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Keywords: Auditing, corruption, information design, regulation

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Frequent Audits and Honest Audits*

Jacopo Bizzotto[†] Alessandro De Chiara[‡]

February 15, 2022

Abstract

A regulator hires an auditor to inspect a firm. Audits serve two purposes: to detect violations and to motivate the firm to invest in compliance. Auditor and firm can collude to hide violations. Honest audits require sufficient monetary incentives for the auditor, and more frequent audits call for larger incentives. We link the optimal audit frequency to the budget constraint faced by the regulator, and to the firm's bargaining power in the collusive agreement. We show that (i) the optimal audit frequency need not be monotonic in the regulator's budget size, (ii) tolerating collusion can foster ex-ante investment, and (iii) a regulator that enjoys more flexibility in designing the auditor's compensation scheme might be less willing to deter corruption.

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1 Introduction

The threat of an audit can encourage compliance of companies and tax payers as well as accountability of public officials. Unfortunately, at times, audited subjects deal with this threat by bribing the auditor. Corruption of auditors is ubiquitous, but arguably most common in low income countries, where limited fiscal efficiency often results in relatively low salaries for public auditors.[1] In this paper, we study the optimal design of audits and, in the spirit of Laffont (2005) and Estache and Wren-Lewis (2009), we focus on situations where the resources available to the regulator are limited. We consider, in particular, (i) how infrequent audits can deter corruption, (ii) under which conditions tolerating corruption can be desirable, and (iii) whether frequent audits are an exclusive privilege of deep-pocketed regulators.

We model a regulator hiring an auditor to verify whether a firm complies with a regulation. The regulator commits at the onset to the frequency with which an audit occurs. The firm can invest to increase the odds of being compliant. After learning whether a violation has occurred, but before the audit (possibly) occurs, the firm has the opportunity to bribe the auditor. The amount of the bribe is bargained between firm and auditor. If a bribe is paid, the auditor commits to overlook a violation. At times, the bribe goes unnoticed; the regulator then pays a salary to the auditor and requires the firm to implement a costly remedy in case the audit is conducted and reveals a violation. If instead the regulator discovers the bribe, then the auditor has to forgo his salary and the firm has to implement the remedy. The amount of resources available to the regulator is proxyed here by the size of the auditor's salary. This amount is, at least initially, exogenously determined.

Our main result amounts to show that the optimal audit frequency is a non-monotonic function of the auditor's salary. A regulator should strategically lower the audit frequency, that is, hold random audits, only for intermediate sizes of the salary. The intuition goes as follows. The salary determines the auditor's opportunity cost of accepting a bribe. The audit frequency determines the non-compliant firm's benefit of paying a bribe. If the salary is not too large, the regulator faces a choice: lowering the audit frequency so as to discourage collusion, or requiring frequent audits and tolerating collusion. We show that, if the salary is sufficiently

[1] Audits in low income countries have been the focus of a large recent literature. Auditing of public officials is studied in Olken (2007), Bobonis, Cámara Fuertes and Schwabe (2016), and Avis, Ferraz and Finan (2018). For tax compliance audits see, among others, Pomeranz (2015), Castro and Scartascini (2015), Kettle, Hernandez, Ruda and Sanders (2016), Bérgolo, Ceni, Cruces, Giaccobasso and Perez-Truglia (2017), and Shimeles, Gurara and Woldeyes (2017). For audits evaluating the compliance of private companies to various regulations, see Duflo, Greenstone, Pande and Ryan (2013) and Gonzalez Lira and Mobarak (2021).

low, the regulator tolerates collusion while for intermediate salary she discourages collusion by lowering the audit frequency. For a high enough salary, the choice is moot: collusion is avoided regardless of the audit frequency. Optimal audit frequency is therefore non-monotonic in the amount of resources available to the regulator.

Why would the regulator tolerate collusion when she can avoid it? The regulator has two goals: (i) to screen the firm for violations *ex-post*, and (ii) to encourage investment in compliance *ex-ante*. Collusion, as one would expect, results in poor screening. Yet, a regulator with limited resources who sets up to deter collusion will also end up deterring investment. This last point requires further explanation. In order to deter collusion, a cash-constrained regulator has to commit to infrequent audits, which in turn, discourage investment, as the firm anticipates that a violation might go undetected. If instead the regulator gives up on collusion deterrence, audits can be frequent. The threat of frequent audits allows the auditor to bargain the price of the bribe up. The firm incurs the cost of a bribe only when in violation, so expensive bribes effectively foster investment in compliance. A regulator with sufficiently limited resources will then ensure fewer violations by tolerating corruption rather than by requesting honest, but sporadic, audits.

Our main result continues to hold if the regulator can choose the auditor's salary. In this case, we proxy the amount of resources available to the regulator with the salary's shadow cost. The idea is that a regulator with fewer resources attaches a higher opportunity cost to each unit of salary. Tolerating collusion, we show, is optimal only if the shadow cost is high. The frequency of the audit, in turn, should be reduced only for intermediate shadow costs, i.e., when the cost is sufficiently low to justify deterring collusion, yet sufficiently high to call for a modest salary. A modest salary, we already established, must be matched with infrequent audits if collusion is to be deterred.

The process by which firm and auditor come to set a price for the bribe is an important component of our model. Many factors might influence the bargaining strength of firm and auditor. We model some factors explicitly, such as salary and audit frequency, but we also account for generic "other factors". Perhaps the most obvious example is the extent to which auditors compete. A monopolistic auditor should be expected to hold a stronger position in the bargaining process then an auditor facing competition. The degree of competition among auditors could influence the bribe size and, as a consequence, also the optimal audit frequency. A comparative statics exercise shows that whatever weakens the auditor bargaining position makes collusion less desirable for the regulator, and calls for infrequent audits.

In our model(s), the auditor's salary is independent of the audit report. A natural question is what would happen if compensation conditional on the report were viable. It turns out that also in this case a regulator faced with a high shadow cost would tolerate collusion. Report-dependent compensation in fact makes tolerating collusion quite an appealing option: the regulator can induce the second-best level of investment at no cost. Tolerating collusion becomes so appealing to the regulator that, for some parameter combinations, collusion is tolerated with report-contingent compensation, but deterred if the salary cannot depend on the report. A consequence of this is that the intermediate region of the shadow cost for which, in our model(s) it was optimal to recommend infrequent audits disappears: with report-dependent compensation it is optimal to audit the firm with certainty.

The rest of the paper is structured as follows. In the next subsection we discuss the related literature. Section 2 presents the baseline model. The analysis can be found in Section 3. In Section 4 we modify the baseline model, first to make the salary endogenous, then to allow report-dependent compensation. Section 5 concludes.

Related literature

Our work shares some features with the literature on collusion in principal-supervisor-agent models that stemmed from the seminal work of Tirole (1986).[2] Within this literature, our work is most closely related to Asseyer (2020). Asseyer (2020) considers, like us, the problem of a regulator that designs the signal available to the supervisor.[3] Asseyer's paper belongs more firmly than ours in this mechanism-design literature in which the principal offers an actual contract to the agent, whereas we focus on incomplete contracts. Another relevant difference is that the type of the agent is exogenous in Asseyer (2020). This assumption is typical in this literature, with some notable exceptions, such as Hiriart, Martimort and Pouyet (2010), Khalil, Lawarrée and Yun (2010), and Ortner and Chassang (2018). While we consider random audits as a way to curb collusion, Ortner and Chassang (2018) focus on the provision of random, and privately known, compensation schemes to the auditor to prevent corruption. Broadly speaking, while random audits reduce the gains from collusion, random compensation makes it harder to share those gains. Our work is also closely related to Khalil *et al.* (2010) in that they show that collusion can help in providing the right incentives to invest ex ante as it acts as a penalty for bad quality. We reinforce this observation by showing that collusion can in fact provide more incentives than honest auditing does. Finally, Che, Huang and Zhang

[2]For a recent review of this literature, see Burguet, Ganuza and Montalvo (2018).

[3]The consequence of different precision of the supervisor's signal has also been considered in Faure-Grimaud, Laffont and Martimort (2003).

(2021) relate the choice of tolerating collusion with the accuracy of the audit. However, in their model, test accuracy is exogenous, and collusion is desirable as it allows the agent to correct a wrong signal.

Beyond the principal-supervisor-agent literature, Perez-Richet and Skreta (2021) and Pollrich and Wagner (2016) have shown that lowering the signal precision can reduce the applicant's incentives to falsify a test. Perez-Richet and Skreta (2021) study the optimal test design by a regulator under the threat of direct falsification by an applicant. In their model, the relationship between the regulator and the applicant is not mediated by a (corruptible) auditor. In Pollrich and Wagner (2016) instead, collusion takes place between a certifier (which plays the role of the auditor in our model) and a sequence of applicants. In their model there is no benevolent regulator who designs the auditing policy and pays transfers to the certifier. Relatedly, Okat (2016) and Gonzalez Lira and Mobarak (2021) make the point that random tests might be more informative that regular ones as the latter make it easy for the agent to learn how to falsify a test. Gonzalez Lira and Mobarak (2021) also provide evidence that random audits of Chilean fish vendors are more effective in finding violations than regular ones. Our results provide a complementary rationale for this observation.[4]

Finally, our regulator faces a simple information design problem, in the spirit of those described in the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011 and Rayo and Segal, 2010). Within this literature, we are mostly related to those models where the design of the test affects the distribution of the state being tested (Farragut and Rodina, 2017, Boleslavsky and Kim, 2020, Saeedi and Shourideh, 2020, Zapechelnyuk, 2020, and Bizzotto and Vigier, 2021).

2 The Model

A regulator (she) hires an auditor (he) to evaluate whether a firm (it) operates in violation of some regulation. The state $\omega \in \{0,1\}$ describes the firm's status: either compliant, $\omega = 0$, or in violation, $\omega = 1$. The firm privately selects $p \in [0,1]$, at a a cost $p^2/2$, so that:[5]

$$Pr(\omega = 0) = p.$$

[4] Random audits are also an equilibrium outcome if the regulator is unable to commit to an audit frequency, as shown, for instance, in Mookherjee and Png (1989), Khalil (1997), Khalil and Lawarrée (2006), and Finkle and Shin (2007).

[5] With an abuse of notation, here we refer to a random variable and to its realization with the same symbol.

While only the firm directly observes its status, an audit results in a potentially informative report $r \in \{0,1\}$. At the onset of the game the regulator selects an audit frequency (or, probability) $\pi \in [0,1]$. In the absence of collusion (more on this below), the frequency of the audit determines its informativeness:

$$Pr(r=\omega|\omega=1)=\pi, \text{ while } Pr(r=\omega|\omega=0)=1,$$

so, if the audit occurs, then $r=\omega$, while if the audit does not occur, then $r=0$. Before the audit, the firm can bribe the auditor. For expositional reasons, we do not model the strategic decision to offer/request a bribe. We assume instead that with probability $\alpha \in [0,1]$, the auditor selects a bribe amount, denoted $b_{a(uditor)} \in \mathbb{R}$, and the firm can only accept or reject it. With probability $1-\alpha$ instead, the firm selects an amount $b_{f(irm)} \in \mathbb{R}$, and the auditor can accept or reject it.[6] We say that collusion occurs whenever the counter-party agrees to the proposed amount. In this case, (i) the firm pays the bribe to the auditor, (ii) the auditor reports $r=0$, and (iii) with some probability $\epsilon \in (0,1)$ the regulator detects the collusive agreement.

If $r=0$ and no collusion is detected, the firm earns a profit of $k \in (0,1)$. If instead either $r=0$ and collusion is detected, or $r=1$, then the firm must provide a remedy. The remedy's cost is equal to the entire firm's profit, k. If collusion is detected, the auditor pays a fine equal to his salary $t \geq 0$.

The timeline is as follows:

1. The regulator publicly announces an audit frequency π;
2. The firm privately chooses p and then observes the realization of ω;
3. Nature determines who sets the bribe; the bribe is set, and it is accepted or rejected;
4. The regulator observes the report r;
5. If a bribe has been paid, it is detected with probability ϵ;
6. Payoffs realize.

In the absence of a remedy, a violation causes a non-verifiable loss of size $H>0$ for the regulator. If the firm takes a remedy, the size of the loss is reduced to $H-\Delta$, where $H \geq$

[6]The choice to not model the strategic decision to offer/request a bribe is without consequences, as a player can always set an amount such that the other player prefers to avoid bribery.

$\Delta > k$. The regulator internalizes costs and benefits of firm and auditor, so her utility can be interpreted as a measure of social welfare. All players are risk-neutral. If (i) $r = 0$, (ii) bribery is not detected, and (iii) player $i \in \{a(uditor), f(irm)\}$ gets to select the bribe amount, then payoffs are:

$$u_{r(egulator)} = k - H \cdot \omega - \frac{p^2}{2};$$
$$u_a = t + b_i \cdot \mathbb{1}_b;$$
$$u_f = k - b_i \cdot \mathbb{1}_b - \frac{p^2}{2};$$

where $\mathbb{1}_b = 1$ if the bribe is paid and $\mathbb{1}_b = 0$ otherwise. If instead (i) either $r = 1$, or bribery is detected, and (ii) player i selects the bribe amount, then:

$$u_r = (\Delta - H) \cdot \omega - \frac{p^2}{2};$$
$$u_a = t \cdot (1 - \mathbb{1}_b) + b_i \cdot \mathbb{1}_b;$$
$$u_f = -b_i \cdot \mathbb{1}_b - \frac{p^2}{2}.$$

We focus on perfect Bayesian equilibria of the game, or equilibria for short. The strategy of the regulator selects a frequency $\pi \in [0, 1]$. The firm's strategy maps: (i) any π into an investment p and (ii) any $\{\pi, \omega\}$ into an amount b_f, and any $\{\pi, \omega, b_a\}$ into a choice to *accept* or *reject*. The auditor's strategy maps any π into an amount b_a, and any $\{\pi, b_f\}$ into a choice to *accept* or *reject*.

2.1 Discussion of the Assumptions.

Audit Frequency. The audit frequency π could alternatively be interpreted as the probability that an audit finds evidence of a violation (conditional on the firm being in violation). Indeed, when conducting an audit, an auditor seeks damning evidence. For instance, an auditor might look for inconsistencies in the firm's income statements or its risk projections, or might check whether industrial facilities pollute beyond the limits. The regulator can affect π in various ways. She can, for instance, determine the amount of resources available to the auditor, or can assign the audit task to a more or less experienced auditor. Regardless of the interpretation, one could reasonably argue that a larger π should result in a larger cost for the auditor, the regulator, or both. We set this cost equal to zero, so as to make more forcefully the point that limiting the informativeness of an audit can be optimal *for strategic reasons*.

Enforceability of side-agreements. In line with the agency theory of corruption, we assume that the collusive agreement between the auditor and the firm is enforceable. Several mechanisms can be invoked here, such as reciprocity, reputation or reliable intermediaries (for a discussion, see Tirole, 1992).

Institutional constraints. We assume that the salary is independent of the auditor's report. This is reasonable as, typically, the legislative branch of the government allocates money to regulatory agencies, and the latter have limited latitude in setting their employees' wages. For instance, the U.S. Congress sets the budget of federal agencies through the federal appropriations process. Similarly, in developing countries, public agencies' budgets require government approval and public officials' salaries follows civil service pay scales (see Estache and Wren-Lewis, 2009 and literature cited therein). We explore the role played by institutional constraints in Section 4.

Violation, remedy, and fines. In the model, a violation causes a welfare loss. This loss could capture the risk of environmental damages, workplace accidents or financial turmoil. The loss can be mitigated by way of some costly remedy. Each dollar spent by a non-compliant firm on this remedy reduces the loss by a factor of $\Delta/k > 1$. As a result, it is socially desirable that a non-compliant firm spends the entire profit k to remedy the loss. We assume limited liability of firm and auditor: the firm cannot be punished beyond the loss of profits, and the auditor cannot be punished beyond the loss of wages. This seems reasonable in most circumstances, and perhaps the more so in the context of a low income country.

Firm's compliance. The firm's investment p captures the resources the firm devotes to internal monitoring so as to avoid employee misconduct or mistakes that can jeopardize the firm's financial security or its compliance with regulation.

Ex-post bribery detection. The probability of detection ϵ can be interpreted as an additional signal. This signal could be an event revealing that the firm has breached a pertinent rule, e.g. a workplace accident, or unexpected environmental damages. Else, the signal might come from an investigation uncovering malfeasance, or from a second auditor disclosing contradicting evidence, as in the models of Kofman and Lawarrée (1993) and Khalil and Lawarrée

(2006).

3 Analysis

We first consider the way firm and auditor bargain around the bribe. If $\omega = 1$, then the firm prefers collusion as long as the bribe amount is not larger than $(\pi - \epsilon)k$, as collusion reduces the odds of a costly remedy from π to ϵ. If instead $\omega = 0$, then no positive bribe amount can induce the firm to collude. The auditor, in turn, considers only bribes not smaller than the opportunity cost of collusion, given by $\epsilon \cdot t$. It is then easy to establish that if in equilibrium collusion never occurs, than it must be the case that $\epsilon \cdot t \geq (\pi - \epsilon)k$, that is, the audit probability cannot exceed $\epsilon(t+k)/k$. If instead collusion occurs, then $\pi \geq \epsilon(t+k)/k$ and collusion occurs only if $\omega = 1$. The following lemma records these observations, as well as the size of the bribe paid in equilibrium.[7]

Lemma 1. *In any equilibrium in which collusion never occurs, the regulator selects a probability $\pi \leq \epsilon(t+k)/k$. In any other equilibrium: (i) $\pi \geq (k+t)\epsilon/k$, (ii) collusion occurs whenever $\omega = 1$, (iii) the auditor demands $b_a = (\pi - \epsilon)k$ whereas the firm offers to pay $b_f = \epsilon \cdot t$ when $\omega = 1$.*

We now consider separately candidate equilibria with and without collusion.

Candidate equilibria without collusion. In any equilibrium in which collusion never occurs, the firm selects $p = p^N(\pi)$, where

$$\begin{aligned} p^N(\pi) &\equiv \arg\max_{p\in[0,1]} \quad \mathbb{E}(u_f), \\ &= \arg\max_{p} \quad p\cdot k + (1-p)(1-\pi)k - \frac{p^2}{2}, \\ &= k\cdot\pi. \end{aligned}$$

A higher audit probability makes it more likely that a firm in violation has to incur a remedy, thus justifying a larger investment in compliance. In any such equilibrium, the regulator sets the audit probability π^N so as to maximize her expected payoff, subject to a "no-collusion"

[7] We are implicitly ruling out equilibria in which, when indifferent, firm and auditor follow a mixed strategy. This will turn out to be without loss of generality.

constraint familiar from Lemma 1:

$$\pi^N = \arg\max_{\pi\in[0,1]} \quad k-(1-p^N(\pi))\left(H-\pi^N(\Delta-k)\right)-\frac{(p^N(\pi))^2}{2},$$
$$\text{s.t.: } \pi \leq \frac{(k+t)\epsilon}{k}.$$

As the maximand is increasing in π, then the regulator sets the highest π that satisfies all constraints, as recorded in the next lemma.

Lemma 2. *In any equilibrium without collusion, the regulator announces* $\pi = \min\{(k+t)\epsilon/k, 1\}$ *and the firm selects* $p = \min\{(k+t)\epsilon, k\}$.

A higher probability of ex-post detection, ϵ, increases here audit screening quality, in line with the seminal work of Becker and Stigler (1974), and with the empirical findings of Di Tella and Schargrodsky (2003).

Candidate equilibria with collusion. Lemma 1 implies that in any equilibrium in which collusion occurs whenever $\omega = 1$, the firm selects $p = p^C(\pi)$, where

$$\begin{aligned} p^C(\pi) &\equiv \arg\max_{p\in[0,1]} \quad p\cdot k+(1-p)\left((1-\epsilon)\cdot k-\alpha(\pi-\epsilon)k-(1-\alpha)\epsilon\cdot t\right)-\frac{p^2}{2}, \\ &= \epsilon\cdot k+\alpha(\pi-\epsilon)k+(1-\alpha)\epsilon\cdot t. \end{aligned}$$

Also in this case a higher audit probability results in a larger investment in compliance. The logic goes as follows. Larger π results in larger b_a (see Lemma 1). The marginal benefit of investment thus increases, as the firm pays the bribe only if $\omega = 1$.

In any equilibrium with collusion, the regulator selects the audit probability π^C that maximizes her expected payoff, subject to a "collusion" constraint.

$$\pi^C = \arg\max_{\pi\in[0,1]} \quad k-(1-p^C(\pi))\left(H-\epsilon(\Delta-k)\right)-\frac{(p^C(\pi))^2}{2},$$
$$\text{subject to } \pi \geq \frac{(k+t)\epsilon}{k}.$$

A $\pi \in [0,1]$ that satisfies $\pi \geq (k+t)\epsilon/k$ exists only if $k \geq t\cdot\epsilon/(1-\epsilon)$. When this condition is violated, no bribe amount small enough to suit the firm can be large enough to suit the auditor. When instead the condition holds, $\pi^C = 1$. The next lemma records these remarks.

Lemma 3. *In any equilibrium in which collusion occurs with positive probability, the regulator announces* $\pi = 1$ *and the firm selects* $p = \alpha k + (1-\alpha)(t+k)\epsilon$*. If* $t > (1-\epsilon)k/\epsilon$*, in equilibrium collusion does not occur.*

The equilibrium. We just established that for sufficiently large t, in equilibrium collusion does not occur. As the regulator is the first mover, for other parameter values, the equilibrium corresponds to the candidate equilibrium that ensures the highest expected payoff to the regulator. The proposition illustrates.

Proposition 1. *There exists a threshold* $\tilde{t} \in [0, (1-\epsilon)k/\epsilon)$*, such that, in equilibrium:*

- *if* $t < \tilde{t}$*, collusion occurs and the firm is audited with probability 1;*
- *if* $\tilde{t} < t < (1-\epsilon)k/\epsilon$*, collusion is avoided and the firm is audited with probability* $(k + t)\epsilon/k < 1$*;*
- *if* $t \geq (1-\epsilon)k/\epsilon$*, collusion is avoided and the firm is audited with probability* 1.

The threshold $\tilde{t}$ *is strictly increasing in* α*.*

The intuition goes as follows. If collusion cannot occur, i.e. if $t \leq (1-\epsilon)k/\epsilon$, auditing with probability 1 ensures efficient screening and efficient investment in compliance. To illustrate the intuition beyond the remaining results, we compare the utility of the regulator in the candidate equilibrium with collusion,

$$E(u_r|\pi^C) = k - (1 - p^C(\pi^C))(H - \epsilon(\Delta - k)) - \frac{(p^C(\pi^C))^2}{2},$$

with her utility in the candidate equilibrium without collusion:

$$E(u_r|\pi^N) = k - (1 - p^N(\pi^N))\left(H - \pi^N(\Delta - k)\right) - \frac{(p^N(\pi^N))^2}{2}.$$

The regulator's utility depend on (i) screening quality and (ii) the firm's investment. Collusion is associated with poorer screening. Specifically, collusion results in a violation being spotted with probability ϵ, while without collusion a violation is spotted with probability $\pi^N \geq \epsilon$. At the same time, collusion is associated with larger levels of investment. This might seem counter-intuitive. Yet, a firm in violation is better off being audited with probability $\pi^N < 1$, rather than paying a bribe to avoid an audit occurring with probability $\pi^C = 1$. Investment has thus a higher marginal benefit in the candidate equilibrium with collusion, and, as a result, $p^C(\pi^C) > p^N(\pi^N)$ for any $\alpha > 0$.[8]

The regulator effectively faces a trade-off between motivating ex-ante investment and ensuring ex-post screening. If t is small, avoiding collusion brings little benefit in terms of screening ($\lim_{t\to 0} \pi^N = \epsilon$). If t is large, collusion brings little benefit in terms of incentives to invest ($\lim_{t\to(1-\epsilon)k/\epsilon} p^N(\pi^N) = p^C(\pi^C)$). Collusion is therefore tolerated for sufficiently small t, and deterred otherwise. This, in turn, makes the optimal audit frequency a non-monotone function of t: if the regulator faces tight budgetary constraints (i.e., t is small) and collusion is tolerated, audits should be frequent; if budgetary constraints are loose (i.e., t is large) collusion can be deterred while mandating frequent audits. In the intermediate case, collusion is deterred, but this requires somewhat infrequent audits. This result is illustrated in Figure 1, in which the solid red line describes the equilibrium auditing probability as a function of t. As long as $t < \tilde{t}$, collusion is tolerated and $\pi = 1$. For $t = \tilde{t}$, deterring collusion becomes optimal and the audit probability drops below 1. For $t > \tilde{t}$, audit frequency increases in t. In the same figure, we also illustrate the firm's and the auditor's expected payoffs as functions of t (the densely dashed blue line denoted F and the loosely dashed green line denoted A, respectively). Interestingly, an increase in t can make the firm better off, and the auditor worse off. The intuition is that as t becomes larger than $\tilde{t}$, the regulator switches to audits; less frequent audits make the firm better off, and deter collusion, ultimately hurting the auditor.

We relate next the optimal audit frequency to the relative bargaining power α. From a policy perspective, this exercise is relevant because a regulator can influence the relative bargaining power of auditors and firms, as we discuss in the next subsection. From a theory perspective, this exercise is interesting because the audit frequency is set with an eye to the size of the gains from collusion, while the relative bargaining power determines the way firm and auditor split these gains.

The greater is the bargaining power of the auditor, the higher is the expected bribe, and the stronger is the incentive to invest in compliance if collusion is anticipated. Thus, the higher α the stronger the case for tolerating collusion. Accordingly, (i) the threshold $\tilde{t}$ is an increasing function of α and (ii) the equilibrium auditing probability is non decreasing in α. The first result is illustrated in Figure 2. The figure shows the regulator's utility under alternative policies, as a function of t. The solid blue line describes the regulator's utility in the candidate equilibrium without collusion. This utility does not depend on α. The dashed

[8] $p^C(\pi^C) = p^C(\pi^N)$ for $\alpha = 0$.

red lines describe the utility in the candidate equilibrium with collusion, for different values of α. When $\alpha = 0$, the regulator is always better off without collusion. As α takes higher values, the threshold value of t above which the regulator prefers to deter collusion increases. Moreover, an increase in the auditor's bargaining power makes the regulator weakly better off.

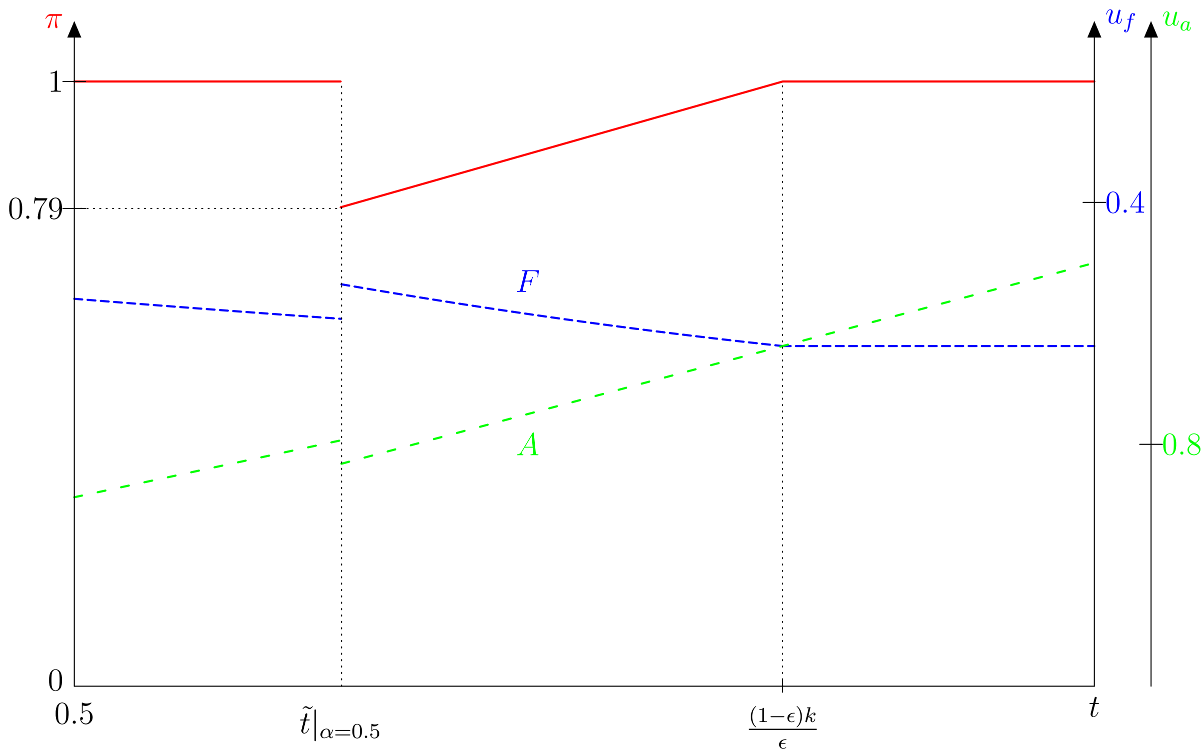

Figure 1: Auditing probability, firm's and auditor's payoff for $\epsilon = 0.4$, $k = 0.75$, $H = 1$, $\Delta = 0.9$, and $\alpha = 0.5$.

Policy implications. Our results imply that auditing policies should be tailored to the institutional framework of the countries where they are implemented. As stressed by Estache and Wren-Lewis (2009), the regulators' limited fiscal efficiency and capacity in less developed countries make it impractical to apply the same regulatory policies as in developed countries. Seen through the lens of our model, these institutional limitations translate into a smaller

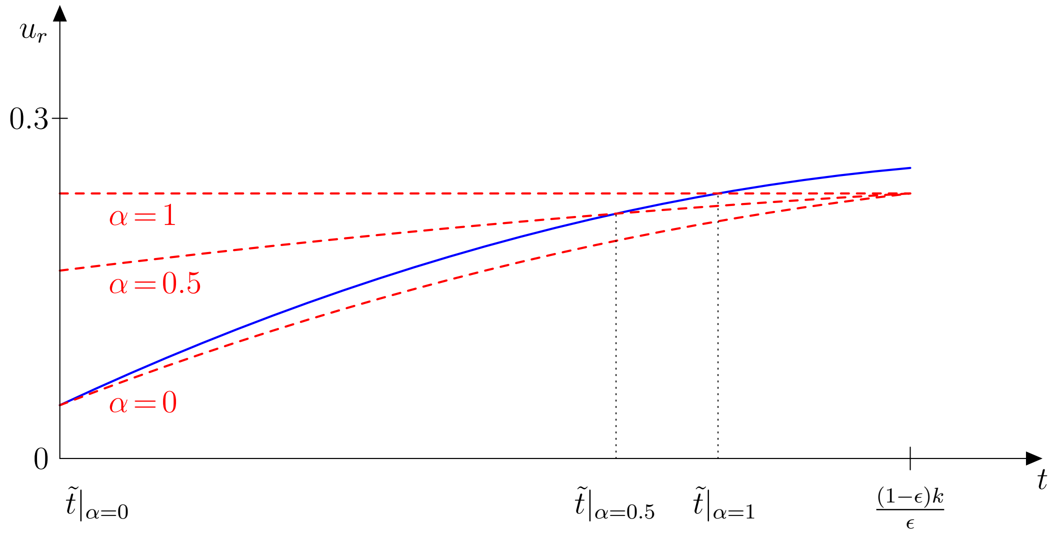

FIGURE 2: REGULATOR'S UTILITY FOR $\epsilon = 0.4$, $k = 0.75$, $H = 1$, AND $\Delta = 0.9$.

budget available to the regulator and, hence, a smaller salary t. Our policy implications are that, in this context, honest audits must be random and that, if collusion is to be tolerated, there is a high return from increasing the regulator's bargaining power, as discussed next.

In practice the regulator can influence the distribution of bargaining power between firm and auditor. One way to do so is to determine how many auditors are entitled to assess the firm's compliance.[9] Competition between auditors (or agencies) is likely to reduce their bargaining power vis-à-vis a firm. As first pointed out by Rose-Ackerman (2013), and extensively discussed by Shleifer and Vishny (1993), officials with overlapping jurisdiction can reduce the bribe to zero.

According to our model, if corruption is tolerated, low bribes should result in frequent violations. This prediction is in line with recent evidence. Burgess, Hansen, Olken, Potapov and Sieber (2012) find that, in the context of the Indonesian logging industry, the higher the number of political jurisdictions from which a firm can (illegally) obtain logging permits, the more extensive the deforestation and the lower the price of timber.[10]

[9]There are several examples of auditors or agencies with overlapping jurisdiction. For instance, *passporting* enables a company that obtains authorization to operate in a state of the European Economic Area (EEA) to do business in any other EEA state without the need for further authorization from that country. This effectively implies that companies can choose which national agency to be audited by. This is especially relevant for financial institutions. In the context of college admission tests, most U.S. colleges accept results from the SAT or the ACT standardized tests. Moreover, in each state, several test centers are authorized to administer either test.

[10]The relationship between bureaucratic competition and corruption has received some attention in the

4 Endogenous Compensation

In this section, we modify the baseline model to let the regulator choose the auditor's salary. In Subsection 4.1, we relax the assumption of a fixed budget and let the regulator set the compensation of the auditor. In Subsection 4.2, we let the compensation depend on the report. A unit of compensation costs $1+\lambda$ to the regulator, where $\lambda \geq 0$. The parameter λ measures the shadow cost of public funds. This cost could come from inefficiencies in raising public funds due to distortionary tax collection (e.g., see Laffont and Tirole, 1993).[11]

4.1 Report-Independent Compensation

We modify here the baseline model by letting the regulator, at the onset of the game, announce a policy $\{\pi, t\}$, where $t \geq 0$. If $r=0$ and bribery is not detected, then $u_r = k - H\cdot\omega - t\cdot\lambda - p^2/2$. If, instead, either (i) $r=0$ and bribery is detected, or (ii) $r=1$, then $u_r = (\Delta - H)\cdot\omega - t\cdot\lambda - p^2/2$.[12] Lemma 1 holds verbatim. We characterize in Appendix B the candidate equilibria with and without collusion. There, we denote with λ^N the threshold value of λ below which $\pi = 1$ when collusion is prevented. The next proposition describes the equilibrium policy.

Proposition 2. *Suppose a policy is a pair $\{\pi, t\}$. There exists a threshold $\tilde{\lambda}$ such that in equilibrium:*

- *if $\lambda > \tilde{\lambda}$, collusion occurs and the firm is audited with probability 1;*
- *if $\underline{\lambda}^N < \lambda < \tilde{\lambda}$, collusion is avoided and the firm is audited with probability $(((H-k)\cdot\epsilon - \lambda)\cdot k + \Delta\cdot\epsilon)/((2\Delta - k)k\cdot\epsilon) < 1$;*
- *if $\lambda \leq \underline{\lambda}^N$, collusion is avoided and the firm is audited with probability* 1.

The threshold $\tilde{\lambda}$ is decreasing in α, and $(\underline{\lambda}^N, \tilde{\lambda}) \neq \emptyset$.

The optimal audit frequency is a non-monotonic function of λ: under "tight budgetary constraints" (large λ), the auditor pays a small t, demands frequent audits and tolerates

theoretical literature (e.g., see Drugov, 2010 and Amir and Burr, 2015). However, in these models corruption is unavoidable, whereas we argue that the audit frequency could be lowered to preempt collusion between a firm and its auditor.

[11] Alternatively, it could capture the cost that the regulator bears due to the social outrage that paying high salaries to civil servants causes, or as the cost borne by the regulator to lobby the government for a more generous budget.

[12] We assume that the fine paid by the auditor when collusion is detected does not allow the regulator to recover the distortionary cost associated with the salary payment.

collusion; for intermediate values of λ, the regulator makes sure that audits are sufficiently infrequent so as to deter collusion. For small λ, the compensation is sufficiently large so as to deter collusion even as the firm is audited with probability 1. A comparison of Proposition 1 and Proposition 2 shows that our main result holds qualitatively unchanged: the audit frequency is in both cases a non-monotonic function of the regulator's budget/resources, as measured by t in the baseline model and λ in the current version.

The comparative statics with respect to the bargaining power also holds unchanged: an increase in α makes collusion less problematic. As a result, the audit probability is non-decreasing in α, and so is the regulator's payoff. We illustrate with a graph the effect of an increase in α. In Figure 3, we show the regulator's utility in the candidate equilibrium in which collusion is prevented (curve NC), as well as her utility in the candidate equilibrium with collusion for different values of α. As in the baseline model, if $\alpha > 0$, and the budget sufficiently tight (i.e., λ sufficiently large), tolerating collusion is optimal for the regulator.

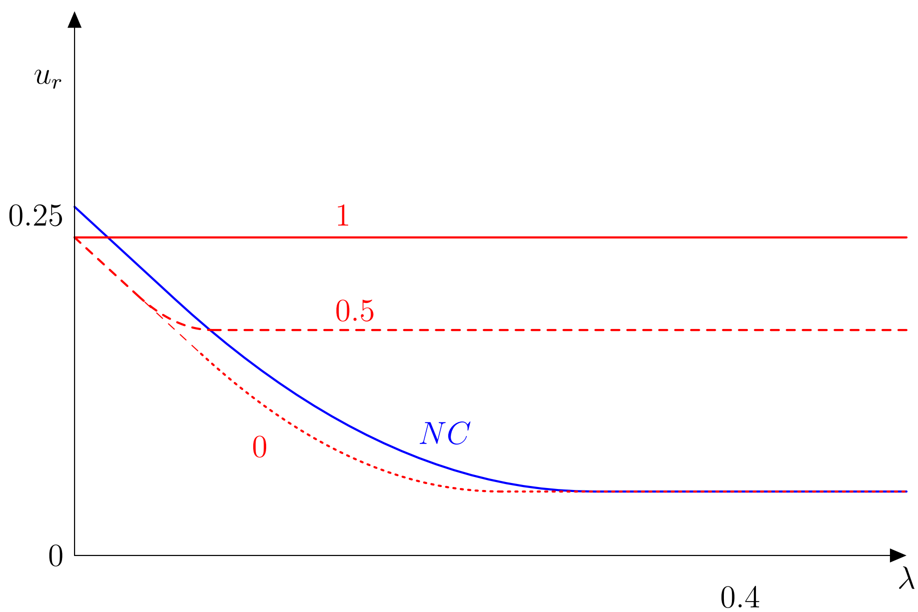

FIGURE 3: REGULATOR'S UTILITY WITH ENDOGENOUS SALARY t, FOR $\epsilon = 0.4$, $k = 0.75$, $H = 1$, AND $\Delta = 0.9$.

4.2 Report-Dependent Compensation

In this subsection, we let the regulator condition the compensation of the auditor on the report. A policy is now a triple $\{\pi, t_0, t_1\}$, where $t_r \geq 0$ is the compensation following report r.[13]

Consider the choice of bribe in equilibrium. As collusion occurs only if $\omega = 1$, the opportunity cost of collusion for the auditor is $\pi \cdot t_1 + (\epsilon - \pi) \cdot t_0$. Arguments akin to those used to prove Lemma 1 lead to the result recorded in the next lemma.

Lemma 4. *Suppose a policy is a triple $\{\pi, t_0, t_1\}$. In any equilibrium in which collusion never occurs, the regulator announces a policy such that $(\pi - \epsilon)k \leq \pi \cdot t_1 + (\epsilon - \pi)t_0$. In any other equilibrium: (i) $\pi \geq \epsilon(t+k)/k$, (ii) a bribe is paid whenever $\omega = 1$, (iii) the auditor demands $b_a = (\pi - \epsilon)k$ whereas the firm offers to pay $b_f = \pi \cdot t_1 + (\epsilon - \pi)t_0$ when $\omega = 1$.*

Candidate equilibria without collusion. In any equilibrium without collusion, the policy solves:

$$max_{\{\pi,t_0,t_1\}\in[0,1]\times\mathbb{R}^2_+} \quad k - \left(1 - p^N(\pi)\right)(H - \pi(\Delta - k)) - \lambda\left(t_0 + \left(1 - p^N(\pi)\right)\pi(t_1 - t_0)\right) - \frac{(p^N(\pi))^2}{2},$$
$$\text{subject to: } (\pi - \epsilon)(k + t_0) \leq \pi \cdot t_1.$$

It is is easy to verify that $\pi \geq \epsilon$ in any solution.[14] Hence, any solution must be such that (i) the constraint binds, and (ii) $t_0 = 0$, i.e., the compensation comes in the form of a bonus associated with reporting a violation. These observations turn the problem of the regulator into a univariate maximization one. Solving the problem yields the result recorded in the next lemma.

Lemma 5. *Suppose a policy is a triple $\{\pi, t_0, t_1\}$. There exist some $0 < \underline{\lambda}^{N'} \leq \overline{\lambda}^{N'}$, and some decreasing function $t^{N'}(\cdot)$ such that, in any equilibrium without collusion, the regulator announces $t_0 = 0$ and*

$$\{\pi, t_1\} = \begin{cases} \{1, (1-\epsilon)k\} & \text{if } \lambda \leq \underline{\lambda}^{N'} \\ \left\{\frac{\epsilon k}{k - t^{N'}(\lambda)}, t^{N'}(\lambda)\right\} & \text{if } \lambda \in (\underline{\lambda}^{N'}, \overline{\lambda}^{N'}) \\ \{\epsilon, 0\} & \text{if } \lambda \geq \overline{\lambda}^{N'} \end{cases},$$

[13] Also in this case, a unit of salary costs $1+\lambda$ to the regulator, where $\lambda > 1$. The payoff of the players are shown in Appendix B.

[14] To see this, note that policy $\{\epsilon, 0, 0\}$ dominates any policy such that $\pi < \epsilon$.

while the firm selects $p = k \cdot \pi$.

Candidate equilibria with collusion. In any equilibrium in which collusion occurs whenever $\omega = 1$, Lemma 4 ensures that the firm selects $p = p^{C'}(\pi, t_0, t_1)$, where

$$\begin{aligned} p^{C'}(\pi, t_0, t_1) &\equiv \arg\max_{p \in [0,1]} p \cdot k + (1-p)((1-\epsilon)k - \alpha(\pi - \epsilon)k - (1-\alpha)(\pi \cdot t_1 + (\epsilon - \pi)t_0)) - \frac{p^2}{2} \\ &= \epsilon \cdot k + \alpha(\pi - \epsilon)k + (1-\alpha)(\pi \cdot t_1 + (\epsilon - \pi)t_0). \end{aligned}$$

The policy in any equilibrium with collusion takes the simple form described in the following lemma.

Lemma 6. *Suppose a policy is a triple $\{\pi, t_0, t_1\}$. In any equilibrium in which collusion occurs with positive probability, the regulator announces $\{\pi, t_0, t_1\} = \{1, 0, (1-\epsilon)k\}$, and the firm selects $p = k$.*

If collusion is tolerated, the regulator never has to compensate the auditor, as (i) $t_0 = 0$, and (ii) the auditor invariably reports $r = 0$. What is more, the firm opts for the second-best level of investment: $p = k$. The intuition goes as follows. As t_1 comes at no cost to the regulator, she sets the highest t_1 among those that result in collusion, i.e., $t_1 = (1-\epsilon)k$. The policy therefore ensures that, in case of violation, the bribe is so large that the firm is as well off paying the bribe as it is not paying it. It follows that the marginal benefit of investment is "as if" collusion did not occur. But then, as the audit is fully informative, the investment is at the second-best level.

Optimal policy. The following proposition builds on Lemmata 5 and 6.

Proposition 3. *Suppose a policy is a triple $\{\pi, t_0, t_1\}$. In equilibrium, the regulator announces $\{\pi, t_0, t_1\} = \{1, 0, (1-\epsilon)k\}$ and*

- *if $\lambda < (\Delta - k)/k$, then collusion is deterred;*
- *if $\lambda > (\Delta - k)/k$, then collusion occurs.*

The proposition establishes that, even if compensation is report-dependent, the regulator opts for tolerating collusion when the cost of raising funds, i.e., λ, is high. When instead λ is relatively small, and/or the benefit of screening, as measured by Δ/k is large, collusion is deterred.

Yet, similarities with the previous models end there. Firstly, Proposition 3 establishes that the regulator never opts for a random audit. This follows from another observation: whenever λ is too large to justify preventing collusion while holding audits with probability one, the regulator tolerates collusion. It can be shown that this result does not hinge on the specific functional form we chose for the firm's investment cost.[15] Secondly, the relative bargaining power of the auditor and the firm is irrelevant for the nature of the optimal policy: as discussed above, the optimal policy in the candidate equilibrium with no collusion (see Lemma 6) eliminates all surplus for the colluding parties. In the absence of surplus, the relative bargaining power is irrelevant.

Report-dependent compensation gives the regulator enough latitude so as to make it sub-optimal to ever reduce the audit frequency. We have shown, for instance, that the regulator can induce the second best level of investment at no cost, if she is willing to tolerate collusion. In fact, perhaps against one's native intuition, there exists parameter combinations for which collusion occurs in equilibrium *only if* the compensation can depend on the report.[16] We conclude that report-dependent compensation need not curb collusion.

5 Concluding Remarks

In our model we have shed light on the costs and benefits of stochastic audits. Stochastic audits may deter collusion between audited firm and auditor, but at a cost of discouraging the firm's investment in compliance. We have shown that a regulator might actually prefer to tolerate collusion and that the optimal frequency of audits need not be a monotonic function of the regulator's budget.

A few final observations are in order. From a theory standpoint, it is worth stressing the role of some modeling assumptions. Firstly, in our model firm and auditor collude *before* knowing whether the audit will take place. This describes a firm cultivating long-term ties with an auditor, so as to be tipped off about an upcoming inspection, or to ensure that an eventual audit will not flag any issues.[17] This type of bribery resembles capture of a regulatory

[15]Calculations showing that the result holds for any cost function of the form $\frac{c \cdot p^2}{2}$ where $c > 0$ are available upon request.

[16]The proof of Proposition 2 shows that collusion does not occur if λ is smaller than some threshold $\underline{\lambda}^C$. For sufficiently large H, the interval $((\Delta - k)/\Delta, \underline{\lambda}^C)$ is not empty. If λ belongs to this interval, collusion is allowed only if compensation can depend on the report.

[17]See Samuel (2009) for another model where collusion occurs before the auditor obtains hard evidence about the agent's misbehavior (Samuel refers to this form of collusion as preemptive bribery).

officer to guarantee leniency.[18] If collusion were to occur exclusively *after* the audit reveals a violation, then the regulator would have no strategic reason to reduce the audit frequency. Secondly, our firm knows whether it is in violation at the time of offering a bribe. We believe our results would be qualitatively identical if the firm were to observe a signal only partially correlated with its compliance status.

In term of policy implications, our results can inform the design of auditing policies, whose sophisticated design is critical to improve enforcement, as stressed by a growing body of empirical evidence (e.g., see the recent Gonzalez Lira and Mobarak, 2021). In particular, we have shown how the optimal audit frequency is non-monotonic in the size of the budget available to the regulator. As regulatory capacity, fiscal efficiency, and other institutional limitations (e.g., the availability of more flexible contractual arrangements with the auditor) are different between developed and less developed countries, it follows that one-size-fits-all solutions should be avoided, as argued by Laffont (2005) and Estache and Wren-Lewis (2009).

[18] For the case of insurance solvency regulation, U.S. state commissioners enjoy a great degree of personal discretion as they order insurers' exams, select the teams that conduct the audit, and, depending on the auditors' findings, decide which steps must be taken. Tenekedjieva (2021) finds that commissioners who join the insurance industry once they leave office are more lenient regulators, thereby highlighting a potential and worrisome exchange of favors.

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Appendix A

Proof of Lemma 1. Proof follows from the argument in the text. ■

Proof of Lemma 2. Proof follows from the argument in the text. ■

Proof of Lemma 3. Proof follows from the argument in the text. ■

Proof of Proposition 1. For $t > (1-\epsilon)k/\epsilon$, we know from Lemma 3 that in equilibrium collusion does not occur, and we know from Lemma 2 that in equilibrium $\pi = 1$.

Let $t \leq (1-\epsilon)k/\epsilon$. The candidate equilibrium without collusion is associated with regulator's expected payoff:

$$\begin{aligned}
E(u_r|\pi^N) &= k - (1 - p^N(\pi^N))\left(H - \pi^N(\Delta - k)\right) - \frac{(p^N(\pi^N))^2}{2} \\
&= k - (1 - (k+t)\epsilon)\left(H - \frac{(k+t)\epsilon}{k}(\Delta - k)\right) - \frac{(k+t)^2\epsilon^2}{2}.
\end{aligned}$$

The candidate equilibrium with collusion is associated with:

$$\begin{aligned}
E(u_r|\pi^C) &= k - (1 - p^C(\pi^C))(H - \epsilon(\Delta - k)) - \frac{(p^C(\pi^C))^2}{2} \\
&= k - (1 - (\alpha k + (1-\alpha)(t+k)\epsilon))(H - \epsilon(\Delta - k)) - \frac{(\alpha k + (1-\alpha)(t+k)\epsilon)^2}{2}.
\end{aligned}$$

We prove a sequence of claims.

Claim 1. *If $t = (1-\epsilon)k/\epsilon$, then $E(u_r|\pi^N) > E(u_r|\pi^C)$.*

Let $t = (1-\epsilon)k/\epsilon$, then:

$$E(u_r|\pi^N) - E(u_r|\pi^C) = (1-k)(1-\epsilon)(\Delta - k) > 0.$$

Claim 2. *The difference $E(u_r|\pi^N) - E(u_r|\pi^C)$ is a concave function of t.*

The difference $E(u_r|\pi^N) - E(u_r|\pi^C)$ is differentiable with respect to t and:

$$\frac{d^2(E(u_r|\pi^N) - E(u_r|\pi^C))}{dt^2} = -\left(\frac{2\epsilon^2}{k}(\Delta - k)\right) - (2-\alpha)\alpha\epsilon^2 < 0.$$

Claim 3. *The difference $E(u_r|\pi^N) - E(u_r|\pi^C)$ is a decreasing function of α.*

The difference $E(u_r|\pi^N) - E(u_r|\pi^C)$ is differentiable with respect to α and:

$$\begin{aligned}\frac{d(E(u_r|\pi^N) - E(u_r|\pi^C))}{d\alpha} &= -((1-\epsilon)k - \epsilon t)(H - \epsilon(\Delta - k) - \alpha k - (1-\alpha)(t+k)\epsilon) \\ &< -((1-\epsilon)k - \epsilon t)(H - \epsilon(\Delta - k) - k) < 0.\end{aligned}$$

Claim 4. *If $t = 0$ and $\alpha = 0$, then $E(u_r|\pi^N) = E(u_r|\pi^C) = 0$.*

Let $t = 0$ and $\alpha = 0$. Then:

$$\begin{aligned} E(u_r|\pi^N) - E(u_r|\pi^C) &= \\ k - (1 - k\cdot\epsilon)\left(H - \epsilon(\Delta - k)\right) - \frac{k^2\epsilon^2}{2} - (k - (1-k\cdot\epsilon)(H - \epsilon(\Delta - k)) - \frac{(k\cdot\epsilon)^2}{2}) &= \\ -(1-k\cdot\epsilon)\left(H - \epsilon(\Delta - k)\right) + (1-k\cdot\epsilon)(H - \epsilon(\Delta - k)) &= 0.\end{aligned}$$

Claim 1 ensures that at $t = (1-\epsilon)k/\epsilon$, collusion is optimally avoided and $\pi = 1$. Claims 1 and 2 together imply that there exists a non-empty convex subset of $[0, (1-\epsilon)k/\epsilon]$ such that (i) $(1-\epsilon)k/\epsilon$ belongs to the set and (ii) collusion is prevented if and only if t belongs to the set. Let $\tilde{t}$ denote the lower bound of this convex set. Continuity of $E(u_r|\pi^N) - E(u_r|\pi^C)$ in α and t ensure that $\tilde{t}$ is a continuous function of α. Claim 3 ensures that $\tilde{t}$ is a strictly increasing function of α (to see this, set $t = \tilde{t}$ and note that a marginal increase in α must result in $t < \tilde{t}$). Claims ensures that $\tilde{t} = 0$ if $\alpha = 0$ and concludes the proof. ∎

Appendix B

Report-Independent Salary: Intermediate Results

We characterize here the candidate equilibria with and without collusion, when the policy is a pair $\{\pi, t\}$.

Candidate equilibria without collusion. In any equilibrium without collusion, the regulator announces a policy that solves:

$$\max_{\{\pi,t\}\in[0,1]\times\mathbb{R}^+} \quad k-(1-p^N(\pi))(H-\pi(\Delta-k))-\frac{(p^N(\pi))^2}{2}-\lambda\cdot t,$$
$$\text{s.t.: } \pi\leq\frac{(k+t)\epsilon}{k}.$$

The solution is recorded in the next lemma.

Lemma B1. *Suppose a policy is a pair $\{\pi, t\}$. There exists some $0<\underline{\lambda}^N<\overline{\lambda}^N$, and some decreasing function $t^N(\cdot)$ such that, in any equilibrium without collusion, the regulator announces*

$$\{\pi,t\}=\begin{cases}\left\{1,\frac{(1-\epsilon)k}{\epsilon}\right\} & \text{if } \lambda<\underline{\lambda}^N,\\ \left\{\frac{(t^N(\lambda)+k)\epsilon}{k},t^N(\lambda)\right\} & \text{if } \lambda\in[\underline{\lambda}^N,\overline{\lambda}^N],\\ \{\epsilon,0\} & \text{if } \lambda>\overline{\lambda}^N,\end{cases}$$

and the firm selects $p=\pi\cdot k$.

Proof. The argument in the text above ensures that in any candidate equilibrium without collusion, the regulator sets $\pi=\min\left\{\frac{(t+k)\epsilon}{k},1\right\}$. A salary t such that $\frac{(t+k)\epsilon}{k}>1$ is clearly suboptimal. So, $\pi=\frac{(t+k)\epsilon}{k}$, and $t=\arg\max_{x\in\left[0,\frac{(1-\epsilon)k}{\epsilon}\right]} u_r^N(x)$, where

$$u_r^N(x)\equiv k-(1-(x+k)\epsilon)\left(H-\frac{(x+k)(\Delta-k)\epsilon}{k}\right)-\lambda\cdot x-\frac{(x+k)^2\epsilon^2}{2}.$$

Function u_r^N is concave, and the first-order condition yields:

$$t=t^N(\lambda)\equiv\frac{1}{(2\Delta-k)\epsilon}\left(k\cdot H+\Delta-k-\frac{k\lambda}{\epsilon}\right)-k.$$

Let

$$\underline{\lambda}^N \equiv \epsilon\left(H-(2\Delta-k)+\frac{\Delta-k}{k}\right) \text{ and } \overline{\lambda}^N \equiv \epsilon\left(H-\epsilon(2\Delta-k)+\frac{\Delta-k}{k}\right).$$

The lemma follows from the observation that (i) $t^N(\cdot)$ is strictly decreasing, (ii) $t^N(\underline{\lambda}^N)=\frac{(1-\epsilon)k}{\epsilon}$, and $t^N(\overline{\lambda}^N)=0$. ■

Candidate equilibria with collusion. With a slight abuse of notation, we now refer to $p^C(\pi)$ as $p^C(\pi,t)$. In any equilibrium with collusion, the policy solves

$$\begin{aligned}\max_{\{\pi,t\}\in[0,1]\times\mathbb{R}_+} \quad & k-(1-p^C(\pi,t))[H-\epsilon(\Delta-k)]-\lambda\cdot t-\frac{(p^C(\pi,t))^2}{2}, \\ \text{s.t.: } & \epsilon\cdot t\leq k(\pi-\epsilon).\end{aligned} \tag{B1}$$

Solving the problems yields the next lemma.

Lemma B2. *Suppose a policy is a pair $\{\pi,t\}$. There exists some $\underline{\lambda}^C\leq\overline{\lambda}^C$ and some decreasing function $t^C(\cdot)$ such that, in any equilibrium with collusion, the regulator announces $\pi=1$, and*

$$t=\begin{cases}\frac{(1-\epsilon)k}{\epsilon} & \text{if } \lambda<\underline{\lambda}^C,\\ t^C(\lambda) & \text{if } \lambda\in[\underline{\lambda}^C,\overline{\lambda}^C],\\ 0 & \text{if } \lambda>\overline{\lambda}^C,\end{cases}$$

and the firm selects $p=\alpha\cdot k+(1-\alpha)(t+k)\epsilon$.

Proof. As established to prove Lemma 3, for $t>\frac{(1-\epsilon)k}{\epsilon}$ in equilibrium collusion does not occur. We focus in the rest of the proof on policies such that $t\leq\frac{(1-\epsilon)k}{\epsilon}$.

The first derivative of the maximand in (B1) with respect to π is equal to $H-\epsilon(\Delta-k)-p^C(\pi,t)$. As we restrict attention to $\pi\leq 1$ and $t\leq\frac{(1-\epsilon)k}{\epsilon}$, then:

$$H-\epsilon(\Delta-k)-p^C(\pi,t)\geq H-\epsilon(\Delta-k)-k>0.$$

Any solution of problem (B1) is thus such that $\pi=1$. So in any equilibrium with collusion $\pi=1$, and $t=\arg\max_{x\in\left[0,\frac{(1-\epsilon)k}{\epsilon}\right]} u_r^C(x)$, where:

$$u_r^C(x)\equiv \quad k-(1-p^C(1,x))(H-\epsilon(\Delta-k))-\lambda\cdot x-\frac{(p^C(1,x))^2}{2}.$$

For $\alpha = 1$, function $u_r^C(x)$ is monotonic, and $\arg\max_{x \in \left[0, \frac{(1-\epsilon)k}{\epsilon}\right]} u_r^C(x) = 0$. For $\alpha < 1$, function $u_r^C(x)$ is concave, and $\arg\max_{x \in \left[0, \frac{(1-\epsilon)k}{\epsilon}\right]} u_r^C(x) = t^C(\lambda)$, where

$$t^C(\lambda) \equiv -\frac{\lambda}{(1-\alpha)^2\epsilon^2} + \frac{(H - \epsilon \cdot \Delta - (1-\epsilon)\alpha \cdot k)}{(1-\alpha)\epsilon}.$$

Let

$$\underline{\lambda}^C \equiv (1-\alpha)\epsilon(H - \epsilon \cdot \Delta - (1-\epsilon)k), \text{ and } \overline{\lambda}^C \equiv (1-\alpha)\epsilon(H - \epsilon \cdot \Delta - (1-\epsilon)\alpha \cdot k).$$

The lemma follows from the observation that (i) $t^C(\cdot)$ is strictly decreasing, (ii) $t^C(\underline{\lambda}^C) = \frac{(1-\epsilon)k}{\epsilon}$, and (iii) $t^C(\overline{\lambda}^C) = 0$. ■

Proof of Proposition 2. Lemma B1 ensures that the candidate equilibrium without collusion is associated with regulator's expected utility

$$E(u_r|\pi^N, t^N) = \begin{cases} u_r^N\left(1, \frac{(1-\epsilon)k}{\epsilon}\right) & \text{if } \lambda \leq \underline{\lambda}^N, \\ u_r^N\left(\frac{(t^N(\lambda)+k)\epsilon}{k}, t^N(\lambda)\right) & \text{if } \lambda \in (\underline{\lambda}^N, \overline{\lambda}^N), \\ u_r^N(\epsilon, 0) & \text{if } \lambda \geq \overline{\lambda}^N, \end{cases}$$

where:

$$u_r^N(\pi, t) = k - (1 - k \cdot \pi)(H - \pi(\Delta - k)) - \lambda \cdot t - \frac{k^2\pi^2}{2}.$$

and

$$t^N(\lambda) = \frac{1}{(2\Delta - k)\epsilon}\left(k \cdot H + \Delta - k - \frac{k \cdot \lambda}{\epsilon}\right) - k.$$

Lemma B2 ensures that the candidate equilibrium with collusion is associated with regulator's expected utility

$$E(u_r|\pi^C, t^C) = \begin{cases} u_r^C\left(1, \frac{(1-\epsilon)k}{\epsilon}\right) & \text{if } \lambda \leq \underline{\lambda}^C \\ u_r^C(1, t^C(\lambda)) & \text{if } \lambda \in (\underline{\lambda}^C, \overline{\lambda}^C] \\ u_r^C(1, 0) & \text{if } \lambda > \overline{\lambda}^C, \end{cases}$$

where

$$u_r^C(1, t) = k - (1 - (\alpha k + (1-\alpha)\epsilon(k+t)))(H - \epsilon(\Delta - k)) - \lambda \cdot t - \frac{(\alpha k + (1-\alpha)\epsilon(k+t))^2}{2},$$

and

$$t^C(\lambda) = -\frac{\lambda}{(1-\alpha)^2\epsilon^2} + \frac{(H-\epsilon\cdot\Delta-(1-\epsilon)\alpha\cdot k)}{(1-\alpha)\epsilon}.$$

Both $E(u_r|\pi^C, t^C)$ and $E(u_r|\pi^N, t^N)$ are (i) continuous functions of α and λ, (ii) weakly decreasing functions of λ and (iii) strictly decreasing whenever t^C (respectively, t^N) is strictly positive. Note that for $\lambda \leq \min\{\underline{\lambda}^C, \underline{\lambda}^N\}$ the two candidate equilibria are associated with the same policy. The equilibrium corresponds to the candidate equilibrium associated with the highest expected payoff for the regulator. We can generically rule out equilibrium multiplicity by noting that if multiple equilibria were to exists, it would be generically the case that one equilibrium, call it Equilibrium A, would be associated with a higher payoff for the regulator than another equilibrium, call it Equilibrium B. Yet it is easy to verify that, by choosing the appropriate policy, the regulator can ensure a payoff ϵ-close to the payoff in any equilibrium, including Equilibrium A. This is a contradiction as we just showed that in Equilibrium B the regulator has a profitable deviation.

We now prove a sequence of claims.

Claim 1. $\overline{\lambda}^N > \overline{\lambda}^C$.

For $\alpha = 0$:

$$\overline{\lambda}^N > \overline{\lambda}^C \Leftrightarrow$$
$$\frac{\epsilon[Hk + \Delta - k - \epsilon k(2\Delta - k)]}{k} > \epsilon(H - \epsilon\Delta) \Leftrightarrow$$
$$(\Delta - k)(1 - \epsilon k) > 0.$$

As the last inequality holds, threshold $\overline{\lambda}^C$ is a decreasing function of α, and $\overline{\lambda}^N$ does not depend on α, we conclude that $\overline{\lambda}^N > \overline{\lambda}^C$ for any α.

Claim 2. *Let $\lambda \geq \overline{\lambda}^N$. If $\alpha > 0$ then $E(u_r|\pi^C, t^C) > E(u_r|\pi^N, t^N)$, while if $\alpha = 0$ then $E(u_r|\pi^C, t^C) = E(u_r|\pi^N, t^N)$.*

It is immediate to check that for $\alpha = 0$, $u_r^C(1,0) = u_r^N(\epsilon, 0)$. Note also that $\partial u_r^N(\epsilon, 0)/\partial\alpha = 0$, while

$$\frac{\partial u_r^C(1,0)}{\partial\alpha} = (1-\epsilon)k[H - \epsilon\Delta - \alpha(1-\epsilon)k] > 0.$$

Claim 3. *If $\lambda \in (\underline{\lambda}^C, \underline{\lambda}^N] \cup [\overline{\lambda}^C, \overline{\lambda}^N]$, then $E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C)$ is a strictly decreasing* *function of λ.*

Note that (i) $t^N = \frac{(1-\epsilon)k}{\epsilon} > t^C$ for $\lambda \in (\underline{\lambda}^C, \underline{\lambda}^N]$, while $t^N > t^C = 0$ for $\lambda \in [\overline{\lambda}^C, \overline{\lambda}^N)$. The claim thus follows from the envelope theorem.

Claim 4. *If $\lambda \leq \underline{\lambda}^C$, then $E(u_r|\pi^N, t^N) > E(u_r|\pi^C, t^C)$.*

As (i) $\left\{\pi^C, t^C\right\} = \left\{1, \frac{(1-\epsilon)k}{\epsilon}\right\}$ for $\lambda \leq \underline{\lambda}^C$, and (ii) for $\{\pi, t\} = \left\{1, \frac{(1-\epsilon)k}{\epsilon}\right\}$ there exists also a continuation equilibrium in which collusion does not occur, then to prove the claim it is sufficient to show that:

$$u_r^N\left(1, \frac{(1-\epsilon)k}{\epsilon}\right) > u_r^C\left(1, \frac{(1-\epsilon)k}{\epsilon}\right) \Leftrightarrow$$
$$-(1-k)(H-(\Delta-k))k - \frac{k^2}{2} - \lambda\cdot\frac{(1-\epsilon)k}{\epsilon} > k-(1-k)(H-\epsilon(\Delta-k)) - \frac{k^2}{2} - \lambda\cdot\frac{(1-\epsilon)k}{\epsilon} \Leftrightarrow$$
$$1 > \epsilon.$$

Claim 5. *Either $E(u_r|\pi^N, t^N) > E(u_r|\pi^C, t^C)$ for all $\lambda \in (\max\left\{\underline{\lambda}^C, \underline{\lambda}^N\right\}, \overline{\lambda}^C)$, or else there exist a $\tilde{\lambda} \in (\max\left\{\underline{\lambda}^C, \underline{\lambda}^N\right\}, \overline{\lambda}^C)$ such that*

- *$E(u_r|\pi^N, t^N) > E(u_r|\pi^C, t^C)$ for $\lambda \in (\tilde{\lambda}, \overline{\lambda}^C)$, and*
- *$E(u_r|\pi^N, t^N) < E(u_r|\pi^C, t^C)$ for $\lambda \in (\max\left\{\underline{\lambda}^C, \underline{\lambda}^N\right\}, \tilde{\lambda})$.*

The envelope theorem ensures that $d(E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C))d\lambda = t^C - t^N$. It is easy to verify that $d(t^C - t^N)/d\lambda < 0$ over the interval $\lambda \in (\max\left\{\underline{\lambda}^C, \underline{\lambda}^N\right\}, \overline{\lambda}^C)$. Function $E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C)$ is thus strictly concave in λ in this interval. We distinguish two cases. If $\underline{\lambda}^C > \underline{\lambda}^N$ the claim is a consequence of Claim 4 and the concavity of $E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C)$. If instead $\underline{\lambda}^C < \underline{\lambda}^N$, then for λ larger but sufficiently close to $\underline{\lambda}^N$ it must be the case that $t^C < t^N$, and therefore $d(E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C))d\lambda < 0$. Concavity of $E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C)$ then ensures that $d(E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C))d\lambda < 0$ for any $\lambda \in (\max\left\{\underline{\lambda}^C, \underline{\lambda}^N\right\}, \overline{\lambda}^C)$. The claim follows.

A consequence of Claims 1-5 is that there exists a $\tilde{\lambda} \in (\underline{\lambda}^N, \overline{\lambda}^N)$ such that collusion occurs only if $\lambda > \tilde{\lambda}$. To conclude the proof it is sufficient to notice that the difference $E(u_r|\pi^N, t^N) - E(u_r|\pi^C, t^C)$ is a (weakly) decreasing function of α. ■

Report-Dependent Compensation: The Model

The payoff in the model with contingent payments are as follows. If $r = 0$ and bribery is not detected, payoffs are:

$$u_{r)} = k - H \cdot \omega - \lambda \cdot t_0 - \frac{p^2}{2};$$
$$u_a = t_0 + b_i \cdot \mathbb{1}_b;$$
$$u_f = k - b_i \cdot \mathbb{1}_b - \frac{p^2}{2};$$

where $\mathbb{1}_b = 1$ is a bribe is paid ($\mathbb{1}_b = 0$ otherwise) and $i \in \{a, f\}$. If instead $r = 1$, or bribery is detected, then:

$$u_r = (\Delta - H) \cdot \omega - \lambda \cdot t_1 \cdot r - \frac{p^2}{2};$$
$$u_a = t_1 \cdot (1 - \mathbb{1}_b) + b_i \cdot \mathbb{1}_b;$$
$$u_f = -b_i \cdot \mathbb{1}_b - \frac{p^2}{2}.$$

Proof of Lemma 4. Proof follows from argument in the text. ■

Proof of Lemma 5: The argument in the text reduces the problem to $\max_{\pi \in \{\epsilon, 1\}} u_r^N(\pi)$, where

$$u_r^{N'}(\pi) \equiv k - (1 - \pi k)\left(H - \pi(\Delta - k) + \lambda(\pi - \epsilon)k\right) - \frac{\pi^2 k^2}{2}.$$

The first order condition yields:

$$\pi = \pi^{N'}(\lambda) \equiv \frac{k \cdot H + (\Delta - k) - \lambda \cdot k(1 + k\epsilon)}{k\,(2\Delta - k - 2\lambda \cdot k)}.$$

Define:

$$\lambda^{\dagger} \equiv \frac{\Delta}{k} - \frac{1}{2};$$
$$\hat{\lambda} \equiv \frac{1}{k(1-k)} \left(k \cdot H + (\Delta - k) - \frac{(1+\epsilon)(2\Delta - k)k}{2} \right).$$

Let $\lambda(\pi) \equiv \frac{k \cdot \pi(2\Delta - k) - k \cdot H - \Delta + k}{k(2\pi \cdot k - 1 - k \cdot \epsilon)}$, so that $\pi^{N'}(\lambda(\pi)) = \pi$ for any $\pi \in [0,1]$. The following properties can be easily verified:

1. $u_r^{N'}$ is strictly concave if $\lambda \in (0, \lambda^{\dagger})$, and convex if $\lambda \geq \lambda^{\dagger}$;
2. $u_r^{N'}(\epsilon) > u_r^{N'}(1) \Leftrightarrow \lambda > \hat{\lambda}$, and $u_r^{N'}(\epsilon) < u_r^{N'}(1) \Leftrightarrow \lambda < \hat{\lambda}$;
3. the following have the same sign: (i) $H - \frac{1+\epsilon(2\Delta - k)}{2}$, (ii) $\tilde{\lambda} - \lambda^{\dagger}$, (iii) $d\pi^{N'}(\lambda)/d\lambda$, (iv) $\lambda(\epsilon) - \lambda^{\dagger}$;
4. $\pi^{N'}(0) > 1$.

Let $H \geq \frac{1+\epsilon(2\Delta - k)}{2}$. Then for any $\lambda < \lambda^{\dagger}$: $u_r^{N'}$ is strictly concave and $\pi^{N'}(\lambda) > 1$, thus the optimal policy has $\pi = 1$. For any $\lambda \geq \lambda^{\dagger}$: $u_r^{N'}$ is convex, thus the optimal policy has $\pi = 1$ if $\lambda \in (\lambda^{\dagger}, \tilde{\lambda})$ and $\pi = \epsilon$ if $\lambda > \tilde{\lambda}$.

Let $H < \frac{1+\epsilon(2\Delta - k)}{2}$. Then for any $\lambda < \lambda^{\dagger}$: $u_r^{N'}$ is strictly concave and $\pi^{N'}(\lambda) > 1$, thus the optimal policy has $\pi = 1$ for $\lambda \in (0, \lambda(1)]$, $\pi = \pi^{N'}(\lambda)$ for $\lambda \in (\lambda(1), \lambda(\epsilon))$ and $\pi = \epsilon$ for $\lambda \in [\lambda(\epsilon)), \lambda^{\dagger})$. For any $\lambda \geq \lambda^{\dagger}$: $u_r^{N'}$ is convex and $\pi^{N'}(\epsilon) > \pi^{N'}(1)$, thus the optimal policy has $\pi = \epsilon$.

Define:

$$\underline{\lambda}^{N'} = \begin{cases} \hat{\lambda} & \text{if } H > \frac{1+\epsilon(2\Delta - k)}{2} \\ \lambda(1) & \text{if } H < \frac{1+\epsilon(2\Delta - k)}{2} \end{cases}, \text{ and } \overline{\lambda}^{N'} = \begin{cases} \hat{\lambda} & \text{if } H > \frac{1+\epsilon(2\Delta - k)}{2} \\ \lambda(\epsilon) & \text{if } H < \frac{1+\epsilon(2\Delta - k)}{2} \end{cases},$$

and

$$t^{N'}(\lambda) \equiv \frac{(\pi^{N'}(\lambda) - \epsilon)k}{\pi^{N'}(\lambda)}.$$

The lemma follows. ■

Proof of Lemma 6. Let $p^{C'}(\pi,t_0,t_1)=\epsilon\cdot k+\alpha(\pi-\epsilon)k+(1-\alpha)(\pi\cdot t_1+(\epsilon-\pi)t_0)$. In any equilibrium with collusion, the regulator selects the policy that maximizes her expected payoff, among the policies that induce collusion. The problem of the regulator is;

$$\max_{\{\pi,t_0,t_1\}\in[0,1]\times\mathbb{R}^2_+} k-\lambda\cdot t_0-(1-p^{C'}(\pi,t_0,t_1))(H-\epsilon(\Delta-k))-\frac{(p^{C'}(\pi,t_0,t_1))^2}{2},$$
$$\text{subject to: } \pi(k+t_0-t_1)\geq\epsilon\cdot(k+t_0).$$

Consider the unconstrained problem, that is the problem without the collusion constraint $\pi(k+t_0-t_1)\geq\epsilon\cdot(k+t_0)$. Any solution of the unconstrained problem must be such that $p^{C'}(\pi,t_0,t_1)=\pi k$ and $t_0=0$. This turns the unconstrained problem to:

$$\max_{\pi\in[\epsilon,1]} \quad k-(1-\pi k)(H-\epsilon(\Delta-k))-\frac{(\pi k)^2}{2}. \tag{P_B}$$

which is maximized by $\pi=1$. We have thus established that any solution of the unconstrained problem must be such that $\pi=1$, $t_0=0$ and $t_1\geq k(1-\epsilon)$. It is easy to verify that only solution $\{\pi,t_0,t_1\}=\{1,0,k(1-\epsilon)\}$ satisfies the constraint. The lemma follows. ■

Proof of Proposition 3. Lemma 6 ensures that in a candidate equilibrium with collusion:

$$\mathbb{E}(u_r)=V\equiv k-(1-k)\left[H-\epsilon(\Delta-k)\right]-\frac{k^2}{2}.$$

The regulator's expected utility in a candidate equilibrium without collusion is:

$$W(\pi)\equiv k-(1-\pi\cdot k)[H-\pi(\Delta-k)+(\pi-\epsilon)\lambda\cdot k]-\frac{(\pi\cdot k)^2}{2}.$$

As stated in Lemma 5, in a candidate equilibrium without collusion:

$$\mathbb{E}(u_r)=\begin{cases} W(1) & \text{if } \lambda\leq\underline{\lambda}^{N'}, \\ W(\pi_N(\lambda)) & \text{if } \lambda\in(\underline{\lambda}^{N'},\overline{\lambda}^{N'}), \\ W(\epsilon) & \text{if } \lambda\geq\overline{\lambda}^{N'}. \end{cases}$$

The following properties can be easily verified:

1. $W(1) > V \Leftrightarrow \lambda < \frac{\Delta - k}{k}$;
2. if $H > \frac{1+\epsilon(2\Delta - k)}{2}$, then $\frac{\Delta - k}{k} < \hat{\lambda}$;
3. if $H < \frac{1+\epsilon(2\Delta - k)}{2}$, then $\frac{\Delta - k}{k} < \lambda(1)$.

Verifying the first property is immediate. To verify the second property, note that:

$$\begin{aligned}
\frac{\Delta - k}{k} &< \hat{\lambda} \Leftrightarrow \\
\Delta - k &< \frac{1}{1-k}\left(k \cdot H + (\Delta - k) - \frac{(1+\epsilon)(2\Delta - k)k}{2}\right)
\end{aligned}$$

We verify that this inequality holds for $H = \frac{1+\epsilon(2\Delta - k)}{2}$ (and hence it holds for larger values of H):

$$\begin{aligned}
(\Delta - k)(1-k) &< k \cdot \frac{1+\epsilon(2\Delta - k)}{2} + (\Delta - k) - \frac{(1+\epsilon)(2\Delta - k)k}{2} \Leftrightarrow \\
(\Delta - k)(-k) &< k \cdot \frac{1+\epsilon(2\Delta - k)}{2} - \frac{(1+\epsilon)(2\Delta - k)k}{2} \Leftrightarrow \\
(\Delta - k)(-2) &< \cdot 1 - (2\Delta - k) \Leftrightarrow \\
2k &< 1 + k \Leftrightarrow \\
k &< 1.
\end{aligned}$$

To verify the third property, note that:

$$\begin{aligned}
\frac{\Delta - k}{k} &< \lambda(1) \Leftrightarrow \\
\frac{\Delta - k}{k} &< \frac{k(2\Delta - k) - k \cdot H - \Delta + k}{k(2k - 1 - k \cdot \epsilon)} \Leftrightarrow \\
0 &< \frac{k(2\Delta - k) - k \cdot H - (\Delta - k)(1 + 2k - 1 - k \cdot \epsilon)}{2k - 1 - k \cdot \epsilon} \Leftrightarrow \\
0 &< \frac{k(2\Delta - k) - k \cdot H - (\Delta - k)(2k - k \cdot \epsilon)}{2k - 1 - k \cdot \epsilon} \Leftrightarrow \\
0 &< \frac{2\Delta - k - H - (\Delta - k)(2 - \epsilon)}{2k - 1 - k \cdot \epsilon} \Leftrightarrow \\
0 &< \frac{\epsilon\Delta + (1-\epsilon)k - H}{2k - 1 - k \cdot \epsilon}.
\end{aligned}$$

The numerator is always negative, while the denominator is negative if and only if $k < 1/(2 - \epsilon)$.

Yet, $H < (1+\epsilon(2\Delta - k))/2$ holds only if $\Delta < (1+\epsilon(2\Delta - k))/2 \Leftrightarrow \Delta < (1-\epsilon \cdot k)/(2(1-\epsilon))$. This condition in turn holds only if $k < (1-\epsilon \cdot k)/(2(1-\epsilon)) \Leftrightarrow k < 1/(2-\epsilon)$. We thus conclude that the last highlighted inequality holds.

Note that $\mathbb{E}(u_r) - W(\pi)$ is a weakly decreasing function of λ, and it is strictly decreasing if $\pi > \epsilon$. Recall also, from the proof of Lemma 5, that $\underline{\lambda}^{N'} = \hat{\lambda}$ if $H > (1+\epsilon(2\Delta - k))$, while $\underline{\lambda}^{N'} = \lambda(1)$ if $H < (1+\epsilon(2\Delta - k))$. The proposition then follows from properties 1-3. ■