# Pulse propagation sustained by noise in arrays of bistable electronic circuits

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One-dimensional arrays of nonlinear electronic circuits are shown to support propagation of pulses when operating in a locally bistable regime, provided the circuits are under the influence of a global noise. These external random fluctuations are applied to the parameter that controls the transition between bistable and monostable dynamics in the individual circuits. As a result, propagating fronts become destabilized in the presence of noise, and the system self-organizes to allow the transmission of pulses. The phenomenon is also observed in weakly coupled arrays, when propagation failure arises in the absence of noise.

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## I. INTRODUCTION

Information propagation is one of the most relevant phenomena in nature. Living organisms need to communicate at all levels, ranging from the microscopic scale of cell signalling up to the macroscopic one of human telecommunication technology. In the former case, the mechanism most frequently used for signal transmission is excitability [1]. A system is said to be excitable when, even possessing a single steady state, it responds to small perturbations of that state in a nontrivial way, abruptly departing away from it and eventually returning to rest after a certain approximately fixed excursion time. In the presence of spatial coupling, this excitation is able to propagate through the system basically undisturbed, in what constitutes an excellent mechanism for transmitting signals through a resting medium. In particular, this process allows the propagation of one-dimensional pulses, two-dimensional spiral waves and three-dimensional scroll rings [2], structures that are all routinely observed in biological excitable media [3].

Soon after the excitable character of electrical signaling in nerve axons was identified [4], electronic transmission lines were proposed that satisfactorily modeled neural communication [5]. Following that trend, last decades have witnessed the development of many electronic circuits exhibiting nonlinear dynamical behavior, among which the so-called Chua circuit has been specially fruitful [6]. In particular, arrays of coupled Chua circuits have been shown to exhibit very rich spatiotemporal dynamics, such as Turing patterns, spiral waves, scroll rings [7], and rotating waves [8]. One of the characteristic features of this circuit is its ability to operate in different dynamical regimes, including chaotic, oscillatory, excitable, and bistable regimes. In this paper, we propose a simplified version of the Chua circuit that exhibits a transition from bistable to excitable dynamics in terms of a single control parameter, and analyze the effect on that transition of random fluctuations affecting this control parameter. We show that such an external noise advances the transition towards excitability, thus enhancing the excitable behavior of the system and allowing the propagation of excitable pulses even under deterministically bistable conditions.

The influence of external noise in the dynamics of spatially distributed systems has been profusely studied in recent years. A great deal of the results obtained point towards a constructive, rather than destructive, role in the system behavior, leading, for instance, to situations of noise-induced order [9]. In the particular case of propagation phenomena, random fluctuations have been reported to sustain propagation of fronts in a chain of bistable diode oscillators [10]. That situation is nevertheless unsuitable for signal transmission purposes, since a propagating front leaves the system in a state different from the original one, and hence a resetting mechanism is needed if a second signal has to be transmitted through the system in the same conditions as its predecessor. Noise-sustained propagation of harmonic signals in simple bistable models has also been reported [11,12], but this kind of signal contains a minimum amount of information. Hence, it would seem that a bistable medium is not adequate to transmit information-carrying pulses. However, recent theoretical investigations indicate that this is not necessarily the case. By way of example, a one-component bistable medium with a convective term has been shown to exhibit pulse propagation sustained by noise [13]. External random fluctuations have also been seen to destabilize front propagation (thus providing a resetting mechanism leading to the transmission of pulses) in two-component bistable media with activator-inhibitor dynamics, by means of both a noise-induced decay from one of the two stable states towards the other [14], and a noise-induced transition from the bistable to the excitable regime [15]. In this paper, we show experimentally that such a noise-enhanced excitability does indeed exist, by using a one-dimensional chain of nonlinear electronic circuits in which global random fluctuations are added to a parameter controlling the transition from bistability to excitability in the individual circuits (recent investigations indicate that effects of local and global noise in this kind of process is very similar [16]). The experimental implementation of the system is shown schematically in Fig.

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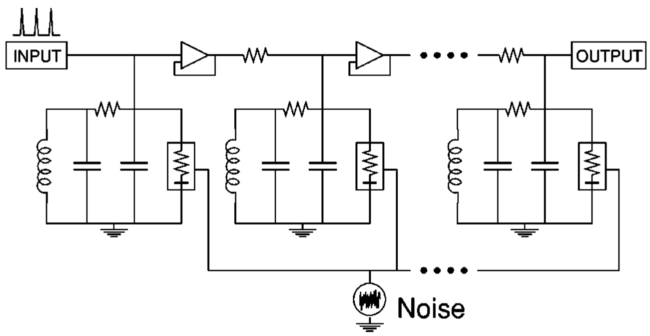


FIG. 1. Schematic representation of the nonlinear circuit array. The boxed element in each unit is a nonlinear resistor whose structure can be seen in the detailed diagram shown in Fig. 2.

1. Twenty nonlinear electronic oscillators are coupled unidirectionally by way of an operational amplifier in series with a resistor. The first circuit is driven by a periodic train of short pulses, and the output at the last circuit is analyzed when all the circuits are affected globally by a controlled source of electronic noise.

The following section describes our implementation of the individual elements forming the array and their isolated behavior in the absence of external noise. In Sec. III, the behavior of the whole circuit array is studied in the presence of noise, and the resulting noise-induced pulse propagation is described.

## II. A SIMPLE BISTABLE/EXCITABLE ELECTRONIC CIRCUIT

Each unit in Fig. 1 consists of four linear electric elements (two capacitors, one resistor, and one inductance), plus a nonlinear electronic element whose structure can be examined in detail in Fig. 2. This kind of circuit, containing an operational amplifier acting as a voltage-controlled voltage source, has been widely studied in last years as a standard generator of chaotic dynamics, in the form of the so-called double scroll attractor [6]. In the implementation and for the parameters of Fig. 2, the system operates nonchaotically in either a bistable or an excitable regime, as will be shown in what follows.

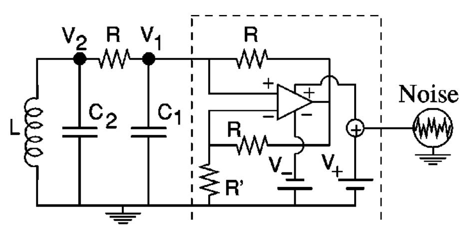


FIG. 2. Implementation of the Chua circuit exhibiting a transition between bistable and excitable regimes. The dashed line encloses the nonlinear resistor represented schematically in Fig. 1. The values of the elements are $R=270\ \Omega$, $L=10$ mH, $C_1=1$ nF, $C_2=10$ nF, $R'=220\ \Omega$, and $V_-=5$ V. $V_+$ is used as a control parameter. The operational amplifier is taken from a TL082 integrated circuit.

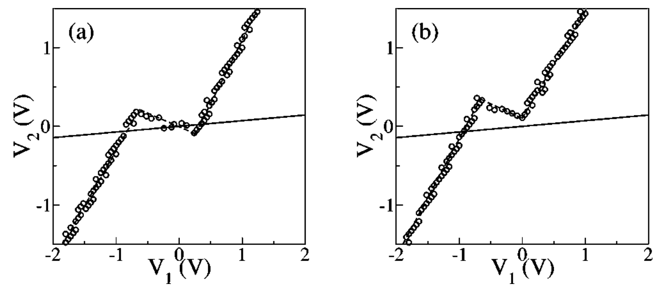


FIG. 3. Nullclines of the isolated circuit in the reduced plane $V_1$-$V_2$ for two different values of the positive supply voltage $V_+$: (a) $V_+=2$ V, bistable dynamics; (b) $V_+=0$ V, excitable dynamics. The solid line corresponds to the $V_2$ nullcline, and the circles to the $V_1$ nullcline.

The dynamics of the circuit shown in Fig. 2 is governed by the equations:

$$C_1\frac{dV_1}{dt}=\frac{V_2-V_1}{R}-g(V_1,V_+), \tag{1a}$$

$$C_2\frac{dV_2}{dt}=\frac{V_1-V_2}{R}+I_L, \tag{1b}$$

$$L\frac{dI_L}{dt}=-V_2-r_LI_L, \tag{1c}$$

where $V_i$ is the voltage drop in the capacitor $i$, $I_L$ the current through the inductance, and $r_L$ its internal resistance ($21\Omega$ in the present setup). The function $g(V_1,V_+)$ represents the characteristic curve of the nonlinear resistor, which is piecewise linear and contains a region of negative resistance.

The nullclines of the system can be drawn in the reduced plane $V_1$-$V_2$ by introducing the $I-L$-nullcline obtained from Eq. (1c), $I_L=-V_2/r_L$, into Eqs. (1a) and (1b). The result is

$$V_2=V_1+Rg(V_1,V_+), \tag{2a}$$

$$V_2=\frac{r_L}{R+r_L}V_1. \tag{2b}$$

These curves are shown in Fig. 3 for two different values of the positive supply voltage $V_+$. The $V_1$ nullcline has been plotted making use of the experimentally measured characteristic $g(V_1,V_+)$ of the nonlinear resistor. As can be seen in the figure, the value of $V_+$ affects strongly the position of the breaking point where the second slope discontinuity of that nullcline occurs. In that way, for large values of $V_+$ [Fig. 3(a)] two steady states exist, corresponding to the two outer crossings between the two nullclines, leading to a bistable situation. On the other hand, for small enough voltage $V_+$ [Fig. 3(b)] the steady state at $V_1>0$ disappears. In this monostable regime, the system behaves in an excitable way, going through large excursions in phase space as a response to sufficiently strong perturbations from its single steady state.

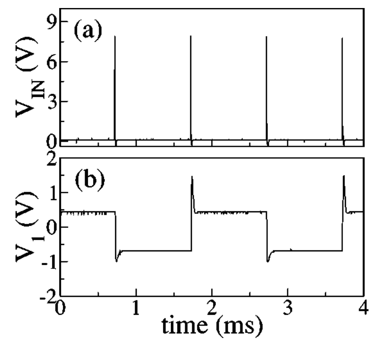


FIG. 4. Response (b) to a pulsed driving (a) of a single, noise-free nonlinear circuit in the bistable regime.

The bistable response of the system for large $V_+$ can be observed by driving the circuit externally with a periodic train of short pulses, such as the one shown in Fig. 4(a). This signal drives a solid-state switch through which the capacitor $C_1$ discharges for a short time ($\sim 5\ \mu s$, much shorter than the characteristic times of the system). This discharge produces a jump from one steady state of the system to the other, as shown in Fig. 4(b). In a biological context, a similar response to periodic trains of pulses has been recently observed in experiments on thalamocortical neurons of cats, an observation that gives evidence of the generic bistable behavior of such cells [17].

## III. NOISE-INDUCED PULSE PROPAGATION

Noise is introduced into the nonlinear elements of all circuits through the positive supply voltage of the operational amplifier, as shown in Fig. 2. The random signal is generated by amplifying the shot noise generated by a *pn* junction diode [10]. We now examine the effect of this external noise on the coupled dynamics of the one-dimensional array of circuits shown in Fig. 1, when only the first element in the chain is driven externally by a train of pulses, in the way described in the preceding section. The supply voltage $V_+$ of all circuits is set to 2 V, corresponding to a deterministic regime of bistable operation [as in Fig. 3(a)]. Under these conditions and for a small amount of noise, each jump between steady states, such as those shown in Fig. 4(b), propagates through the array in the form of a front (which is terminated by the next jump). This behavior is shown in Fig. 5(a), which displays the temporal evolution of $V_1$ for the last circuit of the chain. The periodic train of jumps exhibited by this circuit reflects a propagation of corresponding fronts originated in the first circuit by the input pulse train mentioned above (see Fig. 4).

In fact, noise acts in this system in a way similar to that described in the theoretical analysis of Ref. [15], increasing the effective value of $V_+$. This implies a modification of the nullcline scenario from the one shown in Fig. 3(a) to another one like that of Fig. 3(b). As a consequence, for a large enough intensity of the external noise, the steady state at $V_1 > 0$ becomes destabilized in such a way that the system starts to behave effectively in an excitable regime. The evolution of the last circuit in the chain is shown in Figs. 5(b)

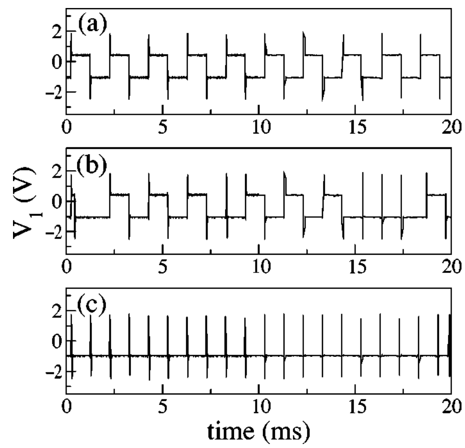


FIG. 5. Response of the last circuit in the chain to a periodic train of pulses applied to the first circuit, for increasing values of the noise intensity: (a) $V_{\mathrm{noise}} = 0.30$ V, (b) $V_{\mathrm{noise}} = 0.42$ V, (c) $V_{\mathrm{noise}} = 0.65$ V. This intensity is computed as the root mean square of the random time series provided by the noise generator.

and 5(c) for increasing noise strengths. For not too large noise, the positive steady state decays only at certain times (and at certain circuits), leading to a mixed train of spiked and squared pulses, such as the one shown in Fig. 5(b). For sufficiently large noise, on the other hand, the positive steady state always decays, and a train of fully excitable pulses propagates through the medium and reaches the last circuit [Fig. 5(c)]. Hence the system behaves as an excitable medium, even though all the circuits have been set in a deterministically bistable regime.

We have characterized the noise-induced transition from a completely bistable to a completely excitable behavior depicted in Fig. 5 by computing the percentage of pulses that reach the last circuit of the chain in the form of a spike, in contrast with the square pulses typical of the bistable regime. Since in the spiked case each circuit is reset after the passage of a bit, one can consider that information transmission is possible in this situation, and thus that percentage represents the *efficiency* of the system as a communication channel. We have plotted in Fig. 6 the value of the communication efficiency for increasing noise strength. The transition from bistability (0% efficiency) to excitability (100% efficiency) as noise increases is clearly observed.

The propagation of the noise-induced excitable pulses described above is reflected in a delay between the time evo-

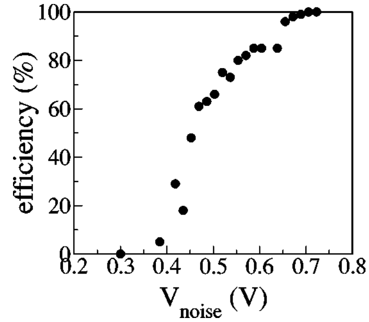


FIG. 6. Efficiency of pulse propagation vs noise strength.

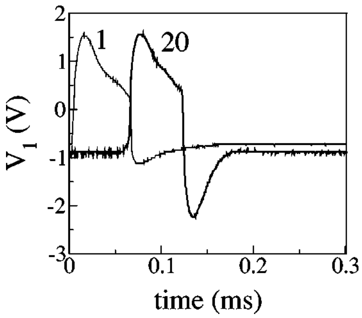


FIG. 7. Simultaneously measured time evolutions of the voltage drops in capacitor $C_1$ for the first and last circuits of the chain. The system operates in the regime corresponding to Fig. 5(c).

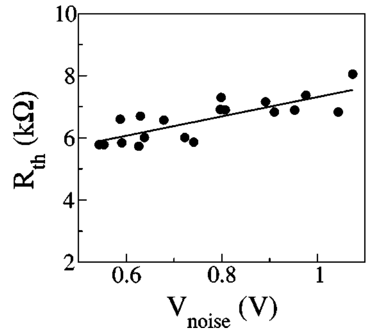


FIG. 8. Threshold value of the coupling resistance between the two last circuits of the chain beyond which propagation fails, as a function of noise intensity.

lutions of the different circuits, as can be seen in Fig. 7, which compares simultaneous measures of the time evolutions of $V_1$ for the first and last circuits of the chain. One can first note that the shapes of the pulses at the two ends of the chain are clearly different, with the pulse at the last circuit being more pronounced than the one at the first circuit. This is a consequence of the wave shaping ability of excitable media [18]. On the other hand, the time spent by the pulses in traveling through the chain can be estimated by measuring the delay between the pulses at the first and the last circuits, which in our case is smaller than one-tenth of ms. This delay is naturally related to the propagation speed of the pulse. In contrast to results found previously in theoretical models of excitable [15] and bistable [19] media, where the propagation speed was seen to increase with noise intensity, in this case we have observed no clear systematic dependence of the pulse delay with the strength of the electronic noise. This behavior still needs to be understood. On the other hand, the propagation delay (and hence the pulse speed) does depend on the coupling resistence between the circuits. As expected, when the coupling resistance increases (i.e., when the coupling strength decreases), the delay also increases (and hence the speed diminishes). For large enough resistance the pulse speed drops to zero, and propagation can no longer occur: it is the so-called regime of *propagation failure* [20].

Experimental results have shown that noise can sustain propagation of fronts under situations of propagation failure in chains of bistable electronic circuits [10]. We have checked whether the same thing occurs in our system. To that end, we have placed a varying resistor in the coupling line between the last and the next-to-last circuits in the chain, and have measured the threshold resistance at which propagation starts to fail. The results for increasing noise intensity are shown in Fig. 8. One can see that the threshold resistance increases with noise, which means that noise helps the signals (either deterministic fronts of noise-induced pulses) to propagate by decreasing the minimum coupling strength necessary for transmission. In other words, for large enough noise signals can be transmitted even when the coupling resistance is so large that propagation fails under deterministic conditions.

## IV. CONCLUSIONS

In this paper we have experimentally verified the constructive role of external noise in the propagation of signals through one-dimensional chains of nonlinear electronic circuits. Random fluctuations in the form of amplified electronic noise are added globally to all circuits in the chain. The individual elements of the array are a simplified version of the so-called Chua circuit exhibiting bistable and excitable behavior as a certain control parameter is varied. When the external noise is added to that control parameter, the transition from bistability to excitability is clearly advanced, in the direction of enlarging the region of excitable behavior. In that way, the system behaves as an excitable medium even though the parameters are set in a deterministically bistable regime. This is reflected in the fact that the system sustains propagation of pulses, instead of fronts, as long as noise of sufficient intensity is added. We have characterized the noise-induced transition between bistability and excitability by means of an efficiency parameter measuring the fraction of pulses that reach the end of the chain in excitable form. We have also analyzed the influence of noise on the propagation speed of the structures (either deterministic fronts or noise-induced pulses) through the system, but have observed no clear-cut dependence of that speed on noise intensity. Finally, we have seen that the noise-induced excitable pulses are able to propagate through the medium even under conditions of propagation failure (i.e., for small coupling between the individual elements). This property constitutes a second expression of the constructive role of fluctuations in this kind of propagating media. Given the standard use of nonlinear electronic circuits as models of signal transmission in neural media [1], we expect that findings as those reported in this paper can shed some light on the mechanisms of information propagation on those systems.

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