The chiral approach to Meson-Baryon interactions with S = -2

Author: Victoria Valcarce Cadenas
Facultat de Física, Universitat de Barcelona, Diagonal 645, 08028 Barcelona, Spain.

Advisor: Volodymyr Magas

Abstract: We show that two Ξ^* poles can be dynamically generated in a coupled-channels chiral model with next-to-leading order terms in Lagrangian, focusing on the study of low-energy meson-baryon scattering in the strangeness $S = -2$ sector and isospin $I = 1/2$. One of the resonances found, which has a mass of about 1600 MeV, couples very strongly to the $\pi\Xi$ channel. Consequently, it can be identified as the $\Xi(1620)$ state with $J^P = 1/2^-$. The second pole detected, near the $\bar{K}\Sigma$ threshold, can be identified as an s-wave $\bar{K}\Sigma$ molecular state, which we can associate with the $\Xi(1690)$ resonance assigning it the same quantum numbers as the previous one.

I. INTRODUCTION

The search for new subatomic particles and the study of their internal structure is fundamental to fulfil the eagerness to answer big questions like how matter is formed or how the Universe has evolved.

Quantum Chromodynamics (QCD) is the theory that describes the strong interaction within the Standard Model, which relates the fundamental interactions as an exchange of the so-called gauge bosons. For the strong interaction, these correspond to gluons. Thus, these interactions are responsible for the grouping of quarks, leading to hadrons. Consequently, hadrons are divided into two groups according to their constituents: mesons, which consist of one quark and one antiquark; and baryons, formed by three quarks.

One of the main goals of Hadronic Physics is the study of the nature of hadrons and the prediction of new states and their properties. At high momentum transfers (above several GeV), strong interactions are described by the scattering of quarks and gluons, and perturbative methods can be applied since the strong coupling is small; quarks behave as if they were free particles. Nevertheless, at lower energies (< 1 GeV), QCD is strongly coupled and a non-perturbative analysis is needed, because a power expansion of the coupling constant would diverge.

The study of the low energy regime is extremely important since this is where most of the hadronic and nuclear processes take place. Because QCD becomes non-perturbative, a high energy independent dynamics would be necessary, where the degrees of freedom of the Lagrangian are no longer quarks and gluons, but hadrons, in order to be able to carry out a perturbative expansion of it. Thus, one must resort to effective theories, such as $SU(3)$ Chiral Perturbation Theory (χPT), which respects the symmetries of QCD, in particular, chiral symmetry and chiral spontaneous symmetry breaking patterns [1].

QCD contains six flavours of quarks with different masses. As a result, they are not strictly interchangeable with each other. However, three of the flavours (up, down and strange) have small and very similar masses, and an approximate $SU(3)$ flavour symmetry for these is considered valid. This is the reason why from now on, we will only focus on light quarks.

At low energies, the χPT describes satisfactorily the meson-meson or meson-baryon interactions. Nevertheless, it fails in the vicinity of a resonance, i.e., baryonic or mesonic excited states, since these are associated with a pole in the scattering amplitude that cannot be reproduced by a perturbative expansion. Unitarized Chiral Perturbation Theory (UχPT), which combines chiral dynamics with unitarization techniques in coupled channels, turns out to be a very powerful solution to this problem, allowing to describe the so-called dynamically generated resonances.

In this study the focus is given to the $\Xi(1620)$ and $\Xi(1690)$ resonances. In the Particle Data Book, a status of one and three stars is attributed to them, respectively. They are both quoted with an isospin $I = 1/2$, but the spin and parity are unknown [2]. Several theoretical and experimental studies point to a value of $J^P = 1/2^-$ [3–5]. In [3], Ramos *et al.* apply UχPT extended to the strangeness $S = -2$ sector and use an effective Lagrangian at lowest order, managing to generate the $\Xi(1620)$ resonance. However, in [6], Bao-Xi Sun *et al.* add the so-called Born terms, generating a resonance that is not consistent with the $\Xi(1620)$ particle. The aim of this project is to show that by incorporating, in addition, next-to-leading order terms which have been studied and fitted to experimental data in the $S = -1$ sector in [7], both resonances can be dynamically generated. This would demonstrate that they can be identified as molecular states.

II. CHIRAL UNITARY APPROACH

A. Lagrangian and interaction kernel

This section provides a development of the coupled-channel formalism employed for describing meson-baryon scattering [for a more detailed explanation see [1]]. The starting point is the $SU(3)$ chiral effective Lagrangian that describes the coupling of the octet of pseudoscalar mesons (π, K, η) to the octet of $1/2^+$ baryons (N, Λ, Σ,

Ξ),

$$\mathcal{L}_{eff} = \mathcal{L}_{\phi B}^{(1)} + \mathcal{L}_{\phi B}^{(2)} \tag{1}$$

which consists of an expansion in powers of momentum where the number in parentheses points to the number of powers in each term. The first term corresponds to the lowest order (LO) contributions, while the second one is the next-to-leading order (NLO) term. Since higher orders only introduce small corrections to the main term and for each higher order contribution introduced the number of low energy constants grows, 2nd order higher terms have been neglected in this work.

Consequently, the relevant contributions to the interaction kernel are diagrammatically represented in Fig. 1. The first three ((i), (ii) and (iii)) are calculated using the LO term of the Lagrangian, while the (iv) contribution comes from considering the NLO term.

FIG. 1: Feynman diagrams of the pseudoescalar meson-baryon octet interaction: Weinberg-Tomozawa term (i), direct (ii) and crossed (iii) Born terms, and NLO terms (iv). Solid (dashed) lines represent the octet baryons (pseudoescalar octet mesons).

At LO $\mathcal{O}(p)$, the most general form of the Lagrangian is given by

$$\mathcal{L}_{\phi B}^{(1)} = \langle \bar{B}(i\gamma_\mu D^\mu - M_0)B\rangle + \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \\ + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle \tag{2}$$

where the first term provides the contact interaction of the meson-baryon, known in the literature as the Weinberg-Tomozawa (WT) term. The other terms associated to the coefficients D and F give a contribution to the s- and u- channel interactions, which are usually called direct Born (DB) term and crossed Born (CB) term, respectively.

The braket $\langle ... \rangle$ stands for the trace of its argument, M_0 is the common baryon octet mass in the chiral limit and the low energy constants D and F are the $SU(3)$ axial vector constants subject to the constraint $g_A = D + F = 1.26 \pm 0.05$, which arises from the determination of neutron and hyperon β decays [8]. Other important definitions are:

$$D^\mu B = \partial^\mu B + \frac{1}{2}\left[\left[u^\dagger, \partial^\mu u\right], B\right] \tag{3}$$

$$u^2(\phi) = U(\phi) = \exp\left(i\frac{\phi}{f}\right) = \mathbb{1} + \frac{i\phi}{\sqrt{2}f} - \frac{\phi^2}{4f^2} + ... \tag{4}$$

$$u^\mu = iu^\dagger \partial^\mu U u^\dagger \tag{5}$$

where f is the meson decay constant in the chiral limit.

The matrices of the pseudoscalar meson and the baryon octet are given as follows

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \tag{6}$$

and

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \tag{7}$$

Developing the WT term of the Lagrangian in Eq. (2) in a non-relativistic limit

$$\mathcal{L}_{\phi B}^{WT} = \langle \bar{B}i\gamma_\mu \left[(\phi\partial^\mu\phi - \partial^\mu\phi\phi)B - B(\phi\partial^\mu\phi - \partial^\mu\phi\phi)\right]\rangle, \tag{8}$$

one can derive the meson-baryon interaction kernel associated (see representation (i) in Fig. 1), which reads:

$$V_{ij}^{WT} = -C_{ij}\frac{1}{4f^2}\mathcal{N}_i\mathcal{N}_j(2\sqrt{s} - M_i - M_j) \tag{9}$$

where the sub-indexes i, j denote the incoming and outgoing meson-baryon channel. $N_i = \sqrt{(M_i + E_i)/2M_i}$ is the normalization factor with $E_i = (s + M_i^2 - m_i^2)/(2\sqrt{s})$ being the energy of the baryon. M_i and m_i are, respectively, the physical masses of the baryon and the meson in channel i, and $\sqrt{s}$ is the total center-of-mass (CM) energy of the system.

As it is seen, the WT term depends only on one parameter - the pion decay constant f, which is well known experimentally, $f_{exp} = 92.4$ MeV. However, in UχPT calculations this parameter is usually ranging from $f = 1.15f_{exp}$ to $f = 1.2f_{exp}$, meaning to be a sort of average over the decay constants of the mesons involved in the various coupled channels [7]. The indices (i, j) cover all the initial and final channels which, in the case of strangeness $S = -2$ and charge $Q = 0$ explored here, amount to six: $\pi^+\Xi^-$, $\pi^0\Xi^0$, $\bar{K}^0\Lambda$, $K^-\Sigma^+$, $\bar{K}^0\Sigma^0$, $\eta\Xi^0$.

The constants C_{ij} are determined by $SU(3)$ Clebsch-Gordan coefficients. We obtained them (see Appendix A, Table IV) by introducing the meson and baryon fields in their matrix form and, then, computing them in the Lagrangian using Mathematica program.

Next in the ChPT hierarchy are the Born terms from Eq. (2). Following Feynman's rules, we constructed the direct and crossed Born diagrams to obtain the interaction kernel. Considering the s-wave projection and the non relativistic assumption, the DB and CB terms reads

$$V_{ij}^{DB} = -\sum_{k=1}^{8} \frac{C_{\bar{i}i,k}^{Born} C_{\bar{j}j,k}^{Born}}{12f^2} \mathcal{N}_i \mathcal{N}_j \cdot \\ \cdot \frac{(\sqrt{s}-M_i)(\sqrt{s}-M_k)(\sqrt{s}-M_j)}{s-M_k^2} \tag{10}$$

and

$$V_{ij}^{CB} = \sum_{k=1}^{8} \frac{C_{\bar{j}k,i}^{Born} C_{\bar{i}k,j}^{Born}}{12f^2} \mathcal{N}_i \mathcal{N}_j \Bigg[\sqrt{s} + M_k \\ - \frac{(M_i+M_k)(M_j+M_k)}{2(M_i+E_i)(M_j+E_j)} (\sqrt{s} - M_k + M_i + M_j) \\ + \frac{(M_i+M_k)(M_j+M_k)}{4q_iq_j} \Bigg\{ \sqrt{s} + M_k - M_i - M_j \\ - \frac{s+M_k^2-m_i^2-m_j^2-2E_iE_j}{2(M_i+E_i)(M_j+E_j)} (\sqrt{s} - M_k + M_i + M_j) \Bigg\} \\ \cdot \ln \frac{s+M_k^2-m_i^2-m_j^2-2E_iE_j-2q_iq_j}{s+M_k^2-m_i^2-m_j^2-2E_iE_j+2q_iq_j} \Bigg], \tag{11}$$

$$\text{with} \quad q_i = \frac{\sqrt{(s-(M_i+m_i)^2)(s-(M_i-m_i)^2)}}{2\sqrt{s}} \tag{12}$$

being the CM three-momentum. We see in Fig. 1 (terms (ii) and (iii)) that the reaction goes $\phi_i B_i \to B_k \to \phi_j B_j$, where B_k is the intermediate baryon. The sum over k extends to all possible baryons of the octet. However, as B_k needs to preserve the same quantum numbers as the incoming and outgoing meson-baryon pair, i.e., $S = -2$ and $Q = 0$, there is just one possibility for the DB term within the field basis considered, the Ξ^0 particle. The Born coefficients, which include the constants D and F, integrate the total effective coupling for each transition $\bar{i}i \to \bar{j}j$ for each baryon k involved in the interaction. They are given by the product of the coupling constants, $C_{\bar{i}i,k}^{Born} C_{\bar{j}j,k}^{Born}$, of each vertex of the Feynmann diagram, and their compilation can be found in Appendix B.

Finally, at NLO $\mathcal{O}(p^2)$, the contributions of the $\mathcal{L}_{\phi B}^{(2)}$ term to meson-baryon scattering are the s-wave ones, and these are:

$$\mathcal{L}_{\phi B}^{(2)} = b_D \langle \bar{B}\{\chi_+, B\}\rangle + b_F \langle \bar{B}[\chi_+, B]\rangle + b_0 \{\bar{B}B\rangle\langle\chi_+\rangle \\ + d_1 \langle \bar{B}\{u_\mu, [u^\mu, B]\}\rangle + d_2 \langle \bar{B}[u_\mu, [u^\mu, B]]\rangle \\ + d_3 \langle \bar{B} u_\mu \rangle \langle u^\mu B\rangle + d_4 \langle \bar{B}B\rangle\langle u^\mu u_\mu\rangle \tag{13}$$

where χ_+ is the term responsible of explicit breaking of chiral symmetry:

$$\chi_+ = -\frac{1}{4f^2}\{\phi, \{\phi, \chi\}\}, \quad \chi = \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & m_K^2 - m_\pi^2 \end{pmatrix}. \tag{14}$$

The parameters preceding each term of Eq. (13) are the low energy constants at NLO. After some algebra, the resulting expression of the potential (see (iv) in Fig. 1) becomes:

$$V_{ij}^{NLO} = \frac{1}{f^2} \mathcal{N}_i \mathcal{N}_j \Bigg[D_{ij} - 2 \Bigg(w_i w_j \\ + \frac{q_i^2 q_j^2}{3(M_i+E_i)(M_j+E_j)} \Bigg) L_{ij} \Bigg] \tag{15}$$

where $w_i = \sqrt{m_i^2 + q_i^2}$ is the meson energy. The D_{ij} and L_{ij} coefficients depend on the NLO parameters and have been determined using Mathematica, in the same procedure as for LO constants. They are given in Appendix C, Table V.

To recap, this model has 10 free parameters $(f, D, F, b_0, b_D, b_F, d_1, d_2, d_3, d_4)$ that have been studied and fitted to the experimental data in the $S = -1$ sector in [7]. These values are given in Appendix D, Table VI. The power of UχPT lies in the fact that we can extend to our sector the same Lagrangian as the one used in such reference. So for $S = -2$, the experimental data is too poor to fit the parameters correctly.

All in all, adding all the equations found in this section, the total interaction kernel up to NLO can be written as:

$$V_{ij} = V_{ij}^{WT} + V_{ij}^{DB} + V_{ij}^{CB} + V_{ij}^{NLO} \tag{16}$$

B. Bethe-Salpeter equation

As previously mentioned, a perturbative treatment of the scattering amplitude can not be employed in an energy region which contains molecular like resonances. Therefore, a non-perturbative resummation is needed. The UχPT consists in solving the Bethe-Salpether (BS) equation in coupled channels for the scattering amplitudes T_{ij} using the potential derived from the chiral Lagrangian, Eq. (16). The BS expression, which accounts for infinite contributions of the coupled channels, corresponds to an infinite sum (see Fig. 2):

$$T_{ij} = V_{ij} + V_{il}G_lV_{lj} + V_{il}G_lV_{lk}G_kV_{kj} + ... = V_{ij} + V_{il}G_lT_{lj} \tag{17}$$

where the sub-indexes $i, j, l, ...$ run over all possible channels and the loop function G_l stands for the propagator of the meson-baryon state of channel l.

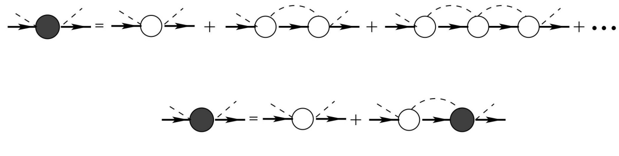

FIG. 2: Schematic illustration of the Bethe-Salpeter equation. The scattering matrix T (interaction kernel V) is represented by the solid (empty) blobs, and the Green function G is denoted by the intermediate meson-baryon propagators (loops).

Developing Eq. (17) (more detailed discussion can be found in [1]), it can be presented in matrix form as follows:

$$T = (1 - VG)^{-1} V \tag{18}$$

where the loop function G stands for a diagonal matrix with elements:

$$G_l = i \int \frac{d^4 q_l}{(2\pi)^4} \frac{2M_l}{(P - q_l)^2 - M_l^2 + i\epsilon} \frac{1}{q_l^2 - m_l^2 + i\epsilon} \tag{19}$$

with P the total momentum of the system. Its logarithmic divergence makes it necessary to apply dimensional regularization (see [9] for details), obtaining as final expression:

$$G_l = \frac{2M_l}{(4\pi^2} \Bigg\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \frac{q_l}{\sqrt{2}} \ln \left[\frac{(s + 2\sqrt{s} q_l)^2 - (M_l^2 - m_l^2)^2}{(s - 2\sqrt{s} q_l)^2 - (M_l^2 - m_l^2)^2} \right] \Bigg\}. \tag{20}$$

This loop functions G_l depend on new free parameters $a_l(\mu)$, the so-called substraction constants, that replace the divergence for a given dimensional regularization scale μ which is taken to be 1 GeV, a characteristic value for this type of physics. Thus, natural-size values of a_l can be deduced and they are found to be approximately of the order of -2 (see [1] for a complete description).

Applying physical masses of hadrons, we can find six physical channels in our sector. However, taking into account isospin symmetry, these can be reduced into four. So, in the $S = -2$ sector and $I = 1/2$, the wave function in the isospin space can be written (just by looking at any Clebsch-Gordan coefficient table) as

$$|\pi\Xi; \frac{1}{2}, \frac{1}{2}\rangle = \sqrt{\frac{2}{3}} |\pi^+ \Xi^-\rangle - \sqrt{\frac{1}{3}} |\pi^0 \Xi^0\rangle , \tag{21}$$

$$|\bar{K}\Lambda; \frac{1}{2}, \frac{1}{2}\rangle = |\bar{K}^0 \Lambda\rangle , \tag{22}$$

$$|\bar{K}\Sigma; \frac{1}{2}, \frac{1}{2}\rangle = -\sqrt{\frac{2}{3}} |K^- \Sigma^+\rangle + \sqrt{\frac{1}{3}} |\bar{K}^0 \Sigma^0\rangle , \tag{23}$$

$$|\eta\Xi; \frac{1}{2}, \frac{1}{2}\rangle = |\eta \Xi^0\rangle . \tag{24}$$

Now, we can proceed to construct the amplitudes of T-matrix in isospin basis from the amplitudes calculated in physical basis (see Appendix E).

In the case of isospin $I = 3/2$, the procedure is the same. After obtaining their scattering amplitudes, it is observed that the meson-baryon interactions in this case are repulsive, therefore, no resonance states would be generated dynamically. Hence, we will neglect this isospin channel in our calculations.

III. DATA TREATMENT

With the new T-matrix in the basis of isospin $I = 1/2$, one can find the resonances. These appear as matrix-singularities, and recalling Eq. (18), they will be the zeros in the complex plane of $(1 - VG)^{-1}$. In the present work, the problem is solved numerically with the Fortran code used in [7], which we modified and adapted for the current situation. Our program looks for pole positions in the second Riemann sheet of the scattering amplitude and couplings using the Steepest Descent Method.

In the proximity of the pole, the scattering matrix on the real axis has the following appearance:

$$T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - z_p} \tag{25}$$

where $z_p = M_R - i\Gamma_R/2$ is the pole position in the complex energy plane, whose real and imaginary parts correspond to its mass (M_R) and half width ($\Gamma_R/2$). The complex coupling constants (g_i, g_j) of the resonance to the corresponding meson-baryon channels are extracted from the residue of the pole.

As a trial run, we set the substraction constants a_l to a value of -2 as in [3, 6] and we generate two resonances in contrast with the results of such references. Thus, we can conclude that NLO term in the Lagrangian has a fundamental contribution in this particular sector.

Now the question is whether varying only the substraction constants within a reasonable range, we can move these poles close to the experimental position of $\Xi(1620)$ and $\Xi(1690)$ resonances (Table I).

	$a_{\pi\Xi}$	$a_{\bar{K}\Lambda}$	$a_{\bar{K}\Sigma}$	$a_{\eta\Xi}$
Set 1	-2.00	-2.00	-2.00	-2.00
Set 2	-1.00	-1.00	-2.00	-2.00
Set 3	-0.80	-0.80	-2.00	-2.00
Set 4	-1.00	-0.50	-2.25	-2.25
Set 5	-1.90	-0.00	-2.25	-2.25

TABLE I: Substraction constants for a $\mu = 1$ GeV.

The experimental measurements of the one-star $\Xi(1620)$ state, which has only been seen to decay into $\pi\Xi$ final states, show values of its mass and decay width of 1600-1645 MeV and 15-55 MeV, respectively. The $\Xi(1690)$ resonance is better known and is rated with three stars. Several experimental studies attribute its mass and width as 1690 ± 10 MeV and 20 ± 15 MeV, respectively. Historically, this resonance was discovered as a threshold enhancement in both the neutral and charged $\bar{K}\Sigma$ mass spectra in $K^- p \to (\bar{K}\Sigma)\pi K$.

Tables II and III show the evolution at the lower and higher mass poles for sets of coupling constants from Table I. The lower pole shows a strong coupling to the $\pi\Xi$ and $\bar{K}\Lambda$ channels, but a very weak one to $\bar{K}\Lambda$ and $\eta\Xi$. This fact can be considered as an argument to identify this resonance as the $\Xi(1620)$. We note that a change in the values of the subtraction constants for $\pi\Xi$ and $\bar{K}\Lambda$ channels does not affect the position of the more massive resonance, since the coupling to these channels is very weak. However, it couples more strongly to $\bar{K}\Sigma$ and $\eta\Xi$ states, indicating clear evidence for the assignment of the pole to the $\Xi(1690)$ resonance.

From these tables we see that the second pole is in a good comparison with experimental data of the $\Xi(1690)$ resonance. While for the $\Xi(1620)$ state the situation is more complicated. Sets 3, 4 and 5 put the mass of this resonance in the experimental observed range, but the width is rather far from the available data. Obviously, to better reproduce the results we could adjust a bit the model parameters of Table VI, but this is outside the purpose of our study. However, only varying substraction constants we got relatively narrow poles in set 5 (almost compatible with experimental width), but the price is to assign to $a_{\bar{K}\Lambda}$ a value of -0.00, which is a bit off the natural-size.

$\Xi(1620)$	$M - i\frac{\Gamma}{2}$	$\lvert g_{\pi\Xi}\rvert^2$	$\lvert g_{\bar{K}\Lambda}\rvert^2$	$\lvert g_{\bar{K}\Sigma}\rvert^2$	$\lvert g_{\eta\Xi}\rvert^2$
Set 1	1513 - i41	2.060	1.280	0.365	0.230
Set 2	1591 - i148	2.700	1.800	0.867	0.288
Set 3	1600 - i178	2.730	1.790	0.911	0.261
Set 4	1610 - i162	2.690	1.850	0.950	0.271
Set 5	1603 - i79	2.270	2.070	0.994	0.306

TABLE II: The pole position (in units of MeV) and corresponding coupling constants for various sets focusing on the elastic $\pi\Xi \to \pi\Xi$ channel.

$\Xi(1690)$	$M - i\frac{\Gamma}{2}$	$\lvert g_{\pi\Xi}\rvert^2$	$\lvert g_{\bar{K}\Lambda}\rvert^2$	$\lvert g_{\bar{K}\Sigma}\rvert^2$	$\lvert g_{\eta\Xi}\rvert^2$
Set 1	1685 - i16	0.721	0.476	2.250	1.290
Set 2	1687 - i21	0.675	0.882	2.290	1.370
Set 3	1688 - i20	0.588	0.941	2.280	1.360
Set 4	1680 - i9	0.403	0.656	1.710	0.881
Set 5	1681 - i7	0.323	0.521	1.630	0.801

TABLE III: The pole position (in units of MeV) and corresponding coupling constants for various sets focusing on the elastic $\bar{K}\Sigma \to \bar{K}\Sigma$ channel.

IV. CONCLUSIONS

In the present work, the interaction in s-wave of the pseudoscalar meson and the baryon octet with strangeness $S = -2$ and isospin $I = 1/2$ has been investigated by employing the Unitarized Chiral Perturbation Theory. Starting from the Weinberg-Tomozawa term of the effective Lagrangian, we have successively added the direct and crossed Born terms and, for the first time in this sector, the contact interactions of second quiral order.

By solving the Bethe-Salpeter equation in coupled channels, using the interaction kernel derived from the chiral Lagrangian, we have searched for poles in the second Riemann sheet of the scattering amplitude. The best-fit results we have observed are:

$$z_1 = (1603 - i79)\ \text{MeV} \qquad z_2 = (1681 - i7)\ \text{MeV} \tag{26}$$

The couplings to the different channels have also been calculated. For the first resonance, there is a strong (weak) coupling to channels $\pi\Xi$ and $\bar{K}\Lambda$ ($\bar{K}\Sigma$ and $\eta\Xi$). While the second one is strongly coupled to $\bar{K}\Sigma$ and $\eta\Xi$ and slightly to $\pi\Xi$ and $\bar{K}\Lambda$. These facts suggest we can identify these two poles with the $\Xi(1620)$ and $\Xi(1690)$ states, indicating clear evidence supporting the assignment of the quantum numbers $J^P = 1/2^-$ to them.

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Appendix A

C_{ij}	$\pi^+\Xi^-$	$\pi^0\Xi^0$	$\bar{K}^0\Lambda$	$K^-\Sigma^+$	$\bar{K}^0\Sigma^0$	$\eta\Xi^0$
$\pi^+\Xi^-$	1	$-\sqrt{2}$	$-\sqrt{\frac{3}{2}}$	0	$-\frac{1}{\sqrt{2}}$	0
$\pi^0\Xi^0$		0	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$\bar{K}^0\Lambda$			0	0	0	$-\frac{3}{2}$
$K^-\Sigma^+$				1	$-\sqrt{2}$	$-\sqrt{\frac{3}{2}}$
$\bar{K}^0\Sigma^0$					0	$\frac{\sqrt{3}}{2}$
$\eta\Xi^0$						0

TABLE IV: C_{ij} coefficients in the Weinberg-Tomozawa contact potential of the pseudoescalar meson and the baryon octet with strangeness $S = -2$ and charge $Q = 0$. The coefficients are symmetric, $C_{ji} = C_{ij}$.

Appendix B

Set of Clebsch-Gordan coefficients present in the DB and CB contributions to the interaction kernel, equations 10 and 11, taken from [10]. The equations must be multiplied by a factor $[1/(2\sqrt{3}f)]$ to recover our model's notation, because it differs from that of the reference taken. It should be noted that the coefficients are symmetric under the combined transformation $B_1 \leftrightarrow B_2$ and $\phi \leftrightarrow \bar{\phi}$, i.e., $C_{\bar{\phi}B_1,B_2} = C_{\phi B_2,B_1}$.

$$C^{(Born)}_{K^-p,\Lambda} = C^{(Born)}_{\bar{K}^0n,\Lambda} = C^{(Born)}_{\eta\Xi^-,\Xi^-} = C^{(Born)}_{\eta\Xi^0,\Xi^0} = -D - 3F$$

$$\sqrt{2}C^{(Born)}_{K^-p,\Sigma^0} = -\sqrt{2}C^{(Born)}_{\bar{K}^0n,\Sigma^0} = C^{(Born)}_{\bar{K}^0p,\Sigma^+} = C^{(Born)}_{\pi^+\Xi^-,\Xi^0} = C^{(Born)}_{\pi^0\Xi^-,\Xi^-} = -\sqrt{2}C^{(Born)}_{\pi^0\Xi^0,\Xi^0} = \sqrt{6}(D - F)$$

$$C^{(Born)}_{\pi^0\Sigma^0,\Lambda} = C^{(Born)}_{\pi^-\Sigma^+,\Lambda} = C^{(Born)}_{\eta\Sigma^+,\Sigma^+} = C^{(Born)}_{\eta\Sigma^0,\Sigma^0} = -C^{(Born)}_{\eta\Lambda,\Lambda} = 2D$$

$$C^{(Born)}_{\pi^+\Sigma^-,\Sigma^0} = -C^{(Born)}_{\pi^-\Sigma^+,\Sigma^0} = C^{(Born)}_{\pi^0\Sigma^+,\Sigma^+} = 2\sqrt{3}F$$

$$C^{(Born)}_{K^+\Xi^-,\Lambda} = C^{(Born)}_{K^0\Xi^0,\Lambda} = -D + 3F$$

$$\sqrt{2}C^{(Born)}_{K^+\Xi^-,\Sigma^0} = -\sqrt{2}C^{(Born)}_{K^0\Xi^0,\Sigma^0} = C^{(Born)}_{\bar{K}^0\Sigma^-,\Xi^-} = C^{(Born)}_{K^-\Sigma^+,\Xi^0} = \sqrt{6}(D + F)$$

(B1)

Appendix C

D_{ij}	$\pi^+\Xi^-$	$\pi^0\Xi^0$	$\bar{K}^0\Lambda$	$K^-\Sigma^+$	$\bar{K}^0\Sigma^0$	$\eta\Xi^0$
$\pi^+\Xi^-$	$2(2b_0+b_D-b_F)m_\pi^2$	0	$\frac{-(b_D-3b_F)\mu_1^2}{\sqrt{6}}$	0	$\frac{(b_D+b_F)\mu_1^2}{\sqrt{2}}$	$\frac{2\sqrt{2}(b_D-b_F)m_\pi^2}{\sqrt{3}}$
$\pi^0\Xi^0$		$4(b_0+b_D-b_F)m_\pi^2$	$\frac{(b_D-3b_F)\mu_1^2}{2\sqrt{3}}$	$\frac{(b_D+b_F)\mu_1^2}{\sqrt{2}}$	$\frac{(b_D+b_F)\mu_1^2}{2}$	$\frac{-2(b_D-b_F)m_\pi^2}{\sqrt{3}}$
$\bar{K}^0\Lambda$			$\frac{2(6b_0+5b_D)m_K^2}{3}$	$\frac{2\sqrt{2}b_D m_K^2}{\sqrt{3}}$	$\frac{-2b_D m_K^2}{\sqrt{3}}$	$\frac{(b_D-3b_F)\mu_2^2}{6}$
$K^-\Sigma^+$				$2(2b_0+b_D+b_F)m_K^2$	$-2\sqrt{2}b_F m_K^2$	$\frac{-(b_D+b_F)\mu_2^2}{\sqrt{6}}$
$\bar{K}^0\Sigma^0$					$2(2b_0+b_D)m_K^2$	$\frac{(b_D+b_F)\mu_2^2}{2\sqrt{3}}$
$\eta\Xi^0$						$\frac{2(2b_0\mu_3^2+b_D\mu_4^2+b_F\mu_5^2)}{3}$

L_{ij}	$\pi^+\Xi^-$	$\pi^0\Xi^0$	$\bar{K}^0\Lambda$	$K^-\Sigma^+$	$\bar{K}^0\Sigma^0$	$\eta\Xi^0$
$\pi^+\Xi^-$	$-d_1+d_2+2d_4$	0	$\frac{\sqrt{3}(d_1-d_2)}{\sqrt{2}}$	$-2d_2+d_3$	$\frac{d_1+3d_2}{\sqrt{2}}$	$\frac{-\sqrt{2}(d_1-3d_2)}{\sqrt{3}}$
$\pi^0\Xi^0$		$-d_1+d_2+2d_4$	$\frac{-(d_1-3d_2)}{2\sqrt{3}}$	$\frac{d_1+d_2}{\sqrt{2}}$	$\frac{-(d_1+d_2)}{2}$	0
$\bar{K}^0\Lambda$			$\frac{d_1+3d_2+2d_4}{2}$	$\sqrt{6}d_2$	$-\sqrt{3}d_2$	0
$K^-\Sigma^+$				0	$-\sqrt{2}d_1$	$\frac{-(d_1+3d_2)}{\sqrt{6}}$
$\bar{K}^0\Sigma^0$					d_2+2d_4	$\frac{d_1+3d_2}{2\sqrt{3}}$
$\eta\Xi^0$						$d_1+3d_2+2d_4$

TABLE V: D_{ij} and L_{ij} coefficients in the NLO potential of the pseudoescalar meson and the baryon octet with strangeness $S=-2$ and charge $Q=0$. The coefficients are symmetric, $D_{ji}=D_{ij}$ and $L_{ji}=L_{ij}$.

For the table below we can add the following definitions:

$$\mu_1^2 = m_K^2 + m_\pi^2$$
$$\mu_2^2 = 5m_K^2 - 3m_\pi^2$$
$$\mu_3^2 = 4m_K^2 - m_\pi^2$$
$$\mu_4^2 = 8m_K^2 - 3m_\pi^2$$
$$\mu_5^2 = 8m_K^2 - 5m_\pi^2$$

Appendix D

f/f_{exp}	D	F	b_0	b_D	b_F	d_1	d_2	d_3	d_4
1.196	0.70	0.51	0.129	0.120	0.209	0.151	0.126	0.299	0.249

TABLE VI: Values of the parameters of the chiral Lagrangian described in the text taken from [7]. The value of the pion decay constant is $f_{exp} = 92.4$ MeV.

Appendix E

Conversion of T-matrix elements, T_{ij}, in the new isospin basis $I = 1/2$, i.e., $(|\pi\Xi\rangle, |\bar{K}\Lambda\rangle, |\bar{K}\Sigma\rangle, |\eta\Xi\rangle)$ originally written in physical basis $(|\pi^+\Xi^-\rangle, |\pi^0\Xi^0\rangle, |\bar{K}^0\Lambda\rangle, |K^-\Sigma^+\rangle, |\bar{K}^0\Sigma^0\rangle, |\eta\Xi^0\rangle)$. The elements are symmetric, $T_{ij} = T_{ji}$.

$$T_{11} = \langle\pi\Xi|T|\pi\Xi\rangle = \tfrac{2}{3}\langle\pi^+\Xi^-|T|\pi^+\Xi^-\rangle - \tfrac{2\sqrt{2}}{3}\langle\pi^+\Xi^-|T|\pi^0\Xi^0\rangle + \tfrac{1}{3}\left\langle\pi^0\Xi^0\middle|T\middle|\pi^0\Xi^0\right\rangle$$

$$T_{21} = \langle\bar{K}\Lambda|T|\pi\Xi\rangle = \sqrt{\tfrac{2}{3}}\langle\bar{K}^0\Lambda|T|\pi^+\Xi^-\rangle - \sqrt{\tfrac{1}{3}}\langle\bar{K}^0\Lambda|T|\pi^0\Xi^0\rangle$$

$$T_{31} = \langle\bar{K}\Sigma|T|\pi\Xi\rangle = -\tfrac{2}{3}\langle K^-\Sigma^+|T|\pi^+\Xi^-\rangle + \tfrac{\sqrt{2}}{3}\langle K^-\Sigma^+|T|\pi^0\Xi^0\rangle + \tfrac{\sqrt{2}}{3}\langle\bar{K}^0\Sigma^0|T|\pi^+\Xi^-\rangle - \tfrac{1}{3}\langle\bar{K}^0\Sigma^0|T|\pi^0\Xi^0\rangle$$

$$T_{41} = \langle\eta\Xi|T|\pi\Xi\rangle = \sqrt{\tfrac{2}{3}}\langle\eta\Xi^0|T|\pi^+\Xi^-\rangle - \sqrt{\tfrac{1}{3}}\langle\eta\Xi^0|T|\pi^0\Xi^0\rangle$$

$$T_{22} = \left\langle\bar{K}\Lambda\middle|T\middle|\bar{K}\Lambda\right\rangle = \left\langle\bar{K}^0\Lambda\middle|T\middle|\bar{K}^0\Lambda\right\rangle$$

$$T_{32} = \langle\bar{K}\Sigma|T|\bar{K}\Lambda\rangle = -\sqrt{\tfrac{2}{3}}\langle K^-\Sigma^+|T|\bar{K}^0\Lambda\rangle + \sqrt{\tfrac{1}{3}}\langle\bar{K}^0\Sigma^0|T|\bar{K}^0\Lambda\rangle$$

$$T_{42} = \langle\eta\Xi|T|\bar{K}\Lambda\rangle = \langle\eta\Xi^0|T|\bar{K}^0\Lambda\rangle$$

$$T_{33} = \left\langle\bar{K}\Sigma\middle|T\middle|\bar{K}\Sigma\right\rangle = \tfrac{2}{3}\langle K^-\Sigma^+|T|K^-\Sigma^+\rangle - \tfrac{2\sqrt{2}}{3}\langle K^-\Sigma^+|T|\bar{K}^0\Sigma^0\rangle + \tfrac{1}{3}\langle\bar{K}^0\Sigma^0|T|\bar{K}^0\Sigma^0\rangle$$

$$T_{43} = \langle\eta\Xi|T|\bar{K}\Sigma\rangle = -\sqrt{\tfrac{2}{3}}\langle\eta\Xi^0|T|K^-\Sigma^+\rangle + \sqrt{\tfrac{1}{3}}\langle\eta\Xi^0|T|\bar{K}^0\Sigma^0\rangle$$

$$T_{44} = \langle\eta\Xi|T|\eta\Xi\rangle = \left\langle\eta\Xi^0\middle|T\middle|\eta\Xi^0\right\rangle$$

(E1)