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CP ASYMMETRIES IN THREE-BODY $B^{\pm}$ DECAYS TO CHARGED PIONS AND KAONS

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CP asymmetries have been measured recently by the LHCb collaboration in three-body B^+ decays to final states involving charged pions and kaons. Large asymmetries with opposite signs at a level of about 60% have been observed in $B^{\pm} \to \pi^{\pm}(\text{or } K^{\pm})\pi^+\pi^-$ and $B^{\pm} \to \pi^{\pm}K^+K^-$ for restricted regions in the Dalitz plots involving $\pi^+\pi^-$ and K^+K^- with low invariant mass. U-spin is shown to predict corresponding $\Delta S = 0$ and $\Delta S = 1$ asymmetries with opposite signs and inversely proportional to their branching ratios, in analogy with a successful relation predicted thirteen years ago between asymmetries in $B_s \to K^-\pi^+$ and $B^0 \to K^+\pi^-$. We compare these predictions with the measured integrated asymmetries. Effects of specific resonant or non-resonant partial waves on enhanced asymmetries for low-pair-mass regions of the Dalitz plot are studied in $B^{\pm} \to \pi^{\pm}\pi^+\pi^-$. The closure of low-mass $\pi^+\pi^-$ and K^+K^- channels involving only $\pi\pi \leftrightarrow K\bar{K}$ rescattering may explain by CPT approximately equal magnitudes and opposite signs measured in $B^{\pm} \to \pi^{\pm}\pi^+\pi^-$ and $B^{\pm} \to \pi^{\pm}K^+K^-$.

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I Introduction

CP asymmetries in two-body and quasi-two-body charmless B and B_s decays have played an important role in testing the Cabibbo-Kobayashi-Maskawa (CKM) framework and, with high precision, may probe virtual effects of heavy new particles. Isospin symmetry, which holds well in strong interactions, has been applied experimentally in two well-known cases to CP asymmetries in strangeness-conserving $B \to \pi\pi, \rho\rho$ and strangeness-changing $B \to K\pi$ decays.

In the first example [1], CP asymmetries measured in $B^0 \to \pi^+\pi^-$ and $B^0 \to \rho^+\rho^-$ provide the most precise source of information for determining $\alpha \equiv \text{Arg}(-V^*_{tb}V_{td}/V^*_{ub}V_{ud})$ [2]. Two inputs of this method, the direct asymmetries $A_{CP}(B^0 \to \pi^+\pi^-)$ and $A_{CP}(B^0 \to \rho^+\rho^-)$ due to interference of tree and penguin amplitudes, may be affected by new heavy particles entering the $b \to d$ penguin loop. This could show up through an inconsistency between this measurement of α and a future improved determination of $\gamma \equiv \text{Arg}(-V^*_{ub}V_{ud}/V^*_{cb}V_{cd})$ in $B \to D^{(*)}K^{(*)}$ [3, 4]. In the second example an isospin sum rule has been proposed [5] combining CP rate differences in all four $B \to K\pi$ decays, $B^0 \to K^+\pi^-, K^0\pi^0$ and $B^+ \to K^+\pi^0, K^0\pi^+$. A violation of this sum rule, rather than merely the nonzero difference observed between CP asymmetries in $B^0 \to K^+\pi^-$ and $B^+ \to K^+\pi^0$ [4] (often named "the $K\pi$ puzzle" [6, 7, 8]), would be unambiguous evidence for new physics in $b \to sq\bar{q}$ transitions. This test requires a substantial improvement in the measurement of $A_{CP}(B^0 \to K^0\pi^0)$ [4].

An application of isospin symmetry to three-body B decays of pions and kaons is too involved for studying CP asymmetries in these decays because *five* independent isospin amplitudes are needed for describing merely the subset of decays to three kaons [9, 10]. In contrast, U-spin symmetry, an SU(2) subgroup of flavor SU(3) under which the pairs of quarks (d, s) and mesons (π^-, K^-) transform like doublets, seems potentially powerful for studying asymmetries in all three-body charged B decays involving charged pions and kaons, $B^+ \to K^+\pi^+\pi^-, K^+K^+K^-, \pi^+\pi^+\pi^-, \pi^+K^+K^-$. All four processes involve only *two* independent U-spin amplitudes [10]. One should be aware of possible U-spin breaking effects of order 30%, expected in hadronic amplitudes and consequently in CP asymmetry relations.

Dalitz-plot analyses of these three-body processes have been carried out by the BaBar and Belle collaborations for $B^+ \to K^+\pi^+\pi^-$ [11, 12], $B^+ \to K^+K^+K^-$ [13, 14, 15, 16], $B^+ \to \pi^+\pi^+\pi^-$ [17, 18], and $B^+ \to \pi^+K^+K^-$ [19]. Three-body charmless B and B_s decays have been studied under various assumptions in Ref. [20].

The LHCb collaboration has recently reported measurements of CP asymmetries for all four three-body B^+ decay modes involving charged pions and kaons. The measurements include two asymmetries in decays to strangeness-one final states [21],

$$
\begin{aligned}
A_{CP}(B^+ \to K^+\pi^+\pi^-) &= +0.032 \pm 0.008(\text{stat}) \pm 0.004(\text{syst}) \pm 0.007(J/\psi K^+) , \\
A_{CP}(B^+ \to K^+K^+K^-) &= -0.043 \pm 0.009(\text{stat}) \pm 0.003(\text{syst}) \pm 0.007(J/\psi K^+) , \quad (1)
\end{aligned}
$$

with significance of 2.8σ and 3.7σ. A very recent BaBar result [16], $A_{CP}(B^+ \to K^+K^+K^-) = -0.017^{+0.019}_{-0.014} \pm 0.014$, is consistent with the LHCb measurement within 1.1σ.

Two other asymmetries have been measured by LHCb in decays to strangeness-zero states [22],

$$
\begin{aligned}
A_{CP}(B^+ \to \pi^+\pi^+\pi^-) &= +0.120 \pm 0.020(\text{stat}) \pm 0.019(\text{syst}) \pm 0.007(J/\psi K^+) , \\
A_{CP}(B^+ \to \pi^+K^+K^-) &= -0.153 \pm 0.046(\text{stat}) \pm 0.019(\text{syst}) \pm 0.007(J/\psi K^+) , \quad (2)
\end{aligned}
$$

with significance of 4.2σ and 3.0σ.

Considerably larger CP asymmetries with same signs as the above were measured in the latter two decay modes for localized regions of phase space. The two regions, corresponding to pairs of $\pi^+\pi^-$ and K^+K^- with low invariant mass, $m^2_{\pi^+\pi^-\,\text{low}} < 0.4$ GeV$^2/c^4$ (the other pion pair obeying $m^2_{\pi^+\pi^-\,\text{high}} > 15$ GeV$^2/c^4$) and $m^2_{K^+K^-} < 1.5$ GeV$^2/c^4$, involve the following asymmetries [22]:

$$
\begin{aligned}
A_{CP}(B^+ \to \pi^+(\pi^+\pi^-)_{\mathrm{low}\,m}) &= +0.622 \pm 0.075 \pm 0.032 \pm 0.007 \ , \\
A_{CP}(B^+ \to \pi^+(K^+K^-)_{\mathrm{low}\,m}) &= -0.671 \pm 0.067 \pm 0.028 \pm 0.007 \ . \qquad (3)
\end{aligned}
$$

Enhancements have also been observed in $\Delta S = 1$ asymmetries (1) for low-mass $\pi^+\pi^-$ and K^+K^- pairs, $0.08\ \mathrm{GeV}^2/c^4 < m^2_{\pi^+\pi^-} < 0.66\ \mathrm{GeV}^2/c^4$, $m^2_{K^+\pi^-} < 15\ \mathrm{GeV}^2/c^4$ and $1.2\ \mathrm{GeV}^2/c^4 < m^2_{K^+K^-\,\mathrm{low}} < 2.0\ \mathrm{GeV}^2/c^4$, $m^2_{K^+K^-\,\mathrm{high}} < 15\ \mathrm{GeV}^2/c^4$ [21]:

$$
\begin{aligned}
A_{CP}(B^+ \to K^+(\pi^+\pi^-)_{\mathrm{low}\,m}) &= +0.678 \pm 0.078 \pm 0.032 \pm 0.007 \ , \\
A_{CP}(B^+ \to K^+(K^+K^-)_{\mathrm{low}\,m}) &= -0.226 \pm 0.020 \pm 0.004 \pm 0.007 \ . \qquad (4)
\end{aligned}
$$

The purpose of this Letter is to study these measured asymmetries theoretically, trying to understand their pattern within the CKM framework. While the inclusive asymmetries (1) and (2) require integration over an entire three-body phase space, the asymmetries (3) and (4) for localized regions of phase space may depend on resonance behavior dominating these regions. Thus different methods may have to be applied for analyzing total and localized asymmetries.

In Section II we show that total CP rate differences for pairs of $\Delta S = 0$ and $\Delta S = 1$ three-body B^+ decay processes, where final states are related to each other by a U-spin reflection, are equal in magnitudes and have opposite signs. We reiterate a general proof presented in Ref. [23], aimed mainly at pairs of two-body and quasi-two-body B and B_s decays (see also [24, 25, 26]), mentioning only briefly the pair $B^+ \to K^+\pi^+\pi^-$ and $B^+ \to \pi^+K^+K^-$. (Ref. [10] mentioned briefly a similar relation for $B^+ \to \pi^+\pi^+\pi^-$ and $B^+ \to K^+K^+K^-$.) Section III studies sources of enhanced CP asymmetries for localized regions of phase space involving low-mass $\pi^+\pi^-$ and K^+K^- pairs, pointing out a possible approximate asymmetry relation following from CPT, while Section IV concludes.

II U-spin relates $\Delta S = 0, 1$ CP rate asymmetries

The low-energy effective weak Hamiltonian describing $\Delta S = 1$ B decays is [27]

$$
\mathcal{H}_{\mathrm{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} \left[V^*_{ub}V_{us} \left(\sum_1^2 C_i(\mu) Q_i^{us} + \sum_3^{10} C_i(\mu) Q_i^s \right) + V^*_{cb}V_{cs} \left(\sum_1^2 C_i(\mu) Q_i^{cs} + \sum_3^{10} C_i(\mu) Q_i^s \right) \right] \ , \qquad (5)
$$

where $C_i(\mu)$ are scale-dependent Wilson coefficients and $Q_i^{qs} (q = u, c), Q_i^q$ are four-quark operators. Suppressing chiral and color structure of these operators and denoting quark charges by e_q, the flavor structure of the operators is given by

$$
\begin{aligned}
Q^{qs}_{1,2} &= \bar{b}q\bar{q}s \ , \ q = u, c \ , \\
Q^s_{3,..,6} &= \bar{b}s \sum_{q'=u,d,s,c} \bar{q}'q' \ , \\
Q^s_{7,..,10} &= \frac{3}{2}\bar{b}s \sum_{q'=u,d,s,c} e_{q'}\bar{q}'q' \ . \qquad (6)
\end{aligned}
$$

Focusing on U-spin properties of these twelve operators we note that, since u, c, b and $\bar{d}d + \bar{s}s$ are U-spin singlets, each of these operators represents an s ("down") component of a U-spin doublet operator, so that one can write in short

$$\mathcal{H}_{\rm eff}^{\Delta S=1} = V_{ub}^* V_{us}\, U^s + V_{cb}^* V_{cs}\, C^s \ , \tag{7}$$

where U and C are two independent U-spin doublet operators. Similarly, the $\Delta S = 0$ effective Hamiltonian $\mathcal{H}_{\rm eff}^{\Delta S=0}$, in which one replaces s by d in Eq. (5), involves d ("up") components of the same two U-spin doublet operators U and C multiplying different CKM factors, $V_{ub}^* V_{ud}$ and $V_{cb}^* V_{cd}$,

$$\mathcal{H}_{\rm eff}^{\Delta S=0} = V_{ub}^* V_{ud}\, U^d + V_{cb}^* V_{cd}\, C^d \ . \tag{8}$$

A very simple implication of Eqs. (7) and (8) is obtained by comparing two decay processes, $\Delta S = 1$ and $\Delta S = 0$, in which initial and final states are each other's U-spin reflections,

$$U_r : d \leftrightarrow s \ . \tag{9}$$

We write the $\Delta S = 1$ amplitude for a generic process $B \to f$ in the form

$$A(B \to f,\ \Delta S = 1) = V_{ub}^* V_{us}\, A_u + V_{cb}^* V_{cs}\, A_c \ , \tag{10}$$

where $A_u \equiv \langle f|U^s|B\rangle$ and $A_c \equiv \langle f|C^s|B\rangle$ are complex amplitudes involving CP-conserving phases. The $\Delta S = 0$ amplitude for the corresponding U-spin reflected process, $U_r B \to U_r f$, is then given by

$$A(U_r B \to U_r f,\ \Delta S = 0) = V_{ub}^* V_{ud}\, A_u + V_{cb}^* V_{cd}\, A_c \ , \tag{11}$$

where we used $\langle U_r f|U^d|U_r B\rangle = \langle f|U^s|B\rangle \equiv A_u$, $\langle U_r f|C^d|U_r B\rangle = \langle f|C^s|B\rangle \equiv A_c$.

In the case of three-body B^+ decays (where B^+ is invariant under U_r) the amplitudes in (10) and (11) depend on the same corresponding final particle momenta. For instance

$$\begin{aligned} A(B^+ \to K^+(p_1)\pi^+(p_2)\pi^-(p_3)) &= V_{ub}^* V_{us}\, A_u(p_1, p_2, p_3) + V_{cb}^* V_{cs}\, A_c(p_1, p_2, p_3) \ , \\ A(B^+ \to \pi^+(p_1)K^+(p_2)K^-(p_3) &= V_{ub}^* V_{ud}\, A_u(p_1, p_2, p_3) + V_{cb}^* V_{cd}\, A_c(p_1, p_2, p_3) \ . \end{aligned} \tag{12}$$

Applying CP-conjugation to (10) and (11), one has

$$\begin{aligned} A(\bar{B} \to \bar{f},\ \Delta S = -1) &= V_{ub} V_{us}^*\, \bar{A}_u + V_{cb} V_{cs}^*\, \bar{A}_c \ , \\ A(U_r \bar{B} \to U_r \bar{f},\ \Delta S = 0) &= V_{ub} V_{ud}^*\, \bar{A}_u + V_{cb} V_{cd}^*\, \bar{A}_c \ . \end{aligned} \tag{13}$$

Here $\bar{A}_{u,c} = A_{u,c}$ for two-body decays and $\bar{A}_{u,c} = A_{u,c}(-\vec{p}_1, -\vec{p}_2, -\vec{p}_3)$ for three-body decays. Thus

$$|A(B \to f)|^2 - |A(\bar{B} \to \bar{f})|^2 = 2\ {\rm Im}(V_{ub}^* V_{us} V_{cb} V_{cs}^*){\rm Im}(A_u^* A_c + \bar{A}_u^* \bar{A}_c) \ , \tag{14}$$

while

$$|A(UB \to Uf)|^2 - |A(U\bar{B} \to U\bar{f})|^2 = 2\ {\rm Im}(V_{ub}^* V_{ud} V_{cb} V_{cd}^*){\rm Im}(A_u^* A_c + \bar{A}_u^* \bar{A}_c) \ . \tag{15}$$

Table I: Branching fractions for three-body B^+ decays to charged pions and kaons [4].

Final state	Branching fraction (10^{-6})
$K^+\pi^+\pi^-$	51.0 ± 3.0
$K^+K^+K^-$	34.0 ± 1.0
$\pi^+\pi^+\pi^-$	15.2 ± 1.4
$\pi^+K^+K^-$	5.0 ± 0.7

Unitarity of the CKM matrix implies [28]

$$\mathrm{Im}(V^*_{ub}V_{us}V_{cb}V^*_{cs}) = -\mathrm{Im}(V^*_{ub}V_{ud}V_{cb}V^*_{cd}) \ , \tag{16}$$

leading to a general U-spin relation [23]

$$|A(B \to f)|^2 - |A(\bar{B} \to \bar{f})|^2 = -[|A(UB \to Uf)|^2 - |A(U\bar{B} \to U\bar{f})|^2] \ . \tag{17}$$

In the case of three-body B^+ decays one integrates this momentum-dependent amplitude relation over three-body phase space to obtain a corresponding relation between CP rate differences. Denoting rates by the final charged particles, the following relations are expected to hold in the U-spin symmetry limit:

$$\begin{aligned}\Gamma(\pi^-K^-K^+) - \Gamma(\pi^+K^+K^-) &= -[\Gamma(K^-\pi^-\pi^+) - \Gamma(K^+\pi^+\pi^-)] \ , \\ \Gamma(\pi^-\pi^-\pi^+) - \Gamma(\pi^+\pi^+\pi^-)] &= -[\Gamma(K^-K^-K^+) - \Gamma(K^+K^+K^-)] \ ,\end{aligned} \tag{18}$$

or

$$\frac{A_{CP}(B^+ \to \pi^+K^+K^-)}{A_{CP}(B^+ \to K^+\pi^+\pi^-)} = -\frac{\mathcal{B}(B^+ \to K^+\pi^+\pi^-)}{\mathcal{B}(B^+ \to \pi^+K^+K^-)} \ , \tag{19}$$

$$\frac{A_{CP}(B^+ \to \pi^+\pi^+\pi^-)}{A_{CP}(B^+ \to K^+K^+K^-)} = -\frac{\mathcal{B}(B^+ \to K^+K^+K^-)}{\mathcal{B}(B^+ \to \pi^+\pi^+\pi^-)} \ . \tag{20}$$

Branching fractions for the four three body B^+ decay modes involving charged pions and kaons are given in Table I [4]. Table II compares U-spin symmetry predictions for ratios of asymmetries using Eqs. (19) and (20) with the LHCb results (1) and in (2). U-spin predicts the two ratios of $\Delta S = 0$ and $\Delta S = 1$ asymmetries to be negative, as measured, and larger than one — inversely proportional to corresponding branching ratios.

The modest violation of U-spin seen in the top line of Table II (currently at 2.0σ) is not surprising given the very different resonant substructure of the two Dalitz plots. Such sources of U-spin breaking are absent in the successful prediction [26] of the large negative asymmetry ratio in $B_s \to K^-\pi^+$ and $B^0 \to K^+\pi^-$ [4],

$$\frac{A_{CP}(B_s \to K^-\pi^+)}{A_{CP}(B^0 \to K^+\pi^-)} = -\frac{\tau(B_s)}{\tau(B^0)}\frac{\mathcal{B}(B^0 \to K^+\pi^-)}{\mathcal{B}(B_s \to K^-\pi^+)} = -3.6 \pm 0.4 \ . \tag{21}$$

This prediction should be compared with the world-averaged asymmetries [4] recently updated by LHCb measurements [29],

$$A_{CP}(B^0 \to K^+\pi^-) = -0.082 \pm 0.006 \ , \quad A_{CP}(B_s \to K^-\pi^+) = 0.26 \pm 0.04 \ , \tag{22}$$

Table II: U-spin predictions for asymmetry ratios (19) and (20) compared with LHCb measurements (1) and (2).

Asymmetry ratio	U-spin prediction	LHCb result
$A_{CP}(B^+ \to \pi^+ K^+ K^-)/A_{CP}(B^+ \to K^+ \pi^+ \pi^-)$	-10.2 ± 1.5	-4.8 ± 2.3
$A_{CP}(B^+ \to \pi^+ \pi^+ \pi^-)/A_{CP}(B^+ \to K^+ K^+ K^-)$	-2.2 ± 0.2	-2.8 ± 1.0

implying $A_{CP}(B_s \to K^- \pi^+)/A_{CP}(B^0 \to K^+ \pi^-) = -3.2 \pm 0.5$ in good agreement with (21). The above prediction is subject to first order U-spin breaking [30, 31, 32, 33, 34] in the sum

$$\frac{A_{CP}(B_s \to K^- \pi^+)}{A_{CP}(B^0 \to K^+ \pi^-)} + \frac{\tau(B_s)}{\tau(B^0)} \frac{\mathcal{B}(B^0 \to K^+ \pi^-)}{\mathcal{B}(B_s \to K^- \pi^+)} = 0.4 \pm 0.6 \ , \tag{23}$$

which should be compared to each of the two terms in this sum whose magnitudes are each around 3 - 4.

III Asymmetries for low mass $\pi^+\pi^-$ and K^+K^- pairs

The strangeness-conserving CP asymmetries exhibited in Eq. (3) have two distinguishing features. (1) They are large, very close to maximal. The relative weak phase of the $\bar{b} \to \bar{d}$ penguin and the $\bar{b} \to \bar{u} u \bar{d}$ tree amplitude is γ, whose sine is very large [2]. (2) They are opposite in sign. The U-spin relations discussed in the previous Section would have then implied smaller asymmetries, but also of opposite signs, for restricted regions of phase space in the $|\Delta S| = 1$ transitions $B^+ \to K^+ \pi^+ \pi^-$ and $B^+ \to K^+ K^+ K^-$. While the opposite sign relation holds between Eqs. (3) and (4), the large value of $A_{CP}(B^+ \to K^+ (\pi^+\pi^-)_{\mathrm{low}\, m})$ exhibits sizeable U-spin breaking relative to $A_{CP}(B^+ \to \pi^+ (K^+K^-)_{\mathrm{low}\, m})$ due to different resonant structures of $\pi^+\pi^-$ and K^+K^-.

The Dalitz plot for $B^\pm \to \pi^\pm \pi^+ \pi^-$ contains a prominent ρ^0 band in the spectrum of the low-mass $\pi^+\pi^-$ pair. This band would be equally populated near both ends in the absence of interference with other partial waves. However, the extremity with high $m^2_{\pi^+\pi^- \ \mathrm{high}}$ is visibly depopulated in comparison with the extremity with low $m^2_{\pi^+\pi^- \ \mathrm{high}}$, strongly suggesting interference with a strong S-wave amplitude. This feature has led to the proposal that such interference is responsible for the pronounced CP asymmetry in the first Eq. (3) [35]. With a suitable relative strong phase, this mechanism could explain both the overall asymmetry and the fact that it is enhanced when selecting events with low $m^2_{\pi^+\pi^- \ \mathrm{low}}$ and high $m^2_{\pi^+\pi^- \ \mathrm{high}}$.

An attempt to account for strong phases through resonant substates is frustrated by incomplete information on the decays $B \to PS$, where P and S denote pseudoscalar and scalar mesons. A recent fit to these decays based on QCD factorization [36] reaches different conclusions depending on which scalar mesons are ascribed to a ${}^3P_0$ $q\bar{q}$ nonet and which are labeled as tetraquarks or mesonic molecules. Relative strong phases *are* available in fits to $B \to PV$ decays, where V denotes a vector meson [37].

To show that a CP asymmetry for $B^\pm \to \pi^\pm \pi^+ \pi^-$ as large as that in Eq. (3) is plausible, we supplement the Dalitz plot analysis of Ref. [18] with an amplitude corresponding to

$f_0(500)\pi^\pm$, where $f_0(500)$ is an S-wave $I=0$ $\pi\pi$ resonance assumed for present purposes to be a $q\bar{q}$ state. We may then relate a penguin amplitude for $B^\pm \to \pi^\pm f_0(500)$ via SU(3) to the penguin amplitude assumed to dominate $B^\pm \to \pi^\pm K_0^*(1430)$.

A $B \to PV$

Following Ref. [37] we may write the relevant amplitudes for $B^+ \to \rho^0\pi^+$ as follows:

$$\begin{aligned} \mathcal{A}(B^+ \to \rho^0\pi^+) &= -\frac{1}{\sqrt{2}}\left(t_V + c_P + p_V - p_P\right) , \\ &= -|\mathcal{P}| - |\mathcal{T}|\, e^{i(\delta_V\ +\ \gamma)} . \end{aligned} \tag{24}$$

The assumption in Ref. [37] is that $p_V = -p_P$ and these amplitudes are chosen to be real as shown in Fig. 2 in Ref. [37]. Also t_V and c_P have zero relative phase, while the relative strong and weak phases of t_V with respect to p_V are $\delta_V \sim -18^\circ$ and $\gamma \sim 65^\circ$. Under these conditions then:

$$\begin{aligned} |\mathcal{P}| &= \sqrt{2}|p_V| = \sqrt{2}|p_P| = \sqrt{2}\times 7.5\ \text{eV} = 1.06\times 10^{-5}\ \text{MeV} , \\ |\mathcal{T}| &= \frac{|t_V|+|c_P|}{\sqrt{2}} = \frac{1}{\sqrt{2}}(30.3+5.3)\ \text{eV} = 2.51\times 10^{-5}\ \text{MeV} . \end{aligned} \tag{25}$$

The CP-conjugate process will thus have the amplitude given as

$$\overline{\mathcal{A}}(B^- \to \rho^0\pi^-) = -|\mathcal{P}| - |\mathcal{T}|\, e^{i(\delta_V\ -\ \gamma)} . \tag{26}$$

In Ref. [37] the strong phase of p_P is taken to be zero and (as a consequence of the assumption $p_V = -p_P$) the strong phase of p_V is taken to be π. This is based on the relative P-wave amplitude between the final-state particles [38]. Following this reasoning, in the $B \to PS$ case, we take $\tilde{p}_S = \tilde{p}_P$.

Using the above we estimate the CP-averaged amplitude and CP asymmetry for the process as follows:

$$\begin{aligned} |\mathcal{A}|^2_{\text{avg}} &= \frac{|\overline{\mathcal{A}}|^2 + |\mathcal{A}|^2}{2} \\ &= \left(|\mathcal{P}|^2 + |\mathcal{T}|^2 + 2|\mathcal{P}||\mathcal{T}|\cos\delta_V\cos\gamma\right) \\ &= (30.9\ \text{eV})^2 \qquad (\mathcal{B}\ \sim\ 9\times 10^6\ [4]) , \end{aligned} \tag{27}$$

$$\begin{aligned} A_{\text{CP}} &= \frac{|\overline{\mathcal{A}}|^2 - |\mathcal{A}|^2}{|\overline{\mathcal{A}}|^2 + |\mathcal{A}|^2} \\ &= \frac{2|\mathcal{P}||\mathcal{T}|\sin\delta_V\sin\gamma}{|\mathcal{A}|^2_{\text{avg}}} \\ &= -0.16 . \end{aligned} \tag{28}$$

The CP asymmetry is very sensitive to changes in the strong phase δ_V. Its value in 2004, to which δ_V was fitted in Ref. [37], was -0.19 ± 0.11. While its slightly different current value [4] $0.18^{+0.09}_{-0.17}$ is not significantly inconsistent with (28), the central value favors $\delta_V \sim +18^\circ$.

B $B \to PS$

We consider $B \to PS$ decays with some inputs from Ref. [36]. The decay $B^+ \to K_0^*(1430)\pi^+$ is a pure penguin $|\Delta S| = 1$ process, and its amplitude may be written as

$$\mathcal{A}(B^+ \to K_0^*(1430)\pi^+) \;=\; \tilde{p}'_P \; . \tag{29}$$

We use the branching fraction quoted in Ref. [36] (average of Belle and BaBar): $\mathcal{B}(B^+ \to K_0^*(1430)\pi^+) = 45.1 \times 10^{-6}$, finding $|\tilde{p}'_P| = 7.2 \times 10^{-5}$ MeV. The corresponding contribution to a $\Delta S = 0$ process is then $\tilde{p}_P = \lambda|\tilde{p}'_P| = 1.66 \times 10^{-5}$ MeV, where we have used $\lambda \equiv \tan\theta_C = 0.23$.

Since $f_0(500)$ is a singlet, we assume that it is $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$. One can then write the amplitude representation for $B^+ \to f_0(500)\pi^+$ as follows:

$$\begin{aligned} \mathcal{A}(B^+ \to f_0(500)\pi^+) &= -\frac{1}{\sqrt{2}}(\tilde{p}_S + \tilde{p}_P - \tilde{t}_S - \tilde{c}_P) \; , \\ &= -\sqrt{2}\; |\tilde{p}_P|\; e^{i\delta_{f_0}} + |\tilde{T}|\; e^{i(\delta_S + \gamma)} \; , \end{aligned} \tag{30}$$

where we have assumed $\tilde{p}_S = \tilde{p}_P$. We have defined $\tilde{t}_S + \tilde{c}_P \equiv \sqrt{2}\,|\tilde{T}|e^{i(\delta_S+\gamma)}$, while δ_{f_0} is the relative strong phase between the penguin contributions in the amplitudes for ρ^0 and f_0 modes. $f_0(500)$ is a wide scalar resonance with mass close to 500 MeV and width close to 540 MeV. In the absence of any reliable estimate for a tree contribution in $B^+ \to f_0(500)\pi^+$, we assume that the amplitude for this process is penguin-dominated. A tree contribution in this process would lead to a 3-body asymmetry by interference with $\mathcal{P}$ which is seen in (25) to be suppressed relative to $\mathcal{T}$.

We may then predict the amplitude and branching fraction for the process:

$$\mathcal{A}(B^+ \to f_0(500)\pi^+) \;\sim\; -\sqrt{2}\; |\tilde{p}_P|\; e^{i\delta_{f_0}} \; , \tag{31}$$

$$|\mathcal{A}(B^+ \to f_0(500)\pi^+)| \;\sim\; \sqrt{2}\; |\tilde{p}_P| \;=\; 2.35 \times 10^{-5} \text{ MeV} \; , \tag{32}$$

$$\mathcal{B}(B^+ \to f_0(500)\pi^+) \;\sim\; 5.1 \times 10^{-6} \; . \tag{33}$$

Our estimate (33) is consistent with the current 90% confidence level upper limit, $\mathcal{B}(B^+ \to f_0(500)\,\pi^+, f_0 \to \pi^+\pi^-) < 4.1 \times 10^{-6}$ [17]. We note the comparable magnitudes of amplitudes $\mathcal{T}$ and $\sqrt{2}\,\tilde{p}_P$ in (25) and (32) which may lead by their interference to a very large asymmetry in $B^+ \to \pi^+\pi^-\pi^+$ for low-mass $\pi^+\pi^-$ pairs.

C $B^\pm \to \pi^\pm\pi^+\pi^-$ Dalitz plot

We will now use an isobar analysis to reproduce the $B^\pm \to \pi^\pm\pi^+\pi^-$ Dalitz plot for low-mass $\pi^+\pi^-$ pairs. In the isobar model with only two contributing components we can write the amplitude for the three-body process in terms of the constituent two-body processes as follows:

$$\mathcal{A}_{B^+\to\pi^+\pi^+\pi^-}(m^2_{\rm low}, m^2_{\rm high}) \;=\; c_\rho\; F_\rho(m^2_{\rm low}, m^2_{\rm high}) + c_{f_0}\; F_{f_0}(m^2_{\rm low}, m^2_{\rm high}) \; , \tag{34}$$

where the c's are complex isobar coefficients and F's are dynamical functions of momenta m^2_{low}, and m^2_{high} which respectively represent the higher and lower invariant masses of the two $\pi^+\pi^-$ pairs in the final state. Since $f_0(500)$ is a scalar we use a simple Breit-Wigner form for F_{f_0} with $m_{f_0} = 500$ MeV and $\Gamma_{f_0} = 540$ MeV. In case of the ρ^0 it is standard to use a more specific Gounaris-Sakurai form, as seen in Ref. [18]. We use the amplitudes in Eqs. (24) and (31) as the isobar coefficients for $B^+ \to \rho^0\pi^+$ and $B^+ \to f_0\pi^+$, respectively. An expression similar to (34) applies to the CP-conjugate process, $B^- \to \pi^-\pi^-\pi^+$, in which the isobar coefficient of $B^- \to \rho^0\pi^-$ is given by (26) while that of $B^- \to f_0\pi^-$ remains the same as (31). In our analysis, we use $\gamma = 65^\circ$ while the relative strong phase δ_{f_0} between the ρ^0 and f_0 modes is an unknown parameter.

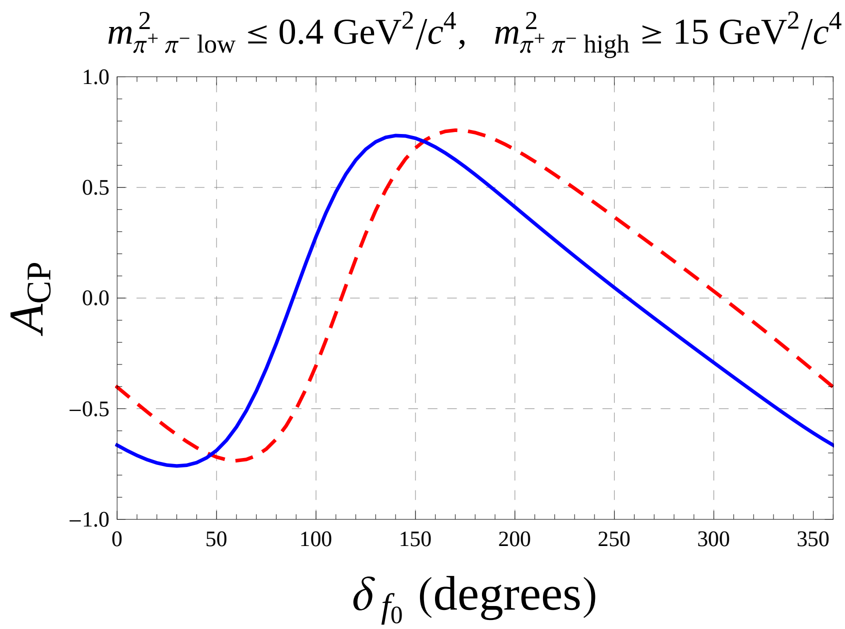

Figure 1: Dependence of $A_{CP}(B^+ \to \pi^+\pi^+\pi^0)$ with $m^2_{\pi^+\pi^-\,\mathrm{low}} < 0.4\ \mathrm{GeV}^2/c^4$ and $m^2_{\pi^+\pi^-\,\mathrm{high}} > 15\ \mathrm{GeV}^2/c^4$ on the strong phase δ_{f_0}, for $\delta_V = -18^\circ$ (solid) or 18° (dashed).

In Fig. 1 we plot A_{CP} for the Dalitz plot region $m^2_{\pi^+\pi^-\,\mathrm{low}} < 0.4\ \mathrm{GeV}^2/c^4$ and $m^2_{\pi^+\pi^-\,\mathrm{high}} > 15\ \mathrm{GeV}^2/c^4$ as a function of δ_{f_0} for $\delta_V = -18^\circ$ as found in the fit of Ref. [37] (solid) or for $\delta_V = 18^\circ$, as suggested by the discussion below Eq. (28) (dashed). For $\delta_V = -18^\circ$, a maximum CP asymmetry of nearly 0.75 is found for $\delta_{f_0} \simeq 140^\circ$, with A_{CP} exceeding 0.5 for a 75° range of δ_{f_0} about this value. The main effect of the reversal of the sign of δ_V is to shift the plot along the δ_{f_0} axis by an amount $2\delta_V$. As a consistency check, one can predict the dependence on δ_{f_0} of the CP asymmetry with $m^2_{\pi^+\pi^-\,\mathrm{low}} < 0.4\ \mathrm{GeV}^2/c^4$ and $m^2_{\pi^+\pi^-\,\mathrm{high}} < 15\ \mathrm{GeV}^2/c^4$. The result is shown in Fig. 2. The measured CP asymmetries for both ranges of $m^2_{\pi^+\pi^-\,\mathrm{high}}$ should be consistent with a single value of δ_{f_0}.

We have checked that Figs. 1 and 2 are not affected significantly by using a simple pole form for the $f_0(500)$ resonance, or by changing its mass and width parameters. We have also studied the effect of a decay amplitude to $f_0(980)\pi^+$ by varying its strength and strong phase relative to the amplitude for $f_0(500)\pi^+$. The effect on the asymmetry for $m^2_{\pi^+\pi^-,\,\mathrm{low}} < 0.4\,\mathrm{GeV}^2/c^4$ was found to be negligible due to the small overlap of the $f_0(980)$ resonance tail with this low invariant mass.

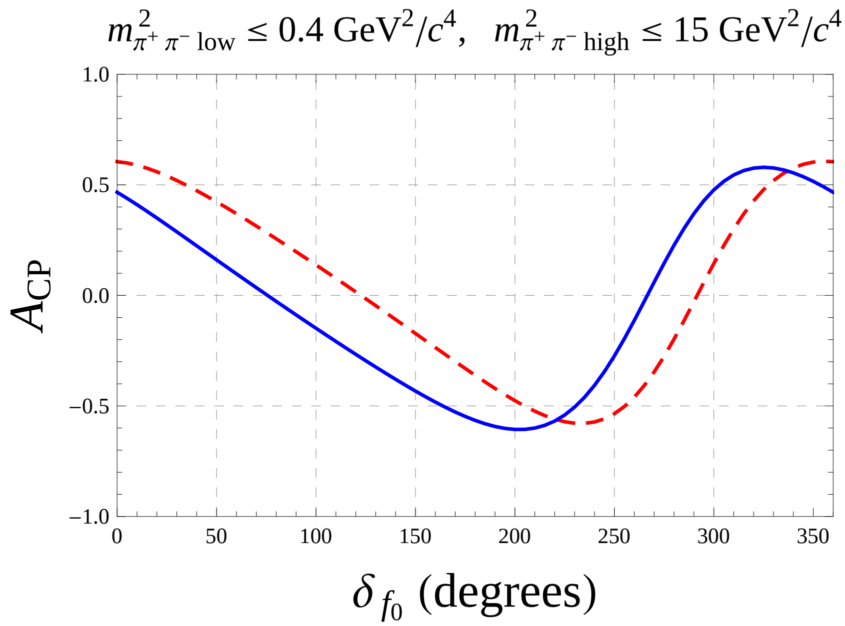

Figure 2: Same as Fig. 1 but for $m^2_{\pi^+\pi^- \text{ high}} < 15$ GeV$^2/c^4$

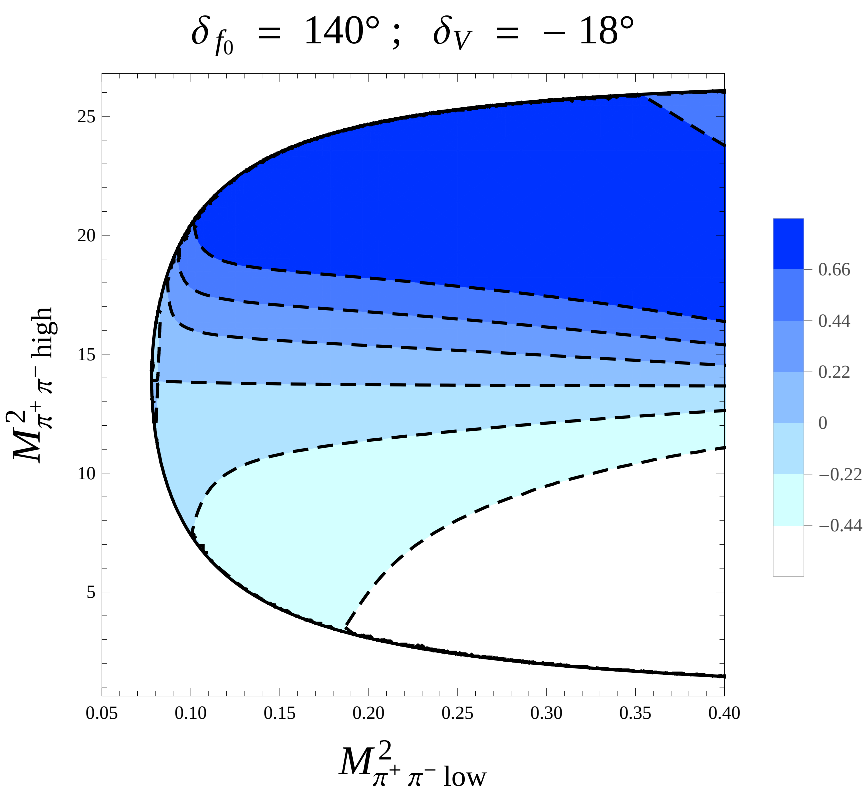

Figure 3: CP asymmetry $A_{CP}(B^+ \to \pi^+(\pi^+\pi^-)_{\text{low m}})$ for restricted region of the Dalitz plot, with $\delta_V = -18°$ and $\delta_{f_0} = 140°$.

As we are introducing only the $\rho\pi$ and $f_0(500)\pi$ final states, we should expect to reproduce only the region of the Dalitz plot with low $m^2_{\rm low}$. The values of $A_{CP}(B^+ \to \pi^+(\pi^+\pi^-)_{\rm low\ m})$ are shown in Fig. 3 for $m^2_{\rm low} \leq 0.4\,{\rm GeV}^2/c^4$, with the choice of parameters $\delta_V = -18^\circ$ and $\delta_{f_0} = 140^\circ$. For this specific choice, the CP asymmetry for very low $m^2_{\rm low}$ is strongly positive for $m^2_{\rm high} > 15$ GeV$/c^2$ and mostly negative for $m^2_{\rm high} < 15$ GeV$/c^2$.

D $B^\pm \to \pi^\pm K^+ K^-$

For the $\Delta S = 0$ decays $B^\pm \to \pi^\pm K^+ K^-$, the B^+ decay shows a prominent feature in $m^2_{K^+K^-}$ between threshold and 1.5 GeV$^2/c^4$. This feature is greatly suppressed in the B^- decay, accounting for the large CP asymmetry in the second of Eqs. (3). The high K^+K^- threshold means that there can be no counterpart of the (on-shell) ρ^0–S-wave interference accounting for the CP asymmetry in $B^\pm \to \pi^\pm\pi^+\pi^-$, and information on P-wave K^+K^- resonant amplitudes above 1 GeV$/c^2$ is fragmentary. The strong phases in the case of $B^\pm \to \pi^\pm K^+ K^-$ can be completely distinct from those in $B^\pm \to \pi^\pm\pi^+\pi^-$, and seem so.

E $B^\pm \to K^\pm \pi^+\pi^-$

The Dalitz plot of $B^+ \to K^+\pi^+\pi^-$ for the low-mass region, $m^2(\pi^+\pi^-) < 0.66\,{\rm GeV}^2/c^4$, contains a ρ^0 band which involves a CP asymmetry [4] $A_{CP}(B^+ \to K^+\rho^0) = 0.37 \pm 0.11$. Other contributions observed in this low-mass region are a nonresonant term and contributions from $K^+ f_0(980)$ and $K^+ f_2(1270)$ [11, 12]. A calculation of the asymmetry for this restricted region, which may account for the large positive asymmetry in the first of Eqs. (4), is beyond the scope of this Letter.

F CPT argument

CPT implies equal partial decay widths for a particle and its antiparticle for a closed set of final states connected among themselves by final state interactions [39]. A simple observation may explain the opposite signs of the asymmetries in (3) if the decays $B^\pm \to \pi^\pm X$ for low $M(X)$ are saturated by $X = \pi^+\pi^-, K^+K^-$. Assuming that the CPT theorem holds locally where rescattering occurs only between these states, an asymmetry in the first of Eqs. (3) must be compensated by an asymmetry in the second. This ignores the effects of neutral pairs in X and of possible multi-particle realizations of X, but it serves at least as a qualitative guide.

The low-mass $\pi^+\pi^-$ and K^+K^- channels are relatively self-contained, with the only important rescatterings involving charge exchange and $\pi\pi \leftrightarrow K\bar{K}$. Rescattering to multi-particle final states occurs only at typical subenergies > 1.6 GeV [40, 41, 42, 43]. The processes giving rise to pairs of final-state neutral particles may be amenable to evaluation using chiral perturbation theory [44]. Thus the asymmetries in $B^\pm \to \pi^\pm\pi^+\pi^-$ and $B^\pm \to \pi^\pm K^+K^-$ Dalitz plots with low $M(\pi^+\pi^-)$ and $M(K^+K^-)$, respectively, could be related to one another through rescattering. The presence of symmetry breaking leads to different thresholds for $\pi\pi$ and $K\bar{K}$ pairs, and imposing different cutoffs on their invariant mass is expected to affect this relation. This is demonstrated by the asymmetries measured in $B^\pm \to K^\pm X$ given in Eq. (4).

IV Conclusion

We have examined the CP asymmetries in three-body decays of $B^{\pm}$ mesons to charged pions and kaons. Predictions of ratios of asymmetries on the basis of U-spin are seen to be obeyed qualitatively, with violations ascribable to resonant substructure differing for $\pi^+\pi^-$ and K^+K^- substates. Larger CP asymmetries for regions of the Dalitz plot involving low effective mass of these substates can be undertood qualitatively in terms of large final-state strong phases; the weak phases are conducive to such large asymmetries, being nearly maximal. We conclude that further resolution of this problem must rely either on a deeper understanding of the resonant substructure in $B \to PPP$ decays, or further understanding of the hadronization process independently of resonances. We have argued that the approximately equal magnitudes and opposite signs measured for asymmetries in $B^+ \to \pi^+\pi^+\pi^-$ and $B^+ \to K^+\pi^+\pi^-$ may follow from the closure of low-mass $\pi^+\pi^-$ and K^+K^- channels involving only $\pi\pi \leftrightarrow K\bar{K}$ rescattering.

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