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Maria Arazo^{1,2,*} , Ricardo Mayol^{1,2} and Montserrat Guilleumas^{1,2} ¹ Departament de Física Quàntica i Astrofísica, Facultat de Física, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain² Institut de Ciències del Cosmos, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain

* Author to whom any correspondence should be addressed.

E-mail: maria.arazo@fqa.ub.edu

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Abstract

We investigate Bose–Einstein condensates in bubble trap potentials in the presence of a small gravity. In particular, we focus on thin shells and study both contact and dipolar interacting condensates. We first analyze the effects of the anisotropic nature of the dipolar interactions, which already appear in the absence of gravity and are enhanced when the polarization axis of the dipoles and the gravity are slightly misaligned. Then, in the small gravity context, we investigate the dynamics of small oscillations of these thin, shell-shaped condensates triggered either by an instantaneous tilting of the gravity direction or by a sudden change of the gravity strength. This system could be a preliminary stage for realizing a gravity sensor in space laboratories.

1. Introduction

The recent progress in microgravity experiments with Bose–Einstein condensates (BECs) [1–4] and the development of new exotic confining potentials [5–10] have fostered a novel field of research on shell-shaped BECs. These hollow condensates were first realized in 2004 [11] and are at present under investigation in the NASA Cold Atom Laboratory (CAL) on the International Space Station [12–14]. Due to Earth's gravity, the atoms sag to the bottom of the trap destroying shell-shaped BECs. Thus, microgravity conditions—in which gravity is small enough to be neglected—ensure that these condensates are realizable in experiments. The observation of a shell-shaped BEC at CAL [12] has spiked the interest in such condensates under microgravity conditions [15–17]. In this paper, however, we are interested in the effect of a small gravity in shell-shaped BECs. Hence, to ensure that shells are realizable, we consider gravities larger than microgravity to study its effect but still some orders of magnitude smaller than the terrestrial gravity. This range of gravities is valid for the parameters we used in the numerical calculations from sections 3 and 4. However, one can extend it easily to other values—see section 5—by considering a different set of parameters, provided that the system is within the mean-field regime and in the thin-shell limit.

BECs in shell-shaped potentials open the possibility to investigate condensation and superfluidity phenomena in new topologies: collective modes [18–20], self-interference effects [17], thermodynamics of shells and curved manifolds [21, 22], quantized vortices [16, 23, 24], topological transitions in curved systems [25], the dimensional reduction to a ring-shaped condensate [26], and the transition from filled to hollow condensates [27, 28], among others. Theoretical work has focused mainly on shell-shaped BECs with contact-interacting atoms rather than with atoms that possess a non-negligible dipolar moment. The latter situation has been examined in the limit of a thin shell [29] and under rotation [30].

While contact interactions are short-range and isotropic, the interaction between particles with a dipolar moment presents a long-range and anisotropic character [31]. This intrinsic feature of dipolar BECs makes these systems especially sensitive to the shape of the trapping potential. Besides, the existence of a privileged direction defined by the dipole polarization might endow dipolar BECs with an interesting sensitivity to small changes in orientation, such as perturbations of gravity.

In this work, we investigate the effects of the dipolar interaction in thin shell-shaped condensates. Moreover, since experiments in microgravity conditions or at CAL facilities might suffer gravity perturbations, we study the dynamics of small oscillations in small-gravity conditions—for both contact and dipolar interacting condensates—which could yield to identify small changes in the direction or magnitude of the gravity.

The paper is structured as follows. Section 2 introduces the shell-shaped potential and the theoretical framework. In section 3, we analyze the ground state configurations—both for contact and dipolar interacting BECs—in the presence of gravitational sag. Then, we discuss two cases: when the gravity is parallel with the z -axis—the polarization direction in dipolar BECs—and when it is slightly misaligned. Section 4 explores the dynamics of small oscillations triggered by a tiny variation in the gravity direction or its strength. In section 5, we extend our study to other sets of parameters and ranges of gravity. Lastly, we summarize our results and provide future perspectives in section 6.

2. Theoretical framework

We consider N dilute and weakly-interacting dipolar bosons at zero temperature confined in a shell-shaped potential $V_{\text{ext}}(\vec{r})$. In the mean-field framework, the Gross–Pitaevskii equation (GPE) provides a good description of a weakly interacting dipolar BEC:

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\text{ext}}(\vec{r}) + g|\Psi(\vec{r}, t)|^2 + V_{\text{dd}}(\vec{r}) \right] \Psi(\vec{r}, t) = i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t}, \quad (1)$$

where $\Psi(\vec{r}, t)$ is the condensate wave function normalized to the total number of particles N . The atom–atom mean-field interaction is characterized by the contact-interacting potential with coupling constant $g = 4\pi\hbar^2 a_s/m$, where a_s is the s -wave scattering length and m the atomic mass, and the dipolar interaction $V_{\text{dd}}(\vec{r}) = \int |\Psi|^2 v_{\text{dd}}(\vec{r} - \vec{r}') d\vec{r}'$. The dipolar interaction potential for a polarized sample of particles with dipolar moment $\vec{\mu}$ oriented along the z -axis is

$$v_{\text{dd}}(\vec{r} - \vec{r}') = \frac{C_{\text{dd}}}{4\pi} \frac{1 - 3 \cos^2 \theta}{|\vec{r} - \vec{r}'|^3}, \quad (2)$$

where $|\vec{r} - \vec{r}'|$ is the relative distance between particles, θ is the angle between $\vec{r} - \vec{r}'$ and the direction of polarization and C_{dd} is $\mu_0\mu^2(d^2/\epsilon_0)$ for a magnetic (electric) dipole moment.

Analogously to contact interactions—which are characterized by the s -wave scattering length—, a dipolar effective length can be introduced for dipole–dipole interactions, $a_{\text{dd}} = C_{\text{dd}}m/(12\pi\hbar^2)$. Then, the relative strength of both interactions is defined as the ratio of these two effective lengths, $\epsilon_{\text{dd}} = a_{\text{dd}}/a_s$, which in the case of magnetic moments is

$$\epsilon_{\text{dd}} = \frac{\mu_0\mu^2 m}{12\pi\hbar^2 a_s}. \quad (3)$$

Shell-shaped BECs have been experimentally realized by employing time-dependent, radio-frequency induced adiabatic potentials within a conventional magnetic trap [5]. In the thin-shell limit, where the thickness of the shell is small compared to its radius, these bubble trap potentials can be approximated by a radially shifted harmonic trap [19, 27]:

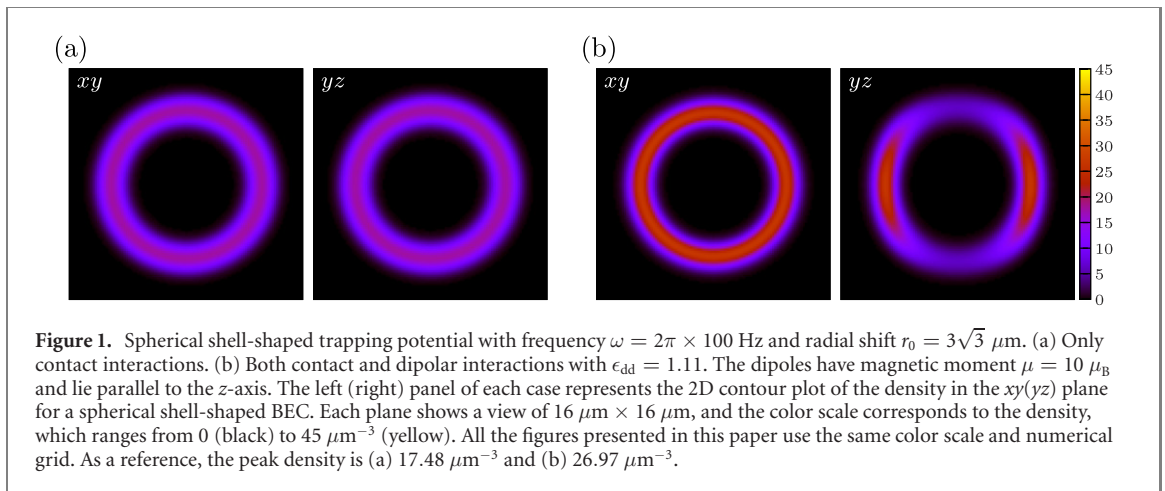
$$V_{\text{ext}}(\vec{r}) = \frac{1}{2} m\omega^2 (r - r_0)^2. \quad (4)$$

This potential defines a spherically symmetric shell of radius r_0 , with $\omega \equiv \omega_x = \omega_y = \omega_z$ and $r^2 \equiv x^2 + y^2 + z^2$.

3. Ground states

From now on, we consider a typical BEC in the mean-field regime: $N = 10^4$ atoms of ^{164}Dy polarized along the z -axis with magnetic dipolar moment $\mu = 10 \mu_B$, scattering length $a_s = 120a_0$, and mass $m = 164$ amu. In this case, the relative strength of the interactions is $\epsilon_{\text{dd}} = 1.11$.

We start by characterizing the ground state of the system with and without gravitational sag. In the spherical-shell geometry described before, we obtain the shell-shaped ground state wave function by solving the time-independent GPE with the imaginary-time propagation method in 3D. The dipolar term transforms the GPE into a more complicated equation. However, one can evaluate the dipolar interaction



integral, $V_{\text{dd}}(\vec{r})$, employing Fourier transform techniques—see [32] and references therein³. In particular, we use the FFTW package [33]. The size of the 3D box shown in all the figures is $16 \mu\text{m} \times 16 \mu\text{m} \times 16 \mu\text{m}$.

3.1. Dipole–dipole interactions

The dipolar interaction deforms the ground state density to minimize the energy of the system. This effect is a consequence of the anisotropic character of the dipolar interactions and depends on the specific trapping potential. It was observed already in the first dipolar condensates as the appearance of new structured biconcave ground states for some particular values of the strength of the dipolar interactions and the harmonic trap anisotropy [31]. Afterward, this feature was proposed to generate a self-induced bosonic Josephson junction in a toroidally confined dipolar condensate [34, 35].

For a spherical shell-shaped confining potential (4), we show in figure 1 the 2D contour plot of the density in the $xy(yz)$ plane in the left (right) panel. Figure 1(a) corresponds to a pure contact interacting BEC and figure 1(b) to a dipolar BEC with the dipoles aligned along the z -axis. Due to the confinement, the condensate has a hollow core and is shell-shaped. The density distribution with only contact interactions—see figure 1(a)—is entirely isotropic, while the addition of dipolar interactions produces a density accumulation around the equatorial region of the bubble—see right panel of figure 1(b). In the equator, the dipoles lie mainly in a head-to-tail configuration, and the resulting interaction is attractive. In the polar regions, the dipoles sit instead side by side, which gives a net repulsive interaction.

This anisotropic effect of the dipolar interaction was already shown in toroidal condensates [36] and, more recently, in spherical shell-shaped potentials [29, 30]. Note that, despite the contour plot in the xy plane is almost the same with and without dipolar interactions—see left panels on figures 1(a) and (b)—, the maximum value of the density is higher in the presence of dipolar interactions than without them. The asymmetry in the density contour plot enhances as the relative strength between dipolar and contact interaction, ϵ_{dd} , increases [30, 36].

3.2. Gravitational sag

The effect of gravity can be accounted for by including an additional potential term in the GPE (1), the gravitational sag potential V_g [19]. To investigate the anisotropic effects of the dipolar interaction, we consider a general case in which the direction of gravity is not aligned with any of the axes of the trap but lies in the xz plane. The gravitational sag potential reads

$$V_g(\vec{r}) = mg(x \sin \theta_0 + z \cos \theta_0), \quad (5)$$

where θ_0 is the angle between the gravity direction and the z -axis. In the particular case where the gravity and the z -axis are aligned ($\theta_0 = 0$), the gravitational sag $V_g(z) = mgz$ is equivalent to adding a vertical displacement to the trap's center [19, 37].

Here we investigate the effects of gravity in the same spherical shell-shaped trapping potentials as in the previous subsection, both in contact-interacting and in dipolar BECs. We restrain our study to small strengths of gravity—with *small* we mean larger than microgravity but smaller than Earth's gravity—since

³ We have checked that the static results do not change when considering a different number of points or size of the numerical grid. This statement holds also for the dynamical results presented in section 4.

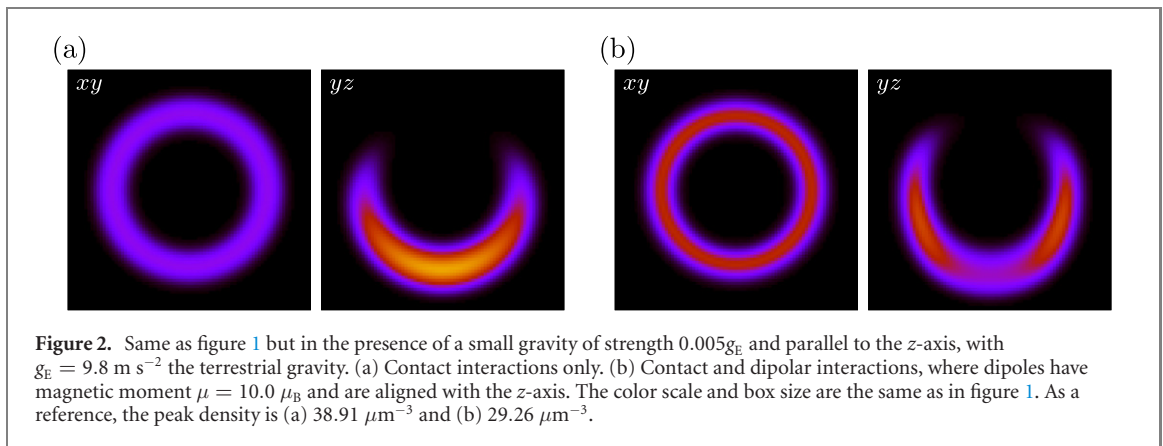


Figure 2. Same as figure 1 but in the presence of a small gravity of strength $0.005g_E$ and parallel to the z -axis, with $g_E = 9.8 \text{ m s}^{-2}$ the terrestrial gravity. (a) Contact interactions only. (b) Contact and dipolar interactions, where dipoles have magnetic moment $\mu = 10.0 \mu_B$ and are aligned with the z -axis. The color scale and box size are the same as in figure 1. As a reference, the peak density is (a) $38.91 \mu\text{m}^{-3}$ and (b) $29.26 \mu\text{m}^{-3}$.

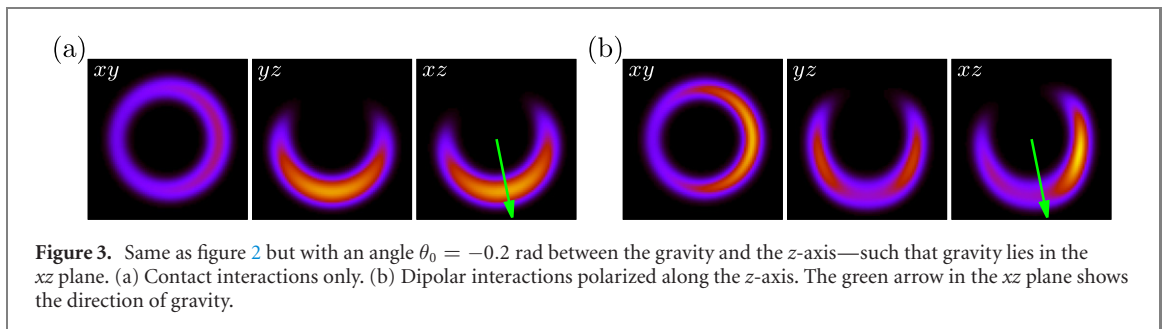


Figure 3. Same as figure 2 but with an angle $\theta_0 = -0.2$ rad between the gravity and the z -axis—such that gravity lies in the xz plane. (a) Contact interactions only. (b) Dipolar interactions polarized along the z -axis. The green arrow in the xz plane shows the direction of gravity.

the terrestrial gravity destroys shell-shaped geometries [12] and microgravity effects are negligible in our case.

3.2.1. Gravity aligned with the z -axis

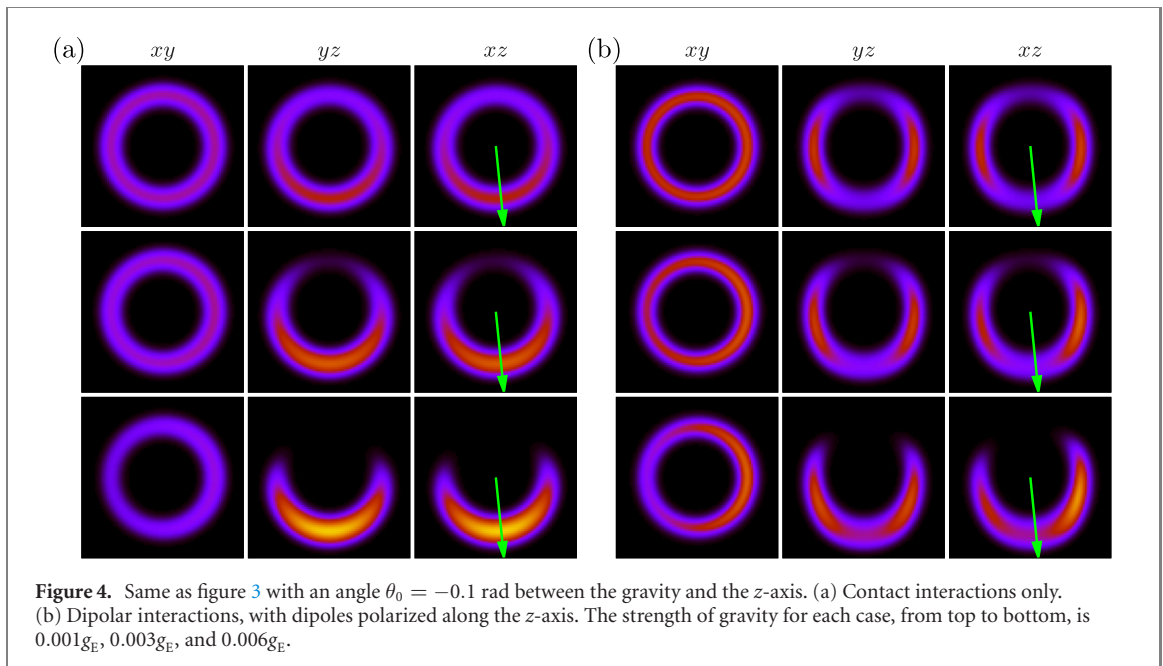
We start by considering a gravity aligned with the z -axis and, for dipolar BECs, parallel with the polarization direction. Figure 2 depicts the numerical results—see figure 1 for comparison without gravity. As we can see in the right panel (yz plane) of the contact interacting case—figure 2(a)—, the atoms fall to the bottom of the shell-shaped potential. The density distribution in the xz plane is the same as in the yz plane due to the axial symmetry of the system: confinement, gravity, and polarization. The distortion of the trap, which results in a partially filled shell, is a clear signature of gravitational sag [19, 38].

In the presence of dipolar interactions—figure 2(b)—, the interplay between their anisotropic character, the confining potential, and the gravitational sag leads to a partially filled shell, like in the contact-interacting case, but with a density depletion in the south region. As discussed in the situation with no gravity, the repulsive interaction between two parallel dipoles produces a significant reduction of the density in the bottom of the condensate—see the right panel of figure 2(b). The maximum density band lies slightly below the equatorial region, depending on the balance between the gravity and the dipolar moment of the atoms.

3.2.2. Misaligned gravity

We now explore a more general situation where the gravity and polarization direction are not aligned. Instead, gravity forms an angle θ_0 with the z -axis and lies in the xz plane. In figure 3, we show the 2D contour density plots in the three planes: xy (left), yz (middle), and xz (right). Figure 3(a) corresponds to a pure contact interacting BEC, and figure 3(b) to a dipolar one. For a contact interacting BEC, the 2D contour plot in the yz plane remains almost unaltered as compared to figure 2(a), but the density's maximum in the xz plane tilts in the direction of gravity—marked with a green arrow in the right panels of figures 3(a) and (b). As one can see in the xy plane, this tilting also produces an accumulation of particles in the right part of the bottom region of the shell.

The situation becomes more amusing for dipolar BECs, though, since the polarization axis fixes a privileged direction that breaks the symmetry when the gravity and the dipoles are not aligned. As a result, the density configurations in the xz and yz planes are now different from the contact-interacting case, as shown in figure 3(b). The density contour plot in the yz plane is also similar to the density configuration when the gravity is parallel to the z -axis—see the right panel of figure 2(b). However, changes in the density in the xz plane are more significant now: the maximum of the density lies in the right lobe of the shell and



at a larger tilting angle compared to the direction of gravity. Within this region, the dipoles mainly lie head-to-tail, which results in an attractive interaction, whereas in the bottom of the shell (south pole), the atoms sit side by side, and hence the net interaction is repulsive.

It is interesting to stress that this symmetry-breaking phenomenon, shown in the xz plane, is produced by the anisotropic character of the dipolar interactions and depends both on the tilting angle θ_0 and on the strength of gravity. In figure 4, we show the 2D contour plots of the density in the three planes (xy , yz and xz) for a condensate with the same shell-shaped potential and for different small values of the gravity ($0.001 \leq g/g_E \leq 0.007$), tilted an angle $\theta_0 = -0.1$ rad from the z -axis. Figure 4(a) corresponds to the numerical results for a contact interacting BEC, and figure 4(b) to a dipolar condensate. For small values of the strength of gravity (below $0.003g_E$), the condensate forms a *full shell* with a higher density on the bottom. For slightly larger values (above $0.004g_E$), the system is no longer a *full shell* due to the sag effect of the gravity; there are practically no atoms at the top of the trap, and the shape of the condensate is a hollow *half shell*. When we include dipolar interactions, their anisotropic character counterbalances the effect of gravity. As a result, the hole that appears at the top of the shell is small compared with the contact interacting case.

4. Dynamics of small oscillations

In this section, we investigate the dynamical response of the system in the regime of small oscillations. To this aim, we trigger the dynamics by an instantaneous change in the tilting angle of gravity or its strength. We obtain the real-time evolution of the system by numerically solving the GPE (1).

In the first scenario—subsection 4.1—, we consider gravity is initially tilted forming a small angle θ_0 with the z -axis but contained in the xz plane, and then it is suddenly aligned with the z -axis at $t = 0$. In the second scenario—subsection 4.2—, the gravity is parallel with the z -axis ($\theta_0 = 0$), and we analyze the dynamics when slightly changing its strength from g_0 to g . To avoid large oscillations and complicated dynamics, we constrain our study to small variations. Table 1 provides a summary of all the particular cases discussed in this section.

Precision of the frequencies calculated. We have calculated some cases with longer evolution times to check the numerical value of the frequencies. For example, for a BEC with only contact interactions, gravity $g = 0.005g_E$ and variation of the angle of 0.1 rad—see figures 5(a) and 6(a)—we obtain the frequency 15.80 Hz for an evolution time $t_f = 0.3$ s, and 15.82 Hz for $t_f = 1.0$ s. We have also checked for other cases that the results start to vary at the second decimal digit. Therefore, the estimated error of the frequencies given in subsections 4.1 and 4.2 is ± 0.05 Hz.

4.1. Variations in the orientation of gravity

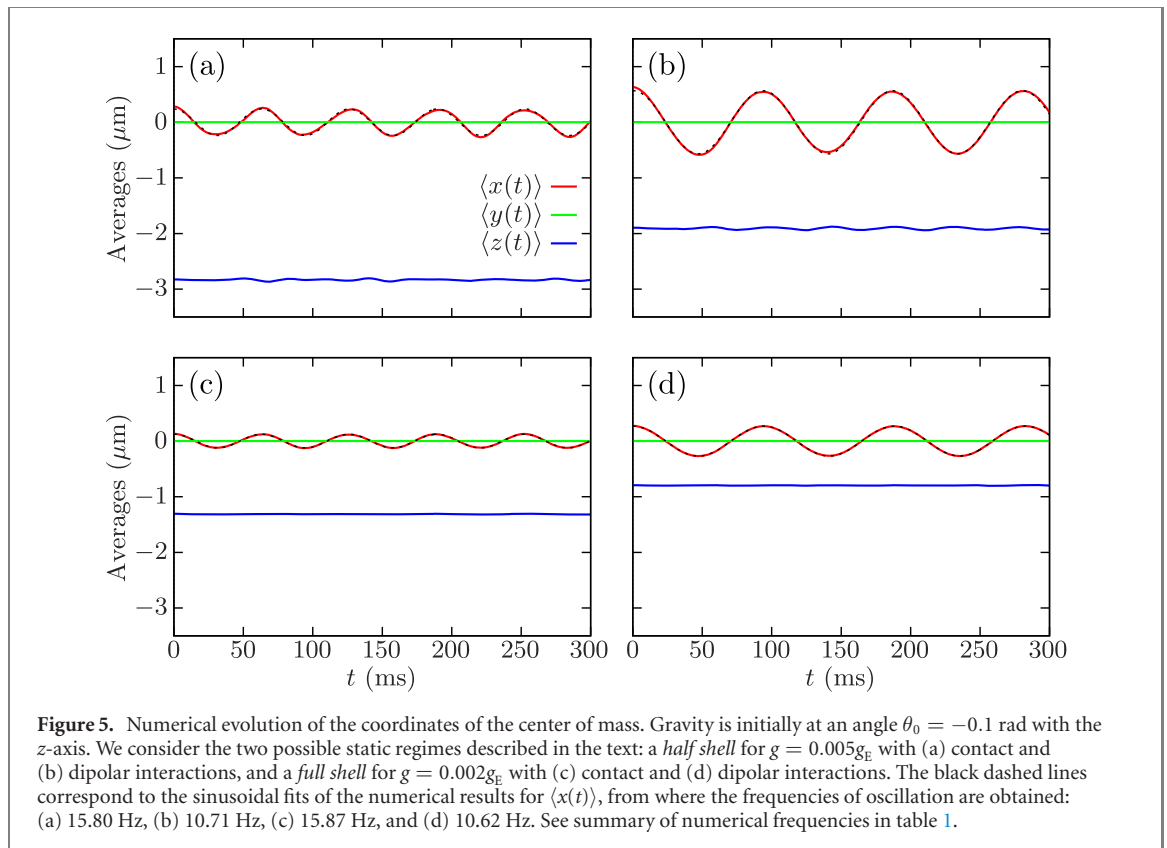
For all the results presented here, we consider $\theta_0 = -0.1$ rad. We have checked that, for a given strength of gravity, the dynamics are the same independently of the sign and value of the initial tilting angle as long as

Table 1. Summary of numerical frequencies obtained from the oscillation of the center of mass for the particular cases studied in subsections 4.1 and 4.2. We indicate the angle—with the z -axis—and strength of gravity and which of them is changed to trigger the dynamics. For each situation, we give the frequency for the BEC with only contact interactions (CI) and with both contact and dipolar interactions (DDI), and we indicate if the shape of the ground state is a *half shell* or a *full shell*.

		Gravity		Frequency (Hz)	
		Angle (rad)	Strength (g_E)	CI	DDI
Variations in the angle ^a	<i>Half shell</i>	$-0.1 \rightarrow 0.0$	0.005	15.80	10.71
	<i>Full shell</i>	$-0.1 \rightarrow 0.0$	0.002	15.87	10.62
Variations in the strength ^b	<i>Half shell</i>	0.0	$0.005 \rightarrow 0.006$	24.69	25.12
	<i>Full shell</i>	0.0	$0.003 \rightarrow 0.002$	16.10	22.20

^aFrequency calculated from the oscillation of $\langle x(t) \rangle$. See subsection 4.1.

^bFrequency calculated from the oscillation of $\langle z(t) \rangle$. See subsection 4.2.



such angle is small. We open this subsection with a detailed study of two particular cases—one with $g > 0.004g_E$ and the other with $g < 0.004g_E$ —to see how the shape of the ground state affects the dynamics. The results we show for discussion are the oscillations of the center of mass—figure 5, see numerical frequencies in table 1—and some snapshots of the density during the first period of the evolution—figure 6. Lastly, we analyze how the oscillation frequency depends on the strength of gravity—see figure 7.

4.1.1. Particular cases

We start with a gravity of strength $g = 0.005g_E$ such that the system resembles a *half shell*, as we discussed in section 3. First, we consider a shell-shaped BEC with only contact interactions. In figure 5(a), we present the time evolution of the coordinates of the center of mass: $\langle x(t) \rangle$, $\langle y(t) \rangle$, and $\langle z(t) \rangle$. Since we prepare the system with a slight misalignment of the gravity, the sudden alignment with the z -axis forces the system to bounce back and forth in the xz plane around the new equilibrium position, the z -axis. This behavior appears as a sinusoidal-like oscillation of $\langle x(t) \rangle$ as a function of time, while the other coordinates remain almost unaltered. The sinusoidal fit of the numerical evolution of $\langle x(t) \rangle$ gives a frequency of 15.80 Hz—we checked that the frequency of oscillation is close to this value when the initial angle $|\theta_0|$ is approximately below 0.15 rad. Figure 6(a) displays a few snapshots—the times shown cover a whole period—of the 2D

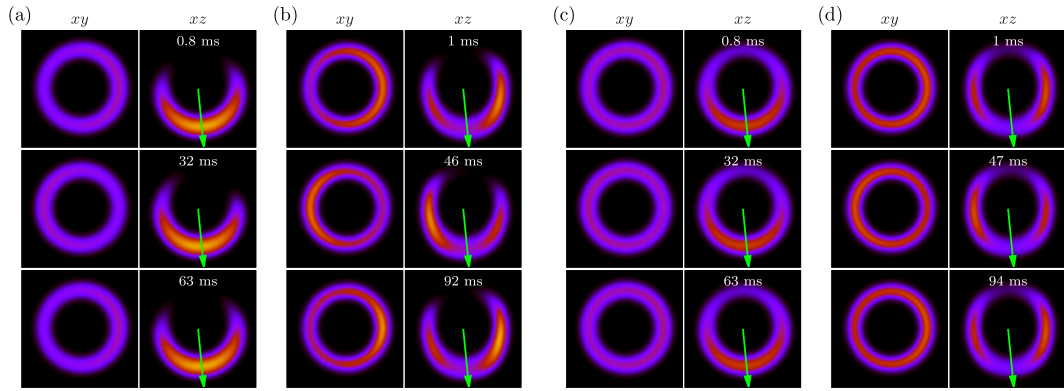


Figure 6. Snapshots of the 2D contour plots of the density in the xy (left panels) and xz (right panels) planes at different times of the evolution. We discarded the yz planes because the densities remain unchanged, as the variation in gravity is constrained to the xz plane. The initial tilt of the gravity is $\theta_0 = -0.1$ rad, and we study the same two situations as in figure 5: a *half shell* for $g = 0.005g_E$ with either (a) only contact interactions or (b) both contact and dipolar interactions, and a *full shell* for $g = 0.002g_E$ with (c) only contact and (d) also dipolar interactions. In both dipolar cases, the dipoles have magnetic moment $\mu = 10 \mu_B$. The green arrow shows the initial direction of gravity, which is later aligned with the z -axis to start the dynamics. See summary of cases in table 1.

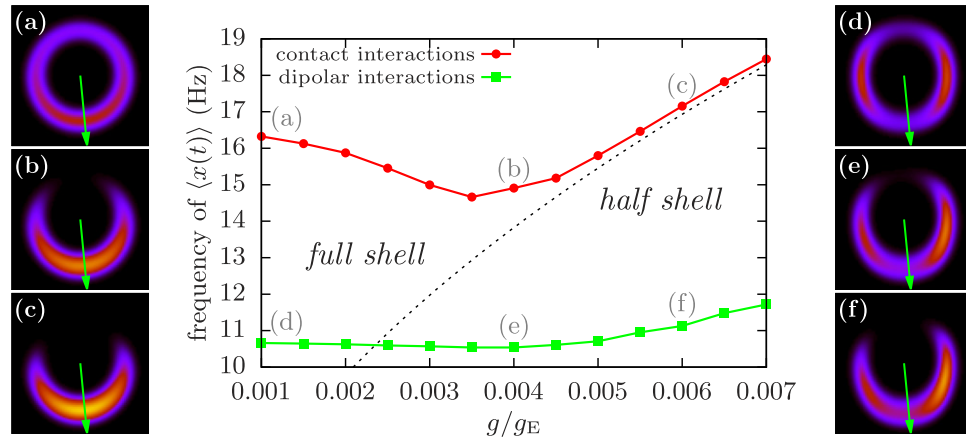


Figure 7. Oscillation frequency of $\langle x(t) \rangle$ as a function of the gravity with contact (red) and dipolar (green) interactions, where the gravity has an initial tilting angle $\theta_0 = -0.1$ rad. We obtain the oscillation frequency by fitting a sinusoidal function to the numerical data. The panels on both sides show the 2D contour plots of the initial density in the xz plane—which contains gravity—for the different values of the gravity labeled from (a) to (f), both for the contact interacting BEC (left panels) and the dipolar one (right panels). The green arrow, as in the previous figures, marks the initial direction of gravity. Lines between data points are added to guide the eye. The dashed line indicates the frequency of a mathematical pendulum, $\sqrt{g/r_0}$, with fixed length r_0 .

contour plots of the density in the two planes where the oscillations are observable, xy and xz . Since the shell-shaped BEC is 3D, the oscillatory behavior of $\langle x(t) \rangle$ —see figure 5(a)—produces symmetric rearrangements of the density in the other directions, as shown in the xy plane of figure 6(a).

Figure 5(b) shows the numerical evolution of the center of mass for a dipolar condensate with an initial tilting angle of gravity $\theta_0 = -0.1$ rad. As we discussed before—see section 3 and figure 4(b)—, the filled region of the shell-shaped potential appears at a larger tilting angle in a dipolar condensate than in a contact interacting one. This feature of the anisotropy of the dipolar interactions produces a larger amplitude of the oscillations of $\langle x(t) \rangle$ in dipolar BECs. The sinusoidal fit of the numerical evolution of $\langle x(t) \rangle$ gives a frequency of 10.71 Hz; as in the contact interacting case, the other components of the center of mass of the system, $\langle y(t) \rangle$ and $\langle z(t) \rangle$, show practically no variations. When the gravity is suddenly aligned, the system oscillates around the z -axis as expected. However, unlike in the contact interacting case, the atoms do not pass over the south pole of the *half shell*, where the net dipolar interaction is repulsive: their movement is instead constrained to the high-density band that appears below the equatorial region. One can see this behavior in figure 6(b), which shows a few snapshots covering one period in the xy and xz planes.

Lastly, we study the situation where the gravity is small enough—in particular, $g = 0.002g_E$ —that the BEC still retains its *full shell* shape. From figures 5(c) and (d), we can see that the oscillations of $\langle x(t) \rangle$ are broader and slower in the dipolar BEC than in the contact interacting one, as in the previous case. The

frequencies of oscillation we obtain from the fit are 15.87 Hz (contact BEC) and 10.62 Hz (dipolar BEC), which resemble those from the previous case. If we compare the oscillations—figures 5(c) and (d)—with those obtained for a heavier gravity—figures 5(a) and (b)—, we observe that the frequencies are similar in both the contact and the dipolar BECs, but the amplitudes of the oscillations are much lower now. From the snapshots of the density shown in figures 6(c) and (d), we can see that in the case of a smaller gravity, as expected, the atoms can move around the whole shell—not just the lower part—, which could explain why the oscillations of the center of mass in the x direction are more restricted. We will explore in more detail the effect of gravity in the dynamics in the following subsection.

4.1.2. The role of gravity

Here, we study the dynamics of small oscillations due to variations in the tilting angle—initially $\theta_0 = -0.1$ rad in all the cases—for different strengths of gravity. In figure 7, we show the oscillation frequency of the x coordinate of the center of mass, $\langle x(t) \rangle$, as a function of the strength of gravity. We consider BECs with both contact and dipolar interactions. As figure 7 shows, the oscillation frequency depends on the strength of gravity, and two different behaviors arise: first, starting from the lowest gravity, the frequency decreases as g increases until it reaches a particular value (between $0.003g_E$ and $0.004g_E$); then it increases again. These two behaviors are related to the two distinct shapes that can be observed in the ground states of the system for different values of gravity, as is shown in figure 4 and discussed in section 3: when the strength of gravity is small, the ground state of the system is a *full shell*, while for heavier values of gravity it resembles a *half shell*.

At small strengths of gravity, the condensate is a *full shell* with a higher density at the bottom of the trap. Then, an increase in gravity drags more atoms to the bottom of the trap, which leads to a decrease in the oscillation frequency. When considering dipolar interactions, though, their anisotropic nature compensates for the effect of gravity; as a result, the oscillation frequency becomes almost invariant to small changes in the strength of gravity.

On the other hand, at larger values of g , the system is no longer a *full shell* but a *half shell*, and the oscillation frequency increases as the strength of gravity does. The angular frequency of a mathematical pendulum is related to gravity g and its length l by $\sqrt{g/l}$. For the contact-interacting case, in particular, we can see that for $g > 0.004g_E$, the frequency approaches this behavior as g grows. For comparison, we show in figure 7 the frequency for a pendulum—see dashed line—assuming a fixed length $l \sim r_0 = 3\sqrt{3} \mu\text{m}$. In the dipolar case, the frequency also grows with gravity for $g > 0.005g_E$. However, now the system does not behave like a pendulum. The atoms bounce from the right to the left lobe, but they never cross the south pole of the shell since they can only move around the high-density band, as we show in figure 6 and discuss in the accompanying text. Therefore, the classical pendulum analogy fails in this case.

4.2. Variations in the strength of gravity

In the previous subsection—4.1—, we discussed the dynamics due to variations in the angle of gravity. Here we fix the angle of gravity with the z -axis to zero ($\theta_0 = 0$) and study the system's response to variations in the strength of gravity. As before, we constrain our study to small oscillations, which now translates to small variations in the strength of gravity. We start by preparing the system under a gravity g_0 aligned with the z -axis, and then, at $t = 0$, we change g_0 to g .

In the first part of this subsection, we study in detail two cases: first, when $g, g_0 > 0.004g_E$, so the corresponding ground states resemble a *half shell*, as discussed in section 3; then, we set $g, g_0 < 0.004g_E$, with both values of gravity laying in the regime where the system is still a *full shell*. See table 1 for a summary of the numerical frequencies obtained and figure 8 for some snapshots of the evolution. For these cases, we choose a large change in gravity ($|g - g_0| = 0.001g_E$) to see the system's dynamics well. In the second part, we fix a smaller value of the variation ($|g - g_0| = 0.0001g_E$) to study small oscillations and compare the frequencies of oscillation obtained for different values of the final gravity g —see figure 9.

4.2.1. Particular cases

In the first case of our study, the initial strength of gravity is $g_0 = 0.005g_E$, and the evolution starts when we abruptly increase it to $g = 0.006g_E$. Within these values of the gravity, the ground state of the system resembles a *half shell*, as we already mentioned—see the last row in figure 4. In figures 8(a) and (b), we plot the densities at different times—the snapshots cover a whole period of the oscillation—to show the dynamics of both the contact and dipolar cases. In the contact-interacting case—figure 8(a)—, the atoms are mainly located at the south pole of the shell, occupying a region that shrinks and grows periodically due to the increase in gravity. This behavior resembles a spring that oscillates vertically. Here, though, the movement of the atoms is confined to the surface of the shell. In the dipolar case—figure 8(b)—, instead, the band of maximum density appears below the equatorial region. Then, the sudden change in gravity

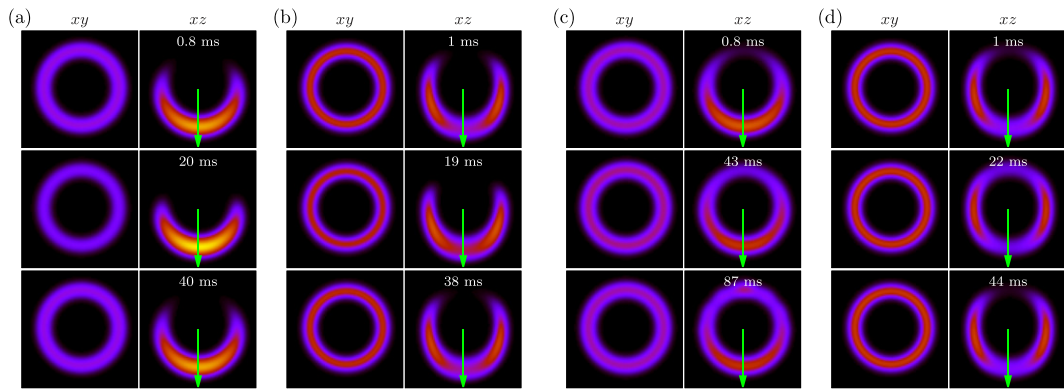


Figure 8. Snapshots of the 2D contour plots of the density in the xy (left) and xz (right) planes at different times of the evolution. Gravity is parallel to the z -axis, and its strength is varied from g_0 to g at $t = 0$. Since the densities in the xz and yz planes are equivalent, we discarded the yz planes. First case, *half shell*: $g_0 = 0.005g_E$ and $g = 0.006g_E$, with (a) contact interactions only and (b) contact and dipolar interactions. Second case, *full shell*: $g_0 = 0.003g_E$ and $g = 0.002g_E$, with (c) contact interactions and (d) dipolar interactions. The dipole moment, $\mu = 10 \mu_B$, is the same for all the cases with dipolar interactions. See summary of cases in table 1. Notice the density accumulation that appears in case (c) at the top of the shell—right panel—of the last row: it is an effect of considering such a large change in gravity, $|g - g_0| = 0.001g_E$. To avoid this, we study the effect of gravity on the oscillation frequency—figure 9—with smaller variations.

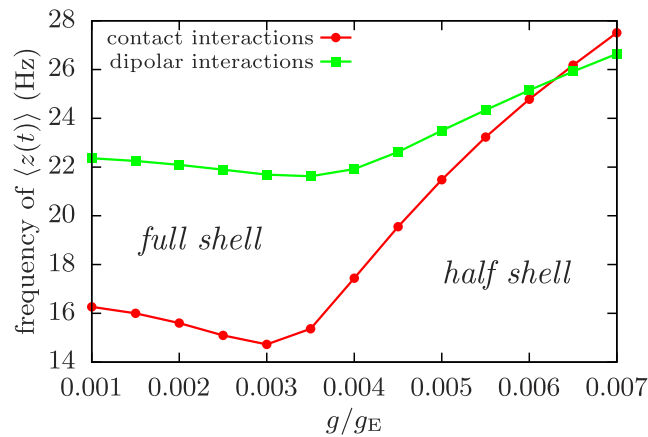


Figure 9. Oscillation frequency of $\langle z(t) \rangle$ as a function of the final gravity g , with initial gravity $g_0 = g + 0.0001g_E$. The red line corresponds to the case with only contact interactions, and the green line to the dipolar interacting BEC. As before, we obtain the oscillation frequency by fitting a sinusoidal function to the numerical data. Lines are added to guide the eye.

causes this band to oscillate along the z direction. Since the gravity is parallel to the z -axis, we study the oscillation frequency of the z coordinate of the center of mass through a sinusoidal fit to the numerical results for $\langle z(t) \rangle$. We obtain a frequency of 24.69 Hz for the contact-interacting BEC and 25.12 Hz for the dipolar one. Unlike in subsection 4.1, here we find that both frequencies are similar.

For the second case, where the gravity is small enough that the system has the shape of a *full shell*, we decrease the initial gravity $g_0 = 0.003g_E$ to $g = 0.002g_E$. The dynamics are very similar to the previous case, as figures 8(c) and (d) shows. In this case, however, the oscillation frequencies of $\langle z(t) \rangle$ for the contact and dipolar BECs are more different; in particular, we find 16.10 Hz for the contact-interacting BEC and 22.20 Hz for the dipolar one.

The oscillations of the center of mass—in both frequency and amplitude—depend on the strength of gravity. These results match those found in subsection 4.1. In contact-interacting BECs, when the gravity is light—and the system is a *full shell*—, we find that the oscillations are slow and broad since the atoms can move around the whole shell. For a heavier gravity, the atoms drop to the bottom of the trap. Then, the amplitude of the oscillation decreases while its frequency increases. The differences found in dipolar BECs come from the anisotropic nature of the dipolar interactions, which counterbalances gravity. The band of maximum density is no longer at the south pole but below the equator. Therefore, compared to the contact-interacting case, the oscillations change much less when different strengths are studied.

4.2.2. The role of gravity

Finally, we study the dynamics of small oscillations induced by a variation in the strength of gravity and how these results differ depending on whether the value of gravity is relatively small—and the ground state resembles a *full shell*—or large—when the system becomes a *half shell*. We choose $|g - g_0| = 0.0001g_E$ to ensure small oscillations. For these values of $g - g_0$, the results we obtain for a given g are equivalent—in frequency and amplitude—either if $g > g_0$ or $g < g_0$. Therefore, we define from now on g_0 such that $g_0 = g + 0.0001g_E$.

In figure 9, we plot the frequency of oscillation of $\langle z(t) \rangle$ as a function of the final gravity g . The results resemble those from figure 7. The frequency increases with the final gravity for large values of the gravity—when the system is a *half shell*—, while it decreases with the final gravity for smaller values—when the system resembles a *full shell*. Since the dipolar interaction compensates for the gravity, the effect of the variation in strength is more noticeable in the contact-interacting BEC than in the dipolar BEC—as in subsection 4.1—for small final gravities.

Comparing the results obtained for the contact-interacting BEC either with changes in strength—see figure 9, red line—and orientation—see figure 7, red line—, we can see that the frequencies lie within a similar range of values in both cases. The frequencies we obtain now for the dipolar BEC, however, are faster. This increase in frequency is an effect of the anisotropy of the dipolar interactions. In the first scenario—see figure 7, green line—the center of mass moves mainly along the x -axis, and all the dipoles point toward the z -axis. Then, an atom that moves in that direction feels a net repulsive interaction from its neighbors, which reduces the frequency of oscillation. In the second scenario—see figure 9, green line—the center of mass moves instead around the z -axis. Since the resulting interaction between dipoles along the direction of motion is attractive and twice as large as in the previous case, the frequency of the oscillation is much larger.

5. Extension to other systems

Writing the time-dependent GPE in dimensionless units in the usual way—using the oscillator length $a_{ho} = \sqrt{\hbar/(m\omega)}$ as the unit length, ω^{-1} as the unit time, and $\hbar\omega$ as the unit energy—one can define three dimensionless constants, which are the coefficients of the gravitational, contact-interacting and dipolar terms of the dimensionless equation: $G = g/(a_{ho}\omega^2)$, $C = 4\pi Na_s/(a_{ho})$, and $D = Nm\mu_0\mu^2/(4\pi\hbar^2 a_{ho})$. We also define the dimensionless radius $\xi_0 = r_0/a_{ho}$. Then, the system can be scaled in terms of these dimensionless constants. In our configuration, the numerical values are: $G = 31.62\tilde{g}$, $C = 1016.46$, $D = 268.27$ and $\xi_0 = 6.62$, where we introduce the gravity in units of the terrestrial gravity $\tilde{g} = g/g_E$ for convenience, and we also fix $\epsilon_{dd} = 1.11$. With these values, one can translate the same physics to another set of parameters that are experimentally accessible in ^{52}Cr [39, 40], ^{164}Dy [41], or ^{168}Er [42].

For example, we consider a condensate of ^{168}Er with $N = 10^4$ atoms, which has a dipolar moment of $7\mu_B$. The configuration proposed with our dimensionless constants can be obtained with a scattering length $a_s \sim 60a_0$ and a trap frequency $\omega \sim 2\pi \times 390$ Hz. Then, the radius of the resulting shell is $r_0 \sim 2.6\mu\text{m}$, and the range of valid gravities—large enough that its effect is noticeable but does not destroy the system—is, in this case, between $0.008g_E$ and $0.050g_E$.

More generally, we can use these dimensionless constants and state that our study can be extended to values of gravity between $0.003g_E$ and $0.023g_E$ in units of $a_{ho}\omega^2$. Consequently, it is possible to explore a different range of gravities by considering another set of parameters for the system.

For instance, going back to the system we consider in this work (^{164}Dy), one could study a range of gravities closer to microgravity by increasing the number of atoms and the radius of the shell, and reducing the frequency of the trap. In particular, with $N \sim 3 \times 10^4$, $\omega \sim 2\pi \times 11$ Hz and $r_0 \sim 15.6\mu\text{m}$, the range of valid gravities lies between $4 \times 10^{-5}g_E$ and $26 \times 10^{-5}g_E$.

6. Summary and outlook

In the present work, we have studied the statics and dynamics of shell-shaped condensates with contact and dipolar interactions in the presence of a small gravity. We have constrained our study to gravity values above microgravity—and thus non-negligible—and below terrestrial gravity, which destroys shells.

First of all, we have analyzed the ground states of the system in three cases: without gravitational sag, with gravity parallel to the z -axis—which is the polarization direction we considered for dipolar BECs—, and with a small gravity misaligned with the z -axis and contained in the xz plane. We have discussed the effect of the dipolar interactions in either of the three cases. Next, we have done a more general analysis of the ground states to examine the effect of gravity's strength. Observing the shape that the system displays,

we have defined two regimes: a *full shell* for small gravities and a *half shell* for larger gravities. These two regimes play a relevant role in the dynamical behavior of the system.

Later, we have studied the dynamics of small oscillations due to changes in the orientation and strength of gravity. For each of those two scenarios, we have studied two particular cases—comparing the *full shell* and *half shell* regimes—, and we have analyzed, more generally, the effect that gravity has in the behavior of the oscillations when the variation—in angle or strength—is fixed and very small. With this, we have seen how the two static regimes translate into two distinct dynamical behaviors: the oscillation frequency increases with gravity for large values of gravity (*half shell*) while it decreases for smaller values (*full shell*). Additionally, we have compared the results obtained for contact-interacting BECs with those obtained for their dipolar-interacting counterparts. We have discussed that dynamics due to changes in angle or strength are equivalent in the contact BECs, but the dynamical behavior differs in dipolar BECs due to the anisotropic nature of their interactions, which counterbalances the effect of gravity. We have shown that even though the dipolar interaction adds a privileged direction to the one already defined by gravity, the resulting shells with gravitational sag and dipolar interactions present a configuration that, compared to contact-interacting one, is less sensitive to misalignments and perturbations in the gravitational sag.

Finally, we have extended our study to other systems, and we have seen that the range of valid gravities depends on the particular system in consideration. Therefore, one could choose a set of parameters—namely the mass, the frequency, and the number of atoms—such that the gravitational effects become non-negligible but non-destructive at much smaller gravities than the ones we have studied here. We want to stress that the physics of shell-shaped condensates under gravitational sag is not limited to dysprosium BECs, as we have considered. It can also be exported to other condensates with controllable contact interactions, either with and without dipolar interactions.

The atomic cloud in this system is not only sensitive to changes in its orientation, but it is also sensitive to small gravitational variations, either in its direction or strength. However, one cannot discern directly from our results—especially in the contact-interacting case—if the cause of the oscillations is a change in gravity's orientation or its strength. Studying instead a more simplistic configuration, such as a ring-shaped BEC, may shed some light on how to discriminate between these two situations. In any case, these findings could pave the way to the experimental realization of a gravity or accelerometer sensor intended for small gravity conditions. Monitoring gravity and its changes from space in satellite missions—see [43] and references therein—is another possible application of this system. To conclude, we want to point out the experimental feasibility of the proposed system. We have used values for the experimental parameters that are currently available in laboratories.

Due to the complexity of the 3D dynamics of this system, a more exhaustive theoretical analysis is beyond the scope of this paper. However, we consider that a new proposal of a gravity sensor with restricted low dimensional dynamics may provide a more analytical insight of the system. Discussing other configurations such as toroidal BECs under gravity conditions will be addressed elsewhere.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

ORCID iDs

Maria Arazo  <https://orcid.org/0000-0001-5413-423X>

Montserrat Guilleumas  <https://orcid.org/0000-0003-2011-1480>

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