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Working Papers

Col·lecció d'Economia E22/431

STACKELBERG COMPETITION IN GROUNDWATER RESOURCES WITH MULTIPLE USES

Julia de Frutos Cachorro

Guiomar Martín-Herrán

Mabel Tidball

ISSN 1136-8365

UNIVERSITAT DE BARCELONA
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Working Papers

UB Economics *Working Paper No. 431*

Title: Stackelberg competition in groundwater resources with multiple uses

Abstract: We study a problem of exploitation of a groundwater resource, mainly used for irrigation, in which a water agency is needed in order to manage an exceptional and priority extraction of water for an alternative/new use (e.g. domestic water). To this goal, we build a two-stage discrete Stackelberg game in which the leader (the water agency) just intervenes when the new use takes place (in the second stage) and the follower is a representative agent of the regular users of the aquifer, i.e. the agricultural users. We study two types of Stackelberg equilibrium, which can arise depending on the agents' commitment behavior, namely open-loop (commitment) equilibrium and feedback (non-commitment) equilibrium. We analyze and compare extraction behaviors of the different agents for the different equilibria and the consequences of these extraction behaviors for the final state of the resource and the agents' profits. For some hypotheses on the parameters, theoretical results show that commitment strategies lead to higher stock levels than non-commitment strategies when the leader's weight assigned to the profits from the agricultural use is lower or equal than the one assigned to the profits from the non-agricultural use. However, performing numerical simulations relaxing previous economic assumptions, we show that there are situations in which non-commitment strategies could be more favorable than commitment strategies not only in terms of final stock of the resource but also in terms of users' profits.

JEL Codes: Q25, C72

Keywords: Groundwater resource, multiple uses, Stackelberg dynamic game, information structures

Authors:

Julia de Frutos Cachorro	Guiomar Martín-Herrán	Mabel Tidball
Universitat de Barcelona and BEAT	IMUVA and Universidad de Valladolid	CEE-M and University of Montpellier
Email:	**Email:**	**Email:**
j.defrutos@ub.edu	guiomar.martin-herran@uva.es	mabel.tidball@inrae.fr

Date: September 2022

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Acknowledgements: This work is supported by the Spanish State Research Agency under project PID2020-112509GB-100 (AEI), by Junta de Castilla y León (Spain) under project VA169P20, by the ANR GREEN-Econ research project with reference ANR-16-CE03-0005 and by the ANR-DFG project CRaMoRes with reference ANR-19-FRAL-0010-01.

1 Introduction

Groundwater depletion is a major challenge nowadays where groundwater resources are playing an increasingly important role, not just for irrigation (the regular use in the majority of aquifers) but also for domestic and industrial purposes. A situation that occurs and illustrates this increase in competition for groundwater resources is the need of an exceptional extraction of groundwater for a non-agricultural use, such as the construction of a water reserve, the transfer between basins or a special need for domestic water in an urban area because of water scarcity, among other cases. For example, as explained in De Stefano et al. (2015), groundwater is used for domestic water supply mostly only during drought periods in the city of Madrid. Because of the exceptional groundwater need and the priority of this new use (e.g. domestic water supply), a benevolent water authority who exceptionally prioritizes this new use over the regular users could be more appropriate to solve the water conflict than a regulator who would be in charge of implementing efficient policies during a certain period of time. More specifically, in this paper, we study a problem of exploitation of a groundwater resource, mainly used for irrigation, who faces exceptionally the entrance of a new use in the system, therefore involving a problem of water scarcity for the farmers. In this context, a benevolent water authority (e.g. a water agency) is needed in order to manage how much groundwater could be extracted for the new (non-agricultural) use in such a way that the profits of the different resource users (agricultural and non-agricultural users) are guaranteed.

Indeed, we step away from the literature that focuses on centralized management of aquifers and consequently seeks to implement the efficient or Pareto solution to the groundwater management problem. However, groundwater quantity issues are mostly addressed in the literature using optimal control theory (see Koundouri et al. (2017) for a review), most of them dealing with different types of uncertainties (see Tsur and Zemel (2014) for a review), especially the problem of water scarcity (e.g. de Frutos Cachorro et al. (2014)). In particular, in de Frutos Cachorro et al. (2014), water scarcity is considered through the modeling of an exogenous shock to the groundwater resource (namely, a decrease in the recharge rate of the aquifer), and subsequently, optimal extraction paths and the social costs of optimal adaptation to the shock are analyzed. In contrast with this literature, our approach importantly introduces the strategic aspect in the behavior of the different decision-makers and treats water scarcity as an endogenous shock to the groundwater resource.

In this context, the use of dynamic games has been largely justified in the literature of water management to deal with problems where dynamic and strategic interactions among decision makers (i.e. the water agency and the farmers) are taken into account (see Madani (2010) for a review). Indeed, game theory literature focuses on cooperative or/and non-cooperative - Nash equilibrium - solutions for irrigation users (e.g. Negri (1989) and Rubio and Casino (2001)) under water scarcity (e.g. de Frutos Cachorro et al. (2019)), or more complex problems due to farmers' heterogeneity (e.g. Saleh et al. (2011)), competition between uses (e.g. de Frutos Cachorro et al. (2021)), and spatial characteristics such as multicell aquifers (e.g. Saak and Peterson (2007)), among others. However, as explained, for example, in Kicsiny (2017), when water conflicts enter the problem, Stackelberg (or leader-follower) equilibria offer a more real representation of the problem in practical cases with respect to previous classic Nash-equilibria. In this paper, it is then appropriate and interesting to consider that the different agents compete à la Stackelberg and play hierarchically; that is, the water agency is the leader and therefore makes extraction decisions of the new (non-agricultural) priority use in the first place, before the regular users of the resource, the farmers. Subsequently, the farmers who can be represented by one agent, which is called the follower in the Stackelberg game, make the resource extraction decisions depending on the actions of the leader.

In fact, general Stackelberg dynamic games could be classified according to their relevance at theoretical or/and empirical levels. The books by Dockner et al. (2000) and Başar and Olsder (1999) are well-known references for Stackelberg dynamic games in continuous or discrete time, respectively. Some studies offer interesting findings from a theoretical perspective, although their application to real cases is restricted (e.g. Nie (2005), Erdlenbruch et al. (2014)). To the best of our knowledge, few of these papers focus on comparing different types of Stackelberg equilibria, specially open-loop and feedback Stackelberg equilibria, which correspond to different agents' commitment behavior. In addition, this is extremely important in practice as each equilibrium concept can be seen as more realistic than the other one depending on the information that is available to each player. On the one hand, the implementation of feedback strategies requires that the current state variable (the stock of water in our model) can be observed. Therefore, in some settings the open-loop strategies are more realistic if the state variable is unobservable or only observable with a delay. On the other hand, in the open-loop equilibrium, the leader makes a commitment about his extraction behavior and the follower believes this commitment and chooses his extraction of the resource under this belief. The problem with Stackelberg open-loop equilibria is that they are generally inconsistent[1]over time and therefore less realistic. The feedback

[1]The problem of temporal inconsistency is due to the fact that, if the leader optimally decides to carry

Stackelberg equilibrium does not have this disadvantage. This equilibrium is consistent over time, and the players do not commit about their extraction behavior over time, but take decisions depending on the state of the resource at the beginning of each period. In particular, Nie (2005) analyzes and compares open-loop (commitment) and feedback (non-commitment) Stackelberg equilibria for a general setting. The study concludes that feedback Stackelberg strategies are more efficient for the leader's objective than open-loop strategies. Turning now to empirical studies, most of these works use general algorithms to find approximate solutions to complex problems, mostly focusing on a specific equilibrium type (e.g. Kicsiny et al. (2014) and Xu et al. (2019)). Concerning the literature combining Stackelberg problems, water scarcity and competition between groundwater uses, Kicsiny et al. (2014) address the problem of a local authority (the leader) who would manage for a given time period the use of a water reserve for different types of uses, by firstly reserving a minimal guaranteed quantity for both uses, and subsequently by assigning a proportion of the water reserve available to domestic use. The follower (a representative agent of the farmers) then decides the proportion of the available reserve left for irrigation after the leader's decision. However, in Kicsiny et al. (2014), just the feedback Stackelberg solution is analyzed, and in contrast with our study, the water conflict is continuous over time and therefore, the local authority fixes a minimum and a maximum quantity for both uses at each period, before taking extraction decisions for the non-agricultural priority use. Moreover, the dynamics in our formulation are in the stock of the aquifer (renewable resource), while in Kicsiny et al. (2014) the dynamics are in the water available, that is, the water remaining from the total reserve (non-renewable resource).

In this paper, we aim to study whether feedback (non-commitment) or open-loop (commitment) Stackelberg strategies could be more profitable for the sustainability of the resource (in terms of final stock levels) and/or for the agents' profits. To the best of our knowledge, this paper is novel in the sense that proposes and compares different Stackelberg equilibria for the resource management in a context of competition between different groundwater uses.

Our paper also differs from the literature in several ways. First and very importantly, we are interested in the modeling of a specific situation, which is the need of a water authority (e.g. a water agency) in order to manage an exceptional and priority extraction of water for a non-agricultural use, as illustrated before in the first paragraph of this section

out a number of actions over several periods, and if other economic agents (in our case, the farmers) believe in this commitment and choose their actions under this belief, then, at some period in the future, the leader would want to deviate from his commitment (Kydland and Prescott (1977)).

by means of the example of groundwater need for domestic water in Madrid mostly only during drought periods. To this goal, we build a two-stage discrete problem, in which the leader just intervenes when it is necessary (here in the second stage of game where the non-agricultural water use takes place), while in Nie (2005) and Kicsiny et al. (2014), a leader-follower approach is applied at each step of the game and both leader and follower are active players at all stages. Second, we assume that the leader (i.e. the water agency) is a benevolent entity who decides how much groundwater could be extracted for the new (non-agricultural) use by considering in his objective the profits from the different uses (agricultural and non-agricultural uses) in order to avoid a water conflict, allowing in addition the possibility of assigning different weights to the different uses. In fact, by making a sensitivity analysis of main results with respect to these weights, we will show that the introduction of this aspect in the modeling is extremely relevant, constituting one of the main drivers of final results. Next, as explained in the previous paragraph, we are particularly interested in analyzing and comparing the extraction behaviors of the agents for different Stackelberg equilibria depending on the type of existing commitment between the decision-makers. With this aim, we analytically solve and compare the open-loop (commitment solution) and feedback (non-commitment) equilibria for some hypotheses on the parameters. Theoretical results show that commitment strategies lead to higher stock levels than non-commitment strategies when the leader's weight assigned to the profits from the agricultural use is lower or equal than the one assigned to the profits from the new (non-agricultural) use. Finally, we make numerical simulations in order to study whether the main results remain valid when relaxing previous economic assumptions, for example, allowing the leader to introduce the sustainability of the resource in his objective function. Numerical results suggest that there are situations in which non-commitment strategies could be more favorable than commitment strategies not only in terms of final stock of the resource but also in terms of users' profits.

The remainder of this paper is organized in the following way. Section 2 describes the Stackelberg game, the model resolution for the two types of commitment behavior and provides sensitivity analyses of the theoretical solutions. Comparisons between theoretical results for open-loop and feedback Stackelberg equilibria are performed in Section 3. In Section 4, we complete our analysis by presenting numerical simulations for a more complex version of the problem. Conclusions are collected in Section 5. All the proofs are relegated to the appendices.

2 The game

We formulate our problem as a two-stage Stackelberg model in discrete time with two decision-makers. First, there is a representative agent for the regular users of the resource (the farmers), namely the follower. Second, there is a water authority (e.g., a water agency), namely the leader of the Stackelberg game, who announces at $t = 0$ that another priority extraction (e.g., domestic water) will take place at the beginning of the second stage, for a non-agricultural use. The stock of the aquifer is the state variable and, at time $t = 0$, this stock is denoted by G_0. Agents' extraction decisions over the two periods, i.e. extraction of the leader for the new (non-agricultural) use in the second period and extractions of the follower for the agricultural use in the first and in the second period, are the control variables of the problem.

2.1 Game formulation

In the first period, the aquifer is exclusively exploited by the follower, and we denote by g_{1f}, the amount of water extracted by the follower in the first period. Hence, the stock of the aquifer at the end of the first period (at time $t = 1$), G_1, reads:

$$G_1 = G_0 - g_{1f} + r, \tag{1}$$

where r denotes the constant recharge of the aquifer over the first period.

In the second period, we give entrance to a second decision-maker in such a way that over this period there is competition between different users (and uses) for the exploitation of the groundwater resource. The new decision-maker is a water agency that has the role of the leader in the Stackelberg game and aims to decide how much water could be extracted for a new (non-agricultural) priority use, considering the fact that follower is also exploiting the aquifer in the second period. In this second period, both agents then make extraction decisions, denoted by g_{2l} the extraction of the leader for the new (non-agricultural) use and by g_{2f} the extraction of the follower for irrigation purposes. Assuming for simplicity that the recharge of the aquifer over the second period is identical to the recharge over the first period, the stock of the aquifer at the end of the second period (at time $t = 2$), G_2, reads:

$$G_2 = G_1 - g_{2f} - g_{2l} + r. \tag{2}$$

The players, leader and follower, make their extraction decisions in order to maximize their objectives. In the case of the follower, the representative agent of the farmers aims to maximize profits over the two periods. The (per period) profits are given as the difference between the net revenues from groundwater exploitation minus the pumping cost over this period. In particular, the (per period) net revenues are represented by a linear function of the extracted quantity of water in this period (g_{tf}) as follows:

$$R_f(g_{tf}) = a_f g_{tf}, \quad t = 1, 2, \tag{3}$$

with a_f a positive parameter (see e.g. Howith (1995) and Buysse et al. (2007) for similar revenue functions). In Section 4, numerical simulations will be computed for the case of non-linear revenues[2], commonly used to represent agricultural revenues from groundwater use in the literature (cf. Rubio and Casino (2001), de Frutos Cachorro et al. (2019), Pereau (2020)), to study whether or not the main results change with respect to the linear revenue case.

As in the previous literature, the (per period) pumping costs depend on the stock of the aquifer at the end of the period, G_t, and the extracted quantity of water in this period, g_{tf}, and reads:

$$C_f(G_t, g_{tf}) = (z - cG_t)g_{tf}, \quad t = 1, 2, \tag{4}$$

with z and c positive parameters. More specifically, z corresponds to the maximum unit (or marginal) cost, i.e., the marginal cost when $G = 0$.[3]

The (per period) profit of the follower is then

$$\Pi_f(g_{tf}, G_t) = R_f(g_{tf}) - C_f(G_t, g_{tf}), \quad t = 1, 2, \tag{5}$$

with functions R_f and C_f given by (3) and (4), respectively.

The leader's objective is to choose the extraction of water in the second period for the new use, g_{2l}, in order to maximize a weighted sum of the follower's profits and profits derived from extractions for the new use, that is,

$$\sum_{t=0}^{1} \theta \Pi_f(g_{(t+1)f}, G_{t+1}) + (1 - \theta)\Pi_l(g_{2l}, G_2), \tag{6}$$

[2]In the non-linear revenue case, we will assume a quadratic and concave function of extraction quantities, i.e. the (per period) net revenues become $R_f(g_{tf}) = a_f g_{tf} - \frac{b_f}{2} g_{tf}^2, \quad t = 1, 2$, with a_f and b_f positive parameters.

[3] In what follows we assume that the marginal pumping costs are positive and check a posteriori in all the numerical simulations that this hypothesis is satisfied. In particular, $G_0 < z/c$ by assumption.

with θ , $0 \leq \theta < 1$, the weight assigned by the leader to the follower's profits, and Π_l, the profits derived from the new use in the second period, defined by

$$\Pi_l(g_{2l}, G_2) = a_l g_{2l} - (z - cG_2)g_{2l}, \tag{7}$$

with $a_l, z, c \geq 0$.

This specification assumes that the profits of the new user of the aquifer can be defined by net revenue and pumping cost functions qualitative similar as those corresponding to the follower's case. However, we assume that parameters of the revenue functions are agent specific in order to correctly describe the difference between the two agents who share the aquifer for different purposes.

2.2 Game resolution under different commitment behaviors

In a general Stackelberg game, or leader-follower game, the leader takes decisions first and the follower takes decisions subsequently depending on the actions of the leader. Different types of Stackelberg equilibria can be computed depending on the commitment behavior between the two agents over the two-period game. In what follows, we analytically solve the game for two types of Stackelberg equilibrium, namely open-loop (commitment solution) and feedback (non-commitment) equilibria. While the former involves time-inconsistent solutions, the latter procures time-consistent solutions (see proofs in Appendix A).

2.2.1 Open-loop Stackelberg equilibrium

In an open-loop Stackelberg equilibrium, the leader commits at $t = 0$ about his path of extraction in the second period. The follower then believes this leader's commitment and chooses subsequently the path of extractions over the two-period game under this belief.

Accordingly, first, the follower decides about extraction behavior in the two periods, g_{1f} and g_{2f}, assuming extraction policy of the leader in the second period, g_{2l}. Since the follower's objective is to maximize the profits over the two periods, the follower is facing the following problem:

$$\begin{aligned} \max_{g_{1f} \geq 0,\ g_{2f} \geq 0} \quad & \sum_{t=0}^{1} \Pi_f(g_{(t+1)f}, G_{t+1}), \\ & \text{s.t: } (1),\ (2) \\ & G_1, G_2 \geq 0 \end{aligned} \tag{8}$$

with function Π_f given by (5). We then obtain the follower's best-reaction functions $\tilde{g}_{1f}(g_{2l})$ and $\tilde{g}_{2f}(g_{2l})$, that is, the follower's extractions over the two periods as functions of the leader's extraction in the second period. Next, the leader chooses the extraction of water in the second period, g_{2l}, in order to maximize a weighted sum of the two-period profits of the follower (taking into account the follower's best-reaction functions $\tilde{g}_{1f}(g_{2l})$ and $\tilde{g}_{2f}(g_{2l})$) and the profits derived from extractions for the new use:

$$\begin{aligned} \max_{g_{2l}\geq 0} \quad & \left\{ \sum_{t=0}^{1} \theta \Pi_f(\tilde{g}_{(t+1)f}, G_{t+1}) + (1-\theta)\Pi_l(g_{2l}, G_2) \right\}, \\ & \text{s.t: (1), (2)} \\ & G_1, G_2 \geq 0 \end{aligned} \tag{9}$$

with functions Π_f and Π_l given by (5), and respectively (7), and $0 \leq \theta < 1$, with θ the weight assigned to the follower's profits. More specifically, the greater the weight θ, the more important the agricultural use of the aquifer will be for the leader; and vice versa, the lower θ, the more prevalent the new use of the aquifer will be for the leader. Solving the previous problem (see Appendix B.1 for details), we obtain g_{2l}^{OL}, where the superscript OL stands for open-loop equilibrium. Finally, substituting g_{2l}^{OL} in the follower's decisions, we obtain g_{1f}^{OL} and g_{2f}^{OL}:

$$g_{1f}^{OL} = \frac{2a_f + 3a_l + 5cG_0 + 3cr - 5z - \theta(5a_f + 3a_l + 8cG_0 + 6cr - 8z)}{6c(2-3\theta)}, \tag{10}$$

$$g_{2f}^{OL} = \frac{(1-\theta)(4a_f - 3a_l + cG_0 + 3cr - z)}{3c(2-3\theta)}, \tag{11}$$

$$g_{2l}^{OL} = \frac{3a_l - 2a_f + cG_0 + 3cr - z - \theta(3a_l - a_f + cG_0 + 3cr - z)}{2c(2-3\theta)}. \tag{12}$$

Once the optimal extraction strategies have been characterized, we can also obtain the states of the aquifer at the end of the two periods for previous extraction behavior:

$$G_1^{OL} = \frac{(3a_l + 5a_f - 10cG_0 - 12cr - 8z)\theta - (2a_f + 3a_l - 7cG_0 - 9cr - 5z)}{6c(2-3\theta)}, \tag{13}$$

$$G_2^{OL} = \frac{(3a_l + 5a_f - cG_0 - 3cr - 8z)\theta - (2a_f + 3a_l - cG_0 - 3cr - 5z)}{3c(2-3\theta)}. \tag{14}$$

Concerning players' profits derived from optimal extractions strategies and states of the aquifer, mathematical expressions are very long and, are therefore relegated to Appendix

B.1.

In this paper, we assume that the agricultural use needs water at each moment for growing its crops. Moreover, we are interested in the problem in which a new use needs water to procure a new activity. Consequently, we are interested in positive extractions of water both for the agricultural use (follower's extraction) and the non-agricultural use (leader's extraction), as well as positive stock of the aquifer at the end of the two periods. In summary, we focus on interior and strictly positive solutions, and hence, corner solutions are not analyzed. In order to guarantee the positivity of extraction decisions and state variables, we assume that the following sufficient conditions are fulfilled.

Condition 1: **A**: $a_f > z$, **B**: $a_l + 3z - 4a_f > 0$ and **C**: $c(G_0 + 3r) > 2(2a_l - a_f - z)$ **D**: $0 \leq \theta \leq 1/2$.
See Appendix B.2.

Please first note that conditions 1.A and 1.B imply $a_l - a_f > 0$, which means that the marginal revenue for the new use is higher than the marginal revenue for the agricultural use. Condition 1.A is an usual requirement which establishes that marginal revenue should be higher than the maximum unit (or marginal) cost. Condition 1.B can also be written as $a_l - a_f > 3(a_f - z)$ and suggests that the difference between marginal revenues for both activities should be sufficiently higher than the (minimum) marginal profit from the agricultural activity. Finally, condition 1.C requires that G_0 is great enough, with $G_0 < z/c$ by assumption to ensure positive marginal costs (see Footnote 3). In summary, sufficient conditions request that the new use or activity that enters in the game in the second period needs to be more profitable than the agricultural use, and the level of the resource at the beginning of the planning horizon must be high enough to procure these activities.

Since the goal of both players is to reach a maximum value in their objectives (problems (8) and (9)), the concavity of the objective functions of the leader and the follower with respect to their decision variables has to be guaranteed. In Appendix B.3 we prove that Condition 1.D ensures this property.

In the remainder of this section, we assume that Condition 1 is satisfied, allowing the following analytical results to be deduced.

Next, as explained before in the introduction of the manuscript, we also assume that the leader is a benevolent water agency who considers in his objective the profits from the different uses, allowing the possibility of assigning different weights to the different uses. This aspect is introduced in the modeling through the parameter θ, the weight assigned by the leader to the follower's profits in his objective. It is therefore interesting to analyze how optimal solutions are influenced by the introduction of this parameter. In particular, we are interested in seeing how a change in θ affects the leader's optimal strategy in the second period, and how this change leads to a modification of the follower's optimal strategies and, consequently, of the state of aquifer at the end of the two periods and total follower's profits. Next proposition collects the results.

Proposition 1 *The extraction of the follower in the first period, g_{1f}^{OL}, and the extraction of the leader in the second period, g_{2l}^{OL}, decrease as θ increases. However, the extraction of the follower in the second period, g_{2f}^{OL}, increases with θ. The total effect on the follower's extraction over the two periods, $g_{1f}^{OL}+g_{2f}^{OL}$, is positive, while the effect on the total extraction in the second period, $g_{2f}^{OL}+g_{2l}^{OL}$, is negative. Furthermore, the state of the aquifer at the end of the first, G_1^{OL}, and second, G_2^{OL}, periods as well as the follower's optimal profits, Π_f^{OL}, increase with θ.*

Proof: See Appendix B.4.

This proposition shows that the greater the weight assigned to the follower's profits by the leader in his objective, the lower the leader's extraction in the second period and the lower the follower's extraction in the first period, but the greater the extraction in the second period. Note that although any change of θ has two opposite effects in the follower's extraction policies, the total effect on the extraction over the two periods is positive, because $\frac{\partial g_{1f}^{OL}}{\partial \theta} + \frac{\partial g_{2f}^{OL}}{\partial \theta} = -\frac{1}{3}\frac{\partial g_{2l}^{OL}}{\partial \theta} > 0$. On the other hand, any increment of θ leads to a lower total extraction in the second period because $\frac{\partial g_{2f}^{OL}}{\partial \theta} + \frac{\partial g_{2l}^{OL}}{\partial \theta} = \frac{1}{3}\frac{\partial g_{2l}^{OL}}{\partial \theta} < 0$. That is, the fall in the leader's extraction in the second period due to an increment of θ more than compensates the rise in the follower's extraction in this period.

Indeed, a change in θ mainly influences the leader's extraction behavior, then it is important to note that this effect is reduced by around 2/3, and respectively 1/3, on the follower's extraction behavior in the second period, and respectively in the first period.

The fact that the leader's extraction in the second period decreases with θ and follower's total extraction over the two periods increases with θ is natural, because θ represents the weight the leader gives to the follower in his objective function (in fact the follower exploits this, by increasing his extraction in the second period).

As expected from the marginal effects of θ on the players' optimal strategies, the state of the aquifer at the end of the two periods increases as θ augments (see equations (35) and (36)), being this effect higher on the resource state at the end of the second period (i.e. when the resource is shared by different uses), than at the end of the first period.

Finally, we study how the players' optimal profits change with θ. From (37) the effect on the follower's optimal profits, Π_f^{OL}, is clearly positive under Conditions 1.C and 1.D. Unfortunately, the effect on the leader's optimal profits cannot be easily determined. We will study this effect later on in this paper through some numerical examples.

2.2.2 Feedback Stackelberg equilibrium

In a feedback Stackelberg equilibrium, the follower chooses at each step his extraction behavior after the leader has decided and announced his strategy. The problem has to be solved through backward induction. As the leader does not extract water in the first stage, there is just one decision-maker, the follower, in the first period. The feedback equilibrium of the problem can be therefore seen as a "degenerated Stackelberg", whose solution can be obtained following a 3-step procedure (see Appendix C.1 for details).

The game is solved backward, and hence, in the first step, the follower decides about extraction behavior in period 2, g_{2f}, assuming the leader's extraction policy in the second period, g_{2l}, and his own extraction in the first period, g_{1f}. The follower then has to solve the following problem

$$\begin{aligned} \max_{g_{2f}\geq 0} \quad & \Pi_f(g_{2f}, G_2), \\ & \text{s.t: (1), (2)} \\ & G_1, G_2 \geq 0, \end{aligned} \tag{15}$$

with function Π_f given by (5). The solution to this problem gives g_{2f} as a function of g_{2l} and g_{1f}, $\hat{g}_{2f}(g_{2l}, g_{1f})$. In the second step, after substituting $\hat{g}_{2f}(g_{2l}, g_{1f})$ in the leader's problem, the leader decides about extraction behavior in period 2, g_{2l}. The leader's problem becomes:

$$\begin{aligned} \max_{g_{2l}\geq 0} \quad & \theta\Pi_f(\hat{g}_{2f}, G_2) + (1-\theta)\Pi_l(g_{2l}, G_2), \\ & \text{s.t: (1), (2),} \\ & G_1, G_2 \geq 0, \end{aligned} \tag{16}$$

with functions Π_f and Π_l given by (5), and respectively (7), and $0 \leq \theta < 1$. The solution to this problem establishes g_{2l} as a function of g_{1f}, i.e. $\hat{g}_{2l}(g_{1f})$. Finally, substituting the leader's reaction function in the second stage in the follower's problem in period 1, the follower's problem to solve becomes:

$$\begin{aligned} \max_{g_{1f}\geq 0} \quad & \Pi_f(g_{1f}, G_1) + \Pi_f\left(\hat{g}_{2f}(\hat{g}_{2l}, g_{1f}), G_2\right), \\ & \text{s.t: } (1), (2), \\ & G_1, G_2 \geq 0, \end{aligned} \tag{17}$$

with function Π_f given by (5). From the solution to this problem, we obtain the follower's extraction strategy in the first period, g_{1f}^{FB}, where the superscript FB stands for feedback equilibrium. Subsequently, we replace the latter value in the reaction functions of the leader and the follower in period 2, and we obtain g_{2l}^{FB}, g_{2f}^{FB}, the optimal strategies of the leader and the follower in the second period. Next we present all these optimal strategies as well as the states of the resource derived from these strategies:

$$g_{1f}^{FB} = \frac{a_f(3\theta(5\theta-6)+5)+2a_l(1-\theta)^2+((22-17\theta)\theta-7)(z-cG_0)+c(4\theta(4\theta-5)+6)r}{c(5\theta-3)(7\theta-5)}, \tag{18}$$

$$g_{2f}^{FB} = \frac{(1-\theta)(2-3\theta)(5a_f-4a_l+cG_0+3cr-z)}{c(5\theta-3)(7\theta-5)}, \tag{19}$$

$$g_{2l}^{FB} = \frac{2(a_f((11-5\theta)\theta-5)+a_l(\theta-1)(11\theta-7)+(\theta(6\theta-7)+2)(c(G_0+3r)-z))}{c(5\theta-3)(7\theta-5)}, \tag{20}$$

$$G_1^{FB} = \frac{a_f(3(6-5\theta)\theta-5)-2a_l(1-\theta)^2+c\left(2G_0(2-3\theta)^2+\left(19\theta^2-26\theta+9\right)r\right)+\left(17\theta^2-22\theta+7\right)z}{c(5\theta-3)(7\theta-5)}, \tag{21}$$

$$G_2^{FB} = \frac{a_f((21-20\theta)\theta-5)+(4a_l-c(G_0+3r))((5-3\theta)\theta-2)+\left(32\theta^2-41\theta+13\right)z}{c(5\theta-3)(7\theta-5)}. \tag{22}$$

Again, mathematical expressions concerning optimal profits are very long, and hence, are relegated to Appendix C.1.

As in the open-loop scenario, we are interested in positive extractions for the agricultural and non-agricultural uses as well as positive stock of the aquifer at the end of the two periods, and hence, we focus on strictly positive and interior solutions. In order to ensure the positivity of extractions and state variables in the feedback case, stronger conditions than in the open-loop case must be imposed. Sufficient conditions ensuring positive resource extractions and stocks are given either by Condition 2.1 or Condition 2.2 below.

Condition 2.1:
A: $a_f > z$, **B**: $a_l + 3z - 4a_f > 0$, **C'**: $c(G_0 + 3r) > \frac{12a_l + 20a_f - 32z}{3}$ and **D'**: $0 \leq \theta \leq 1/2$.

Condition 2.2:
A: $a_f > z$, **B**: $a_l + 3z - 4a_f > 0$, **C"**: $\frac{8a_l + 5a_f - 13z}{2} > c(G_0 + 3r) > 2(2a_l - a_f - z)$ and **D"**: $\bar{\theta_1} < \theta \leq 1/2$, with $0 < \bar{\theta_1} < 1/2$.
See Appendix C.2.
Note that in comparison with Condition 1 (the equivalent set of sufficient conditions in the open-loop case that ensure positive extractions and stocks) we must either impose a greater initial resource (**C'** instead of **C**) or avoid small values of θ (**D"** instead of **D**).

As in the open-loop equilibrium, the concavity of the objective functions of the leader and the follower with respect to the corresponding decision variables in the three steps of the game resolution (i.e. in problems (17), (15) and (16)) has to be guaranteed. In Appendix C.3 we prove that either Condition 2.1.D' or Condition 2.2.D" ensures the concavity of the different objective functions.

In the remainder of this section, we assume that either Condition 2.1 or Condition 2.2 is satisfied, allowing the following analytical results to be deduced.

A sensitivity analysis of the variables in the feedback case with respect to change in θ could be performed, and is presented in the next proposition. These results are qualitatively similar to those obtained when the players use open-loop strategies.

Proposition 2 *The extraction of the follower in the first period, g_{1f}^{FB}, and the extraction of the leader in the second period, g_{2l}^{FB}, decrease as θ increases. However, the extraction of the follower in the second period, g_{2f}^{FB}, increases with θ. The total effect on the follower's extraction over the two periods, $g_{1f}^{FB} + g_{2f}^{FB}$, is positive, while the effect on the total extraction in the second period, $g_{2f}^{FB} + g_{2l}^{FB}$, is negative. Furthermore, the state of the aquifer at the end of the first, G_1^{FB}, and second, G_2^{FB}, periods as well as the follower's optimal profits, Π_f^{FB}, increase with θ.*

Proof: See Appendix C.4.

As in the case of open-loop strategies, the effect of an increment in θ on the leader's optimal profits cannot be determined analytically and we postpone this study to the numerical examples. We corroborate that the main tendencies, which were found in the open-loop case, regarding the effect of θ on extractions, the state of the aquifer and the follower's profit are maintained for the feedback case.

In the next sections, we compare the output variables for the different commitment behaviors. In Section 3, we focus on theoretical results for the case in which the weight assigned for the agricultural use is lower than or equal to the weight of the non-agricultural use, $\theta \leq 1/2$. A numerical analysis for other leader's weight, θ, and economic assumptions will be performed in Section 4.

3 Theoretical results: Open-loop vs. Feedback Stackelberg equilibria

In this section, we compare extraction behavior of both agents, the states of the resource and the players' profits for the different types of equilibria (see equations (10) to (14) for the open-loop case and (18) to (22) for the feedback case). Results for both equilibria can be compared if Condition 1 and either Condition 2.1 or Condition 2.2 are satisfied. As previously shown, Condition 2.1 and Condition 2.2. ensures the fulfillment of Condition 1. Therefore, these conditions are assumed in what follows. We remain that our attention is restricted to the case of the leader's weight for the new use greater or equal to the weight for the agricultural use, $\theta \leq 1/2$.

We first compare extraction behavior of the follower and leader for each period depending on the type of commitment behavior.

Proposition 3 *Individual follower's extraction in the first period, g_{1f}, and the leader's extraction in the second period, g_{2l}, is greater in the feedback than in the open-loop case. The opposite is obtained for the follower's extraction behavior in the second period, g_{2f}.*

Proof: See Appendix D.1.

One of the results indicating that the leader's extraction behavior in the second period is more aggressive in the feedback than in the open-loop case can be explained by the fact that the leader has an extra information about stock levels at the beginning of each period in the feedback case (also called non-commitment solution) than in the open-loop case (also called commitment solution), and consequently, he could better adapt to the fact that he is only using the resource in the second period by increasing extractions. In addition, as explained in the introduction, we can interpret the entrance of a new use in the second period, and therefore the leader's extraction, as an endogenous shock to the groundwater resource, implying a problem of water scarcity for the follower. In the literature about shocks in optimal groundwater management, de Frutos Cachorro et al. (2014) treats water scarcity as an exogenous shock to the groundwater resource and shows that the higher the intensity of the shock (which could be equivalent here to a higher leader's extraction), the higher the impatience effect and therefore the extractions before the shock occurrence (which could be equivalent here to the follower's extraction in the first period). The same reasoning can then be applied in this work to explain that, $g_{2l}^{FB} > g_{2l}^{OL}$ implies $g_{1f}^{FB} > g_{1f}^{OL}$. Furthermore, the follower in the feedback case earlier adapts to anticipated extraction losses of the second period (i.e. $g_{2f}^{FB} < g_{2f}^{OL}$) due to competition with the leader, by increasing extractions in the first period in comparison with the open-loop case (i.e. $g_{1f}^{FB} > g_{1f}^{OL}$).

Next, we compare the difference in extraction behavior of the follower in the first and the second period, which we name "the jump", for the open-loop ($d^{OL} = g_{1f}^{OL} - g_{2f}^{OL}$) and the feedback ($d^{FB} = g_{1f}^{FB} - g_{2f}^{FB}$) cases. First, we are interested in studying the sign of d^{OL} and d^{FB}. This gives us important information about how the follower reacts to the entrance of the new use in the second period. Possible cases are summarized in the following (see description and computation details in Appendix D.2.1).

- If $a_l - a_f > 2cr/3$:

  Case 1: $d^{OL} > 0, d^{FB} > 0$.

- If $a_l - a_f < 2cr/3$, two different cases can arise depending on additional conditions (see Appendix D.2.1 for details):

  Case 2: $d^{OL} < 0, d^{FB} < 0$.

  Case 3: $d^{OL} < 0, d^{FB} > 0$.

We are also interested in the comparison of the "jumps" between the possible equilibria, i.e. the sign of $d = |d^{FB}| - |d^{OL}|$. This sign will depend on the previous study cases as described in the next proposition.

Proposition 4 *If Case 1 applies, the difference in extraction behavior of the follower between the two periods is higher when feedback strategies are used than under open-loop commitment behavior, i.e.* $d > 0$*. The opposite is true, i.e.* $d < 0$*, if Case 2 is fulfilled. The result is ambiguous under Case 3.*

Proof: See Appendix D.2.2.

Indeed, despite the commitment behavior considered, **Case 1** can be explained by the fact that the higher the marginal revenue of the new use with respect to the marginal revenue of the agricultural use (i.e. the greater the value of $a_l - a_f$), the higher the leader's interest in extracting to accumulate gains[4] (i.e. the greater the value of g_{2l}), and therefore from the interpretation of Proposition 3 the higher the follower's interest to anticipate future extraction losses by increasing his extraction in the first period, i.e. $g_{1f} - g_{2f} > 0$. The opposite reasoning can be applied to explain **Case 2**. However, **Case 3** depends on other parameters of the model (see Appendix D.2.1 for specific conditions) and therefore results are not conclusive. Furthermore, interpretation of Proposition 4 for Cases 1 and 2 is immediate from results obtained in Proposition 3.

We now compare total extractions for both players over the two periods under the two scenarios concerning the players' behavior (open-loop and feedback). Using the notation $\text{Total} = g_{1f} + g_{2f} + g_{2l}$, we obtain the following proposition.

Proposition 5 *The total amount of resource extracted by the follower and the leader over the two periods is higher in the feedback case,* $Total^{FB}$*, than in the open-loop case,* $Total^{OL}$*.*

Proof: See Appendix D.3.

Focusing now on the impact of extraction decisions on the states of the resource, the following corollary is immediate from Propositions 3 and 5.

Corollary 1 *The state of the resource both after the first and second periods, i.e.* G_1 *and respectively* G_2*, is higher in the open-loop case than in the feedback case.*

This means that players' commitment about extraction behavior over the two periods is positive for the state of the resource with respect to the non-commitment case.

[4]This could be also seen directly by studying the sign of the derivatives of expressions in (12) and (20) with respect to a_l.

This is in line with results obtained in the literature characterizing Nash equilibria (Negri (1989), Rubio and Casino (2001), de Frutos Cachorro et al. (2019)). Indeed, the strategic externality, which appears in feedback non-commitment solutions due to competition between the different uses for the available stock, exacerbates the exploitation of the resource.

Finally, players' profits for each solution type and for any θ will be compared through numerical simulations in Section 4. However, for the specific cases $\theta = 1/2$ and $\theta = 0$, we can show the following proposition.

Proposition 6 *The leader's and follower's total profits are always higher in the open-loop case than in the feedback case for $\theta = 1/2$ and $\theta = 0$. Moreover, while in the first period the follower's profits are higher in the feedback case than in the open-loop case, the opposite applies in the second period (see Appendix D.4 for the proof).*

Indeed, for these specific cases, open-loop (commitment) equilibrium procures a higher profitability than feedback (non-commitment) solutions for the agents. This result goes also in line with previous literature that looks for Nash equilibria (e.g. de Frutos Cachorro et al. (2019)), but contrasts with Nie (2005), which compares different commitment behavior in the case of Stackelberg equilibria. However, we remind that our specific problem could be seen as a "degenerated Stackelberg" as the leader does not extract in the first period. Our results then are closer to the results obtained in the Nash case (de Frutos Cachorro et al. (2019)) than to the results obtained in the Stackelberg case (Nie (2005)). In addition, if we focus on the follower's profit in the first period, the opposite result is obtained and feedback strategy procures higher profitability than open-loop strategies. As explained before, this could be interpreted by the fact that the follower in the feedback case adapts earlier to the entrance of the second use by augmenting extractions in the first period and therefore by increasing profits, in comparison to the open-loop case. In any case, previous results should not necessarily be maintained for other values of θ or/and for non-linear revenues for the agricultural use. In what follows, we run some numerical simulations to test if these results remain unchanged under other economic assumptions.

4 Numerical results

In this section, we perform numerical simulations in order to analyze whether or not the main results obtained in the previous sections concerning agents' extraction behavior and stock and profit implications remain unchanged relaxing previous economic assumptions.

Parameter	Description	Value
a_f	Coefficient of revenue agricultural use (linear term)	4.5
a_l	Marginal revenue from alternative use	6
z	Marginal pumping cost intercept	4
c	Marginal pumping cost slope	0.281
G_0	Initial stock level	10
r	Natural recharge rate	5
b_f	Coefficient of revenue agricultural use (non-linear term)	$b_f \in \{0, 0.01, 0.1\}$
θ	Weight assigned by the leader to agricultural use	$\theta \in \{0.5, 0.581, 0.655\}$
A	Coefficient of the leader's valuation of final stock	$A \in \{0, 0.07\}$

Table 1: Parameter values of the model.

For this purpose, we use values of the parameters which are listed in Table 1. More specifically, we fix values corresponding to model parameters above the horizontal line and run several simulations with respect to parameters below the line, i.e., for different non-linear revenue functions of the agricultural use, with $R_f(g_{tf}) = a_f g_{tf} - \frac{b_f}{2} g_{tf}^2$, or more specifically, for different coefficient (non-linear term) of the revenue function (b_f), and for different leader's weight of the agricultural use (θ). Moreover, we will address the case in which the leader, aiming at analyzing the sustainability of the resource, includes in his objective function the value of the stock at the end of the planning horizon (i.e. by adding the term AG_2 with $A > 0$ in the leader's objective function, equations (9) and (16), for the open-loop and feedback scenarios, respectively). In fact, we have obtained analytical solutions for these general cases, and run numerical simulations for different parameter values with Maple.

In what follows, we take as a benchmark scenario, the case in which the revenue function of the agricultural use is linear ($b_f = 0$), the leader equally weighs the profits derived from both uses of the aquifer ($\theta = 1/2$), and no value is assigned to the final stock ($A = 0$), for which theoretical results have been shown in the preceding section[5]. The benchmark case is compared with the case in which the weight assigned by the leader to the profits of the agricultural use is higher than $\frac{1}{2}$ (i.e., $\theta > \frac{1}{2}$); the case of non-linear revenues of the agricultural use (i.e., $b_f > 0$); and the case in which the value of the final stock is considered in the leader's objective ($A > 0$).

[5]We note that parameter values of Table 1 satisfy Condition 2.2 and therefore ensure the positivity of extraction and state variables in the open-loop and feedback cases.

We first confirm that results of the sensitivity analysis of the optimal strategies and resource stocks with respect to parameter θ (Propositions 1-2) are maintained for parameter values in all numerical simulations. In the following subsections, we study whether results on extraction decisions and stock implications (Propositions 3-5) hold up in these new scenarios (Tables 2, 3 and 5) and we analyze profit results per period and use (Tables 4 and 5).

4.1 Equal $\theta = 1/2$ vs. different $\theta > 1/2$ weights

First of all, focusing on the scenario of linear revenues of the agricultural use ($b_f = 0$), we compare numerical results for the benchmark case $\theta = \frac{1}{2}$ with the case $\theta > \frac{1}{2}$, where the leader assigns a higher weight to the profits from the agricultural use than to the profits from the alternative/new use (see columns 1-4 in Tables 2 and 4 and column 1 in Table 3).

Column		1	2	3	4	5	6	7	8	9	10	11	12
		$b_f = 0$				$b_f = 0.01$				$b_f = 0.1$			
		Follower		Leader		Follower		Leader		Follower		Leader	
Period		1	2	2	Total	1	2	2	Total	1	2	2	Total
$\theta = 1/2$	A=0	+	−	+	+	+	−	+	+	+	−	+	+
	A=0.07	+	−	+	+	+	−	+	+	+	−	+	+
$\theta = 0.581$	A=0	−	+	+	−	−	−	+	+	+	−	+	+
	A=0.07	−	+	+	−	−	−	+	+	+	−	+	+

Table 2: Sign of differences between the extraction results in the feedback and the open-loop cases: + means $FB > OL$, – means $FB < OL$.

Column		1	2	3
		$b_f = 0$	$b_f = 0.01$	$b_f = 0.1$
$\theta = 1/2$	A=0	+	+	+
	A=0.07	+	+	+
$\theta = 0.581$	A=0	−	−	+
	A=0.07	−	−	+

Table 3: Sign of the difference between the "jump" in the feedback and the open-loop cases, the "jump" being previously defined by the difference between the follower's extractions in the first and second periods: + means $FB > OL$, – means $FB < OL$.

Concerning extraction behavior, for a great value of θ (e.g. $\theta = 0.581$), extraction strategies of the follower in both periods are reversed with respect to the benchmark case (compare penultimate and first row in columns 1 and 2). In other words, in the feedback case, we observe a more conservative extraction behavior for the resource by the follower at the end of the first period, and a more aggressive extraction behavior at the end of the second period, in comparison with the open-loop case. This implies lower extraction differences by the follower between periods (see column 1 Table 3) and lower total extractions by both agents over the two periods (see column 4 in Table 2) in the feedback than in the open-loop case. Indeed, we remind that lower total extractions means higher stock levels at the end of the second period. Therefore, when $\theta = 0.581$, we note that higher stock levels are obtained in the feedback (non-commitment) case than in the open-loop (commitment) case, after the entrance of the alternative use. The interpretation behind this result seems therefore intuitive. As already noted, when the leader equally weights both uses, the follower seems to adapt earlier, i.e. in the first period, in the non-commitment than in the commitment case, to anticipate extraction losses in the second period due to competition with the other use. When θ increases, that is, the agricultural use becomes more and more important for the leader with respect to the new use, this "anticipation" or fear for water shortage is reduced in the feedback case and consequently, the representative agent for agricultural users focus on extracting more in the second period than in the first period. However, this does not compensate extraction behavior in the first period and total extractions over the two periods become more important in the open-loop than in the feedback case.

Column		1	2	3	4	5	6	7	8	9	10	11	12
		$b_f = 0$				$b_f = 0.01$				$b_f = 0.1$			
		Follower			Leader	Follower			Leader	Follower			Leader
Period		1	2	Total	Total	1	2	Total	Total	1	2	Total	Total
	A=0	+	−	−	−	+	−	−	−	+	−	−	−
$\theta = 1/2$	A=0.07	+	−	−	−	+	−	−	−	+	−	−	−
	A=0	−	+	−	−	−	−	−	−	+	−	−	−
$\theta = 0.581$	A=0.07	−	+	−	−	−	−	−	−	+	−	−	−

Table 4: Sign of differences between the profit results in the feedback and the open-loop cases: + means $FB > OL$, – means $FB < OL$.

Next, simulated results concerning the follower's profits reproduce the previous extraction results (see two first columns Table 4). Opposite to the benchmark case, for higher values of θ ($\theta = 0.581$) lower follower's profits (respectively greater follower's profits) are now observed for the feedback case at the end of the first period (respectively at the end of the second period). However, same tendencies are observed for the total follower's and leader's profits over the two periods (see columns 3-4 Table 4).

Simulated results then suggest that when the leader assigns a higher weight to the profits from the agricultural use than to the profits from the alternative use, commitment strategies remain more profitable than non-commitment strategies, although non-commitment strategies could be more favorable than commitment strategies in terms of final stock levels.

4.2 Linear ($b_f = 0$) vs non-linear ($b_f > 0$) revenues from the agricultural use

We now compare the cases of linear and non-linear revenues from the agricultural groundwater use. Similar results to those of the benchmark case are observed in linear and non-linear cases, when θ is equal to $\frac{1}{2}$ (e.g. compare columns 9-12 with columns 1-4 for the first row in Table 2).

Moreover, for $\theta = 0.581$, while extraction strategies of the follower in both periods are reversed with respect to the benchmark case (i.e. $b_f = 0$ and $\theta = 1/2$) in the case $b_f = 0$, extraction strategies become more and more similar to the benchmark case when b_f increases, (e.g. compare penultimate row column 9 with first row column 1 in Table 2).

As for the follower's profits at the end of both periods (see Table 4), same trends than for the follower's extraction decisions are now observed, when comparing the linear and non-linear revenue cases for different values of θ.

Numerical results indicate that a decrease in the follower's marginal revenue (i.e. an increase in b_f) might compensate the effect of an increase in the leader's weight assigned to the profits from the agricultural use (i.e. an increase in θ) on extraction strategies and profit results.

4.3 Valuation ($A > 0$) vs no valuation ($A = 0$) of the final stock levels

We finally compare extraction decisions and profit results for the case in which the final stock value is included in the leader's objective function ($A > 0$) to the case in which it is not included ($A = 0$). We note that extraction behavior and profit results show similar trends in both scenarios for simulated values in Tables 2, 3 and 4, i.e. for values of θ between $\frac{1}{2}$ and 0.581 (e.g. compare two first rows in Table 2).

However, if we perform additional simulations for a much greater value of θ, i.e. for the case $\theta = 0.655$ (see Table 5), surprising results are observed concerning the total follower's profits, when $A = 0.07$ (see last row column 7).

Column	1	2	3	4	5	6	7	8
	Extractions				Profits			
	Follower		Leader		Follower			Leader
Period	1	2	2	Total	1	2	Total	Total
A=0	–	+	–	–	–	+	–	–
A=0.07	–	+	–	–	–	+	+	–

Table 5: Sign of differences between the extraction and profit results in the feedback and the open-loop cases, $b_f = 0.1$ and $\theta = 0.655$: + means $FB > OL$, – means $FB < OL$.

Simulated results show that, in some particular cases, non-commitment extraction behavior could be more profitable for the follower and at the same time, more favorable in terms of final stock levels than the commitment case. This could be obtained when the leader assigns a great weight to the agricultural use profits with respect to the alternative use profits and takes into account the final stock value in his objective function. Indeed, when the leader values the sustainability of the resource, extractions for the new use are reduced to preserve stock levels, entailing then less competition for water in the second period and subsequently less strategic interactions in the feedback (non-commitment) case. In other words, this "conservative extraction behavior for the resource" gives more possibilities for the representative farmer to extract and to accumulate profits in the second period in the non-commitment case, achieving higher profits with respect to the commitment case.

5 Conclusions and extensions

In this paper, we study a problem of exploitation of a groundwater resource, mainly used for irrigation, who faces exceptionally the entrance of a new priority use in the system, therefore involving a problem of water scarcity for the farmers. A water agency is therefore needed in order to manage how much groundwater could be extracted for the new (non-agricultural) priority use. To model this situation, we build a two-stage discrete Stackelberg game in which the leader (the water agency) just intervenes when the new use takes place (in the second stage) and the follower is a representative agent for regular users of the resource, the farmers. The aim of the paper is to analyze and compare extraction behaviors of the different agents (water agency and farmers) for different Stackelberg equilibria, representing different commitment behaviors, and the consequences of these extraction policies for the final state of the resource and the agents' profits. In particular, we compute and compare the open-loop (commitment solution) and feedback (non-commitment) equilibria.

First, theoretical results are provided for the case in which the leader assigns a lower (or equal) weight to the profits from the agricultural use than to the profits from the alternative/new use. We show analytically that the leader's extraction behavior in the second period is more aggressive in the feedback than in the open-loop case. The follower in the feedback case hence earlier adapts to anticipated extraction losses of the second period due to competition with the leader, by augmenting extractions in the first period in comparison with the open-loop case. In other words, the follower's extraction behavior is more aggressive (respectively less aggressive) in the feedback case than in the open-loop case in the first period (respectively in the second period). This leads to lower total extractions, or equivalently, higher final stock levels in the commitment than in the non-commitment case. These theoretical results go in the same line of the existing literature that considers simultaneous play and characterizes Nash equilibria (e.g. Negri (1989), Rubio and Casino (2001), de Frutos Cachorro et al. (2019)).

Next, we perform numerical simulations to study if previous results are maintained for the case of non-linear revenues of the agricultural use and for the case in which the leader assigns now a higher weight to the profits from the agricultural use than to the alternative (non-agricultural) use. Simulated results show that an increase in the weight assigned to profits from the agricultural use could provide higher final stock levels in the non-commitment than in the commitment case. This is mainly due to a reduction of the fear for water shortage from the representative agent of agricultural users in the non-commitment case and consequently, a decrease in total extractions over the two periods with respect to the commitment case. Moreover, a decrease in the marginal revenue from agricultural use could compensate the effect of an increase in the weight assigned to the profits from the agricultural use on simulated results.

Finally, the two-period model could be seen as a limitation of the study with regard to the analysis of the sustainability of the resource. To cope with this, we introduce in the modeling the possibility that the leader values the final state of the resource in his objective (i.e. the addition of a scrap value function) and perform additional simulations for great values of the leader's weight assigned to profits from the agricultural use. In this context, simulated results show that non-commitment strategies could be not only more favorable for the sustainability of the resource (i.e. higher stock levels at the end of the second period) but could be also more profitable for the agricultural use, in comparison with commitment strategies. This interesting result could be explained by the fact that a more conservative extraction behavior is performed by the leader when the sustainability of the resource is valued, hence giving more possibilities for the representative agent of the farmers to extract and to accumulate profits in the non-commitment case with respect to the commitment case. For the leader's profits, Nie (2005) obtains a similar result. However, as our problem can be seen as a "degenerated Stackelberg", in contrast with Nie (2005), higher leader's profits are always obtained in the commitment than in the non-commitment case, and therefore, in this sense, results are closer to the results that are often found in the case of Nash equilibria (e.g. de Frutos Cachorro et al. (2019)).

There are several possible extensions to our paper. First of all, in this work, we consider that the follower is a representative agent for agricultural users. It could be therefore interesting to consider different followers, such as for example several farmers with possible heterogeneities, who play simultaneously à la Nash between them, and à la Stackelberg with the leader. We could also compute the corresponding efficient solution to our problem, in which the leader could be a regulator who takes all the extraction decisions considering the same objective function, in order to derive and analyze the possible policy implications of our study. Finally, we could apply our theoretical model to a real case.

A Study of time-consistency

In order to verify the time-inconsistency of the open-loop equilibrium and the time-consistency of the feedback equilibrium for our model, we use the pure definition of time-consistency described in Kydland and Prescott (1977) and adapted to our setup. A policy plan g_{2l} is consistent if, for $t = 2$, g_{2l} maximizes the objective function of the leader, taking as given previous decisions, and the strategy selected coincides with the optimal decision rule.

In the open-loop case, the problem the leader is facing at $t = 2$ is described in (9), where the follower's strategies can be expressed as functions of g_{2l} as follows:

$$g_{1f} = \tilde{g}_{1f}(g_{2l}), \tag{23}$$

$$g_{2f} = \tilde{g}_{2f}(g_{2l}). \tag{24}$$

Denoting the leader's objective function by a function $\bar{\Pi}_l^{OL}(g_{1f}, g_{2f}, g_{2l})$, the leader aims to find g_{2l} that maximizes his objective subject to restrictions (23) and (24).

Necessary condition for an interior solution is then:

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0 \iff \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} \frac{\partial \tilde{g}_{1f}}{\partial g_{2l}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2f}} \frac{\partial \tilde{g}_{2f}}{\partial g_{2l}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0. \tag{25}$$

If past decision, i.e. g_{1f}, is given, the previous necessary condition becomes:

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0 \iff \cancel{\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} \frac{\partial \tilde{g}_{1f}}{\partial g_{2l}}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2f}} \frac{\partial \tilde{g}_{2f}}{\partial g_{2l}} + \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{2l}} = 0. \tag{26}$$

The leader's strategy is time-consistent if conditions in (25) and (26) coincide, i.e. if

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} \underbrace{\frac{\partial \tilde{g}_{1f}}{\partial g_{2l}}}_{\neq 0} = 0 \iff \frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} = 0.$$

From equations (9), (5) and (7), we can easily show that:

$$\frac{\partial \bar{\Pi}_l^{OL}}{\partial g_{1f}} = -\theta c(g_{1f} + g_{2f}) - (1-\theta) c g_{2l} < 0.$$

We conclude that the open-loop equilibrium described in this paper cannot be time-consistent.

In the feedback case, the problem to solve for the leader at $t = 2$ is described in (16), where the follower's strategy at $t = 2$ can be expressed as a function of g_{1f} and g_{2l} as follows:

$$g_{2f} = \hat{g}_{2f}(g_{1f}, g_{2l}). \tag{27}$$

Denoting the leader's objective function by a function $\hat{\pi}_l^{FB}(g_{1f}, g_{2f}, g_{2l})$, the leader aims to find g_{2l} that maximizes his objective subject to restriction (27).

Necessary condition for an interior solution now reads:

$$\frac{\partial \hat{\pi}_l^{FB}}{\partial g_{2l}} = 0 \iff \frac{\partial \hat{\pi}_l^{FB}}{\partial g_{2f}} \frac{\partial \hat{g}_{2f}}{\partial g_{2l}} + \frac{\partial \hat{\pi}_l^{FB}}{\partial g_{2l}} = 0 \tag{28}$$

It is easy to see that this expression coincides with the necessary condition when g_{1f} is given. Thus, the leader's plan is time-consistent under feedback information structure.

B Open-loop Stackelberg equilibrium

B.1 Derivation of the open-loop Stackelberg equilibrium

The follower's objective function in (8), the sum of the profits over the two periods, once G_1 and G_2 have been replaced by their expressions in (1) and (2) reads:

$$\bar{\Pi}_f(g_{1f}, g_{2f}, g_{2l}) = g_{2f}(a_f + c(G_0 + 2r - g_{1f} - g_{2f} - g_{2l}) - z) + (a_f - (z - c(G_0 - g_{1f} + r)))g_{1f}.$$

Assuming an interior solution, the maximization of $\bar{\Pi}_f(g_{1f}, g_{2f}, g_{2l})$ with respect to g_{1f} and g_{2f} gives the follower's best-reaction functions

$$\tilde{g}_{1f}(g_{2l}) = \frac{a_f + c(G_0 + g_{2l}) - z}{3c}, \tag{29}$$

$$\tilde{g}_{2f}(g_{2l}) = \frac{a_f + c(G_0 - 2g_{2l} + 3r) - z}{3c}. \tag{30}$$

The optimal two-period profits of the follower are:

$$\begin{aligned} \bar{\Pi}_f(\tilde{g}_{1f}(g_{2l}), \tilde{g}_{2f}(g_{2l}), g_{2l}) &= \frac{1}{3c}\{(a_f - z)(a_f + c(2G_0 - g_{2l} + 3r) - z) \\ &\quad + c^2\left(G_0^2 + 3r^2 + (3r - g_{2l})(G_0 - g_{2l})\right)\}. \end{aligned}$$

The leader's objective in (9) becomes:

$$\theta\bar{\Pi}_f(\tilde{g}_{1f}(g_{2l}), \tilde{g}_{2f}(g_{2l}), g_{2l}) + (1 - \theta)\Pi_l(g_{2l}, G_2), \tag{31}$$

where

$$\Pi_l(g_{2l}, G_2) = \frac{1}{3} g_{2l}(3a_l - 2a_f + c(G_0 - 2g_{2l} + 3r) - z).$$

Assuming an interior solution, the maximization of (31) with respect to g_{2l}, gives the leader's optimal strategy in (12). The follower's optimal strategies in (10) and (11) are obtained replacing g_{2l} by the expression in (12) into the follower's best-reaction functions in (29) and (30).

Replacing the optimal extraction strategies in the agents' profit functions, we obtain the optimal profits of the leader and the follower:

$$\begin{aligned}\Pi_l^{OL} =& -\frac{1}{12c(3\theta-2)} \left[2\theta\left(2a_f^2 + a_f(9a_l + 13cG_0 + 27cr - 13z) - 9a_l^2 + 9a_l(z - c(G_0+3r))\right.\right.\\ &\left.+2\left(c^2\left(G_0^2 - 3r^2\right) - 2cG_0 z + z^2\right)\right) - \theta^2\left(11a_f^2 + a_f(6a_l + 28cG_0 + 48cr - 28z) - 9a_l^2\right.\\ &\left.\left.-12a_l(cG_0 + 3cr - z) + 4(cG_0 - z)(2cG_0 + 3cr - 2z)\right) + (2a_f - 3a_l - c(G_0+3r) + z)^2\right],\end{aligned}$$

$$\begin{aligned}\Pi_f^{OL} =& \frac{1}{12c(2-3\theta)^2}\left[a_f^2(\theta(43\theta - 68) + 28) + 2a_f\left(-12a_l(\theta-1)^2 + c(G_0(\theta(31\theta - 44) + 16)\right.\right.\\ &\left.+3(\theta(13\theta - 20) + 8)r) + ((44 - 31\theta)\theta - 16)z\right) + 9a_l^2(\theta-1)^2 - 6a_l(\theta-1)^2(c(G_0+3r) - z)\\ &+4\theta^2(c^2\left(7G_0^2 + 15G_0 r + 9r^2\right) - cz(14G_0 + 15r) + 7z^2) - 2\theta\left(c^2\left(19G_0^2 + 42G_0 r + 27r^2\right)\right.\\ &\left.\left.-2cz(19G_0 + 21r) + 19z^2\right) + 13c^2G_0^2 + 30c^2G_0 r + 21c^2r^2 - 26cG_0 z - 30crz + 13z^2\right].\end{aligned}$$

B.2 Positivity conditions

In what follows we derive the conditions ensuring the positivity of the players' optimal strategies and the states of the aquifer over the two periods. We characterize these conditions under the assumption that Condition 1.D ($\theta \leq 1/2$) is satisfied.

- $g_{2l}^{OL} > 0$ if and only if $\theta \in (0, \theta_1)$, with

  $$\theta_1 = \frac{2a_f - 3a_l - cG_0 - 3cr + z}{a_f - 3a_l - 2cG_0 - 6cr + 2z}.$$

  Under Condition 1, both the numerator and denominator of θ_1 are negative, and hence, $\theta_1 > 0$. Furthermore, the same conditions guarantee $\theta_1 > 1/2$.

- $g_{1f}^{OL} > 0$ if and only if $(2a_f + 3a_l + 5cG_0 + 3cr - 5z) - \theta(5a_f + 3a_l + 8cG_0 + 6cr - 8z) > 0$. Under Condition 1, specifically, $a_l > z$ and $a_f > z$, the two terms in brackets are positive. Then, $g_{1f}^{OL} > 0$ if and only if $\theta < \theta_2$, with

  $$\theta_2 = \frac{2a_f + 3a_l + 5cG_0 + 3cr - 5z}{5a_f + 3a_l + 8cG_0 + 6cr - 8z}.$$

Under Condition 1.C it can be proved that $\theta_2 > \theta_1$.

Therefore, $\theta < \theta_1$ implies $\theta < \theta_2$.

- Under Condition 1, g_{2f}^{OL} is always positive.

- $G_1^{OL} > 0$ if and only if $(2a_f + 3a_l - 7cG_0 - 9cr - 5z) + \theta(-5a_f - 3a_l + 10cG_0 + 12cr + 8z) < 0$. Under Condition 1, the first term in brackets is negative and the second term is positive. Therefore, $G_1^{OL} > 0$ if and only if $\theta < \theta_3$, with

$$\theta_3 = \frac{2a_f + 3a_l - 7cG_0 - 9cr - 5z}{5a_f + 3a_l - 10cG_0 - 12cr - 8z}.$$

  Under Condition 1 it can be proved that $\theta_3 > 1/2$. Hence, $G_1 > 0$ for all $\theta \leq 1/2$.

- $G_2^{OL} > 0$ if and only if $(2a_f + 3a_l - cG_0 - 3cr - 5z) + \theta(-5a_f - 3a_l + cG_0 + 3cr + 8z) < 0$. If Condition 1.C fulfills, then the second term in brackets is positive. If the first term in brackets was positive, G_2^{OL} could never be positive. Hence, we impose this term to be negative, or equivalently,

$$(G_0 + 3r)c > 3a_l + 2a_f - 5z. \tag{32}$$

  Under condition (32), $G_2^{OL} > 0$ if and only if $\theta < \theta_4$, with

$$\theta_4 = \frac{2a_f + 3a_l - cG_0 - 3cr - 5z}{5a_f + 3a_l - cG_0 - 3cr - 8z}.$$

  Under Conditions 1.A and 1.B it can be easily proved that $\theta_4 > 1/2$. Therefore, if condition (32) is fulfilled, then $G_2 > 0$ for any $\theta \leq 1/2$.

  Finally, Conditions 1.B and 1.C imply condition (32).

B.3 Concavity conditions

The concavity of the follower's objective function in (8) with respect to his decision variables g_{1f} and g_{2f} is ensured if the quadratic form associated with the Hessian matrix is negative definite. The entries of this matrix are $h_{11} = -2c, h_{12} = -c, h_{21} = -c, h_{22} = -2c$, and therefore, the quadratic form is negative definite, and the follower's objective function is strictly concave.

The best response of the follower to g_{2l} is given by (29) and (30) provided that these expressions are positives. (29) is always positive under condition 1.A and (30) is positive if $g_{2l} < \frac{a_f - z + c(G_0+3r)}{2c}$. As the leader is interested in positive extractions of the follower, he maximizes (31) under this last condition. The concavity of the leader's objective function with respect to his decision variable g_{2l} is ensured if the second derivative of this function with respect to g_{2l} is negative. The sign of this derivative is given by the sign of $(-2+3\theta)$. Therefore, the concavity of the leader's objective function requires $\theta < 2/3$ (this condition is ensured if Condition 1.D is satisfied) and $g_{2l}^{OL} < \frac{a_f - z + c(G_0+3r)}{2c}$.

B.4 Proof of Proposition 1

$$\frac{\partial g_{2l}^{OL}}{\partial \theta} = -\frac{1}{2}\frac{4a_f - 3a_l + cG_0 + 3cr - z}{c(2-3\theta)^2}. \tag{33}$$

The derivative in (33) is negative under Condition 1.C, and hence, any increment of θ leads to a reduction of the optimal extraction of the leader.

Furthermore, the derivatives of the follower's optimal strategies with respect to θ read:

$$\frac{\partial g_{1f}^{OL}}{\partial \theta} = \frac{1}{3}\frac{\partial g_{2l}^{OL}}{\partial \theta} < 0, \quad \frac{\partial g_{2f}^{OL}}{\partial \theta} = -\frac{2}{3}\frac{\partial g_{2l}^{OL}}{\partial \theta} > 0. \tag{34}$$

The signs of the derivatives in (34) come from (33).

The effects of a change in θ on the state of the aquifer at the end of the two periods read:

$$\frac{\partial G_1^{OL}}{\partial \theta} = -\frac{1}{3}\frac{\partial g_{2l}^{OL}}{\partial \theta} = -\frac{\partial g_{1f}^{OL}}{\partial \theta} > 0, \tag{35}$$

$$\frac{\partial G_2^{OL}}{\partial \theta} = -\frac{2}{3}\frac{\partial g_{2l}^{OL}}{\partial \theta} = -2\left(\frac{\partial g_{2f}^{OL}}{\partial \theta} + \frac{\partial g_{2l}^{OL}}{\partial \theta}\right) = 2\frac{\partial G_1^{OL}}{\partial \theta} > 0. \tag{36}$$

The effect on the follower's optimal profits, Π_f^{OL}, is clearly positive under Conditions 1.C and 1.D:

$$\frac{\partial \Pi_f^{OL}}{\partial \theta} = -\frac{1}{3}\frac{(1-\theta)(4a_f - 3a_l + cG_0 + 3cr - z)}{2-3\theta}\frac{\partial g_{2l}^{OL}}{\partial \theta} = -3c(1-\theta)(2-3\theta)\frac{\partial g_{1f}^{OL}}{\partial \theta}\frac{\partial g_{2f}^{OL}}{\partial \theta} > 0. \tag{37}$$

C Feedback Stackelberg equilibrium

C.1 Derivation of the feedback Stackelberg equilibrium

The feedback Stackelberg equilibrium is characterized using backward induction.

In the first stage the follower decides the extraction in period 2 and solves the problem in (15). Once G_1 and G_2 have been replaced by their expressions in (1) and (2), the follower's objective function in the second period reads:

$$\widetilde{\Pi}_{2f}(g_{1f}, g_{2f}, g_{2l}) = g_{2f}(a_f + c(G_0 - g_{1f} - g_{2f} - g_{2l} + 2r) - z).$$

Assuming an interior solution, the maximization of $\widetilde{\Pi}_{2f}(g_{1f}, g_{2f}, g_{2l})$ with respect to g_{2f} gives the follower's second-period best-reaction function

$$\hat{g}_{2f}(g_{2l}, g_{1f}) = \frac{af + c(G_0 - g_{1f} - g_{2l} + 2r) - z}{2c}. \tag{38}$$

The follower's optimal second-period profits are:

$$\widetilde{\Pi}_{2f}(g_{1f}, \hat{g}_{2f}(g_{2l}, g_{1f}), g_{2l}) = \frac{(a_f + c(G_0 - g_{1f} - g_{2l} + 2r) - z)^2}{4c}.$$

In the second step, the leader decides the extraction in period 2, taking into account the follower's extraction in this period given in (38). Therefore, the leader's objective in (16) becomes:

$$\theta\widetilde{\Pi}_{2f}(g_{1f}, \hat{g}_{2f}(g_{2l}, g_{1f}), g_{2l}) + (1-\theta)\Pi_l(g_{2l}, G_2), \tag{39}$$

where $\Pi_l(g_{2l}, G_2)$ once G_1, G_2 and g_{2f} have been replaced by their expression in (1), (2) and (38), respectively, reads:

$$\widetilde{\Pi}_l(g_{2l}, g_{1f}) = 2g_{2l}(a_f - 2a_l - c(G_0 - g_{1f} - g_{2l} + 2r) + z).$$

Assuming an interior solution, the maximization of (39) with respect to g_{2l}, gives the leader's extraction in the second period as a function of the follower's extraction in the first period:

$$\hat{g}_{2l}(g_{1f}) = \frac{a_f - 2a_l(1-\theta) + (2\theta - 1)(c(G_0 - g_{1f} + 2r) - z)}{c(3\theta - 2)}. \tag{40}$$

In the third and final step, the follower decides the extraction in period 1 taking into account the leader's reaction function in the second period given in (40). The follower's objective function in the first period becomes:

$$\widetilde{\Pi}_{1f}(g_{1f}) = \frac{(\theta-1)^2(-3a_f + 2a_l + c(-G_0 + g_{1f} - 2r) + z)^2}{4c(2-3\theta)^2} + g_{1f}(a_f - (z - c(G_0 - g_{1f} + r))).$$

Assuming an interior solution, the maximization of $\widetilde{\Pi}_{1f}(g_{1f})$ with respect to g_{1f}, gives the optimal strategy in (18). The optimal strategy in (20) is obtained replacing g_{1f} by the expression in (18) into the leader's second-period best-reaction function in (40). Finally, the optimal strategy in (19) is obtained replacing g_{1f}, and respectively g_{2l}, by the expressions in (18), and (20), respectively, into the follower's second-period best-reaction function in (38).

Replacing the optimal extraction strategies in the agents' profit functions, we obtain the optimal profits of the leader and the follower:

$$\begin{aligned}\Pi_l^{FB} =& \frac{-1+\theta}{c(5-7\theta)^2(3-5\theta)^2}\left[(2-3\theta)^2(\theta-1)\theta(5a_f-4a_l+cG_0+3cr-z)^2-2(a_f((11-5\theta)\theta-5)\right.\\ &+a_l(\theta-1)(11\theta-7)+(\theta(6\theta-7)+2)(cG_0+3cr-z))(a_f((21-20\theta)\theta-5)\\ &\left.+a_l(\theta(23\theta-26)+7)+(\theta-1)(3\theta-2)(cG_0+3cr-z))\right],\end{aligned}$$

$$\begin{aligned}\Pi_f^{FB} =& \frac{1}{c(5\theta-3)(7\theta-5)}\left[a_f^2(3\theta(5\theta-8)+10)+a_f\left(-10a_l(\theta-1)^2+c(2G_0(2\theta(5\theta-7)+5)\right.\right.\\ &+(\theta(25\theta-38)+15)r)-2(2\theta(5\theta-7)+5)z)+4a_l^2(\theta-1)^2-2a_l(\theta-1)^2(c(G_0+3r)-z)\\ &+\theta^2\left(c^2\left(9G_0^2+19G_0r+11r^2\right)-cz(18G_0+19r)+9z^2\right)-2\theta\left(c^2\left(6G_0^2+13G_0r+8r^2\right)\right.\\ &\left.\left.-cz(12G_0+13r)+6z^2\right)+4c^2G_0^2+9c^2G_0r+6c^2r^2-8cG_0z-9crz+4z^2\right].\end{aligned}$$

C.2 Positivity conditions

The optimal strategy of the follower's extraction in the first period, g_{1f}, can be rewritten as:

$$g_{1f}^{FB} = \frac{(a_f+cG_0-z+cr)(4\theta(4\theta-5)+6)+(2a_l-z+cG_0-a_f)(\theta-1)^2}{c(5\theta-3)(7\theta-5)}.$$

Therefore, either Condition 2.1 or 2.2, specifically $a_l > z$ and $a_f > z$, for any $0 \leq \theta \leq 1/2$, g_{1f}^{FB} is positive.

Moving to the second period, the denominator of g_{2f}^{FB} is positive under Condition $0 \leq \theta \leq 1/2$ and the first two factors of the numerator are negative. Therefore, g_{2f}^{FB} is positive if and only if $5a_f - 4a_l + cG_0 + 3cr - z > 0$. Conditions 2.1.A and 2.1.C' or Conditions 2.2.A and 2.2.C" imply that this last inequality is fulfilled.

The optimal strategy of the leader's extraction g_{2l} can be rewritten as:

$$g_{2l}^{FB} = \frac{2((a_l-a_f)(\theta-1)(11\theta-7)+(\theta(6\theta-7)+2)(a_f+c(G_0+3r)-z))}{c(5\theta-3)(7\theta-5)}.$$

Therefore, under Conditions A and B in Condition 2.1 and Condition 2.2, for any $0 \leq \theta \leq 1/2$, g_{2l}^{FB} is positive.

The positivity of G_2^{FB} is given by the positivity of the following second-order polynomial in variable θ:

$$(3c(G_0+3r)-12a_l-20a_f+32z)\theta^2-(5c(G_0+3r)-20a_l-21a_f+41z)\theta+(2c(G_0+3r)-8a_l-5a_f+13z). \quad (41)$$

When $\theta = 1/2$, $G_2^{FB} = 1/4(c(G_0 + 3r) - 4a_l + 2a_f + 2z) > 0$ under Conditions A and B in Condition 2.1 and Condition 2.2, and Condition C' and Condition C" in Condition 2.1 and Condition 2.2, respectively.

When $\theta = 0$, $G_2^{FB} = (2c(G_0+3r)-8a_l-5a_f+13z) > 0 \iff \frac{8a_l+5a_f-13z}{2} < c(G_0+3r)$. The coefficient of the quadratic term is positive if and only if $c(G_0+3r) > \frac{12a_l+20a_f-32z}{3}$.

Under condition A in Condition 2.1 and Condition 2.2

$$2(2a_l - a_f - z) < \frac{8a_l + 5a_f - 13z}{2} < \frac{12a_l + 20a_f - 32z}{3}.$$

The minimum or maximum of the polynomial in (41) is attained at $\bar{\theta}$ such that

$$\bar{\theta} - 1/2 = \frac{2c(G_0 + 3r) - 8a_l - a_f + 9z}{2(3c(G_0 + 3r) - 12a_l - 20a_f + 32z)}. \quad (42)$$

We have that

- Condition 2.1 implies $2c(G_0 + 3r) > 8a_l + a_f - 9z$ and the numerator in (42) is positive, and therefore expression (42) is positive. Then, under this condition the quadratic polynomial in (41) is convex, taking positive values at 0 and 1/2, with a minimum value at $\bar{\theta} > 1/2$. Then $G_2^{FB} > 0$ for all $\theta > 0$ or $G_2^{FB} > 0$ for all $0 < \theta < \tilde{\theta}$ with $\tilde{\theta} > 1/2$.

- Under Condition 2.2 the quadratic polynomial in (41) is concave, taking a negative value at 0 and a positive value at 1/2 with a maximum value at $\bar{\theta}$. Then, there exist two values $\bar{\theta}_1$ and $\bar{\theta}_2$ with $\bar{\theta}_1 \leq 1/2 < \bar{\theta}_2$ such that $G_2^{FB} > 0$ in $[\bar{\theta}_1, \bar{\theta}_2]$.

C.3 Concavity conditions

In the second period, the follower's objective function in (15) is strictly concave with respect to his decision variable g_{2f}, because $\frac{\partial^2 \Pi_f}{\partial g_{2f}^2} = -2c < 0$. The best response of the follower is given by (38) provided it is positive. Using the fact that we ask for positive solutions (in extractions and water levels), $G_2 = G_0 - g_{1f} - g_{2l} - g_{2f} + 2r > 0$, that is, $G_0 - g_{1f} - g_{2l} + 2r > g_{2f}$. As $g_{2f} > 0$ and $a_f > z$, we have that (38) is positive.

In the second period, the concavity of the leader's objective function in (16) with respect

to his decision variable g_{2l} requires $\frac{\partial^2 \Pi_l}{\partial g_{2l}^2} = c(-2+3\theta) < 0$. Therefore, this concavity condition reduces to $\theta < 2/3$. The best response of the leader is given by (40) provided that this expression positive. The denominator of (40) is negative then (40) is positive if the numerator $a_f - a_l + (2\theta-1)(a_l - z + c(G_0 - g_{1f} + 2r))$ is negative (this is verified under the assumptions $a_l > a_f > z$, $G_1 > 0$ and $\theta \leq \frac{1}{2}$).
In the first period, the follower's objective function in (17) is strictly concave with respect to his decision variable g_{1f} if and only if $(\theta-1)^2 - 4(2-3\theta)^2 < 0$. This inequality can be rewritten as $(5\theta-3)(7\theta-5) > 0$, and hence, this condition is fulfilled if either $\theta < 3/5$ or $\theta > 5/7$.
Therefore, either Condition 2.1 or Condition 2.2 ensures the concavity of both stages.

C.4 Proof of Proposition 2

First, results of the leader's extraction behavior with respect to a change in θ can be expressed as:

$$\frac{\partial g_{2l}^{FB}}{\partial \theta} = -2\frac{(5a_f - 4a_l + cG_0 + 3cr - z)(31\theta^2 - 40\theta + 13)}{c(3-5\theta)^2(5-7\theta)^2} < 0. \quad (43)$$

Condition C' or Condition C" in Condition 2.1 and Condition 2.2, respectively, ensures that the derivative in (43) is always negative.

Furthermore, the effects of a change in θ on the follower's optimal strategies are given by

$$\frac{\partial g_{1f}^{FB}}{\partial \theta} = 2\frac{(2-3\theta)(1-\theta)}{31\theta^2 - 40\theta + 13}\frac{\partial g_{2l}^{FB}}{\partial \theta} < 0, \qquad \frac{\partial g_{2f}^{FB}}{\partial \theta} = -\frac{1}{2}\frac{(37\theta^2 - 50\theta + 17)}{31\theta^2 - 40\theta + 13}\frac{\partial g_{2l}^{FB}}{\partial \theta} > 0. \quad (44)$$

Condition (43) and $0 \leq \theta \leq 1/2$ imply the sign of the derivatives in (44).

As in the case of open-loop strategies, the effect of an increment in θ on the follower's extraction over the two periods is positive

$$\frac{\partial g_{1f}^{FB}}{\partial \theta} + \frac{\partial g_{2f}^{FB}}{\partial \theta} = -\frac{(3-5\theta)^2}{2(31\theta^2 - 40\theta + 13)}\frac{\partial g_{2l}^{FB}}{\partial \theta} > 0,$$

while the effect on the total extraction in the second period is negative

$$\frac{\partial g_{2f}^{FB}}{\partial \theta} + \frac{\partial g_{2l}^{FB}}{\partial \theta} = \frac{(3-5\theta)^2}{2(31\theta^2 - 40\theta + 13)}\frac{\partial g_{2l}^{FB}}{\partial \theta} < 0.$$

As a consequence, both the state of the aquifer at the end of the first and the second periods increase with an increment in θ:

$$\begin{aligned}\frac{\partial G_1^{FB}}{\partial\theta} &= -2\frac{(2-3\theta)(1-\theta)}{31\theta^2-40\theta+13}\frac{\partial g_{2l}^{FB}}{\partial\theta} = -\frac{\partial g_{1f}^{FB}}{\partial\theta} > 0,\\ \frac{\partial G_2^{FB}}{\partial\theta} &= -\frac{1}{2}\frac{(37\theta^2-50\theta+17)}{31\theta^2-40\theta+13}\frac{\partial g_{2l}^{FB}}{\partial\theta} = \frac{\partial g_{2f}^{FB}}{\partial\theta} > 0.\end{aligned}$$

Any increment of θ unequivocally leads to greater optimal profits for the follower:

$$\frac{\partial F_f^{FB}}{\partial\theta} = -\frac{(2-3\theta)(1-\theta)}{31\theta^2-40\theta+13}\frac{\partial g_{2l}^{FB}}{\partial\theta} > 0.$$

D Open-loop vs. Feedback Stackelberg equilibria

In this section, all proofs have been performed under Condition 1, which is less restrictive than Conditions 2.1 and 2.2. Therefore the proofs remain valid under Conditions 2.1 or 2.2, which ensure the comparison between the different equilibria.

D.1 Proof of Proposition 3

$$g_{1f}^{FB}-g_{1f}^{OL} = \frac{(\theta-1)(a_f(\theta(95\theta-109)+30)-3a_l(\theta(23\theta-26)+7)+(2\theta-1)(13\theta-9)(cG_0+3cr-z))}{6c(3\theta-2)(5\theta-3)(7\theta-5)}. \quad (45)$$

Under condition $0 \leq \theta \leq 1/2$ the denominator is negative, and hence, the sign of $g_{1f}^{FB}-g_{1f}^{OL}$ is the opposite to the sign of the numerator. Because $\theta-1<0$, then the sign of $g_{1f}^{FB}-g_{1f}^{OL}$ coincides with the sign of the following expression

$$a_f(\theta(95\theta-109)+30)-3a_l(\theta(23\theta-26)+7)+(2\theta-1)(13\theta-9)(cG_0+3cr-z).$$

Last expression can be rewritten as a quadratic polynomial in variable θ as follows:

$$A_2\theta^2+A_1\theta+A_0,$$

where

$$\begin{aligned}A_2 &= 95a_f-69a_l+26(cG_0+3cr-z),\\ A_1 &= -109a_f+78a_l-31(cG_0+3cr-z),\\ A_0 &= 3(10a_f-7a_l+3(cG_0+3cr-z)).\end{aligned}$$

It can be easily proved that Condition 1 implies $A_2 > 0, A_1 < 0$ and $A_0 > 0$. Therefore, the sign of $g_{1f}^{FB} - g_{1f}^{OL}$ is identical to the sign of a quadratic polynomial in variable θ with positive second-order and independent terms and negative first-order term. If $A_1^2 - 4A_0A_2 < 0$, then $g_{1f}^{FB} - g_{1f}^{OL} > 0$. If $A_1^2 - 4A_0A_2 = 0$, then the vertex of the parabola under Condition 1 is greater than $1/2$, and hence, $g_{1f}^{FB} - g_{1f}^{OL} > 0$ for any $\theta \in [0, 1/2]$. If $A_1^2 - 4A_0A_2 > 0$, then the equation $A_2\theta^2 + A_1\theta + A_0 = 0$ has two real positive roots. The smallest root can be proved to be greater than $1/2$ under Condition 1, and therefore, $g_{1f}^{FB} - g_{1f}^{OL} > 0$ for any $\theta \in [0, 1/2]$.

$$g_{2f}^{FB} - g_{2f}^{OL} = \frac{(\theta-1)\left(3(2-3\theta)^2(5a_f-4a_l+cG_0+3cr-z)-(5\theta-3)(7\theta-5)(4a_f-3a_l+cG_0+3cr-z)\right)}{3c(3\theta-2)(5\theta-3)(7\theta-5)}. \tag{46}$$

As before, under condition $0 \le \theta \le 1/2$, the sign of the difference $g_{2f}^{FB} - g_{2f}^{OL}$ coincides with the sign of the following expression

$$3(2-3\theta)^2(5a_f-4a_l+cG_0+3cr-z)-(5\theta-3)(7\theta-5)(4a_f-3a_l+cG_0+3cr-z).$$

Last expression can be rewritten as a quadratic polynomial in variable θ as follows:

$$B_2\theta^2 + B_1\theta + B_0,$$

where

$$\begin{aligned} B_2 &= -5a_f - 3a_l + 8(-cG_0 - 3cr + z), \\ B_1 &= 2(2a_f + 3a_l + 5cG_0 + 15cr - 5z), \\ B_0 &= -3(a_l + c(G_0 + 3r) - z). \end{aligned}$$

Conditions 1.A and 1.B imply $B_2 < 0, B_1 > 0$ and $B_0 < 0$. Therefore, the sign of $g_{2f}^{FB} - g_{2f}^{OL}$ is identical to the sign of a quadratic polynomial in variable θ with negative second-order and independent terms and positive first-order term. Furthermore, $B_1^2 - 4B_2B_0 = (a_f + cG_0 + 3cr - z)(4a_f - 3a_l + cG_0 + 3cr - z) > 0$, and hence, the inverted U-shaped parabola $B_2\theta^2 + B_1\theta + B_0$ cuts the horizontal axis in two points. The smallest root can be proved to be greater than $1/2$ under Conditions 1.A and 1.B, and therefore, $g_{2f}^{FB} - g_{2f}^{OL} < 0$ for any $\theta \in [0, 1/2]$.

$$g_{2l}^{FB} - g_{2l}^{OL} = \frac{(\theta-1)(a_f((31-25\theta)\theta-10)+a_l(\theta(27\theta-34)+11)+(\theta-1)(2\theta-1)(cG_0+3cr-z))}{2c(3\theta-2)(5\theta-3)(7\theta-5)}. \tag{47}$$

Under condition $0 \leq \theta \leq 1/2$, the sign of the difference $g_{2l}^{FB} - g_{2l}^{OL}$ coincides with the sign of the following expression

$$a_f((31-25\theta)\theta-10)+a_l(\theta(27\theta-34)+11)+(\theta-1)(2\theta-1)(cG_0+3cr-z).$$

Last expression can be rewritten as a quadratic polynomial in variable θ as follows:

$$C_2\theta^2 + C_1\theta + C_0,$$

where

$$\begin{aligned} C_2 &= -25a_f + 27a_l + 2cG_0 + 6cr - 2z, \\ C_1 &= 31a_f - 34a_l - 3cG_0 - 9cr + 3z, \\ C_0 &= -10a_f + 11a_l + cG_0 + 3cr - z. \end{aligned}$$

Conditions 1.A and 1.B imply $C_2 > 0, C_1 < 0$ and $C_0 > 0$. Repeating the same reasoning as below in the analysis of the sign of the difference $g_{1f}^{FB} - g_{1f}^{OL}$ and showing that either the smallest root or the vertex of the parabola is greater than 1/2 under Conditions 1.A and 1.B, we can conclude that $g_{2l}^{FB} - g_{2l}^{OL} > 0$ for any $\theta \in [0, 1/2]$.

D.2 Study of d^{OL} and d^{FB} and proof of Proposition 4

D.2.1 Possible case studies

For the open-loop case:

- If $a_l - a_f > 2cr/3$, then $d^{OL} > 0$ for all $\theta \in [0, 1/2]$.
- If $3a_l - 2a_f + cG_0 - cr - z < 0$ (this last condition implies $a_l - a_f < 2cr/3$), then $d^{OL} < 0$ for all $\theta \in [0, 1/2]$.
- If $3a_l - 2a_f + cG_0 - cr - z > 0$ and $a_l - a_f < 2cr/3$, then

  * $d^{OL} > 0$ for all $\theta \in [0, \tilde{\theta}^{OL})$.

  ** $d^{OL} \leq 0$ for all $\theta \in [\tilde{\theta}^{OL}, 1/2]$, where $\tilde{\theta}^{OL} = \frac{3a_l - 2a_f + cG_0 - z - cr}{3a_l - a_f + 2cG_0 - 2z}$.

For the feedback case:

- If $a_l - a_f > cr/2$, then $d^{FB} > 0$ for all $\theta \in [0, 1/2]$.
- If $a_l - a_f < cr/2$, then

* $d^{FB} > 0$ for all $\theta \in [0, \tilde{\theta}^{FB})$.

** $d^{FB} \leq 0$ for all $\theta \in [\tilde{\theta}^{FB}, 1/2]$, where $\tilde{\theta}^{FB} = \frac{2a_l - a_f - z + cG_0}{2a_l + 2cG_0 + cr - 2z}$.

Proof.

$$d^{OL} = g_{1f}^{OL} - g_{2f}^{OL} = \frac{-\theta(a_f - 3a_l - 2cG_0 + 2z) + 2a_f - 3a_l - cG_0 + cr + z}{2c(3\theta - 2)}. \tag{48}$$

The partial derivative of d^{OL} with respect to θ reads:

$$\frac{\partial d^{OL}}{\partial \theta} = \frac{-4a_f + 3a_l - cG_0 - 3cr + z}{2c(3\theta - 2)^2},$$

and is negative under Condition 1. Therefore, if d^{OL} is positive for $\theta = 1/2$, then is positive too for any $\theta \in [0, 1/2]$.

$$d^{OL} \mid_{\theta=1/2} = \frac{3(a_l - a_f) - 2cr}{2c} > 0 \iff a_l - a_f > \frac{2cr}{3}. \tag{49}$$

Consequently, if $a_l - a_f > 2cr/3$, then $d^{OL} > 0$ for $\theta \in [0, 1/2]$. Moreover,

$$d^{OL} \mid_{\theta=0} = -\frac{2a_f - 3a_l - cG_0 + cr + z}{4c} < 0 \iff 2a_f - 3a_l - cG_0 + cr + z > 0. \tag{50}$$

Consequently, if $-2a_f + 3a_l + cG_0 - cr - z < 0$, then $d^{OL} < 0$ for $\theta \in [0, 1/2]$. Note that this condition implies $a_l - a_f < 2cr/3$.
In the last situation, i.e., $-2a_f + 3a_l + cG_0 - cr - z > 0$ and $a_l - a_f < 2cr/3$, there exists $0 < \tilde{\theta}^{OL} = \frac{3a_l - 2a_f + cG_0 - z - cr}{3a_l - a_f + 2cG_0 - 2z} < 1/2$, where the sign of d^{OL} changes from positive to negative.

For the feedback case:

$$d^{FB} = g_{1f}^{FB} - g_{2f}^{FB} = \frac{a_f - 2a_l - cG_0 + z + \theta(2a_l + c(2G_0 + r) - 2z)}{c(5\theta - 3)}. \tag{51}$$

The partial derivative of d^{FB} with respect to θ reads:

$$\frac{\partial d^{FB}}{\partial \theta} = \frac{-5a_f + 4a_l - cG_0 - 3cr + z}{c(3 - 5\theta)^2},$$

and is negative under Condition 1. Therefore, if d^{FB} is positive for $\theta = 1/2$, then is positive too for any $\theta \in [0, 1/2]$.

$$d^{FB} \mid_{\theta=1/2} = \frac{2(a_l - a_f) - cr}{c} > 0 \iff a_l - a_f > \frac{cr}{2}. \tag{52}$$

Consequently, if $a_l - a_f > cr/2$, then $d^{FB} > 0$ for $\theta \in [0, 1/2]$. Moreover

$$d^{FB}\mid_{\theta=0} > 0, \tag{53}$$

by Condition 1. This implies that if $a_l - a_f < cr/2$ there exists $0 < \tilde{\theta}^{FB} = \frac{2a_l - a_f - z + cG_0}{2a_l + 2cG_0 + cr - 2z} < 1/2$, where the sign of d^{FB} changes from positive to negative. ■

From the previous analysis, taking into account the possible signs of d^{OL} and d^{FB} just three cases are possible. These cases are summarized below:

- If $a_l - a_f > 2cr/3$:

  Case 1: $d^{OL} > 0, d^{FB} > 0$.

- If $a_l - a_f < 2cr/3$:

  Case 2: $d^{OL} < 0, d^{FB} < 0$.

  Case 3: $d^{OL} < 0, d^{FB} > 0$.

Indeed, the case $d^{OL} > 0$ and $d^{FB} < 0$ is unfeasible. In fact, $d^{FB} < 0$ requires conditions $a_l - a_f < cr/2$ and $\theta \in [\tilde{\theta}^{FB}, 1/2]$. If $a_l - a_f < cr/2$, $d^{OL} > 0$ requires $\theta \in [0, \tilde{\theta}^{OL}]$. But, under condition $a_l - a_f < cr/2$, we have $\tilde{\theta}^{FB} > \tilde{\theta}^{OL}$ (more precisely $a_l - a_f < cr$ if and only if $\tilde{\theta}^{FB} > \tilde{\theta}^{OL}$).

D.2.2 Proof of Proposition 4

- When $d^{OL} > 0$ and $d^{FB} > 0$ (**Case 1**),

  $$d = d^{FB} - d^{OL} = \frac{(\theta-1)(\theta(5a_f - 3a_l + 2cG_0 + 6cr - 2z) - 2a_f + a_l - cG_0 - 3cr + z)}{2c(3\theta - 2)(5\theta - 3)}. \tag{54}$$

  For $\theta \in [0, 1/2]$, the sign of $d^{FB} - d^{OL}$ is the opposite to the sign of the following first-order polynomial in θ, $\theta(5a_f - 3a_l + 2cG_0 + 6cr - 2z) - 2a_f + a_l - cG_0 - 3cr + z)$. Under Condition 1 it can be easily shown that the independent term is negative and the linear term is positive. Furthermore, under this condition the value of θ for which this polynomial is null is greater than $1/2$. Therefore, for any $\theta \in [0, 1/2]$, the polynomial takes negative values, and hence, $d^{FB} - d^{OL}$ is positive.

- When $d^{OL} < 0$ and $d^{FB} < 0$ (**Case 2**), $d = -(d^{FB} - d^{OL})$. Therefore, following the previous reasoning, the opposite result is obtained and $d < 0$.

- The case $d^{OL} < 0$ and $d^{FB} > 0$ (**Case 3**) is more difficult to analyze in detail. First, we prove that the sign of d can change. As described in Appendix D.2.1, Case 3 requires $a_l - a_f < 2cr/3$, implying $\tilde{\theta}^{FB} > \tilde{\theta}^{OL}$. Next we prove that $d\mid_{\theta=\tilde{\theta}^{FB}}$ and $d\mid_{\theta=\tilde{\theta}^{OL}}$ have different signs.

$$d\mid_{\theta=\tilde{\theta}^{FB}} = \frac{(a_l - a_f - cr)(a_f - z + cG_0 + cr)}{2c(-2a_l - z + 3a_f + cG_0 + 2cr)}.$$

If $a_l - a_f < 2cr/3$, (note that the same sign is obtained in the more restrictive situation $a_l - a_f < cr/2$), then

$$-2a_l - z + 3a_f + cG_0 + 2cr > -2a_l - z + 3(a_l - \frac{2cr}{3}) + cG_0 + 2cr = a_l - z + cG_0 > 0.$$

Therefore, in this case, $d\mid_{\theta=\tilde{\theta}^{FB}} < 0$.
We also compute

$$d\mid_{\theta=\tilde{\theta}^{OL}} = -\frac{(a_l - a_f - cr)(a_f - z + cG_0 + cr)}{c(-6a_l - z + 7a_f + cG_0 + 5cr)}.$$

If $a_l - a_f < 2cr/3$, (note that the same sign is obtained in the more restrictive situation $a_l - a_f < cr/2$), then

$$-6a_l - z + 7a_f + cG_0 + 5cr > -6a_l - z + 7(a_l - \frac{2cr}{3}) + cG_0 + 5cr = a_l - z + cG_0 + \frac{cr}{3} > 0.$$

Therefore, in this case, $d\mid_{\theta=\tilde{\theta}^{OL}} > 0$. We conclude that there exists $\bar{\theta} \in (\tilde{\theta}^{OL}, \tilde{\theta}^{FB})$ with $d(\bar{\theta}) = 0$.
We can also prove that $d(\theta)$ is a quadratic polynomial of θ and that the coefficient in θ^2 is positive. This fact and taking into account that $d\mid_{\theta=\tilde{\theta}^{FB}} < 0$, $d\mid_{\theta=\tilde{\theta}^{OL}} > 0$ with $\tilde{\theta}^{FB} > \tilde{\theta}^{OL}$, we conclude that $d\mid_{\theta=0} > 0$. We also compute

$$d\mid_{\theta=1/2} = \frac{7}{2c}\left(-\frac{4}{7}cr + a_l - a_f\right).$$

If $a_l - a_f < 4cr/7$, we have $\bar{\theta} \in (\tilde{\theta}^{OL}, \tilde{\theta}^{FB})$ with $d(\bar{\theta}) = 0$ and the other root of this last equation is greater than $1/2$. Therefore, $d(\theta) > 0$ if $\theta \in (0, \bar{\theta})$ and $d(\theta) < 0$ if $\theta \in (\bar{\theta}, 1/2)$.
If $a_l - a_f > 4cr/7$, then the other root of $d(\theta) = 0$ is $\hat{\theta} \in (\tilde{\theta}^{FB}, 1/2)$. Then $d(\theta) > 0$ if $\theta \in (0, \bar{\theta}) \cup (\hat{\theta}, 1/2)$ and $d(\theta) < 0$ if $\theta \in (\bar{\theta}, \hat{\theta})$.

D.3 Proof of Proposition 5

The difference of the total extractions under the open-loop and feedback scenarios reads:

$$\text{Total}^{OL} - \text{Total}^{FB} = \frac{(1-\theta)(L_1\theta^2 + L_2\theta + L_3)}{3c(-2+3\theta)(-3+5\theta)(-5+7\theta)}, \tag{55}$$

where

$$\begin{aligned} L_1 &= 5a_f + 3a_l + 8(cG_0 + 3cr - z) > 0, \\ L_2 &= -2(2a_f + 3a_l + 5(cG_0 + 3cr - z)) < 0, \\ L_3 &= 3(a_l + cG_0 + 3cr - z) > 0. \end{aligned}$$

The sign of the expressions above come from Conditions 1.A and 1.B. Under condition $0 \leq \theta \leq 1/2$, the denominator of the expression in (55) is negative, and hence, the sign of the difference $\text{Total}^{OL} - \text{Total}^{FB}$ is the opposite to the sign of the numerator, that coincides with the sign of the quadratic polynomial $L_1\theta^2 + L_2\theta + L_3$. The expression $L_2^2 - 4L_1L_3$ can be simplified as follows:

$$L_2^2 - 4L_1L_3 = (a_f + cG_0 + 3cr - z)(4a_f - 3a_l + cG_0 + 3cr - z).$$

Condition 1.A states that the first factor is positive, while condition 1.C implies that the second factor is positive too. Therefore, the U-shaped parabola $L_1\theta^2 + L_2\theta + L_3$ cuts the horizontal axis at two positive values of θ denoted by $\underline{\theta}^T$ and $\bar{\theta}^T$, with $\underline{\theta}^T < \bar{\theta}^T$. On the one hand, it can be easily proved that $\underline{\theta}^T > 1/2$ if and only if $3(a_f - a_l)(5a_f + 3a_l + 8cG_0 + 24cr - 8z) < 0$. This last condition is fulfilled under conditions 1.A and 1.B.

D.4 Proof of Proposition 6

For the **case** $\boldsymbol{\theta = 1/2}$, the difference on the leader's profits in the open-loop and feedback cases is:

$$\begin{aligned} \Pi_l^{OL} - \Pi_l^{FB} = {} & \frac{4c(2G_0(a_f + a_l - 2z) + 3r(2a_l - a_f - z)) + 8(a_f - z)(a_l - z) - 13(a_f - a_l)^2}{72c} \\ & + \frac{4c^2 G_0(2G_0 + 3r)}{72c}, \end{aligned} \tag{56}$$

To prove that this last expression is positive, we evaluate this difference at $a_l = a_f$ and we check that it is positive. Then, we compute the derivative of the difference with respect to a_l and verify that it is also positive. Note that under Condition 1

$$\Pi_l^{OL} - \Pi_l^{FB} \mid_{a_l=af} = \frac{cG_0^2}{9} + \left(\frac{2}{9}a_f - \frac{2}{9}z + \frac{1}{6}cr\right) G_0 + \frac{1}{18c}(a_f - z)(3cr + 2a_f - 2z) > 0,$$

and

$$\begin{aligned}
\frac{\partial(\Pi_l^{OL} - \Pi_l^{FB})}{\partial a_l} &= \frac{1}{36c}(12cr + 4cG_0 + 17a_f - 4z - 13a_l) \\
&= \frac{1}{9c}(3cr + cG_0 + \frac{17}{4}a_f - z - \frac{13}{4}a_l) \\
&= \frac{1}{9c}((3cr + cG_0 - 4a_l + 2a_f + 2z) + (\frac{3}{4}a_f - 3z + \frac{9}{4}a_l)) > 0.
\end{aligned}$$

We conclude that

$$\Pi_l^{OL} - \Pi_l^{FB} > 0.$$

Now, the difference on the follower's profits in the open-loop and feedback cases is:

$$\Pi_f^{OL} - \Pi_f^{FB} = \frac{(a_l - a_f)(9a_f - 7a_l + 2cG_0 + 6cr - 2z)}{12c}.$$

Following the same reasoning as before this expression is greater than zero under Condition 1.

For the **case** $\theta = \mathbf{0}$, the difference on the leader's profits in the open-loop and feedback cases is:

$$\Pi_l^{OL} - \Pi_l^{FB} = \frac{-100a_f^2 + (60r + 20G_0)c + 220a_L - 20z)a_f + B}{1800c},$$

where

$$B = \frac{99\left(r + \frac{G_0}{3}\right)^2 c^2 - (66z - 6a_l)\left(r + \frac{G_0}{3}\right)c - 109a_L^2 - 2za_L + 11z^2}{1800c}.$$

The idea to prove that $\Pi_l^{OL} - \Pi_l^{FB} > 0$ is similar to the above reasoning. We can prove that

$$\frac{\partial(\Pi_l^{OL} - \Pi_l^{FB})}{\partial a_f} = \frac{1}{1800c}(20c(G_0 + 3r) + 220a_l - 200a_f - 20z) > 0,$$

that is $\Pi_l^{OL} - \Pi_l^{FB}$ increases with a_f. To complete the proof we must now verify that this difference is positive for $a_f = z$.

$$\Pi_l^{OL} - \Pi_l^{FB}|_{af=z} = f(a_l),$$

where $f(a_l)$ is a quadratic inverted-U function of a_l, with roots at a_{l1} and a_{l2} with $a_{l1} = z - c(G_0 + 3r)\frac{20\sqrt{3}-1}{109} < a_{l2} = z + c(G_0 + 3r)\frac{20\sqrt{3}+1}{109}$. When $a_f = z$ we know by Condition 1.C that $a_l < \frac{c(G_0+3r)}{4} + z < a_l^2$. This prove that for a_l in its feasible interval, $z < a_l < \frac{c(G_0+3r)}{4}$, one has $f(a_l) > 0$.
The same kind of reasoning shows that $\Pi_f^{OL} - \Pi_f^{FB} > 0$.

Next, denoting by Π_{1f}, Π_{2f} the follower's profit in the first and the second period respectively, for $\theta = 1/2$, we obtain

$$\begin{aligned}\Pi_{2f}^{OL} - \Pi_{2f}^{FB} &= -4(\Pi_{1f}^{OL} - \Pi_{1f}^{FB}) = \frac{(a_l - a_f)(2c(G_0 + 3r) - 7a_l + 9a_f - 2z)}{9c} \\ &= \frac{(2c(G_0 + 3r) - 8a_l + 4a_f + 2z) + (a_l + 5a_f - 4z)}{9c} > 0\end{aligned}$$

by Condition 1, then $\Pi_{1f}^{OL} - \Pi_{1f}^{FB} < 0$.
For $\theta = 0$ the same condition implies

$$\begin{aligned}\Pi_{2f}^{OL} - \Pi_{2f}^{FB} &= \frac{(c(G_0 + 3r) + a_L - z)(9c(G_0 + 3r) - 31a_l + 40a_f - 9z)}{900c} > 0, \\ \Pi_{1f}^{OL} - \Pi_{1f}^{FB} &= -\frac{(3c(G_0 + 3r) - 7a_L + 10a_f - 3z)(7c(G_0 + 3r) - 23a_l + 30a_f - 7z)}{3600c} < 0.\end{aligned}$$

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