Statistics for life insurance with R

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1. Introduction to the lifecontingencies R package

The lifecontingencies R package (Spedicato, 2013) is an actuarial package that performs life insurance statistical and mathematical calculations. Here, we present a summary of the most important functions of this package with many examples and exercises (with solutions).
1.1. Creating lifetable objects

In order to calculate survival and death probabilities with lifecontingencies, the first thing that we have to do is to install and load the lifecontingencies R package:

```r
#install.packages("lifecontingencies")
library(lifecontingencies)
```

## Package: lifecontingencies
## Authors: Giorgio Alfredo Spedicato [aut, cre]
##     (<https://orcid.org/0000-0002-0315-8888>),
##   Christophe Dutang [ctb] (<https://orcid.org/0000-0001-6732-1501>),
##   Reinhold Kainhofer [ctb] (<https://orcid.org/0000-0002-7895-1311>),
##   Kevin J Owens [ctb],
##   Ernesto Schirmacher [ctb],
##   Gian Paolo Clemente [ctb] (<https://orcid.org/0000-0001-6795-4595>),
##   Ivan Williams [ctb]
## Version: 1.3.8
## Date:     2022-01-07 13:53:00 UTC
## BugReport: https://github.com/spedygiorgio/lifecontingencies/issues

Then, the second step is to create a lifetable object, that will contain the life table that we are going to use. Life tables collect the annual probabilities of survival and/or death by age and gender, as well as additional biometric functions, such as the life expectancy. To create a lifetable we need two basic elements:

1) A sequency of ages 0, 1, ..., \( \omega \), where \( \omega \) is the terminal age.
2) The vector \( l_x \) (number of individuals alive at the beginning of age \( x \)), or survival/mortality rates (also by age).

There are three ways how to create a lifetable:

1) Directly from \( x \) and vector \( l_x \).
2) Importing \( x \) and \( l_x \) from an existing data.frame.
3) By using raw mortality/survival probabilities.

Let's see some examples of each of them:

1) Directly from \( x \) and vector \( l_x \)

```r
x1<-seq(from=0, to=19, by =1)
lx1<-c(1000,965,950,914,850,768,740,705,680,634,600,586,550,487,400,367,310,256,139,50)
Lt<-new("lifetable",x=x1, lx=lx1,name="Example of life table")
print(Lt)
```

## Life table Example of life table
##
## x  lx  px  ex
## 1 0 1000 0.965000 10.951000
## 2 1 965 0.984456 10.3481865
<p>| | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
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<td>7</td>
<td>705</td>
<td>0.9645390</td>
<td>7.1758865</td>
</tr>
<tr>
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<td>634</td>
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<td>5.9069401</td>
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<tr>
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<td>10</td>
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<td>5.2416667</td>
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<tr>
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<tr>
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<td>18</td>
<td>139</td>
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</tr>
</tbody>
</table>

We see that $p_x$ is obtained by calculating $l_{x+1}/l_x$, therefore $p_0 = 965/1000 = 0.965$, and so on. The last column is the residual life expectancy (column $e_x$, that represent the expected number of years that a person aged $x$ is going to survive). It can be obtained as:

$$e_0 = \frac{(965+950+...+50)}{1000} = 10.951,$$

$$e_1 = \frac{(950+914+...+50)}{965} = 10.348,$$

and so on.

2) By using an existing data.frame (the most frequent situation). The lifecontingencies package has some data.frames that let us to create life tables (see more details in Spedicato, 2013):

<table>
<thead>
<tr>
<th>Data</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF92Lt</td>
<td>Life table UK AF92 (United Kingdom, Females, 1992)</td>
</tr>
<tr>
<td>AM92Lt</td>
<td>Life table UK AM92 (United Kingdom, Males, 1992)</td>
</tr>
<tr>
<td>demoChina</td>
<td>Mortality rates of China downloaded from the website of SOA (Society of Actuaries)</td>
</tr>
<tr>
<td>demoIta</td>
<td>Italian life tables, such as RG48 or IPS55</td>
</tr>
<tr>
<td>demoJapan</td>
<td>Mortality rates of Japan downloaded from the website of SOA</td>
</tr>
<tr>
<td>demoUsa</td>
<td>US Social Security life tables</td>
</tr>
<tr>
<td>demoFrance</td>
<td>French life tables, 1990 and 2002</td>
</tr>
<tr>
<td>soa08</td>
<td>Illustrative SOA life table</td>
</tr>
<tr>
<td>soa08Act</td>
<td>Illustrative SOA actuarial table</td>
</tr>
</tbody>
</table>

We create a lifetable from demoUsa:
data("demoUsa", package="lifecontingencies")
is.data.frame(demoUsa)
## [1] TRUE
US07Male<-demoUsa[,c("age","USSS2007M")]
names(US07Male)<-c("x","lx")
US07MaleLt<-as(US07Male,"lifetable")
print(US07MaleLt)
## Life table COERCED
##
##        x     lx        px        ex
## 1     0 100000 0.9926200 74.881620
## 2     1  99262 0.9995064 74.438355
## 3     2  99213 0.996875 73.475119
## 4     3  99182 0.997580 72.498084
## 5     4  99158 0.997983 71.515632
## 6     5  99138 0.998184 70.530059
## 7     6  99120 0.998386 69.542867
## 8     7  99104 0.998486 68.554095
## 9     8  99089 0.998587 67.564472
## 10    9  99075 0.998991 66.574020
## 11    10 99065 0.999092 65.580740
## 12    11 99056 0.999091 64.586698
## 13    12 99047 0.998486 63.592567
## 14    13 99032 0.997476 62.602199
## 15    14 99007 0.996061 61.618007
## 16    15 98968 0.994342 60.642288
## 17    16 98912 0.992822 59.676622
## 18    17 98841 0.991198 58.719489
## 19    18 98754 0.989874 57.771219
## 20    19 98654 0.988546 56.829779
## 21    20 98541 0.987112 55.894947
## 22    21 98414 0.985876 54.967078
## 23    22 98275 0.985042 54.044823
## 24    23 98128 0.984918 53.125785
## 25    24 97980 0.985099 52.206032
## 26    25 97834 0.985588 51.283940
## 27    26 97693 0.985874 50.357958
## 28    27 97555 0.986059 49.429194
## 29    28 97419 0.986142 48.498199
## 30    29 97284 0.985918 47.565499
## 31    30 97147 0.985795 46.632577
## 32    31 97009 0.985465 45.698915
## 33    32 96868 0.985134 44.765433
## 34    33 96724 0.984699 43.832079
## 35    34 96576 0.984158 42.899250
## 36    35 96423 0.983510 41.967321
## 37    36 96264 0.982548 41.036639
## 38    37 96096 0.981581 40.108381
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<td>87</td>
<td>25024</td>
<td>0.8705643</td>
<td>4.392543</td>
</tr>
</tbody>
</table>
US07MaleLt@name<-"USA MALES 2007"
getOmega(US07MaleLt)

## [1] 111

The function getOmega returns the terminal age of the life table, that is to say, the maximum age of the life table.

3) By using raw survival/mortality probabilities

#we create a vector with the survival probabilities
probsup<-seq(0.9,0,by=-0.01)
probsup

## [1] 0.90 0.89 0.88 0.87 0.86 0.85 0.84 0.83 0.82 0.81 0.80 0.79 0.78 0.77 0.76
## [16] 0.75 0.74 0.73 0.72 0.71 0.70 0.69 0.68 0.67 0.66 0.65 0.64 0.63 0.62 0.61
## [31] 0.60 0.59 0.58 0.57 0.56 0.55 0.54 0.53 0.52 0.51 0.50 0.49 0.48 0.47 0.46
## [46] 0.45 0.44 0.43 0.42 0.41 0.40 0.39 0.38 0.37 0.36 0.35 0.34 0.33 0.32 0.31
## [61] 0.30 0.29 0.28 0.27 0.26 0.25 0.24 0.23 0.22 0.21 0.20 0.19 0.18 0.17 0.16
## [76] 0.15 0.14 0.13 0.12 0.11 0.10 0.09 0.08 0.07 0.06 0.05 0.04 0.03 0.02 0.01
## [91] 0.00
# with the function probs2lifetable we create the life table

```r
LifeT <- probs2lifetable(probsup, type = "px", name = "Life table obtained by using raw probabilities")

LifeT
```

## Life table obtained by using raw probabilities

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<th>lx</th>
<th>px</th>
<th>ex</th>
</tr>
</thead>
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<td>0.90</td>
<td>5.84474214</td>
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<td>7.048800e+03</td>
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## 90  89  1.485716e-36  0.01  0.01000000
1.2. Creating an actuarialtable object

Once we create a lifetable, we can also create an actuarialtable. An actuarialtable is similar to a lifetable, but it also incorporates an interest rate that will be necessary in order to calculate the actuarial present value (APV) of life insurance products. As an example, here we show how to create an actuarialtable by using an interest rate of 3 percent. Columns \( D_x \), \( N_x \), \( C_x \) and \( M_x \) are specific functions of \( l_x \) and the interest rate (see more details in Spedicato, 2013):

```r
US07MaleAct<-new("actuarialtable",x=US07MaleLt@x,lx=US07MaleLt@lx,interest=0.03,name="USA MALES 2007 ACT TABLE")
print(US07MaleAct)
```

```
## Actuarial table USA MALES 2007 ACT TABLE interest rate 3 %
##
##       x  l1x  Dx  Nx  Cx  Mx
## 1     0 100000 3.000556e+06 716.50485437 1.260516e+04 7.864341e+05
## 2     1  99262 2.900556e+06 46.18719955 1.188865e+04 7.838290e+05
## 3     2  99213 2.804185e+06 28.36939144 1.184247e+04 7.619403e+05
## 4     3  99182 2.710668e+06 21.32368915 1.181410e+04 7.500979e+05
## 5     4  99158 2.619902e+06 17.25217569 1.179277e+04 7.382838e+05
## 6     5  99138 2.531801e+06 15.07471662 1.176045e+04 7.264910e+05
## 7     6  99120 2.446284e+06 13.00946418 1.174744e+04 7.147155e+05
## 8     7  99104 2.363273e+06 11.84113851 1.174744e+04 7.092955e+05
## 9     8  99089 2.282692e+06 10.72983425 1.173560e+04 6.912076e+05
## 10    9  99075 2.204470e+06  7.44093915 1.172487e+04 6.794720e+05
## 11    10 99065 2.128538e+06  6.50179149 1.171742e+04 6.677471e+05
## 12    11 99056 2.054824e+06  6.31248248 1.171092e+04 6.560297e+05
## 13    12 99047 1.983264e+06  6.10231217 1.170461e+04 6.443188e+05
## 14    13 99032 1.913794e+06  5.89231407 1.169440e+04 6.326142e+05
## 15    14 99007 1.846358e+06  5.68261595 1.167787e+04 6.209198e+05
## 16    15 98968 1.78003e+06  5.47304860 1.165284e+04 6.092419e+05
## 17    16 98912 1.717379e+06  5.2636765  1.161794e+04 5.924199e+05
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<td>3.758963e-02</td>
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</tr>
</tbody>
</table>

Note that it is possible to create a data.frame from an actuarialtable, simply by doing the following:

```R
US07MaleActDf <- as(US07MaleAct, "data.frame")
```
1.3. Basic functions for the calculation of probabilities

In this table there are the basic functions available in lifecontingencies for calculating survival and mortality probabilities (you can check the complete list of functions in Spedicato, 2013).

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>dxt</td>
<td>number of deaths between x and x+t</td>
</tr>
<tr>
<td>pxt</td>
<td>survival probability of an individual aged x between x and x+t</td>
</tr>
<tr>
<td>qxt</td>
<td>probability of death for an individual aged x between x and x+t</td>
</tr>
<tr>
<td>mxt</td>
<td>central mortality rate</td>
</tr>
<tr>
<td>exn</td>
<td>life expectancy between x and x+n</td>
</tr>
</tbody>
</table>

Here there are some examples, where we use the lifetable US07MaleLt:

```r
# probability that an individual aged 20 survives until 21
pxt(US07MaleLt, 20, 1)
## [1] 0.9987112

# probability that an individual aged 30 dies in next 2 years
qxt(US07MaleLt, 30, 2)
## [1] 0.002871936

# probability that an individual aged 30 dies between age 33 and 35
pxt(US07MaleLt, 30, 3) * qxt(US07MaleLt, 33, 2)
## [1] 0.003098397
```

1.4. Assumptions for fractional ages

The functions pxt, pxyzt, qxt and qxyzt can be used with a specific argument called fractional (that can be equal to linear, hyperbolic or constant force) that let us to calculate probabilities for fractional ages or periods (see more details in Ayuso et al., 2007 and Spedicato, 2013). The assumptions are:

a) Linear: Linear interpolation between consecutive ages (assume a uniform distribution for the yearly deaths). In that case, \( l_{x+t} = l_x - t \cdot d_x \), therefore, \( t \cdot q_x = t \cdot q_x \). This is the default assumption.

b) Hyperbolic: Also called Balducci assumption or harmonic interpolation. In that case, \( l_{x+t} = (l_x \cdot l_{x+1})/(t \cdot l_x + (1-t) \cdot l_{x+1}) \) and therefore, \( t \cdot q_x = (t \cdot q_x)/(1 - (1-t) \cdot q_x) \).

c) Constant force of mortality: It assumes a constant force of mortality, also known as exponential interpolation. In that case, \( t \cdot q_x = 1 - \exp(-t \cdot \mu_x) \) where
\( \mu_x \) is the force of mortality, that is assumed to be constant between age \( x \) and \( x + t \). Based on that, for \( t = 0.5 \), it can be proved that \( t q_x = 1 - \sqrt{p_x} \).

Examples:

```r
# example linear
qxt(US07MaleLt, 20, t=0.5)
## [1] 0.0006444018
0.5*qxt(US07MaleLt, 20, t=1)
## [1] 0.0006444018

# example hyperbolic
qxt(US07MaleLt, 20, t=0.5, "hyperbolic")
## [1] 0.0006448173
0.5*qxt(US07MaleLt, 20, t=1)/(1-0.5*qxt(US07MaleLt, 20, t=1))
## [1] 0.0006448173

# example constant force
qxt(US07MaleLt, 20, t=0.5, "constant force")
## [1] 0.0006446096
1-sqrt(pxt(US07MaleLt, 20, t=1))
## [1] 0.0006446096
```

1.5. Exercises

**Exercise 1:** Create a lifetable R object called `FranceH02Lt` from the lifetable `demoFrance` (columns `age` and `TH00_02`) available in the package `lifecontingencies`. The lifetable must be called `FranceH02Lt` and must have the label "France Males 2002".

By using this table, you will calculate:

a) Life expectancy of a men aged 35.

b) Probability that an insured aged 35 would die in next 5 years.

c) Probability that an insured aged 35 reaches age 40.

d) Probability that an insured aged 35 dies at the age of 40. Make the calculation in three different ways.

e) Probability that an insured aged 35 dies between 40 and 50 years old.

f) Probability that an insured aged 35 dies in next 9 months (constant force assumption).
Solution

data(demoFrance)
FranceH02<-demoFrance[,c("age","TH00_02")]
names(FranceH02)<-c("x","lx")
FranceH02Lt<-as(FranceH02,"lifetable")
FranceH02Lt@name<="France Males 2002"

#a) e35
exn(FranceH02Lt,35)
## [1] 41.58141

#b) 5q35
qxt(FranceH02Lt,x=35,t=5)
## [1] 0.009048936

#c) 5p35
1-qxt(FranceH02Lt,x=35,t=5)
## [1] 0.9909511

# or
pxt(FranceH02Lt,x=35,t=5)
## [1] 0.9909511

#d) 5/35
pxt(FranceH02Lt,x=35,t=5)*qxt(FranceH02Lt,x=40,t=1)
## [1] 0.002344497

# or
pxt(FranceH02Lt,x=35,t=5)-pxt(FranceH02Lt,x=35,t=6)
## [1] 0.002344497

# or
qxt(FranceH02Lt,x=35,t=6)-qxt(FranceH02Lt,x=35,t=5)
## [1] 0.002344497

#e) 5/10q35
pxt(FranceH02Lt,x=35,t=5)*qxt(FranceH02Lt,x=40,t=10)
## [1] 0.03735771

#f) 0.75q35
qxt(FranceH02Lt,x=35,t=0.75, "constant force")
## [1] 0.001149332
Exercise 2: By using the lifetable US07MaleLt that we have just created, make the following calculations:

a) Probability that an insured aged 55 reaches age 63.

b) Probability that an insured aged 55 dies in next 6 months (linear assumption).

c) Probability that an insured aged 55 dies in his first year of retirement (that starts at age 65).

d) Probability that an insured aged 55 dies in the first three years of retirement.

# Solution

#a) 8p55
pxt(US07MaleLt, x=55, t=8)
## [1] 0.9198051

#b) 0.5q55
qxt(US07MaleLt, x=55, t=0.5)
## [1] 0.003987902

#c) 10/q55
pxt(US07MaleLt, x=55, t=10)*qxt(US07MaleLt, x=65, t=1)
## [1] 0.01493223

#d) 10/3q55
pxt(US07MaleLt, x=55, t=10)*qxt(US07MaleLt, x=65, t=3)
## [1] 0.04786602

2. Calculating two-lives probabilities with lifecontingencies

This section focuses on the calculation of two-lives survival and death probabilities with lifecontingencies. The first thing that we have to do is to create joint life tables.

2.1. Creating joint life tables with lifecontingencies

A joint life table is a list object built with individual life tables (for example, one for man and another for woman). Example:

US07Female<-demoUsa[,c("age","USSS2007F")]
names(US07Female)<-c("x","lx")
US07FemaleLt<-as(US07Female,"lifetable") # it is a life table object
US07FemaleLt@name<="USA FEMALES 2007"
US07List<-list(US07MaleLt,US07FemaleLt) #joint life table
2.2. Basic functions for calculating two-lives probabilities

Once the joint life table is created, we can use the following basic functions for calculating two-lives probabilities with `lifecontingencies`:

<table>
<thead>
<tr>
<th>Función</th>
<th>Descripción</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pxyt$</td>
<td>joint-life probability (two lives)</td>
</tr>
<tr>
<td>$pxyzt$</td>
<td>joint-life probability (more than two lives)</td>
</tr>
<tr>
<td>$qxyt$</td>
<td>probability of death (two lives)</td>
</tr>
<tr>
<td>$qxyzt$</td>
<td>probability of death (more than two lives)</td>
</tr>
</tbody>
</table>

All these functions have a specific argument called `status`. The `status` let us indicate if we want to calculate the joint-life, last-survivor, etc.. probabilities. Note that the joint status exists while all the members of the group are alive, while the last status exists until the last member of the group dies. Specifically, we have that:

- $pxyt$ with `status = joint` let us calculate the joint-life probability (in the Spanish terminology, called “supervivencia conjunta” or “no disolución”).
- $pxyt$ with `status = last` let us calculate the non last-survivor probability (in Spanish, called “al menos uno vive” or “no extinción”).
- $qxyt$ with `status = joint` let us calculate the non joint-life probability (in Spanish called “disolución”).
- $qxyt$ with `status = last` let us calculate the last-survivor probability (in Spanish called “extinción”).

By default, `status` is equal to `joint`.

Examples:

```r
# joint-life probability of a couple (man aged 55, woman aged 50) during next 3 years
pxyzt(US07List, x=c(55, 50), t=3)
## [1] 0.9642736

# non last-survivor probability of a couple (man aged 55, woman aged 50) during next 3 years
pxyzt(US07List, x=c(55, 50), t=3, status="last")
## [1] 0.9997322

# non joint-life probability of a couple (man aged 55, woman aged 50) during next 3 years
qxyzt(US07List, x=c(55, 50), t=3)
## [1] 0.03572644
```
# Last-survivor probability of a couple (man aged 55, woman aged 50) during next 3 years

\[ q_{x+y+z}(US07List, x=c(55, 50), t=3, \text{status} = \text{"last"}) \]

## [1] 0.0002678365

It is clear that the non joint-life probability is complementary to the joint-life probability, therefore:

\[ 1 - p_{x+y+z}(US07List, x=c(55, 50), t=3) \]

## [1] 0.03572644

and

\[ q_{x+y+z}(US07List, x=c(55, 50), t=3) \]

## [1] 0.03572644

give the same result.

On the other hand, the last-survivor probability is complementary to the non last survivor probability, therefore:

\[ 1 - q_{x+y+z}(US07List, x=c(55, 50), t=3, \text{status} = \text{"last"}) \]

## [1] 0.9997322

equals

\[ p_{x+y+z}(US07List, x=c(55, 50), t=3, \text{status} = \text{"last"}) \]

## [1] 0.9997322

## 2.3. Exercises

**Exercise 1:** By using the table **usa07List** calculate the joint-life probability of a couple (man aged 65 and woman aged 63), during next two years. Do the calculation also by using the survival probabilities of man and woman and check that you get the same result.

#solution

\[ p_{x+y+z}(US07List, x=c(65, 63), t=2) \]

## [1] 0.9472972

\[ p_{x}(US07MaleLt, 65, 2) * p_{x}(US07FemaleLt, 63, 2) \]  

#same result

## [1] 0.9472972

**Exercise 2:** By using the table **usa07List** calculate the non last-survivor probability (at least one is alive) of a couple (man aged 65 and woman aged 63), during next 2
years. Do the calculation in three different ways and check that you get the same result.

#solution
pxyzt(US07List, x=c(65,63), t=2, status="last")
## [1] 0.9993508

#we get the same result
pxt(US07MaleLt, 65, 2)*pxt(US07FemaleLt, 63, 2)+qxt(US07MaleLt, 65, 2)*pxt(US07FemaleLt, 63, 2)+
    +pxt(US07MaleLt, 65, 2)*qxt(US07FemaleLt, 63, 2)
## [1] 0.9993508

#we get the same result
1-qxyzt(US07List, x=c(65,63), t=2, status="last")
## [1] 0.9993508

Exercise 3: By using the table usa07List calculate the last-survivor probability of a couple (man aged 65 and woman aged 63), during next 2 years. Do the calculation in two different ways and check that you get the same result.

#solution
qxyzt(US07List, x=c(65,63), t=2, status="last") #last-survivor
## [1] 0.0006492309

qxt(US07MaleLt, 65, t=2)*qxt(US07FemaleLt, 63, t=2) #both die, same result
## [1] 0.0006492309

Exercise 4: By using the table usa07List calculate the non joint-life (at least one die) probability of a couple (man aged 65 and woman aged 63), during next 2 years.

#solution
qxyzt(US07List, x=c(65,63), t=2, status="joint")
## [1] 0.05270281

1-pxyzt(US07List, x=c(65,63), t=2) #same result
## [1] 0.05270281

Exercise 5: Do the following calculations for a couple (man aged 45, woman aged 42) by using the life table US07List:

a) Joint-life probability of the couple during next 12 years.

b) Non joint-life probability (at least one die) of the couple during next 12 years. Do the calculation in two different ways.
c) Last-survivor probability (all die) of the couple during next 12 years. Do the
calculation in two different ways.

d) Probability that exactly one member of the couple survives after 12 years. Do
the calculation in two different ways.

e) Probability that both members of the couple survive 12 years and after that, at
least one of them dies in the following year.

f) Probability that the couple survive 12 years and after that, both of them die in
the following year.

g) Deferred last-survivor probability of the couple (deferral = 12 years, period = 1
year). Do the calculation in two different ways.

#a) 12p45_42
pxyzt(US07List, x=c(45, 42), t=12)
## [1] 0.9016853

#b) 12q45_42
1 - pxyzt(US07List, x=c(45, 42), t=12)
## [1] 0.09831468

# or
qxyzt(US07List, x=c(45, 42), t=12, status="joint")
## [1] 0.09831468

#c) 12q45_42-
qxyzt(US07List, x=c(45, 42), t=12, status="last")
## [1] 0.002210673

# or
qxt(US07MaleLt, x=45, t=12)*qxt(US07FemaleLt, x=42, t=12)
## [1] 0.002210673

#d) 12q45_42 [1]
qxyzt(US07List, x=c(45, 42), t=12, status="joint") - qxyzt(US07List, x=c(45, 42), t=12, status="last")
## [1] 0.09610401

# or
qxt(US07MaleLt, x=45, t=12)*pxt(US07FemaleLt, x=42, t=12) + 
pxt(US07MaleLt, x=45, t=12)*qxt(US07FemaleLt, x=42, t=12)
## [1] 0.09610401
3. Calculating the actuarial present value APV with lifecontingencies (one life)

In this section, we will see how to calculate the actuarial present value (APV) of life insurance contracts, where the payment of the benefit depends on the survival of just one individual.

3.1. Basic functions for calculating the APV

In the next table, we show the basic functions for calculating the actuarial present value (APV) of life insurance contracts (one life) with lifecontingencies.

<table>
<thead>
<tr>
<th>Funcion</th>
<th>Descripcion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axn</td>
<td>one-life insurance, pays 1 m.u. at the end of the year of death</td>
</tr>
<tr>
<td>AExn</td>
<td>n-year endowment, it pays 1 m.u. at the end of the year of death or after n years, if the insured survives</td>
</tr>
<tr>
<td>Exn</td>
<td>pure endowment, it pays 1 m.u. after a period of n years, if the insured survives</td>
</tr>
</tbody>
</table>
### Función Descripción

<table>
<thead>
<tr>
<th>Función</th>
<th>Descripción</th>
</tr>
</thead>
<tbody>
<tr>
<td>IAxn</td>
<td>increasing life insurance contract (it starts paying 1 m.u. if death occurs in the first year of coverage and after that, the benefit is increased by 1 m.u. every year)</td>
</tr>
<tr>
<td>DAxn</td>
<td>decreasing life insurance (it starts paying n m.u. if death occurs in the first year of coverage and after that, the benefit is decreased by 1 m.u. every year)</td>
</tr>
</tbody>
</table>

Some examples:

**Example 1:** By using the table `soa08Act` calculate the APV of a contract that pays 1 m.u. at the end of the death year. The insured is 25 years old and the coverage period is 40 years.

```r
data(soa08Act, package="lifecontingencies")
Axn(actuarialtable=soa08Act, x=25, n=40)
## [1] 0.04797088
```

**Example 2:** By using the table `soa08Act` calculate the APV of a contract that pays 50000 m.u. at the end of the death year. The insured is 25 years old and the coverage period is 40 years.

```r
data(soa08Act, package="lifecontingencies")
50000*Axn(actuarialtable=soa08Act, x=25, n=40)
## [1] 2398.544
```

**Example 3:** By using the table `soa08Act` calculate the APV of a contract that pays 50000 m.u. at the end of the death year. The insured is 25 years old and the coverage period is 40 years. In case that he survives 40 years, he will also receive 50000 m.u.

```r
data(soa08Act, package="lifecontingencies")
50000*AExn(actuarialtable=soa08Act, x=25, n=40)
## [1] 6227.437
```

**Example 4:** By using the table `soa08Act` calculate the APV of an insurance that pays an increasing benefit if the insured (who is 25 years old) dies during next 2 years. If he dies during the first year, the benefit will be 1 m.u. and if he dies during the second year, the benefit will be 2 m.u.. In both cases, the benefit will be paid at the end of the death year. Do the calculation in two different ways and check that you get the same results. The technical interest rate of `soa08Act` is 6%.

```r
#solution
IAxn(actuarialtable=soa08Act, x=25, n=2)
## [1] 0.003417766
```
Example 5: By using the table soa08Act calculate the APV of a contract that pays 1 m.u. at the end of the year of death. The insured is 25 years old and the coverage period is 1 year. Do the calculations in two different ways and check that you get the same result.

\[
\text{Axn}(\text{actuarialtable}=\text{soa08Act}, x=25, n=1) + 2\times pxt(\text{soa08Act}, 25, 1)\times \text{Axn}(\text{actuarialtable}=\text{soa08Act}, x=26, n=1)\times (1.06)^{-1}
\]

\[
\text{Axn}(\text{actuarialtable}=\text{soa08Act}, x=25, n=1)
\]

\[
qxt(\text{soa08Act}, 25, 1)\times (1.06)^{-1} \text{ same result}
\]

\[
qxt(\text{soa08Act}, 25, 1)\times (1.06)^{-1} \text{ same result}
\]

3.1. Exercises

Exercise 1: Do the same calculation as in Example 5, in two different ways, for a coverage period of two years.

\[
\text{Axn}(\text{actuarialtable}=\text{soa08Act}, x=25, n=2)
\]

\[
0.00228577
\]

\[
qxt(\text{soa08Act}, 25, 1)\times (1.06)^{-1} + pxt(\text{soa08Act}, 25, 1)\times qxt(\text{soa08Act}, 26, 1)\times (1.06)^{-2}
\]

\[
0.00228577
\]

Exercise 2: By using the table soa08Act calculate the APV of a contract that pays 1 m.u. if the insured (who is 25 in the moment when he underwrites the contract) survives until the retirement (65 years old). Do the calculation in two different ways and check that you get the same result.

\[
\text{Exn}(\text{soa08Act}, 25, 40)
\]

\[
0.07657786
\]

\[
pxt(\text{soa08Act}, 25, 40)\times (1.06)^{-40} \text{ same result}
\]

\[
pxt(\text{soa08Act}, 25, 40)\times (1.06)^{-40} \text{ same result}
\]

Exercise 3: By using the table soa08Act calculate the APV of a contract that pays 1 m.u. if the insured (who is 25 when he underwrites the contract) survives until the
retirement (65 years old). If he dies before the retirement, the payment will be 1 m.u. at the end of the year when the death occurs. Do the calculation in two different ways and check that you get the same results.

#solution
AExn(soa08Act, 25, 40)
## [1] 0.1245487
Exn(soa08Act, 25, 40) + Axn(soa08Act, 25, 40)
## [1] 0.1245487

#same result

Exercise 4: By using the table soa08Act calculate the APV of an insurance contract that pays an increasing benefit if the insured (who is 25 when he underwrite the contract) dies during next three years. If he dies during the first year, the payment will be 1 m.u., if he dies during the second year, the payment will be 2 m.u., and if he dies during the third year, 3 m.u. In all cases, the benefit will be paid at the end of the death year. Do the calculation in two different ways and check that you get the same result.

#solution
IAxn(actuarialtable = soa08Act, x = 25, n = 3)
## [1] 0.006756478
Axn(actuarialtable = soa08Act, x = 25, n = 1) +
   + 2 * pxt(soa08Act, 25, 1) * Axn(actuarialtable = soa08Act, x = 26, n = 1) * (1.06^(-1))
   + 3 * pxt(soa08Act, 25, 2) * Axn(actuarialtable = soa08Act, x = 27, n = 1) * (1.06^(-2))
## [1] 0.006756478

Exercise 5: By using the table soa08Act calculate the APV of an insurance policy that pays a decreasing benefit if the insured (who is 25 years old when he underwrites the contract) dies during the next three years. If he dies during the first year, the payment will be 3 m.u., if he dies during the second year, the payment will be 2 m.u., and if he dies during the third year, 1 m.u. In all cases, the benefit will be paid at the end of the death year. Do the calculation in two different ways and check that you get the same result.

#solution
DAxn(actuarialtable = soa08Act, x = 25, n = 3)
## [1] 0.006838219
3 * Axn(actuarialtable = soa08Act, x = 25, n = 1) +
   + 2 * pxt(soa08Act, 25, 1) * Axn(actuarialtable = soa08Act, x = 26, n = 1) * (1.06^(-1))
   + pxt(soa08Act, 25, 2) * Axn(actuarialtable = soa08Act, x = 27, n = 1) * (1.06^(-2))
Exercise 6: By using the table soa08Act calculate the APV of an insurance policy that pays an increasing benefit if the insured (who is 25 years old when he underwrites the contract) dies in next 10 years. If he dies in the first year, the company will pay 100000 m.u. From that moment on, the benefit will be increased by 5000 m.u. per year (this means that if he dies during the second year the benefit will be 105000 m.u., in the third year, 110000 m.u., and so on).

#solution

95000*Axn(actuarialtable=soa08Act,x=25,n=10)+
5000*IAxn(actuarialtable=soa08Act,x=25,n=10)

## [1] 1325.207

4. Calculating the APV with lifecontingencies (two-lives)

In the last section, we will see how to calculate the APV of life insurance contracts, where the payment of the benefit depends on the survival of a group of two individuals.

4.1. Basic function for calculating the APV (two lives)

In the next table there are some basic actuarial functions for calculating the APV available in lifecontingencies. In all cases, the function returns the APV of the corresponding insurance contract.

<table>
<thead>
<tr>
<th>Función</th>
<th>Descripción</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axyn</td>
<td>Two-lives life insurance contract (status = joint for non joint-life case and status = last for last-survivor case)</td>
</tr>
<tr>
<td>Axyzn</td>
<td>Life insurance contract for more that two lives (status = joint for non joint-life case and status = last last-survivor case)</td>
</tr>
</tbody>
</table>

Some examples of these functions:

Example 1. Calculate the APV of a life insurance contract underwritten by a couple (man aged 60 and woman aged 50) that pays a benefit when the first death occurs during the next year. The benefit is 1 m.u., at it will be paid (in the non joint life case), at the end of the year. The interest rate is 1.5%. Do the calculation in two different ways.

Axyzn(US07List,x=c(60,50),i=0.015, n=1, status="joint")

## [1] 0.01441865

qxyzt(US07List,x=c(60,50),t=1,status="joint")(1.015)^(-1) # same result

## [1] 0.01441865
Example 2. Calculate the APV of a life insurance contract underwritten by a couple (man aged 60 and woman aged 50) that pays a benefit when the first death occurs during the next year. The benefit is 50000 m.u., at it will be paid, in the non joint life case, at the end of the year. The interest rate is 1.5%. Do the calculation in two different ways.

\[
50000 \times \text{A}_{\text{xyzn}}(\text{US07List}, x=c(60, 50), i=0.015, n=1, \text{status}="\text{joint}")
\]

## [1] 720.9323

\[
50000 \times \text{q}_{\text{xyzt}}(\text{US07List}, x=c(60, 50), t=1, \text{status}="\text{joint}")*(1.015)^{-1}
\]

# same result

## [1] 720.9323

Example 3. Calculate the APV of a life insurance that pays a benefit when all members of a couple (man aged 60 and woman aged 50) die during the next year. The benefit is 50000 m.u., at it will be paid, in the last-survivor case, at the end of the year. The interest rate is 1.5%. Do the calculation in two different ways.

\[
50000 \times \text{A}_{\text{xyzn}}(\text{US07List}, x=c(60, 50), i=0.015, n=1, \text{status}="\text{last}")
\]

## [1] 1.830881

\[
50000 \times \text{q}_{\text{xyzt}}(\text{US07List}, x=c(60, 50), t=1, \text{status}="\text{last}")*(1.015)^{-1}
\]

# same result

## [1] 1.830881

4.1. Exercises

Exercise 1: Calculate the APV of a life insurance that pays a benefit when the first death (non joint life case) occurs in a couple (man aged 60 and woman aged 50) during the next two years. The benefit is 1 m.u., at it will be paid at the end of the year when the first death occurs. The interest rate is 1.5%. Do the calculation in two different ways.

#solution

\[
\text{A}_{\text{xyzn}}(\text{US07List}, x=c(60, 50), i=0.015, n=2, \text{status}="\text{joint}")
\]

## [1] 0.0295044

\[
\text{q}_{\text{xyzt}}(\text{US07List}, x=c(60, 50), t=1, \text{status}="\text{joint}")*(1.015)^{-1} + \text{p}_{\text{xyzt}}(\text{US07List}, x=c(60, 50), t=1, \text{status}="\text{joint}")*\text{q}_{\text{xyzt}}(\text{US07List}, x=c(61, 51), t=1, \text{status}="\text{joint}")*(1.015)^{-2}
\]

# same result

## [1] 0.0295044
**Exercise 2:** Calculate the APV of a life insurance that pays a benefit when the last death occurs (last-survivor case) of a couple (man aged 60 and woman aged 50) during the next two years. The benefit is 1 m.u., at it will be payd at the end of the year when the last death occurs. The interest rate is 1.5%. Do the calculation in two different ways.

```r
#solution
Axyzn(US07List, x=c(60,50), i=0.015, n=2, status="last")
## [1] 0.0001551046
```

**#otra manera de hacer el mismo cálculo**

```r
cqxyzt(US07List, x=c(60,50), t=1, status="last")* (1.015)^(-1) + (pxyzt(US07List, x=c(60,50), t=1, status="last") - pxyzt(US07List, x=c(60,50), t=2, status="last")) * (1.015)^(-2)
## [1] 0.0001551046
```

**Exercise 3:** By using the life table `US07List` calculate the APV of the following contracts:

- a) Life insurance contract that pays a benefit when the first death occurs, underwritten by a couple (man aged 48 and woman aged 45), the coverage period is 5 years. The benefit is 15000 euros and it is payd at the end of the year when the first death occurs. Interest rate: 1%

```r
15000*Axyzn(US07List, x=c(48,45), i=0.01, n=5, status="joint")
## [1] 579.8566
```

- b) Life insurance contract that pays a benefit when the last death occurs (last-survivor case), underwritten by a couple (man aged 57 and woman aged 55), the coverage period is 10 years. The benefit is 180000 euros and it is payd at the end of the year when the last death occurs. The interest rate is 1%

```r
180000*Axyzn(US07List, x=c(57,55), i=0.01, n=10, status="last")
## [1] 1379.742
```

**References**
