B T d/c\hbar \gg 1$. Quantum effects vanish in front of the thermal ones in this regime and hence, it can be understood as a *classical limit* [4] in which $\hbar\omega \ll k_B T$ with $\omega \sim c/d$. To obtain the force under these conditions, we start from the general expression (11). The dominant values of the sum are those with a small exponent and therefore small $n$. We use the integration variable $x = qn$ instead of $q$ in (11) and keep only the first term ($n = 0$), yielding

$$P = \frac{4\pi^2 (k_B T)^4}{\hbar^3 c^3} \int_0^\infty \frac{x^2 \mathrm{d}x}{\left(\frac{\epsilon_{10}+1}{\epsilon_{10}-1}\right)^2 \exp\left(\frac{4\pi k_B T d}{\hbar c} x\right) - 1}, \quad (16)$$

where we have assumed identical materials such that $\epsilon_{10} = \epsilon_j(0)$, $j = 1, 2$. This integral gives the approximate result of

$$P = \frac{k_B T}{8\pi d^3} \left(\frac{\epsilon_{10} - 1}{\epsilon_{10} + 1}\right)^2. \quad (17)$$

Notice that this result does not depend on $\hbar$. Moreover, we obtain a dependence $d^{-3}$ in the separation, as in the case with non-retardation effects. This is a clear difference between this general theory and the formula developed by Casimir.

## IV. NUMERICAL CALCULATION

The last part of this project is intended to verify the limits of the general formula (11) of the force. In order to do so, the DQAG subroutine of the QUADPAC library was used to numerically compute the integrals. The method is a global adaptive Gauss-Krond quadrature made by Robert Piessens and Elisse Doncker. The two materials studied are Au and SiC, to compare the force in a metallic and polar material. The permittivity of SiC is described by the Drude-Lorentz model and that of Au by a Drude model of the form [9]

$$\epsilon_{\mathrm{SiC}}(\omega) = \epsilon_\infty \frac{\omega_L^2 - \omega^2 - i\Gamma\omega}{\omega_T^2 - \omega^2 - i\Gamma\omega}, \quad \epsilon_{\mathrm{Au}}(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}, \quad (18)$$

where $\epsilon_\infty$ is the high frequency dielectric constant, $\omega_L$ longitudinal optical frequency, $\omega_T$ transverse optical frequency, $\omega_p$ is the plasma frequency and $\Gamma$ is the dissipation rate.

Firstly we will compare the force for a system of two slabs of Au, two slabs of SiC, and the original Casimir force at low and high temperatures.

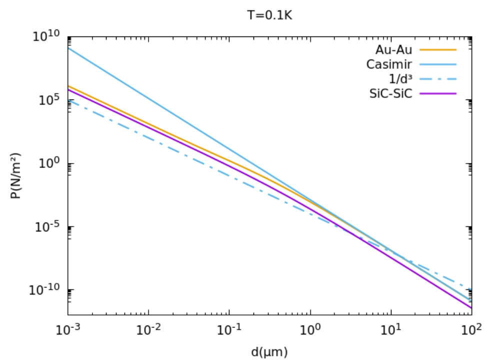


FIG. 2: Casimir force(blue), Casimir-Lifshitz force for Au(orange) and SiC(lilac) and a reference force $P \sim d^{-3}$ at a temperature of 0.1K

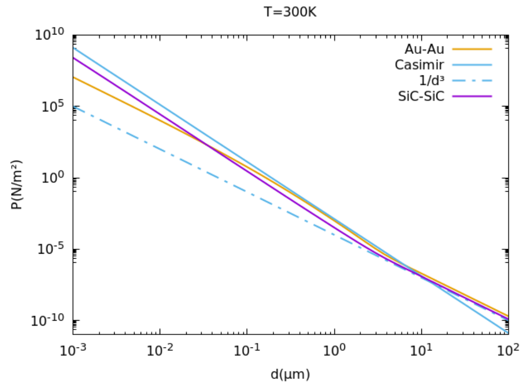


FIG. 3: Casimir force(blue), Casimir-Lifshitz force for Au(orange) and SiC(lilac) and a reference force $P \sim d^{-3}$ at a temperature of 300K

In both figures we have a reference function $d^{-3}$ and also the Casimir force that as shown by (15) is only $d^{-4}$ dependent. In FIG. 2 we can observe that for small distances the force for both materials follows (13) and at large distances equation (14) as expected. We have verified the limits for $k_B T d / c\hbar \ll 1$. In FIG. 3, we have something different. Firstly, for large distances both materials obey (17) as the $d^{-3}$ dependence is clearly observed. For small distances at room temperatures we have to take into account the nature of the material. Au is a metallic material, and thus, temperature dependence is small enough for (13) to be valid. SiC is polar, and expression (14) is valid and we can see that it is parallel to Casimir force ($d^{-4}$). At intermediate distances we can observe how the force for Au is approximately equal equal to the Casimir force as expected.

Now we will study the dependencies on distance for different temperatures for both materials separately.

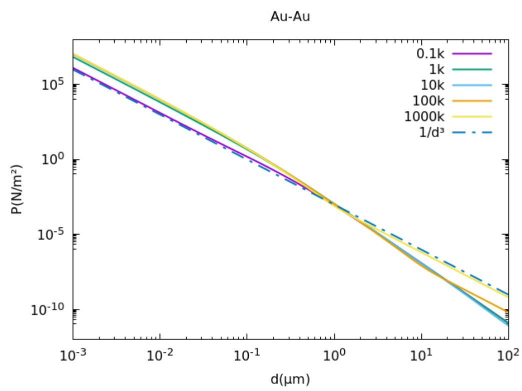


FIG. 4: Casimir-Lifshitz forces for 5 different temperatures differing in one order of magnitude for Au.

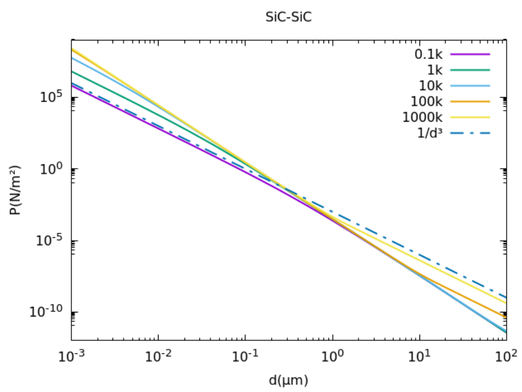


FIG. 5: Casimir-Lifshitz forces for 5 different temperatures differing in one order of magnitude for SiC.

In FIG. 4 (Au) we only deviate from (15) at temperatures around $10^3$ K for large distances, which exemplifies that temperature dependence is almost non-existing for metals at low temperatures. We can see that for intermediate $d$, the force has a non-trivial behaviour that gives an important difference at small distance. Other than that, for small separation we have $d^{-3}$. In FIG. 5 (SiC) we can see a much softer transition between the different behaviours for small distances. For the two highest $T$, we obtain the limit (14), and for the lower ones a gradual transition to (13). For large $d$, is very similar to Au, we only obtain (17) at very high temperatures, and that results in forces that are very close to zero.

## V. CONCLUSIONS

We have presented a complete study of the Casimir-Lifshitz force arising between two parallel slabs for two different materials, gold and silicon carbide. Firstly, we have obtained the general expression for the force (11) with the scattering-matrix approach and the use of the FDT (7). The formula obtained expresses the dependency on the material in the values of the Matsubara frequencies and the optical characteristics, and also its temperature dependence.

Additionally, the expressions for the forces in the small distance regime corresponding to the limit of vanishing temperatures was obtained. In this limit, we obtained the original Casimir force, and also the quasi-static limit. We also relaxed the low-temperature condition and saw how the distance dependence on the force changes for small and large separations. Also in this last case, it has been shown the importance of the material to obtain $d^{-3}$ or $d^{-4}$. The limit of large separations has been obtained as well, leading to a force vanishing as $d^{-3}$.

Finally, we have done a numerical calculation for gold and silicon carbide, to verify the limits theoretically obtained for specific functions for the permittivity. We have numerically obtained the different behaviors the force has in its separation dependence. All of them correctly matched the theoretical formulas obtained. It has been shown how the force at large temperatures and distances does not depend on the material and follows a $d^{-3}$ law, and also the transition between low and high temperatures limits for metallic and polar materials.

In this project, we have studied a system with a specific geometry and in equilibrium. Even so, the inclusion of special geometries as well as an out of equilibrium theory gives the outcome not only of attractive force but also of repulsive [8]. Remarkably, experimental research has been done in this research area [11, 12], and the field of Casimir-Lifshitz phenomena is very active nowadays.

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