Density profile of halos in a specific background

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Abstract: The density profiles of dark matter halos is a subject that has been addressed by numerical and analytic means. One interesting result of simulations is the finding that the inner properties of halos depend on their background density, which is interpreted as due to the different merger rates suffered by halos in different environments, the so-called "assembly bias". In this work, we use the analytic CUSP formalism to derive the density profiles of purely accreting halos in different backgrounds, which shows that mergers are not the origin of that effect.

I. INTRODUCTION

Dark matter clustering plays a crucial role in the formation and evolution of structure in the Universe. In this respect, the typical properties, such as the density profile, of dark matter halos provide very valuable information on the way these objects grow.

Both theory and observations show that dark matter halos increase their mass and size in two complementary ways: by accreting surrounding material or undergoing major mergers with other similarly massive halos. Indeed, their collapse from the primordial density fluctuation which acts as a seed is neither monolithic nor spherical; it is rather clumpy and ellipsoidal. The difficulty to study this kind of collapse by analytic means is the reason why this issue has been traditionally addressed through numerical simulations. Cosmological simulations lead to a very accurate description of the final halo properties, but they are not well suited to understand how those properties emerge, and their results may be somewhat biased due to technical details such as the complex selection procedure one must apply to the raw data [1].

Simulations have shown the density profile of halos of a given mass slightly depends on their particular background density. This effect is called "assembly bias" since it is commonly believed that it is caused by the different frequency of mergers suffered by halos growing in different environments. This explanation relies on the idea that the density profile of halos retains the memory of their past mergers so that should be, in particular, a slight difference between the profile of purely accreting halos and halos of the same mass formed in a merger. Nonetheless, the analytic derivation of halo density profiles by means of the CUSP formalism [1] has demonstrated that both density profiles must be identical due to the violent relaxation produced in mergers which erases the memory of the halo past history. According to CUSP, the "assembly bias" found in simulations would not be the result of mergers, but would also affect purely accreting halos. Indeed, the different density profile of halos in different backgrounds would already be imprinted in the initial field of density fluctuations. More specifically, the density profile of the halo seed determining that of the final halo would already depend on its background density. Unfortunately, that claim is still to be proven.

In this paper we address this issue. Using the CUSP formalism, we derive the density profile of purely accreting halos from seeds lying in different background densities, and show that the density profile of the final halos depends, indeed, on the evolved background density, which shows that the assembly bias can be obtained without making appeal to the action of mergers. To begin with, we will briefly explain how CUSP allows one to derive the density profile of an halo from the density profile of its seed. Then we will modify that derivation so as to deal with the seeds embedded in different background densities. The accurate derivation in the most general conditions requires a complex numerical treatment. To avoid those complications we adopt some practical approximations and simplifying assumptions which allow us to perform the calculations fully analytically. This way, the treatment is simpler and better shows what is fundamental in the derivation.

II. CUSP FORMALISM

The ConflUent System of Peak trajectories is a rigorous formalism that can correctly describe and explain the origin of dark matter halo properties "from the statistics of peaks (secondary maxima) in the filtered Gaussian random field of the density fluctuations" [1]. Its predictions agree successfully with the results of N-body simulations. One interesting result of CUSP is that it allows one to prove that the density profile of purely accreting halos does not depend on their particular more or less clumpy growth, i.e. it does not depend on whether or not the halo has undergone major mergers. Thus to derive the typical density profile of halos of a given mass one can safely assume the halo evolving by pure accretion from a typical seed in the primordial density field.

Next, we remind the main lines of that derivation.

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A. Halo-peak correspondence

A well known result of structure formation theory is that, in the simplest case of spherical collapse, all peaks with a positive density contrast δ^{th} (index "th" means that we are considering a simple top-hat filtering window) at any scale R^{th} collapse at the same time t in the density field at t_i,

$$\delta^{th}(t,t_i) = \delta_c^{th}(t)\frac{D(t_i)}{D(t)} \tag{1}$$

where $\delta_c^{th}(t)$ is the critical linear density contrast for collapse at t, which, in the Einstein-de Sitter cosmology is constant and equal to 1,686, and $D(t)$ is the so-called linear growth factor, which in that cosmology simply equals the scale factor $a(t)$. For simplicity, we hereafter adopt that cosmology, which is a very good approximation for the real flat cosmology holding for our Universe at high enough redshifts (low enough cosmic times t).

On the other hand, the mass M of halos at t is related to the scale R^{th} of the peak at t_i, giving it rise through

$$R^{th}(M,t_i) = \left[\frac{3M}{4\pi\rho_c(t_i)}\right]^{1/3} \tag{2}$$

with $\rho_c(t)$ being the mean cosmic density at t.

Thus, in top-hat spherical collapse there is a one-to-one correspondence between halos and peaks given by those two relations.

However, in the real Gaussian random density field, peaks are triaxial and collapse ellipsoidally. In this more realistic case, the collapsing time for an halo with M and t depends not only on the density contrast and scale R of the corresponding peak, but also on its ellipticity e, prolateness p, and curvature x. This means that, contrarily to what happens in spherical collapse, the collapsing time of peaks with the same density contrast $\delta(t_i)$ will be different in general because depending on other characteristics of the peak. Fortunately, the probability distribution functions of e, p and x of peaks with δ at scale R when one uses the Gaussian window (notice that we drop index "th" in this case) are very sharp at their maximum values, allowing us to take them fixed to these maximum values. Then, all peaks with density contrast δ on different scales R collapse at the same time t

$$t = t_i + t_c[R,\delta,e_{max},p_{max},x_{max}] \tag{3}$$

into halos of different masses M.

Thanks to this result, a one-to-one correspondence can also be defined [1] between halos with mass M at time t and Gaussian filtered triaxial peaks collapsing ellipsoidally with density contrast δ and scale R in the initial density field, given by

$$\delta(t,t_i) = r_\delta(t)\delta^{th}(t,t_i) \tag{4}$$

$$R(M,t,t_i) = r_R(M,t)R^{th}(M,t_i) \tag{5}$$

where $r_\delta(t)$ is approximately unity in the Einstein-de Sitter universe ($a(t) = D(t)$), and the function $r_R(M,t)$ is given below in terms of the Gaussian and top-hat spectral moments on the respective scales corresponding to the mass M.

Indeed, taking profit of the fact that the power spectrum of density perturbations in the real Universe around any given scale is approximately of the power form, the corresponding spectral moment of jth order, σ_j^f, on scale R_f for a filtering window f with Fourier transform $W_f(kR_f)$ takes the form

$$(\sigma_j^f)^2(R_f) \approx \frac{C}{2\pi^2 R_f^{n+3+2j}}\int_0^\infty dx\; x^{n+2(1+j)}W_f^2(x). \tag{6}$$

Thus, isolating $r_R(M,t)$ from Eq. (5) and using the relation between the scale corresponding to M and the 0th order spectral moments for both the Gaussian and top-hat filters, we arrive at

$$r_R(M,t) = [Q_0 r_\sigma^{-1}(M,t)]^{\frac{2}{n+3}}, \tag{7}$$

where constant Q_j^2 is defined as $\frac{\int_0^\infty dx\; x^{n+2(1+j)}W_{Gauss}^2(x)}{\int_0^\infty dx\; x^{n+2(1+j)}W_{th}^2(x)}$, and $r_\sigma(M,t)$ can once again be approximated by unity in the Einstein-de Sitter universe.

Altogether allows us to write the relation (5) in the simple approximate form

$$R(M,t_i) \approx Q_0^{\frac{2}{n+3}}\left[\frac{3M}{4\pi\rho_c(t_i)}\right]^{1/3} \tag{8}$$

The values n and Q_0 at the scales of galactic halos are $\sim -1,5$ and $\sim 0,5$, respectively.

B. Peak trajectories

Given the one-to-one correspondence between halos with M at t and their associated peaks with δ on scale R at the initial time t_i, as a halo accretes and its mass grows, the associated peak traces a continuous trajectory $\delta(R)$ in the δ-R plane.

Taking advantage of the relation

$$\frac{\partial\delta(\mathbf{r},R)}{\partial R} = R\nabla^2\delta(\mathbf{r},R) \equiv -x(\mathbf{r},R)\sigma_2(R)R \tag{9}$$

satisfied by the density contrast with varying scale for a Gaussian filter, where $x(\delta,R)$ is the curvature of the peak on scale R. Consequently, the mean peak trajectory $\delta(R)$ tracing the mass growth $M(t)$ by accretion of typical halos satisfies the differential equation

$$\frac{d\delta}{dR} = -\bar{x}(R,\delta)\sigma_2(R)R \tag{10}$$

where $\bar{x}(R,\delta)$ is the mean curvature x of peaks with δ on scale R, related to the mean instantaneous accretion rate of halos with M at t. Note that, in the previous derivation, the inverse of the mean inverse curvature of peaks has been replaced by the simple mean peak curvature. This is indeed a very good approximation because of the above mentioned sharp distribution of peak curvatures around the maximum value.

For halos of galactic scales, $\bar{x}(R,\delta)$ can be approximated by $\gamma\nu$ with $\gamma = \frac{\sigma_1^2}{\sigma_0\sigma_2}$ and $\nu = \frac{\delta(t)}{\sigma_0(R)}$. With that approximation equation (10) takes the simple form

$$\frac{d\ln\delta}{d\ln R} \approx -\left[\frac{\sigma_1(R)}{\sigma_0(R)}\right]^2 R^2 \tag{11}$$

Thus, given that the power spectrum of density fluctuations is approximately of the power-law form $P(k) \approx Ck^n$, we have

$$\frac{d\ln\delta}{d\ln R} \approx k, \tag{12}$$

whose solution is also of the simple power-law form

$$\delta(R) \approx DR^k \tag{13}$$

with index $k = [(n+3)/2]^{3/2} \sim 0.65$, and D an integration constant fixing the particular trajectory corresponding for the halo with M_0 at t_0.

C. Halo density profile

The typical *unconstrained* density profile of halos with current mass M_0 at the final time t_0 can be derived from the previous results. Indeed, taking into account that accreting halos grow inside-out [1], the time evolution of the mass and radius of halos, $M(t)$ and $r(t)$, determine in the parametric form (with t as parameter t), their mass profile $M(r)$. And by differentiating it, we can obtain the desired the density profile $\rho(r)$.

The typical mass growth $M(t)$ of accreting halos with M_0 at t_0 can be readily obtained from their associated typical peak trajectory $\delta(R)$, Eq. (13) using the relations $R(M,t_i)$ and $\delta(t,t_i)$ given by Eqs. (8) and (4). We then obtain

$$\delta(t,t_i) \approx DQ_0^{\frac{2k}{n+3}}\left[\frac{3M(t)}{4\pi\rho_c(t_i)}\right]^{k/3}, \tag{14}$$

which, by isolating the mass of the halo scaled to its current value and expressing $\delta(t,t_i)$ in terms of $\delta_c^{th}(t)$ (Eq. [1]), leads to

$$\frac{M(t)}{M_0} \approx \left[\frac{\delta_c^{th}(t)}{\delta_c^{th}(t_0)}\frac{a(t)}{a(t_0)}\frac{D^2(t_0)}{D^2(t)}\right]^{3/k}. \tag{15}$$

On the other hand, the evolution of the typical (virial radius of halos with M at t, $r = \{3M/[4\pi\Delta_{vir}(t)\rho_c(t)]\}^{1/3}$, scaled to its current value r_0 leads to

$$\frac{r(t)}{r_0} = \left[\frac{M(t)\Delta_{vir}(t_0)\rho_c(t_0)}{M_0\Delta_{vir}(t)\rho_c(t)}\right]^{1/3}. \tag{16}$$

Thus, the density profile $\rho(r)$ in the parametric form is given by

$$\rho(t) = \frac{\Delta_{vir}(t_0)\rho_c(t_0)}{4\pi}\frac{d(M(t)/M_0)}{dt}\left[\frac{d([r(t)/r_0]^3)}{dt}\right]^{-1} \tag{17}$$

and $r(t)$ from equation (16).

Differentiating the relation (15) and taking into account the Friedman equation in the Einstein-de Sitter universe at the matter-dominated era,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\frac{\rho_c(t_0)}{a^3(t)}, \tag{18}$$

we arrive at

$$\frac{d(M/M_0)}{dt} \approx -\frac{3}{k}\frac{M(t)}{M_0}\frac{\dot{a}}{a(t)}. \tag{19}$$

To derive Eq. (19) we have used the relations $\delta_c^{th}(t) = 1.686$ and $D(t) = a(t)$ holding for the Einstein-de Sitter universe [4].

On the other hand, differentiating the relation (16), we are led to

$$\frac{d(r/r_0)^3}{dt} \approx \frac{3M(t)}{kM_0}\left[\frac{\Delta_{vir}(t_0)}{\Delta_{vir}(t)}\right]^2 a^2(t)\dot{a}\left[k - \frac{\Delta_{vir}(t)}{\Delta_{vir}(t_0)}\right], \tag{20}$$

where, in the Einstein-de Sitter universe $\Delta_{vir}(t)$ is constant and equal to $18\pi^2$ [4].

Plugging these expressions in Eq. (17) we finally obtain

$$\rho(t) = \frac{9\pi\rho_c(t_0)}{2a^3(t)}(1-k)^{-1} \tag{21}$$

defining, together with $r(t)$ in Eq. (16), the desired approximate unconstrained halo density profile $\rho(r)$ in the parametric form.

To illustrate the goodness of this kind of derivation we show in Figure 1 the accurate unconstrained typical (mean) halo density profile predicted by CUSP for halos of $M = 10^{13}M_\odot$ in a realistic universe (with the best current values of the cosmological parameters), compared to the corresponding density profile found in simulations (we actually represent two different analytic fits to it commonly used).

III. DENSITY PROFILE OF HALOS WITH BACKGROUND

Let us now turn to the main objective of this work: the derivation of the density profile of halos with M_0 *constrained to lie at* t_0 *on a specific background.*

To do that we will follow exactly the same derivation above, but for that particular kind of halos. The condition to lie on a specific background will translate on the corresponding seeds which will thus be peaks also located in a specific background evolving into the final one

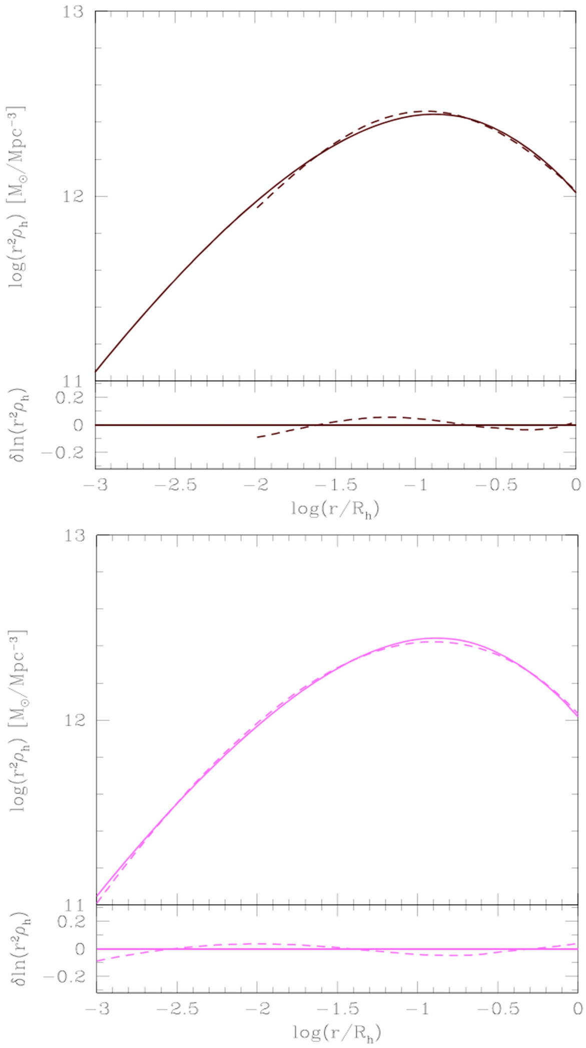

FIG. 1: [1] Mean unconstrained density profiles for halos of $M_{vir} = 10^{13} M_\odot$ derived by means of CUSP with no free parameter (solid lines), compare to the the results of simulations (dashed lines). Top panel is for the NFW analytic fit and the bottom is for the Einasto analytic fit. In each panel we also represent the deviations which show the same S-shape as found in the fits with those analytic functions, with the results of numerical simulations.

at the final time t_0. In fact, since the background does not collapse at t_0 we can simply monitor its evolution of its density contrast in the linear regime, given by the simple expression [6]

$$\delta_b(t) = \delta_b(t_i)\frac{D(t)}{D(t_i)} \tag{22}$$

As $D(t_0)$ and $D(t_i)$ are known, $\delta_b(t_i)$ can be readily related to the final value $\delta_b(t_0)$. Note that the scale R_b of the background also varies with time in such a way that at every moment the background enclosing (according to the simple top-hat expression) the same constant mass M_b.

Following the same steps leading to the typical peak trajectory associated to typical accreting halos, but now with the constraint that peaks along the trajectory lie on the varying background with δ_b on scale R_b, we obtain a differential equation for the density contrast $\delta(R, R_b)$ of the constrained peaks identical to that for uncosntrained ones, Eq. (10), but with the curvature of peaks at each point of the trajectory satisfying that constraint, $x(\delta, R|\delta_b, R_b(\delta_b, M_b))$. However, the approximation $\langle x\rangle \approx \gamma\nu$ used above is still valid now [1], though with slightly different values of the functions γ and ν, hereafter denoted with tilde, having the influence of the background [2].

Their detailed expression is

$$\tilde{\gamma}^2 = \gamma^2\left[1 + \epsilon^2\frac{(1-r_1)^2}{(1-\epsilon^2)}\right] \tag{23}$$

$$\tilde{\nu} = \frac{\gamma}{\tilde{\gamma}}\frac{(1-r_1)}{(1-\epsilon^2)}\left[\nu\frac{(1-\epsilon^2 r_1)}{(1-r_1)} - \epsilon\nu_b\right] \tag{24}$$

in terms of the spectral parameters,

$$\epsilon \equiv \frac{\sigma_{0h}^2}{\sigma_0\sigma_{0b}}, \qquad r_1 \equiv \frac{\sigma_{1h}^2\sigma_0^2}{\sigma_{0h}^2\sigma_1^2}$$

with $\sigma_{jh}(R, R_b)$ defined just as σ_j Eq. (6) and replacing R for the rms average scale $R_h \equiv [(R^2 + R_b^2)/2]^{1/2}$. Here, we must emphasise the reliability of the results since for $\epsilon \to 0$, $\tilde{\gamma} = \gamma$ and $\tilde{\nu} = \nu$ we come back to the same functions as in the no-background scenario.

That accurate expression for the mean curvature greatly complicates the explicit form of that equation. Instead of Eq. (11), we now have

$$\frac{d\delta}{dR} = -\frac{\delta(\sigma_{0b}^2\sigma_1^2 - \sigma_{0h}^2\sigma_{1h}^2) - \delta_b(\sigma_{0h}^2\sigma_1^2 - \sigma_{1h}^2\sigma_0^2)}{\sigma_2(\sigma_0^2\sigma_{0b}^2 - \sigma_{0h}^4)}\sigma_{2h}R_h \tag{25}$$

Solving analytically that equation requires, besides adopting the simple Einstein-de Sitter like ever, neglecting R in front of R_b. Then, expression (25) becomes,

$$\frac{d\delta}{dR} \approx -\frac{B}{A}\left(\delta - \delta_b 2^{\frac{n+3}{2}}\right)2^{\frac{n+6}{2}}\frac{R^{n+5}}{R_b^{n+6}} \tag{26}$$

where A and B are two constants defined as, $A \equiv \int_0^\infty dx\, x^{n+2}W_f^2(x)$ and $B \equiv \int_0^\infty dx\, x^{n+4}W_f^2(x)$, respectively, for the Gaussian filter. Eq. (26) can be reexpressed in the simpler form similar to Eq. (12)

$$\frac{d\ln\delta}{d\ln R} \approx -\frac{B}{A}\left(1 - \frac{\delta_b}{\delta}\cdot 2^{\frac{n+3}{2}}\right)\left(\frac{2^{1/2}}{R_b/R}\right)^{n+6}. \tag{27}$$

Taking into account that, in linear regime, all fluctuations evolve in the same way with time from t_i to t_0 (the only difference for the fluctuation collapsing into the halo is that, at that time, it reaches the critical density contrast for collapse), we have that $\delta_b(t)/\delta(t)$ is a constant, hereafter denoted as Δ_δ. And the same is true for the ratio $R/R_b = (M_0/M_b)^{1/3}$, hereafter denoted as Δ_R, Eq. (27) can be readily integrated. Interestingly enough, the resulting peak trajectory

$$\delta(R) \approx DR^{\tilde{k}}, \tag{28}$$

in the present conditioned case is a power-law like that in the previous unconditioned one, but with the new index where in this case,

$$\tilde{k} = -\frac{B}{A}\left(1 - \Delta_\delta \cdot 2^{\frac{n+3}{2}}\right)\left(\Delta_R 2^{1/2}\right)^{n+6} \tag{29}$$

with both constant values Δ_δ and Δ_R smaller than unity. In other words, the trajectory of peaks subject to lie on a given background density is the same as for unconditioned peaks but with slightly different power index.

This is a very interestingly result because it means that, using that peak trajectory and following the same derivation above for the unconditioned density profile of halos, we will obtain a conditioned profile of the same form as (21) but replacing k by $\tilde{k}$ and, therefore, we come back to the same definition as before. We thus conclude that the typical (mean) density profile $\rho(r)$ of *purely accreting halos lying on any specific background* is similar, though not identical, to that of unconditioned accreting halos of the same mass at the same cosmic density, the difference being only encoded in the slightly different value, dependent on Δ_δ and Δ_R, of index k ($\tilde{k}$) appearing in the expression of $\rho(t)$, Eq. (21).

IV. CONCLUSIONS

The purpose of this paper was to find an analytical expression for the the typical (mean) density profile of purely accreting halos subject to the condition of lying in a specific background, and see whether or not it depends on the background. If it depended on it, this would mean that the different typical (mean) density profiles found in simulations for halos in different backgrounds can be explained without making appeal to mergers, as usually believed, the so-called assembly bias.

Instead of following an accurate derivation, we have preferred to derive simple analytic expressions, which better show where the different background enters in the final result. To do that we have adopted the simple case of an Einstein-de Sitter univers and used some approximations. But these approximations and simplifying assumptions should not alter the main conclusion of the work.

We have found a very simple result showing that the density of purely accreting profile of halos slightly depends, indeed, on the background density. This result is thus fully consistent with the claim reached by means of CUSP that the density profile (and all other properties) of halos of a given mass at a given cosmic time does not depend on the amount and frequency of the mergers they have suffered. They only depend on the properties of their respective collapsing seeds (peaks) in the primordial field of density fluctuations.

One last point, personally this thesis has been extremely enriching and has helped me to open my mind and gain a deeper point of view on the subject of structure (and galaxy) formation in the Universe which was given as well by my advisor.

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