“Daily Growth at Risk: financial or real drivers?
The answer is not always the same”

Helena Chuliá, Ignacio Garrón and Jorge M. Uribe
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Abstract

We estimate Growth-at-Risk (GaR) statistics for the US economy using daily regressors. We show that the relative importance, in terms of forecasting power, of financial and real variables is time varying. Indeed, the optimal forecasting weights of these types of variables were clearly different during the Global Financial Crisis and the recent Covid-19 crisis, which reflects the dissimilar nature of the two crises. We introduce the LASSO and the Elastic Net into the family of mixed data sampling models used to estimate GaR and show that these methods outperform past candidates explored in the literature. The role of the VXO and ADS indicators was found to be very relevant, especially in out-of-sample exercises and during crisis episodes. Overall, our results show that daily information for both real and financial variables is key for producing accurate point and tail risk nowcasts and forecasts of economic activity.

JEL classification: E27, E44, E66.

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Helena Chuliá (corresponding author): Riskcenter, Institut de Recerca en Economia Aplicada (IREA), Departament d'Econometria, Estadística i Economia Aplicada, Universitat de Barcelona (UB). Email: hchulia@ub.edu

Ignacio Garrón: Departament d'Econometria, Estadística i Economia Aplicada, Universitat de Barcelona (UB). Email: igarron@ub.edu

Jorge M. Uribe: Faculty of Economics and Business Studies, Open University of Catalonia. Email: juribeg@uoc.edu

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1. Introduction

There is a large body of recent studies that analyze the predicting power of financial conditions on real economic activity in times of crisis. One group emphasizes the significant role of financial indicators in forecasting low quantiles of the GDP growth rate (e.g. Giglio et al., 2016; Adrian et al., 2019), while others report that after controlling for real variables, financial indicators have little to add to the mix (e.g. Reichlin et al., 2020; Plagborg-Møller et al., 2020). Indeed, some studies claim the opposite, i.e., that real variables (except for unemployment claims) have little to add after financial variables have been incorporated into the forecasting equation (i.e. Carriero et al., 2020).

This disagreement in results is because forecasting real economic activity (or any fragment of the growth distribution for that matter) using financial variables is a unique and challenging problem due to two facts. First, financial and real variables are generally sampled at different frequencies, being the former of a considerably higher frequency than the latter. Second, quantifying how much financial variables add in terms of forecasting power for predicting economic activity, seems to be rather a causal question (in opposition to a predictive one), because it is related to the prolonged controversy in economics about the dichotomy between nominal and real variables, and how (and to what extent) the former influence the latter. Furthermore, this second issue highlights the tension that exists between pure “predictive” and pure “causal” tasks in social science, and in particular, in economics (Athey, 2017). In short, this theoretical distinction is not crystal clear when it comes to macroeconomic studies, which are observational out of necessity, and in which forecasting can generally be improved by using domain knowledge, which is causal. Moreover, at the same time, forecasting exercises are generally expected to improve our understanding about the causal mechanisms in the economy. In short, we trust forecasts more than we can understand.

Due to this complex relationship and the multiplicity of aims that a researcher or policy maker may have for making a forecast, we recommend using an eclectic approach. In this approach both financial and real variables should ideally be used to forecast economic crisis episodes, while letting the data show freely the relative importance of each set of variables on a time-varying basis. Following this approach we make two important contributions to the field. First, we show that the informational content of financial and real economy indicators is different across time. Thus, in some circumstances, forecasting accuracy is due greatly to financial
indicators, such as the equity market volatility index (VXO); however, in other circumstances, real economic indicators, such as the ADS daily business cycle index (Aruoba et al., 2009), are more useful for improving the forecast. Our results clearly show the time-varying importance of real and financial variables. We estimate the optimal weight that our forecasting models assign to ADS or to finance variables, and we show that in periods like the aftermath of the Global Financial Crisis (GFC), financial indicators play a far greater role than ADS, while the opposite is true for the recent Covid-19 crisis. This is in agreement with the general consensus in the macro-financial literature that highlights the financial nature of the GFC, when financial markets and intermediaries acted as amplifiers of shocks to the system (Isohätälä et al., 2016; Brunnermeier and Sannikov, 2016; Gertler and Gilchrist, 2018). However, this literature recognizes that the Covid-19 crisis was simply a product of the supply restrictions imposed to contain the pandemic, which were real and supply-side in nature, albeit with repercussions for the aggregate demand (Guerrieri et al., forthcoming). Note that understanding the mechanisms underlying a crisis episode is a purely causal task, which can assist the researcher to interpret the results of the forecasting exercise and also to improve the actual forecasting.

Second, we also make a contribution regarding the abovementioned issue when real economic activity is forecast using financial variables, which are sampled at higher frequencies (e.g. daily, weekly) than GDP and other real variables (e.g. quarterly, annually). Unlike most of the literature that employs weekly indicators to forecast GDP growth (or lower-frequency financial indicators), we estimate our models using daily right-hand-side variables. Thus, our results use more information in time than most of the extant literature. There are some exceptions in which daily data is also used (i.e. Ferrara et al. 2021); however, in these cases, real variables have been neglected and the number of financial indicators included is rather limited. Therefore, our results are also supported by richer cross-sectional information at the intended frequency than previous studies, which is reflected in the greater accuracy of our models compared to previous studies.

An important concern needs to be addressed when working with daily predictors and without excluding variables (i.e. financial or real) a priori from our set of predictors, as we do. The number of parameters to estimate increases quickly, compared to using lower frequencies and fewer variables. In this case, shrinkage, regularization and dimensionality reduction techniques, such as those provided by LASSO, the Elastic Net method or PCA, become essential. Thus, we introduce these methods into MIDAS-Quantile models for estimating Growth-at-Risk. We use
quantile regression for high dimensional spaces as proposed by Belloni and Chernozhukov (2011) and PCA to reduce the dimensions of our problem even further. In line with what has been emphasized by Lima et al. (2020) and Lima and Meng (2017), parameter-reduction techniques are crucial at high frequencies. LASSO-Quantile and Elastic Net-Quantile models tend to outperform other alternatives explored in past literature (i.e., the traditional quantile MIDAS, where the vector of high-frequency terms takes an arbitrary form, either estimated by frequentist (Ghysels et al., 2016) or Bayesian methods (Mogliani and Simoni, 2021; Ferrara et al., 2021)), both in and out-of-sample. In addition, in line with Stock and Watson (2004), Andreou et al. (2013), and Ferrara et al. (2021) we show that combined forecasts using all indicators are more accurate, especially out-of-sample.

We validate our conclusions using a battery of statistics that come from the different fields of forecasting and quantitative risk-management. Rather than relying on a single statistic coming from the forecasting literature which, for instance, may not take into account that when the Value at Risk of a series is estimated (i.e. a low or high conditional quantile), two properties need to be achieved: Unconditional coverage and independence (Christoffersen, 1998). This is important because, as shown by Brownless and Souza (2021) in a multicountry setting, original indicators of Growth at Risk frequently do not pass basic tests designed in finance to measure the precision of VaR estimates. This, in turn, may call into question the usefulness of the entire enterprise.

We show that this is not the case for our indicators. In fact, on the vast majority of occasions, daily financial information together with daily information on real activity are very useful for anticipating bad scenarios for GDP growth. In addition, our GaR statistics are adequate and meet the performance expected from them.

The rest of this document is organized as follows: Section 2 presents our data and section 3 our methodology. Section 4 presents our main results, while section 5 concludes.

2. Data

We use quarterly data on the US GDP growth rate from the Federal Reserve of the St. Louis FRED database as the dependent variable. In our exercise, we include 11 daily predictors to evaluate the GaR forecast (10 financial variables and 1 real variable). From this set, eight series are the same as in Pettenuzzo et al. (2016): i) the ADS daily business cycle variable of Aruoba et
al. (2009), ii) the interest rate spread between the 10-year government bond rate and the federal fund rate (ISPREAD), iii) the change in the effective federal funds rate (EEFR), iv) the BAA-AAA corporate bond yield credit spread (CSPREAD), v) the excess return on the market (RET), vi) the returns on the portfolio of small minus big stocks (SMB), vii) the returns on the portfolio of high minus low book-to-market ratio stocks (HML), and viii) the returns on a winner minus loser momentum spread portfolio (MOM). The detailed description of the indicators is shown in Table 1. We take advantage of the previous results reported by Pettenuzzo et al. (2016) for these daily series in two ways. First, these authors document that introducing these daily variables through a restricted MIDAS approach (using the Almon lag polynomial) improves in-sample and out-of-sample density forecasts of the industrial production index (IPI) growth rate. Second, they show that the models that include the ADS indicator have the best predictive power among the rest of the daily series. In addition, the results of Lima et al. (2020) are qualitatively identical to those reported by Pettenuzzo et al. (2016) and show that the ADS index has the best predictive power among all other daily series to forecast the industrial production index. These findings are especially relevant for the discussion in the results section. In addition to this group, we add three financial indicators: the equity market volatility index VXO, which has been used as a price of risk indicator; the spread between the yield of 10-year constant maturity Treasury bonds and of 3-month Treasury bills (TERM), as a proxy of credit risk, and the spread between 3-month Libor based on US dollars and 3-Month Treasury Bill spread (TED), as a predictor of US recessions. Finally, our data sample spans the period from 1986Q1 to 2020Q4 and is restricted by the availability of data for all indicators. We include one year of daily lags of the high frequency indicator in all our specifications.

1 Rey (2015) shows that this indicator comoves with global capital flows, global credit growth, and global asset prices. Longstaff et al. (2011) also document that the price of sovereign risk is strongly correlated with VXO.
2 Gunay (2020) show that the TED spread is superior to credit default swap indexes as an early warning indicator for the credit market.
3 Estrella and Hardouvelis (1998) and subsequent literature has shown the forecasting power of the term spread for recessions.
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3. Methodology

To estimate tail risks in GDP growth, we extend the formulation of Adrian et al. (2019) to account for daily high-frequency predictors. To clarify our approach, in this section we show concisely how we adapt the standard Growth at Risk (GaR) to incorporate daily financial and real indicators, using mixed data sampling. To this end, we compare the performance of a traditional MIDAS (with Almon lag polynomials), Bayesian MIDAS, LASSO and Elastic Net methods. We also show how we implement combination forecasts, which have been proven to provide more accurate predictions compared to the alternatives. Finally, we present the tools we use to evaluate tail risk forecasts.

3.1. Growth at Risk framework

As in the standard framework of quarterly Growth at Risk pioneered by the works of Giglio et al. (2016) and Adrian et al. (2019), we rely on quantile regressions (Koenker and Bassett, 1978). Specifically, we assess the combined effect of past GDP growth \( y_{t-h} \) and a given financial condition indicator \( x_{t-h} \), at quarter \( t \) and forecast horizon \( h \), on the current output growth \( y_t \).

At this point it is important to recall that even though \( x_{t-h} \) is observed daily, it is aggregated by taking its quarter average. The baseline quantile regression is given by:

\[
 y_t = \beta_0(\tau) + \beta_1(\tau)y_{t-h} + \beta_2(\tau)x_{t-h} + \epsilon_t(\tau),
\]

where \( \beta(\tau) = (\beta_{0l}(\tau), \beta_0(\tau), \beta_1(\tau))^t \) denotes the vector of parameters corresponding to the \( \tau \)-th quantile, and \( \epsilon_t(\tau) \) is a random noise. The model in Equation 1 solves the following optimization problem:

\[
 \hat{\beta}(\tau) = \arg\min_{\beta(\tau)} E[\rho_t(y_t - \beta_0(\tau) - \beta_1(\tau)y_{t-h} - \beta_2(\tau)x_{t-h})],
\]

where \( \rho_t(\cdot) \) is the check loss function, given by \( \rho_t(\epsilon) = (1 - \tau)I_{\{\epsilon < 0\}}|\epsilon| + \tau I_{\{\epsilon > 0\}}|\epsilon| \), with \( I_{\{\epsilon < 0\}} \) taking the value of 1 when the subscript is true and 0 otherwise. The mathematical
formulation in Equation 2 leads to the solution of a linear programming optimization problem that we have not included here. Its basic structure and the counterpart algorithm solution can be found in Koenker (2005).

The predicted value from that regression is the quantile of \( y_{T-h} \), which is conditional on the information available up to \( T-h \),

\[
Q_t(y_{T}, y_{T-h}, x_{T-h}) = \beta_0(\tau) + \beta_1(\tau) y_{T-h} + \beta_2(\tau) x_{T-h},
\]

Koenker and Bassett (1978) further prove that the predicted value \( Q_t(y_{T}, y_{T-h}, x_{T-h}) \) is a consistent linear estimator of the conditional quantile function of \( y_t \). In this setting, we are particularly interested in the 10 percent quantile forecast as the measure of tail risk (see Figueres et al., 2020, Ferrara et al., 2021), namely \( \text{GaR (10%)} = Q_{t=}^{0.10}(y_{T}, y_{T-h}, x_{T-h}) \).

This last equation can be interpreted as the 10% quantile of GDP growth, which is conditional on the information set available up to \( T-h \) for the predictors. On one hand, there is a lot of literature that documents that financial conditions have strong predictive information for the lower quantiles of future GDP growth (see e.g., Adrian et al. 2019; Prasad et al., 2019; Brownlees and Souza, 2021; Figueres et al., 2020; Ferrara et al., 2021). However, on the other hand, Plagborg-Møller et al. (2020) and Reichlin (2020) state that controlling for real factors is necessary to accurately measure the real-time effect of financial indicators on real activity. We acknowledge these two results in our framework by incorporating the ADS daily business cycle variable of Aruoba et al. (2009), in addition to the financial indicators, as a high-frequency indicator of the real sector. We then follow a combination approach to produce a better point forecast and check the optimal weights of individual high frequency predictors, following previous literature in this field (see Stock and Watson, 2004; Andreou et al., 2013; Pettenuzzo et al., 2016; Ferrara et al., 2021).

3.2. Adapting the standard GaR approach to high-frequency indicators

The problem of the formulation in Equation 1 is that the aggregation of the high-frequency indicator prevents the model from responding to daily shocks. Consequently, similar to Ferrara et al. (2021), we adapt Equation 1 to account for the daily information flow of the high-frequency

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4 Alternatively, Adrian et al. (2019) and Carriero et al. (2020), use the 5 percent quantile forecast as the measure of tail risk. However, due to a shorter sample compared to these works, we opt to use the 10 percent quantile.
indicator up to the latest available observation (minus \( h_d \) days), based on the following regression:

\[
y_t = \beta_0(\tau) + \beta_1(\tau)y_{t-1} + X^D_{t-h_d}' \phi(\tau) + \epsilon_t(\tau),
\]

(4)

where \( \phi(\tau) \) is a \( p \times 1 \) vector of daily parameters and \( X^D_t = (x^1_t, x^2_t, \ldots, x^p_t)' \) is a \( p \times 1 \) matrix of the high-frequency variable available on a daily basis, with \( x^j_t, j = (1, 2, \ldots, p) \), which is updated \( d \) times between quarter \( t \) and \( t - 1 \). In this setup, we consider that \( y_t \) is affected by up to one year (\( q = 4 \) quarters) of past daily shocks and past GDP growth, giving a total number of parameters (including the constant) approximately equal to \( K = q \times d + 2 = 4 \times 60 + 2 = 242 \), assuming a five-day working week (\( d = 60 \) days); that is, \( X^D_t = (x^1_t, x^2_{t-60}, \ldots, x^{240}_{t-239})' \).

Our estimation window is wider than that of Ferrara et al. (2021), as they consider a lag window of 60 days for the high-frequency indicator. This enables the model to capture up to one year of daily information. In our case, \( K \) is relatively larger than the total number of observations \( T \), so we are faced with a parameter proliferation problem, which invalidates the standard estimation procedure of the quantile regression. Thus, in what follows we discuss the four alternative methods used in our results section to estimate the above regression.

### 3.2.1. MIDAS-Q

The mixed data sampling quantile (MIDAS-Q) model offers an effective solution to incorporating high-frequency indicators into Equation 4. This relies on a restriction of the form in which the distributed lags of the high frequency variable are included in the regression. While Ghysels et al. (2016) propose the Beta lag polynomial function for the quantile weighting function, we consider the Almon lag polynomial\(^5\) as in other recent works (Lima et al., 2020; and Mogliani and Simoni, 2021; Ferrara et al., 2021). Let \( X^D_t \) in Equation 5 follows the Almon lag polynomial given by:

\[
B(L^a; \theta_j(\tau)) = \sum_{i=0}^{p-1} b(i; \theta_j(\tau)) L^{ai},
\]

(5)

\(^5\) Since at least Pettenuzzo et al. (2016) many works in the literature have chosen to use the Almon lag polynomial for MIDAS, since it is parsimonious and linear in the parameters (Pettenuzzo et al., 2016; Lima et al., 2020; Mogliani and Simoni, 2021; Ferrara, 2021).
where $L^i_t x_t^j = x_t^j \frac{t_j}{-a_i}$ works as a daily lag operator and $b(i; \theta_j(\tau)) = \sum_{k=0}^{c-1} \theta_{k,j} i^k$ is the weighting function that depends on the vector of parameters $\theta_j(\tau)$ and the lag order $i = (1, 2, ..., p - 1)$. We apply a third-degree Almon lag polynomial by setting $c = 3$ and using the unrestricted expression to transform the matrix of predictors included in Equation 5 to $\bar{X}_t^D = Q X_t^P$, with $Q$ being the polynomial weighting matrix of size $c \times p$. In particular, the $i$-th row element of $Q$ is equal to $(1^3, 1^2, 2^3, ..., (p - 1)p)$. Thus, the MIDAS-Q regression is:

$$y_t = \beta_0(\tau) + \beta_1(\tau) y_{t-1} + \bar{X}_{t-h_d}^D \phi(\tau) + \epsilon_t(\tau),$$

where $\bar{X}_t^D$ is a $c \times 1$ matrix representing the transformed high-frequency predictor. Note that the number of parameters considered for the estimation is reduced substantially to $K = c \times 1 + 2 = 5$. In this setting, our GaR (10%) for this model is the predictive value of $Q_{\tau=0.10}(y_T | y_{T-1}, \bar{X}_{t-h_d}^D)$.

### 3.2.2. BMIDAS-Q

The Bayesian version of the MIDAS quantile (B-MIDASQ), based on the Asymmetric Laplace Density (ALD) estimation approach pioneered by Yu and Moyeed (2001), offers a convenient alternative for estimating Equation 4. Yu and Moyeed (2001) showed that the minimization problem of quantile regressions (see Equation 2) is equivalent to maximizing a likelihood using the asymmetric Laplace distribution (ALD) for the error term $\epsilon_t$. We use the Gibbs sampler implementation of Kozumi and Kobayashi (2011), alongside their mixture representation.

In this framework, the error term $\epsilon_t$ in Equation 6 can be represented as a location scale mixture of normal distributions in which the mixing distribution follows an exponential distribution (for details see, Kozumi and Kobayashi, 2011).

This implies that Equation 4 can be expressed as:

$$y_t = \beta_0(\tau) + \beta_1(\tau) y_{t-1} + X_{t-h_d}^D \phi(\tau) + \varphi(\tau) v_t + \varphi_2(\tau) \sqrt{\sigma v_t} u_t,$$

where $\varphi_1$ and $\varphi_2$ are fixed parameter functions of the quantile $\tau$, $v_t = \sigma z_t$ in which $z_t$ is a standard exponential function, and $u_t$ is a standard normal function. For the priors, we use a normal distribution for the parameters $\beta(\tau) = (\beta_0(\tau), \beta_0(\tau), \phi(\tau))' \sim N(\bar{\beta}, \bar{V})$ and an inverse gamma distribution for $\sigma \sim InvGamma(\bar{n}_0, \bar{s}_0)$. Similar to Ferrara et al. (2021), we specify
standard uninformative priors on the coefficient vector to have a mean of 0 and a variance that takes a diagonal equal to 100, except for the autoregressive lag of GDP, whose prior mean and variance are set to 0.5 and 0.2, respectively. The scale and shape parameters of the inverse gamma function are set to 0.01. The Gibbs sampler is used to estimate the model parameters with 10,000 repetitions (for computation efficiency), after a burn-in period of 1000 iterations, using the normal approximation (Yang et al., 2015).

Similar to Carriero et al. (2020), we compute the GaR (10%) = \( Q_{\tau=0.10}(y_T | y_{T-1}, \bar{X}_{t-h_d}) \) using the posterior mean of the coefficient vector \( \phi(\tau) \), as we are interested in the point forecasts.

### 3.2.3. LASSO-Q

One caveat of the restricted MIDAS approach presented above is the predetermined choice of the weighting function, which may lead to a lag structure for the high-frequency predictor that does not maximize forecast accuracy. We therefore propose estimating GaR by using either the LASSO or Elastic Net penalized methods for choosing a lag structure for the high-frequency predictors (Bai and Ng, 2008; Lima et al., 2020).

In the first step, we select the lags of the high frequency variable, based on the LASSO quantile (LASSO-Q) algorithm proposed by Belloni and Chernozhukov (2011). The model can be summarized as follows,

\[
\min_{\beta, \phi} E \left[ \rho_{\tau}(y_t - \beta_0(\tau) - \beta_1(\tau)y_{t-1} - X_{t-h_d}'(\tau)) \right] + \lambda_\tau \left[ \frac{\sqrt{T(1-\tau)}}{m} \right] \sum_{i=1}^{p} |\phi(\tau)_i|, \tag{8}
\]

where the optimization problem is the sum of the standard quantile minimization function (as in Equation 2) and a penalty function given by a scaled \( l_1 \)-norm of the daily vector of parameters \( \phi(\tau)_i \). The overall penalty is given by \( \lambda_\tau [\sqrt{T(1-\tau)}/T] \), where \( T \) is the sample size. The optimal level of \( \lambda_\tau \) is calculated as in Belloni and Chernozhukov (2011). The LASSO penalty has the distinctive feature of making the coefficients of insignificant predictors exactly equal to zero, keeping only the informative predictors for the forecast.

In the second step, we consider the approach of soft and hard threshold methods applied in a forecasting problem with many predictors, similar to Lima et al. (2020) and Bai and Ng (2008). First, we estimate principal components from the \( K^* = 30 \) most informative predictors selected.

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6 See Kozumi and Kobayashi (2011) for details on the estimation procedure.
by LASSO (Bai and Ng, 2008). Then, we select the optimal number of factors \( (f^*) \) using the
eigen ratio (Ahn and Horenstein, 2013). Finally, we estimate a standard quantile regression using
the factors as regressors to get the GaR \( (10\%) = Q_{\tau=0.10}(y_{T-1}, f^*_{T-h_d}) \). We only use the
factors with associated p-values lower than 0.01.

3.2.4. \( \text{EN-Q} \)
The Elastic Net (EN) estimator of Zou and Hastie (2005) aims to solve two potential drawbacks
of the original LASSO. First, if \( >T \), LASSO can select \( T \) variables at most. Second, if there is
a group of variables with high pairwise correlation coefficients, LASSO tends to select only one
variable from the group and does not care which one. Both LASSO and EN shrink the estimates
and perform model selection. However, while the LASSO penalty is convex, the EN penalty is
strictly convex, which means that predictors must be grouped to have similar coefficients.
The EN objective function is given by:

\[
\min_{\beta, \phi} E \left[ \rho_\tau(y_t - \beta_0(\tau) - \beta_1(\tau)y_{t-1} - X_{t-h_d}^d \phi(\tau)) \right] + \\
\lambda_1 \Sigma_{i=1}^p |\phi(\tau)| + \lambda_2 \Sigma_{i=1}^p \phi(\tau)^2,
\]

where \( \lambda_1 \) and \( \lambda_2 \) are two tuning parameters that satisfy \( \frac{\lambda_2}{\lambda_1 + \lambda_2} > 0 \). This restriction implies
that the EN is strictly convex, so it forces high pairwise correlated predictors to have similar
coefficients. As a result, EN stretches the net that retains all the important predictors, even if
they are highly correlated.

Following Bai and Ng (2008), we reformulate the EN objective function as a LASSO problem.
Therefore, we can use the Belloni and Chernozhukov (2011) procedure to estimate EN. We
implement this representation by defining \( y_t^+(\tau) = (y_t O_p)' \) and \( X_t^+(\tau) = \\
\frac{1}{\sqrt{1 + \lambda_2}} \left( X_t^p \sqrt{\lambda_2 I_p} \right)' \) where \( O_p \) represents a \( p \times 1 \) vector of zeros and \( I_p \) is a \( p \times p \) identity
matrix. The sample size is now equal to \( T + p \), which enables EN to select all \( p \) high-frequency
predictors in all cases. Similar to Lima et al. (2020), the optimal value of \( \lambda_2 \) is obtained by
minimizing the mean cross-validated errors of the model, with the EN mixing parameter set to
\( \alpha = 0.10 \).

In short, in the first step, for the transformed \( y_t^+ \) and \( X_t^+ \) we solve the EN minimization function
based on the LASSO algorithm proposed by Belloni and Chernozhukov (2011). In the second
step, we follow the same procedure as in LASSO to get the \( \text{GaR} (10\%) = Q_{\tau=0.10}(y_T | y_{T-1}, f^*_{T-h_d}) \), using the lags of the high frequency variable selected by EN to construct \( f^* \).

### 3.3. Forecast Combination

There is extensive literature that documents that forecast combinations provide a superior performance, as they use the information from all the underlying models rather than relying on one specific model (e.g., Stock and Watson, 2004; Andreou et al, 2013; Pettenuzzo et al. 2016; Ferrara et al., 2021). Indeed, selecting just one model can be misleading and highly inconvenient in the presence of misspecification (Hansen et al. 2011). While there are different methods for implementing forecast combinations, we rely on the discounted mean squared forecast error combinations approach (Stock and Watson, 2004; Andreou et al, 2013), using the Tick Loss function.

The weights are computed recursively in the following way:

\[
w_{i, T-h_d} = \frac{w_{i, T-h_d}^{-1}}{\sum_i w_{i, T-h_d}^{-1}} \text{ where } w_{i, T-h_d} = \sum_{s=T_0}^{T_f} \delta^{T_f-s} T L_{i,s},
\]

where \( w_{i, T-h_d} \) is the weight of model \( i \), which depends on the discounted Tick Loss function with discount factor \( \delta = 0.9 \), and where \( T_0 \) is the point at which the first prediction is computed, and \( T_f \) is the point at which the most recent prediction can be evaluated with the high-frequency indicator up to the latest available observation (minus \( h_d \) days).

### 3.4. GaR evaluation

We evaluate tail risk forecasts by using a battery of indicators developed in the literature on forecast and risk management. Our main tool for assessing GaR point forecasts is the Tick Loss (TL) function, which has been shown to be particularly suitable when the object of interest is the forecast of a certain quantile of the dependent variable’s conditional distribution (see Giacomini and Komunjer, 2005, Gneiting and Raftery 2007; Gneiting and Ranjan 2011, Manzan, 2015). In particular, Carriero et al. (2020) use it to evaluate the predictive capacity of their models for quantifying tail risks.

The TL for \( \tau = 0.10 \) is specified as follows,
\[ TL_{T=0.10} = \frac{1}{T} \sum_{t=1}^{T} (y_t - GaR(10\%)) \ast (\tau - 1(y_t < GaR(10\%))), \]  

(11)

where \( GaR(10\%) \) refers to the 10\% quantile of GDP growth conditioned by the information set available up to \( T - h \) for the given model and the high-frequency predictor. The indicator function \( 1(y_t < GaR(10\%)) \) takes a value of 1 if it is below the 10\% forecast quantile and 0 otherwise. To assess the statistical significance, we estimate Diebold and Mariano (1995) t-tests for equality predictability of models, with the loss function defined as the TL.

In addition, we employ tests commonly used in the risk management literature to assess interval forecasts. Following Christoffersen (1998), the problem of assessing the adequacy of a Value-at-Risk (VaR) model can be reduced to the problem of determining whether the indicator of excess sequence (i.e., the \( 1(y_t < GaR(10\%)) \)) has two properties: i) an unconditional coverage property, and ii) an independence property. In this setting, GaR forecasts are evaluated using the TL, which is a loss function generally used to assess the accuracy of VaR predictions (Giacomini and Komunjer, 2005). We evaluate these two conditions with the Unconditional Coverage (UC), and the Dynamic Quantile (DQ) tests (Engle and Manganelli, 2004), respectively. Specifically, the DQ is estimated using four lags of the excess sequence indicator (see Engle and Manganelli, 2004). Brownlees and Souza (2021) follow a similar approach for a multi-country GaR evaluation. Note that these two criteria can be achieved by more than one model, so at the end the TL is used in the final selection of the best performing model.

4. Results

We perform an in-sample exercise by using the full sample to evaluate the overall adequacy of GaR predictions from 1986Q1 to 2020Q4. Then, we implement an out-of-sample exercise for the one-step ahead prediction from 2001Q1 to 2020Q4.

4.1. In-sample analysis

First, we estimate the LASSO-Q, EN-Q, MIDAS-Q and BMIDAS-Q models for each individual high-frequency indicator and forecast horizon to obtain insights into their predictive ability. All the models are estimated using a sample from 1986Q1 to 2020Q4. Second, for each model we construct a combined-GaR forecast using all the variables (which we refer to as Y_ALL) including the ADS indicator, and using only financial variables (Y_FIN) without including the ADS indicator.
As a starting point, we show the in-sample combined-GaR forecasts of our LASSO-Q model\textsuperscript{7}. Figure 1 shows the quarterly growth rate of the US along with Y_ALL and Y_FIN, for different forecast horizons, from 1986Q1 to 2020Q4. Overall, we observe that the large negative growth rates that appear during recession periods, such as the Global Financial Crisis (GFC) in 2008–2009 and the Covid-19 pandemic (that started in 2020), are captured by our combined-GaR at different forecast horizons. In particular, our measures follow these sharp drops relatively well, especially in the case of Y_ALL during the Covid-19 pandemic. Note that Y_ALL includes the information of the ADS indicator, which provides significant evidence regarding the recovery of economic activity in the third quarter of 2020. However, this is not fully captured by the combined-GaR that only considers financial variables, Y_FIN. In that quarter, the US economic output increased at the fastest pace as businesses began to reopen and customers came back to stores. The rebound was fueled in part by extensive federal economic assistance provided to households and businesses.

Figure A1 shows the in-sample combined-GaR forecasts of all the models for the last two US recessions at different daily horizons. While these offer similar predictions in the cases of both Y_ALL and Y_FIN during the GFC, they differ greatly for the Covid-19 crisis. In this last recession, penalized models give a better prediction for the combined-GaR that uses only financial variables than the traditional MIDAS models. That is, the Y_FIN prediction for the first models (LASSO-Q and EN-Q) displays a steeper decrease than for the last set of models (MIDAS-Q and BMIDAS-Q). As pointed out by Lima et al. (2020), this difference is ultimately due to the restrictions imposed by MIDAS models on the daily lag matrix, which may not be aligned with the forecast maximization.

\textsuperscript{7} We chose to show the in-sample combined-GaR forecasts of the LASSO-Q model as, in general, both LASSO-Q and EN-Q have lower TL for Y_ALL and Y_FIN than MIDAS-Q and BMIDAS-Q. In addition, the forecasts of the former models are qualitatively similar. Nonetheless, Figure A1 plots the forecasts of all the models during the last two US recession periods at different daily horizons.
Figure 1. In-sample LASSO-Q combined-GaR results for different horizons

Sources: FRED database and authors’ computations.
Note: Time span 1986Q1 to 2020Q4. The red shaded area represents NBER recessions at the end of the period.
Figure 2 presents the summary of the results for all in-sample GaR forecasts at $h_d = 0$, which considers all the daily information up to the last day of the quarter. For each predictor and model, it reports the relative Tail Loss compared to the benchmark model (given by a simple QAR(1)) (panel a), and the Diebold-Mariano (DM) probability test (panel b). A value lower than one in the TL implies that the model outperforms the benchmark (black line), while for the DM test we set a probability level of 10% (red line). Notably, the vast majority of models outperform the benchmark and reject the hypothesis of equality of forecasts according to the DM test, which gives strong evidence of the benefits of introducing daily indicators into the GaR framework. In particular, we find that GaR forecasts that include the ADS indicator (i.e. ADS and Y_ALL) have a lower TL than the ones that do not include this variable. This result is in line with Pettenuzzo et al. (2016), since the models of Aruoba et al. (2009) that include ADS outperform those that incorporate other high-frequency indicators. These results are similar for larger forecast horizons (see Appendix A).
Figure 2. Summary of in-sample results for $h=0$

Note: Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). The red vertical line in panel b) represents the 0.10 probability level. The forecast horizon equal to zero implies that all the information contained in the high frequency variable up to the last day of the current quarter is used.

To gain insights into how adding daily indicators to the model helps improve GaR forecasts at $h_d = 0$, Figure 3 provides a recursive plot of the optimal combination weights assigned to different indicators in the combined-GaR. During the GFC, LASSO-Q and EN-Q assign larger weights to credit spread (CREDIT) and interest spread (TED), while MIDAS-Q and BMIDAS-Q assign more weight to VXO, ISPREAD and CSPREAD. In the latter crisis, which refers to the Covid-19 pandemic, traditional MIDAS models give more weight to ADS while LASSO and EN give a high weight to financial indicators in addition to ADS. Overall, our optimal weights support the time-varying importance of including real and financial daily indicators to improve GaR forecasts. The results are similar for larger forecast horizons (see Appendix A).
Figure 3. In-sample weights for daily predictors on the combined-GaR for $h = 0$

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at $h=0$.

4.2. Out-of-sample analysis

We recursively estimate all the above mentioned specifications for each quarter from 1986Q1 to 2000Q4 and construct out-of-sample GaR forecasts starting from 2001Q1 to 2020Q4. As with the in-sample analysis, we compute the combined-GaR forecasts using all the variables (Y_ALL) or only financial variables (Y_FIN) without the ADS real indicator.

We show the forecasting performance of our LASSO-Q model\(^8\) to offer insights into the out-of-sample predictive ability of the combined-GaR. Figure 4 shows the US quarterly growth rate along with the combined-GaR forecast of Y_ALL and Y_FIN, for the out-of-sample forecast period 2001Q1-2020Q4. Remarkably, during the 2008–2009 global financial crisis, as well as

\(^8\) We chose to show the out-of-sample combined-GaR forecast of the LASSO-Q model as it performs better up to a 20-day horizon. Nonetheless, Figure B1 plots the forecasts for all the models during the last two US recessions at different daily horizons.
During the Covid-19 pandemic, the out-of-sample combined-GaR forecasts can satisfactorily track the downside risks of GDP growth. Although during the GFC, both Y_FIN and Y_ALL produce similar forecasts, Y_ALL performs notably better during the Covid-19 period. This implies that the ADS indicator is a strong predictor (up to a 60 day-horizon) of the downside risks to GDP growth during this last crisis.

Indeed, in Figure B1 we elaborate this idea further by presenting all the models out-of-sample GaR forecasts for the same indicators (Y_ALL and Y_FIN) during the last two US recessions. First, during the GFC we observe that the combined indicator of financial variables, Y_FIN, provides timely guidance on the magnitude of the downside risks of GDP growth, whereas the improvement in the forecast of adding ADS is negligible. In contrast, during the Covid-19 pandemic, financial indicators alone largely underestimate the expected deterioration of GDP growth, while ADS dramatically improves both the sign and the magnitude of the tail risk estimation. Our work improves the precision of the estimated downside risks to GDP by introducing both financial and real high-frequency indicators. Interestingly, related works by Ferrara et al. (2021) and De Santis and Van der Veken (2020), which only include daily financial variables in the GaR framework, fail to capture the real magnitude of the risks during the Covid-19 period. This evidence indicates that financial variables alone play only a modest role in gauging the effect of this last recession. Lastly, as suggested earlier, forecasts from penalized models can track the downside risks better than traditional MIDAS, due to the above-mentioned limitations of these more traditional frameworks.
Figure 4. Out-of-sample LASSO-Q combined-GaR results for different horizons

Sources: FRED database and authors’ computations.

Note: Time span 2001Q1 to 2020Q4. Red shaded area represents NBER recessions at the end of the period.
Again, Figure 5 presents the summary of the results for the out-of-sample GaR forecasts at $h_d = 0$, which considers all the daily information up to the last day of the quarter. For each predictor and model, it reports the relative Tail Loss compared to the benchmark model (given by a QAR(1) model) (panel a), the Diebold-Mariano probability test (panel b), the UC probability test (panel c), and the DQ probability test (panel d). A value lower than one in the TL means that the model outperforms the benchmark (black line), while for the other tests we set a probability level of 10% (red line). First, we compare the models according to TL. Remarkably, the combined-GaR forecast obtained with LASSO-Q or EN-Q using real and financial variables, Y_ALL, outperforms the rest of the models. Regarding the daily indicators, again we find that GaR forecasts that introduce the ADS indicator, i.e. ADS and Y_ALL, outperform those that do not include ADS, similar to Pettenuzzo et al. (2016) and Lima et al. (2020). In addition, the LASSO-Q and EN-Q models have a lower TL for the combined-GaR forecasts than the MIDAS-Q and BMIDAS-Q models. We provide evidence that LASSO and EN lag selection improves forecast accuracy while imposing fewer restrictions than traditional MIDAS models. Second, all the models that include a combination of variables, or VXO and ADS as individual variables, reject the hypothesis of equality of forecasts in the DM test, while the results are mixed for the models with other individual variables. Finally, regarding the UC and DQ tests of GaR adequacy, we observe that combined-GaR forecasts (Y_ALL and Y_FIN) provide adequate GaR forecasts while GaRs using individual series tend to fail these tests.

The results for larger forecast horizons are presented in Appendix B, and in what follows we highlight three main results. First, GaR forecasts that include the ADS indicator are adequate (in terms of TL, DM, UC and DQ) in general up to a 60-day horizon. This result provides robust evidence of the usefulness of introducing real variables into the GaR framework. Second, for the models that include only financial indicators, in general, as the daily forecast horizon increases, the forecast accuracy decreases and it becomes difficult to pass the UC and DQ tests. This evidence is consistent with Plagborg-Møller et al. (2020) and Ferrara et al. (2021), who found that daily indicators can provide accurate and timely GaR information in the short term, while their contribution fades away as the horizon increases. Finally, we show that the combined-GaR using real and financial variables (Y_ALL) with LASSO-Q or EN-Q has a lower TL than the same with MIDAS-Q or BMIDAS-Q, up to a 40-day horizon. In addition, LASSO-Q and EN-Q also improve forecasts in a quantile framework.
Figure 5. Summary of out-of-sample results for $h=0$

Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one in order to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano $t$-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). Panel c) shows the unconditional coverage test (UC) of Kupiec (1995), which reports the probability of the null hypothesis that the exceedance is equal to the quantile (not rejecting is better). Panel d) displays the dynamic quantile test (DQ) of Engle and Manganelli (2004), in which the null hypothesis is that the exceedance indicator (Hit) is an i.i.d. process (not rejecting is better). The vertical red line in panels b), c), and d) represents the 0.10 probability level.

Now we turn to the issue of how we manage to obtain more accurate out-of-sample GaR forecasts at $h_d = 0$ by adding daily indicators to our models. Figure 6 provides a recursive plot of the combination weights assigned to the various indicators in our models. We can observe that both real and financial indicators are important to produce accurate GaR forecasts. First, we observe that during the global financial crisis that started in 2008, the ADS, VXO and credit spread (CSPREAD) indicators receive a relatively high weight across all models. In contrast, during the last crisis, which refers to the Covid-19 pandemic, all models assign higher weights to ADS and VXO. On one hand, the former is in agreement with the general consensus in the
macro-financial literature that highlights the financial nature of the GFC, in which financial intermediaries and markets amplified the shocks to the real economy (Isohätälä et al., 2016; Brunnermeier and Sannikov, 2016; Gertler and Gilchrist, 2018). On the other hand, the latter results recognize that the Covid-19 crisis was a product of the supply restrictions imposed to contain the pandemic, which were real and supply-side in nature (Guerrieri et al., forthcoming). Consequently, our estimated optimal weights suggest that it is fundamental to include real and financial daily indicators to improve GaR forecasts. The results are similar for larger forecast horizons (see Appendix B).

**Figure 6. Out-of-sample weights for daily predictors on the combined-GaR for h=0**

![Graph showing out-of-sample weights for daily predictors on the combined-GaR for h=0](image)

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at h=0.

Overall, the evidence we present here supports the time-varying importance of both the daily financial and real indicators for estimating GaR. Our results are consistent with those for the Euro Zone reported by Ferrara et al. (2020) and for the US reported by De Santis and Van der Veken (2020), in that financial variables provide policymakers with timely warnings about the
downside risks of GDP. Nevertheless, we provide further and clearer evidence in the way suggested by Pettenuzzo et al. (2016), emphasizing the usefulness of incorporating a real high-frequency indicator in the forecasting regressions, such as the ADS. This evidence ultimately suggests that financial variables alone play a limited role in gauging the downside risk for the GDP of the Covid-19 pandemic, and acknowledges the complex ways in which real and financial variables interconnect and determine economic growth, which is causal in nature.

5. Conclusions
We show that on a daily frequency both real and financial variables provide valuable information for forecasting vulnerable periods of economic activity. Our key contribution is that we evaluate the role of these two types of variables from a time-varying perspective, making it possible to obtain a richer set of regressors than those in the existing literature. Our flexible approach allows us to emphasize the important role of economic theory and economic intuition when the results of forecast combination exercises are interpreted and also to improve the point forecast itself. By acknowledging the complexity of the forecasting task in macroeconomics, especially when using high frequency data, we contribute to a better understanding of the economic signals that can be extracted from this daily information to anticipate economic declines. In this sense, we show that during the GFC and the Covid-19 pandemic, the optimal forecasting weights of real and financial variables clearly changed. In the former period, financial indicators were fundamental; however, they failed to capture the magnitude of the GDP decline observed during the Covid-19 quarters. This, in turn, is explained by the different natures of the two crises, that we can only grasp because we understand (up to some extent) the economic mechanisms behind these two crises (i.e. their causes).

Interestingly, within the set of financial variables, VXO is especially relevant across models in out-of-sample exercises, which highlights the prominent role of uncertainty in determining economic outcomes. However, as mentioned above, the financial indicators alone were unable to forecast GDP low quantiles during Covid-19. Indeed, only by including the ADS index did we managed to gauge both the sign and the magnitude of the downside GDP risk in this period.

Our tail-risk models performed satisfactorily in out-of-sample experiments, (which contrasts with the results of previous literature that uses daily variables). Importantly, we base our conclusions on a battery of indicators that are more appropriate for evaluating quantile forecasting than traditional tests originally designed to evaluate forecasts of the average.
Therefore, our results are encouraging from a policy making perspective, as policymakers need good, accurate and timely forecasts.

More technically, we have assessed the performance of two competing dimension reduction techniques, namely MIDAS and shrinkage methods. We have compared four different models and 11 high frequency predictors using a forecast combination approach with time-varying optimal weights. We found that shrinkage models (LASSO and EN) tend to outperform other models previously used in the literature, such as traditional MIDAS, both in terms of in-sample and out-of-sample forecast accuracy at different forecasting horizons. This is probably a consequence of traditional MIDAS restrictions on the lag structure of the high frequency indicator, which do not necessarily improve forecast accuracy. Therefore, our results give further support to past evidence, in that shrinkage models should ideally be used to select the number of lags of the high-frequency predictors.

We use a single indicator for capturing the role of real economic activity, namely the ADS, basically because it is the only indicator that exists at the daily frequency. Nevertheless, we believe that more indicators gauging the informational content of different facets of economic activity and the credit markets will prove to be fundamental in the future, not only to achieve greater forecasting accuracy in real time, but also to understand the causes of ongoing crises, even before the true causal mechanisms are clear to the professionals. In this sense, we see our models as a first step in the direction of anticipating and understanding economic dangers while they are occurring.

References


Appendix A: In-sample results

Figure A1. In-sample forecasts for the two last US recession episodes

b. Covid-19 pandemic

Sources: FRED database and author’s computation.

Note: The red shaded area represents NBER recessions at the end of the period.
Figure A2. Summary of in-sample results for h=10

Note: Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). The red vertical line in panel b) represents the 0.10 probability level. The forecast horizon equal to zero implies that all the information contained in the high frequency variable up to the last day of the current quarter is used.
Figure A3. Weights of each individual model results for h=10

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at h=10.
Figure A4. Summary of in-sample results for h=20

Note: Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). The red vertical line in panel b) represents the 0.10 probability level. The forecast horizon equal to zero implies that all the information contained in the high frequency variable up to the last day of the current quarter is used.
Figure A5. Weights of each individual model results for $h=20$

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at $h=20$. 
Figure A6. Summary of in-sample results for $h=40$

Note: Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). The red vertical line in panel b) represents the 0.10 probability level. The forecast horizon equal to zero implies that all the information contained in the high frequency variable up to the last day of the current quarter is used.
Figure A7. Weights of each individual model results for $h=40$

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at $h=40$. 
Figure A8. Summary of in-sample results for h=60

Note: Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). The red vertical line in panel b) represents the 0.10 probability level. The forecast horizon equal to zero implies that all the information contained in the high frequency variable up to the last day of the current quarter is used.
Figure A9. Weights of each individual model results for $h=60$

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at $h=60$. 
Appendix B: out-of-sample analysis

Figure B1. Out-of-sample forecasts for the two last US recession episodes

b. Covid-19 pandemic

Sources: FRED database and author’s computation.

Note: The red shaded area represents NBER recessions at the end of the period.
Figure B2. Summary of out-of-sample results for $h=10$

Note: This figure shows different forecast adequacy tests for each model. The forecast horizon refers to the relatively short forecast horizons of the high frequency financial indicator (up to 10 business days). Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). Panel c) shows the unconditional coverage test (UC) of Kupiec (1995), which reports the probability of the null hypothesis that the exceedance is equal to the quantile (not rejecting is better). Panel d) displays the dynamic quantile test (DQ) of Engle and Manganelli (2004), in which the null hypothesis is that the exceedance indicator (Hit) is an i.i.d. process (not rejecting is better). The vertical red line in panels b), c), and d) represents the 0.10 probability level.
Figure B3. Weights of each individual model results for out-of-sample h=10

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at h=10.
Figure B4. Summary of out-of-sample results for h=20

Note: This figure shows different forecast adequacy tests for each model. The forecast horizon refers to the relatively short forecast horizons of the high frequency financial indicator (up to 20 business days). Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). Panel c) shows the unconditional coverage test (UC) of Kupiec (1995), which reports the probability of the null hypothesis that the exceedance is equal to the quantile (not rejecting is better). Panel d) displays the dynamic quantile test (DQ) of Engle and Manganelli (2004), in which the null hypothesis is that the exceedance indicator (Hit) is an i.i.d. process (not rejecting is better). The vertical red line in panels b), c), and d) represents the 0.10 probability level.
Figure B5. Weights of each individual model results for out-of-sample h=20

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at h=20.
Figure B6. Summary of out-of-sample results for h=40

Note: This figure shows different forecast adequacy tests for each model. The forecast horizon refers to the relatively short forecast horizons of the high frequency financial indicator (up to 40 business days). Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) to test equality of forecasts, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). Panel c) shows the unconditional coverage test (UC) of Kupiec (1995), which reports the probability of the null hypothesis that the exceedance is equal to the quantile (not rejecting is better). Panel d) displays the dynamic quantile test (DQ) of Engle and Manganelli (2004), in which the null hypothesis is that the exceedance indicator (Hit) is an i.i.d. process (not rejecting is better). The vertical red line in panels b), c), and d) represents the 0.10 probability level.
Figure B7. Weights of each individual model results for out-of-sample h=40

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at h=40.
Figure B8. Summary of out-of-sample results for h=60

Note: This figure shows different forecast adequacy tests for each model. The forecast horizon refers to the relatively short forecast horizons of the high frequency financial indicator (up to 60 business days). Panel a) gives the ratio of Tick Loss (TL) for the indicated model to the benchmark model QAR(1) (lower is better). The vertical black line is equal to one to illustrate the relative performance of each model. Panel b) reports the Diebold and Mariano t-test (DM) for equality of the TL, conducted on a one-sided basis, such that the alternative hypothesis is that the indicated forecast is more accurate than the benchmark (a rejection of the null is preferred). Panel c) shows the unconditional coverage test (UC) of Kupiec (1995), which reports the probability of the null hypothesis that the exceedance is equal to the quantile (not rejecting is better). Panel d) displays the dynamic quantile test (DQ) of Engle and Manganelli (2004), in which the null hypothesis is that the exceedance indicator (Hit) is an i.i.d. process (not rejecting is better). The vertical red line in panels b), c), and d) represents the 0.10 probability level.
Figure B9. Weights of each individual model results for out-of-sample h=60

Note: This figure plots the optimal weights for different daily predictors used in the combined-GaR at h=60.