Master in Actuarial and Financial Sciences

MASTER THESIS

Title: Segmentation applied to the Bonus-Malus System through the Norberg's formula.

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Abstract

The aim of this research is to study the implementation of segmentation in bonus-malus systems that base its analysis in the random variable number of claims. To this end, the theoretical framework proposed by Norberg for the construction of this *a posteriori* pricing method, for both not segmented and segmented case, is presented. This formulation has allowed the computation of the relative pure premiums and the premiums when unitary claim amount is assumed under different examples and scenarios. Then the different paths of premiums for certain insureds from simulated datasets are assessed. It is also evaluated this framework to surcharge premiums.

Keywords: Bonus-malus system, segmentation, Norberg's formula, a *posteriori* pricing, pure premium, motor insurance.

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1. Introduction

An insurance contract is the specific product that allows the coverage of risks. It allows the transformation of the uncertain events that could happen to the goods or persons into probabilities and expected values that could be hold by a firm (Antonio Alegre *et al.*, 2017). All insurance products require an insurance pricing system that ensures that the pure premium of the period, that is the quantity that the policyholder must pay for its risk, matches with the claim amounts that the insurer must satisfy. This premium, the pure premium, is not the same as the total premium, that is the one that the insured finally pay. To obtain the total premium it must be added to the pure premium the corresponding expenses, the taxes, the margin of benefit and other surcharges (Gómez, 2020). Indeed, according to the risk theory, insurance companies should always add a security surcharge to the pure premium to be solvent.

Moreover, in some insurance branch it may be important that the used pricing system has the capacity of classifying the policyholders into homogeneous risk groups. These insurance contracts are the ones that have the particularity that the insurer could access to some of the information that partly defines the policyholders when it is underwritten, remaining unknown other characteristics of the individuals. In case that an insurance company does *a priori* segmentation, it must ensure that all the policyholders with the same initial risk characteristics pay the same quantity at the beginning. Then, when the other part of the information that defines the insured is revealed, it is important that the pricing system could update the premium according to the risk that represents each individual.

The pure premium calculated through a specific pricing system must satisfy three principles, that are described in Boj *et al.* (2020). Firstly, the premiums must be *equitable*, which happens when the premium paid corresponds with the quantity of risk related to the individual. Also, the amounts paid by the insurers must allow the stability of the insurance company in the long run, what happens when the premiums are sufficient. That is, they must satisfy the *solvency* principle. The third one is the *solidarity* principle, which states that the less risky policyholders should pay a higher premium than the one that corresponds to their risk to compensate the lower premium paid by the insureds with higher risk. Nevertheless, the equity and solidarity principles could be contradictory. While the first one states that everyone must pay the quantity that reflects their risk, the second suggest that for an optimal distribution of the risk and the premiums, the insurers should not always pay exactly what corresponds to their risk.

For a non-life insurance company and specifically if it issues motor insurance contracts, it would be ideal to know the exact distribution of the claims and the associated risk parameter (δ) for each insured in the portfolio at every period. If this information were available, the company would be able to compute the exact premium that an insured should pay to make *equitable* premiums. Unfortunately, in practice it is not possible to obtain the exact value of it. Hence, usually insurers determine what is the theoretical distribution that best fits the number of claims of the policyholders, according to certain variables. When the insured holds a policy, these variables could be the age of the insured, his wealth, the job position… and allow the calculation of a pure premium for those policyholders for which no data about the number or severity of claims was gathered (Boj *et al.*, 2020). This gives rise to a *priori pricing*, under which the same premium is assigned to the insureds within a risk group, that is, those who have identical characteristics when they underwrite the insurance product.

However, the individuals allocated in a risk group generally have differentiable characteristics that cannot be known when they access the portfolio, or the observation of which could be very expensive. These specific characteristics of each individual modify the initially assumed probabilities of having an accident and its severity. Examples of these are the driving skills, driving habits, as the propension to over speeding, the consumptions of alcohol and drugs, etc. Hence, it exists heterogeneity between policyholders initially allocated in risk groups that were considered homogeneous at the beginning. Therefore, the price that the insured drivers must pay initially should be updated as the information about the number of accidents, the cost of it or other variables is reveled. This premium requires *a posteriori* pricing process for its computation, being this aspect the principal issue under study in this research.

1.1. *A priori pricing*

A current practice for the obtention of the *a priori* premium is to fit a distribution from the Generalized Linear Model (GLM) to detect the influence of the initial observable factors in explaining the level of *a priori* risk. That is, how the different values that these variables could take, determine the future number of claims and its severity. The GLM constitute an extension of the Linear Regression Models (LRM) where the conditioned dependent variable does not necessarily follow a normal distribution. The most common GLM distributions applicated in insurance *a priori* pricing are the Poisson and the negative binomial. However, some extensions of them as the quasi-Poisson, zero-inflated Poisson and zero-inflated negative binomial fits better the data in some contexts (Kleiber & Zeileis, 2008). The insurance company could have data about the annual number of claims within a similar portfolio as the one where the new policyholders would be included. Alternatively, it might be the case that the portfolio under study already exists before the new insureds entered and the data in it would be able. Hence, with either of these two data sources, the significant risk factors, and its influence in the level of risk of a new policyholder in the portfolio, can be identified. Once the significant risk factors are computed, they could be used to segment the sample of the new insureds into different risk groups.

Factor variables, also known as categorical variables, are those that can only take a finite number of values. Some of these variables as the type of vehicle or the age of the individual have as realization qualitative levels. If it is observed that there exist significant differences in the number of claims, depending on the class that exhibits the individuals in a specific factor variable, the dataset could be split. Thus, each policyholder should be classified in a different group depending on the value that it takes in the categorical variable. Moreover, if there exist more than one factor variable with significant difference across their categories, the interaction of the different factor that can take each of these variables, define the risk groups. Then, the insurance company could determine the expected claim frequency of a risk group, λ_k, as the mean of the risk groups obtained from the experience. This λ_k could be used as the *a priori* pure premium that the insureds must pay in each class.

1.2. *A posteriori pricing*

In *a posteriori* pricing system, the initial premium obtained *a priori* is updated periodically considering the claim experience of the individuals, of the portfolio or a combination of both sources of data. Following Boj *et al.* (2020) some of the models that face with this requirement are:

- Return of benefits for low claim frequency.
- Limited fluctuation credibility theory.
- Maximum precision credibility theory:
  - Exact credibility
  - Credibility of free distribution
- Markovian Bonus-Malus System (BMS).

The most widely used methodology in motor insurance pricing is the last one. In essence, a BMS is a system of classes or levels, that considering the Markov theory updates the premiums when the information about the *a priori* unknown risk factors is revealed. Indeed, the aim of this research is to assess the impact in the *a posteriori* premiums of taking into account or not the segmentation, initially performed to obtain the *a priori* premiums, in the construction of the BMS. This supposes a greater exploitation of the information that the insurance company has *a priori*. Moreover, the consideration of the existence of multiple risk groups within a portfolio is performed in some credibility methods. Therefore, the developed methodology in this project could also be useful for the comparison of these two types of *a posteriori* pricing systems.

The obtention of a segmented BMS could be carried out under the framework purposed by Norberg (1976). This author proposes a methodology that allows the introduction of segmentation in this *a posteriori* ratemaking to obtain the Relative Pure Premium (RPP), associated with each level of the scale (r_i), that accomplish a condition of optimality. Denuit & Charpentier (2009) developed the procedure to obtain these RPPs, both considering and not segmentation, through conditional probabilities and the Bayes rule. Moreover, he demonstrates that the developed methodology leads to the same results as the Norberg's formula.

This thesis is structured in 7 sections. After the introductory explanation of the project, Section 2, provides a brief introduction to the BMS with the principal characteristics of the methodology. In Section 3, the hypothesis and the procedure for the obtention of the conditional probabilities and the Norberg's formula is explained for the not segmented case. The same is done Section 4 but for the segmented case. The RPPs obtained under different assumptions, through both the segmented and not segmented case, are compered in Section 5. In Section 6, the segmented BMSs are implemented to a simulated dataset composed by the number of claims of random individuals. These results are compared with those obtained when segmentation is not considered. Finally, in Section 7, the conclusions, some limitations of the analysis and future lines of research are provided.

2. Brief explanation of the Markovian Bonus-Malus System

According to Lemaire (1995) a BMS is a mechanism in which the insurers who do not report any claim during the year are regarded with a reduction in the premium to pay, while those that declare accidents are penalized with an increase in the amount to pay to be covered in the next period. Thus, following the *equitable* principle, a BMS must achieve the objective of making each insurer pay the amount that correspond with their risk (Gil *et al.*, 2003). The BMS is a flexible *a posteriori* pricing system, that could present a variety of different structures and different hypothesis and methodologies could be applied for its construction.

2.1. Notation and assumptions

Once presented the context in which the BMS is used for and its basic working mechanism, in this section it is going to be identified the principal assumptions and notation that will be used in the thesis. Following Gil *et al.* (2003), let $\mathit{\Delta}$ be a random variable called structural variable and δ the possible values that could take $\mathit{\Delta}$. Hence, δ is considered the risk parameter of a randomly taken insured in the portfolio. The distribution of the random variable $\mathit{\Delta}$ is designed as the structure function, $U(\Delta)$.

Let N_t denote the stochastic process number of claims of the policyholder in the different periods (normally years). The random variables that constitute N_t are considered independent and identically distributed (iid) between them. Thus, it could be denoted $N|\mathit{\Delta} = \delta$ the random variable number of claims per insured for a specific period. Also, the claim amount per insured $\{\mathbb{X}_i\}_{i=1}^{\infty}$ are assumed to be iid random variables, so we express them by simply $\mathbb{X}$. What is more, *N* and $\mathbb{X}$ are also independent of each other.

In practice the insurance companies do not consider the claim amount in the construction of a BMS. The main reason is the long period of time required, after the accident take place, to compute the cost of the claim. Therefore, insurance companies are used to only contemplate the number of claims, leaving the BMS that consider the claim amount to the theoretical field (Boj *et al.*, 2020). In this research, in accordance to the most extended practice in the industry, will be also evaluated the number of claims. Specifically, a BMS considers only the number of guilty claims since the accidents that are not fault of the driver do not define its risk.[1] This aspect gives rise to a widely covered dilemma by the actuarial literature called *Bonus Hunger*, for which the insureds have incentives to not report to the company the occurrence of a claim if its amount is relatively small (Park *et al.*, 2018).

The required conditions for the existence of a BMS are defined in Lemaire (1995) and these are:

- All the policies within a portfolio could be classified in a finite number of classes. Each insured is allocated to a unique level of the scale during a certain period (usually one year). Normally, the larger premiums are assigned to the higher numbered classes and the smaller to the lower ones.

[1] Whenever is mentioned in this thesis that an insured report or have a claim, it refers to the fact that the insured driver has notified to the insurance company a guilty claim.

- The class in which the policyholder stays define the premium that an insured must pay in a particular period. Thus, the worst drivers would be placed in the higher classes.

- Following the *Markovian Condition*, the class currently occupied by an insured depends only on the number of claims during the recent period and the class where he was allocated in the previous one.

A BMS is usually presented in a tabular form named "system table" that allows the easy visualization of all the components of the BMS. The premiums are sorted in it in descending order. Table 1 illustrates the structure of a BMS.

Table 1: Scheme of system table of a BMS

Class	Relative Pure Premiums	Classes after *n* claims			
i	r_i	0	1	2	3 or more
s	r_s				
⋮	⋮				
e	r_e				
⋮	⋮				
1	r_1				

Source: Own elaboration

The elements that define a BMS that are placed in the "system table" are:

- A quantity of classes s.

- The entry class, e. That is, the class where the new policyholders of the portfolio are initially assigned. Under the Norberg (1976) criteria the entry class is determined arbitrarily.

- A scale of RPPs r_i i=1, …, s where (r_1,…, r_s) correspond with the percentage of the Base Premium (BP) that the insureds in each class i must pay. This is the magnitude to compute with the framework under study.[2]

- The transition rules are the laws that determine the transition from one class, i, to a different one, j, after a period depending on the claims that are reported in the term. These transition rules are defined under third and further columns of the "system

[2] The system table sometimes presents the level of premiums, b_i, instead of the relative pure premiums, r_i. The b_i is obtained from the product of r_i and BP.

table” and are also contained in the transition matrix, M, which is a sxs square matrix. These rules sometimes could be represented with a simple law of the form - B/+M where -B is the reduction in the class number occupied by the insured in the next period if he communicates 0 claims in current season and +M the increment in the number of classes for each reported claim in the period[3]. Some examples of these BMS are -1/+1, -1/+2, -1/+3, etc. For instance, these kind of BMS are applied in Iran or Brazil. However, this does not happen in a classical BMS of United Kingdom or a BMS of Ireland (Boj *et al.*, 2020). Both, cannot be described in this form.

Table 2: United Kingdom classical BMS (e=7)

Class	Relative Pure Premiums	Classes after n claims			
i	r_i	0	1	2	3+
7	100	6	7	7	7
6	75	5	7	7	7
5	65	4	6	7	7
4	55	3	5	7	7
3	45	2	5	7	7
2	40	1	4	6	7
1	35	1	4	6	7

Source: Own elaboration from Lemaire (1995)

As it could be appreciated in Table 2, the transition rules of this United Kingdom classical BMS could not be described with a simple law. This is due to the fact that depending on the level of the scale that the individual occupies, the increase in the number of classes for reporting claims and the reduction in it for not declaring any accident will be different.

Hence, considering the nomenclature defined above a BMS could be expressed in short form as $BMS = (M, r_i, e)$ (Norberg, 1976).

2.2. Markov finite chains

The BMSs are constructed considering the Markov condition. A Markov Process can be defined as a stochastic process in which its future development is determined by its present state, rather than the history or the way that it was reached the current state (Lemaire, 1995). If this typology of process is applied in the context of the methodology under study, the class occupied by a policyholder for the next period does not depend directly on his historical claims and only considers the claims of the current period and the class of the recent period.

Furthermore, a Markov chain is said to be finite if, being $(X_t)_{t \in N}$ a process in discrete time, the set of states is a finite part $\mathbb{I}$ of $\mathbb{N}$ (Naturals numbers). That is, a finite Markov chain, $(X_t)_{t \in N}$, must satisfy the following expression:

[3] In some academic research it can be found that +M = Top. In this case it expresses that when a claim is reported during the period, the insured is assigned directly to the most penalized class.

$$Pr(X_{n+1} = i|X_0 = i_0, \dots, X_n = i_n) = Pr(X_{n+1} = i|\, X_n = i_n)$$

$$\text{for n} \in \mathbb{N}\ ;\ (i_0, i_1, \dots, i_n, i) \in \mathbb{I}^{n+2}.$$

This formula contains the transition probabilities, that could be defined as the probability that the chain is at state i after $n+1$ periods if at period n it was in the state i_n. In addition, if these probabilities are the same for all the periods $n = 1,2, \dots$ it is said that the Markov chain has stationary transition probabilities, noted as π_i (García, 2002), a concept that will be defined below. Thus, when the stationary condition is achieved gives rise to the expression

$$Pr(X_{n+1} = i|\, X_n = i_n) = \pi_i \qquad \forall n.$$

Moreover, if $Pr(X_{m+n} = j|\, X_m = i)$ is independent of m then $(X_t)_{t\in N}$ is said to be homogeneous. If this condition is met, the probability of being at the state j at period $m+n$ only depends on the among of time between m and $m+n$ and not on which is the current period (m).

Taking into account the Markov theory, it is going to be stated the most important concepts of the bonus-malus experience rating system:

- The probabilities P_{ij} of an insured being changed from one class i to another level of the scale j in one period are collected in a sxs transition matrix, $M = (P_{ij})_{ij\in\mathbb{I}}$.
- The initial probabilities are $\left(P_j^{(0)}\right)_{j\in\mathbb{I}}$. They are contained in a vector of dimension s, called vector of initial probabilities, $P^{(0)}$, where all positions contain a 0 except the one that corresponds with the entry class, that has a 1.

The transition matrix and the initial probabilities can be used to calculate the law of the process $(X_t)_{t\in N}$ in the following way:

$$Pr(X_0 = i_0, X_1 = i_1, \dots, X_n = i_n) = P_{i_0}^{(0)} \cdot P_{i_0,i_1} \cdot \dots \cdot P_{i_{n-1},i_n},$$

where P_{i_0,i_1} is the probability of being transferred from class i_0 to i_1 in one period.

- The probabilities of being moved from class i to class j in $m+n$ periods are obtained from

$$P_{ij}^{(m+n)} = \sum_{\forall k\in E} P_{ik}^m \cdot P_{kj}^n\,,$$

  where P_{ik}^m is the probability of being transferred from the class i to the k in m periods and P_{kj}^n contains the probability of being transferred from class k to class j in n periods. This expression is known as Chapman-Kolmogorov equation (Karush, 1961).

- The transition matrix in n periods is $M^{(n)} = M^n$.

- The probability of a policyholder being in the state j at period n could be derived from

$$P_j^{(n)} = \sum_{i\in E} P_i^{(0)} \cdot P_{ij}^{(n)} \quad for\ i = 1, \dots, s.$$

- Finally, the probability that the individual will be in the different classes after n periods are contained in a vector of dimension s that could be obtained from

$$P^{(n)} = M^T \cdot P^{(n-1)} = (M^n)^T \cdot P^{(0)}, \qquad (1)$$

  where the superscript T denote the transposed matrix.

Following Gil *et al.* (2003), the chain theory of Markov ensures that a BMS has a stationary distribution $(\pi_i)_{i=1,\dots s}$ if the chain $(X_t)_{t\in n}$ is ergodic. That is, when it is always possible to come to class j from a concrete state i after a finite number of periods n. Therefore, the BMSs under study will be finite, homogeneous and ergodic. Furthermore, the stationary distribution contains the probabilities, π_i, of being at each one of the levels of the system after a sufficiently large number of periods that ensure that the probabilities will not change for the next periods. The period in which the stationary distribution is achieved is the steady state. The idea of stationary probabilities is summarized in (2),

$$\pi_i = \lim_{n\to\infty} P_i^{(n)}, i = 1, \dots, s. \qquad (2)$$

The computation of the stationary probabilities could be executed in different ways:

- Directly applying the definition of stationary probabilities. That is, considering the expression $P^{(n)} = (M^n)^T \cdot P^{(0)}$ for n such that the probabilities remain constant. This leads to the calculation of the nth power of M and selecting the row that coincides with the number of the entry class.

- Let π be the vector of dimension s that contains the stationary probabilities π_i. Then, from (1) and (2),

$$\pi = M^T \cdot \pi. \quad (3)$$

  Therefore, π is the eigen vector of the M^T associated with the eigen value 1. Moreover, it is straightforward that $\sum_{i=1}^{s} \pi_i = 1$. Hence, the stationary probabilities in the vector π could be obtained from the following equation system with s unknown variables and $s + 1$ equations (Boj *et al.*, 2020),

$$\pi = M^T \cdot \pi$$
$$1 = \varepsilon^T \cdot \pi,$$

  where ε is $sx1$ vector in which all its components are 1. Normally, the above equation system must be solved numerically.

- To avoid the calculation using numerical methods it is deduced from (3), the eigen vector of eigen value 1 of the transpose of the transition matrix. From it, is computed the real part standardized at sum one, that results in the stationary probabilities.

- Rolsky et. al. (1999) proposes $\pi^T = \varepsilon^t \cdot (\boldsymbol{I} - M + \boldsymbol{E})^{-1}$ as an explicit expression to obtain π. Where I is the identity matrix and $\boldsymbol{E}$ is an sxs matrix in which all its components are 1.

- Simulation could also be applied to approximate the stationary distribution. For its implementation, in first place it is assumed an entry class. Then, it must be simulated the number of claims from a large enough sample of insureds throughout a sufficient number of periods. Finally, it is obtained an approximation of the stationary distribution by simply calculating the proportion of policyholders in each class[4].This method only provides an approximation and requires more computational time.

3. Construction of a not segmented BMS

An insurer commonly does not know the value that will take the risk parameter of each policyholder, δ. This is because the observable and obtainable variables about the drivers, when the underwriting take place, do not collect all the risk of these insureds. Hence, for a risk collective that display the same results on the *a priori* observable variables, the total claim frequency could be defined by $\delta = \lambda\theta$. The λ coefficient collects the part of the risk of having an accident that could be defined through the information obtained *a priori*. For the moment, it is considered that this parameter is equal for all members of the *a priori* homogeneous collective. Nevertheless, this assumption will be modified in the following section and some aggrupation of the collective are going to be made. Furthermore, θ contains the specific risk of each insured that could not be detected when the policyholder enters in the portfolio. Under this research the risk parameters are going to consider only the risk of having a claim rather than its amount, in accordance with the current practice within the insurance sector.

When the expected value of a count variable as the number of claims is equal to its variance the Poisson distribution fits well the data. Indeed, Denuit & Charpentier (2009) assume that the random variable number of claims *N* is distributed as $MPoi(\lambda, \Theta)$[5] being Θ an unknown continuous random variable whose realizations are θ. Thus, it is going to be considered this hypothesis for the construction of all the BMSs under study in this research. Poisson regression assumes that the mean and variance coincide. However, in modeling the number of claims as well as in other contexts this equality is not always in evidence. When the variance is lower than the mean it is said that there is infradispersion in the regression and if it is larger exist overdispersion, being this last the most common situation. To deal with overdispersion alternative count data distributions might be applied as the negative binomial or the quasi-Poisson. However, the distributions of λ or Θ can be stated in a way that assume that there exist infradispersion or overdispersion in

[4] For the analysis performed in next sections for this simulation is considered 100,000 insureds and 50 periods.
[5] This is the abbreviation of the Poisson mixture distribution.

the data. Another aspect that can be considered is when the data has many zeros. In that case a zero-inflate Poisson or a zero-inflate negative binomial, if there is also overdispersion, might be the best option (Kleiber & Zeileis, 2008).

Taking into account the assumption performed by Denuit & Charpentier about the distribution of N, the probabilities defined in the previous section for the construction of the BMSs, can be derived. The conditional probability of being transferred from class i to class j in one period could be expressed as

$$P_{ij}(\lambda\Theta) = Pr(I_{n+1}=j|I_n=i,\, \lambda\Theta).$$

Where I_n contains the class where the policyholder is allocated after n terms. These probabilities could be easily derived from the probability density function (pdf) of a Poisson distribution.

With these probabilities the transition matrix M can be built

$$M = \boldsymbol{P}(\lambda\Theta) = \begin{pmatrix} P_{00}(\lambda\Theta) & \cdots & P_{0s}(\lambda\Theta) \\ \vdots & \ddots & \vdots \\ P_{s0}(\lambda\Theta) & \cdots & P_{ss}(\lambda\Theta) \end{pmatrix}.$$

Moreover, the conditional probability that an insured will be changed from class i to class j after m periods is given by

$$P_{ij}^{(m)}(\lambda\Theta) = Pr(I_{n+m}=j|I_n=i,\, \lambda\Theta).$$

And the conditional probability that this policyholder will be in class j after n terms now is

$$P_j^{(n)}(\lambda\Theta) = Pr(I_n=j|\, \lambda\Theta).$$

Therefore, the vector $P^{(n)}(\lambda\Theta)$, that contains the above probabilities $\left(P_j^{(n)}(\lambda\Theta)\right)_{j=1,\dots,s}$, can be defined. This vector can be computed from

$$P^{(n)}(\lambda\Theta) = \boldsymbol{P}^T(\lambda\Theta) \cdot P^{(n-1)}(\lambda\Theta) = (\boldsymbol{P}(\lambda\Theta)^n)^T \cdot P^{(0)}(\lambda\Theta).$$

Then, considering expression (2) under this framework, that is, $\pi_i(\lambda\Theta) = \lim_{n\to\infty} P_i^{(n)}(\lambda\Theta)$, in both sides of the former equation, the stationary probabilities could be derived.

$$\lim_{n\to\infty} P^{(n)}(\lambda\Theta) = \boldsymbol{P}^T(\lambda\Theta) \cdot \lim_{n\to\infty} P^{(n-1)}(\lambda\Theta)$$
$$\pi(\lambda\Theta) = \boldsymbol{P}^T(\lambda\Theta) \cdot \pi(\lambda\Theta).$$

Moreover, if it is denoted I the level occupied when the steady state has been reached, the conditional probability of being in a specific class i at that period is

$$Pr[I = i|\lambda\Theta] = \pi_i(\lambda\Theta) \quad for\ i = 1, \dots, s.$$

3.1. Norberg method in a not segmented case

The specific structure of a BMS could be due to many different reasons. The decision of choosing particular transition rules and the premium paid at each class could be made considering commercial reasons but also as consequence of finding the condition of optimality. In this sense, Norberg (1976) proposes a framework for the obtention of optimal BMSs in the steady state. He assumes that the number of claims declared by a random insured during a year is distributed as $MPoi(\lambda, \Theta)$, where $E[\Theta] = 1$. Then the RPP to pay in each class (r_i) must be computed as to minimize $Q = E[(\Theta - r_i)^2]$. This implies computing r_i, as the estimator of Θ that minimizes the least square error.[6]

Firstly, the expression of Q must be expanded

$$Q = \sum_{I=0}^{s} E[(\Theta - r_i)^2 | I = i] \cdot Pr[I = i] = \sum_{I=0}^{s} \int_{\theta>0} (\theta - r_i)^2 \cdot u(\theta|i)\, d\theta \cdot Pr[I = i] \quad (4)$$

where $u(\theta|i) = Pr[\Theta = \theta | I = i]$ is the pdf of Θ given that $I = i$. The $u(\theta|i)$ can be found applying the Bayes rule

$$u(\theta|i) = Pr[\Theta = \theta | I = i] = \frac{Pr\,[I=i|\Theta=\theta] \cdot Pr[\Theta=\theta]}{Pr[I=i]} = \frac{Pr\,[I=i|\Theta=\theta] \cdot u(\theta)}{Pr[I=i]}. \quad (5)$$

Thus, by substituting (5) in (4), Q is obtained

$$Q = \sum_{I=0}^{s} \int_{\theta>0} (\theta - r_i)^2 \cdot \frac{Pr[I = i|\Theta = \theta] \cdot u(\theta)}{Pr[I = i]}\, d\theta \cdot Pr[I = i] =$$
$$= \sum_{I=0}^{s} \int_{\theta>0} (\theta - r_i)^2 \cdot Pr[I = i|\Theta = \theta] \cdot u(\theta)\ d\theta$$
$$= \sum_{I=0}^{s} \int_{\theta>0} (\theta - r_i)^2 \cdot \pi_i(\lambda\theta) \cdot u(\theta)\ d\theta.$$

The RPP that minimizes Q must satisfy[7] $\frac{\partial Q}{\partial r_i} = 0$. Hence,

$$0 = \int_{\theta>0} (\theta - r_i) \cdot \pi_i(\lambda\theta) \cdot u(\theta)\ d\theta.$$

Finally, by isolating r_i, the optimal RPP that Norberg proposes for the not segmented case arises

$$r_i^* = \frac{\int_{\theta>0} \theta \cdot \pi_i(\lambda\theta) \cdot u(\theta)\ d\theta}{\int_{\theta>0} \pi_i(\lambda\theta) \cdot u(\theta)\ d\theta}. \quad (5)$$

[6] Remember that the random variable Θ define the part of the risk that remains unobservable when the policy is underwritten. This variable is continuous, however for the practical implementation of the framework under study it is going to be discretized in following sections.

[7] It is not necessary to compute the second derivative since the only optimal in a quadratic distribution, as the once under study, is a minimum.

It is straightforward to verify that $r_i^* = E[\Theta|I = i]$,

$$E[\Theta|I = i] = \int_{\theta>0} \theta \cdot u(\theta|i)\ d\theta = \int_{\theta>0} \theta \cdot \frac{Pr[I = i|\Theta = \theta] \cdot u(\theta)}{Pr[I = i]}\ d\theta$$
$$= \frac{\int_{\theta>0} \theta \cdot \pi_i(\lambda\theta) \cdot u(\theta)\ d\theta}{Pr[I = i]} = \frac{\int_{\theta>0} \theta \cdot \pi_i(\lambda\theta) \cdot u(\theta)\ d\theta}{\int_{\theta>0} \pi_i(\lambda\theta) \cdot u(\theta)\ d\theta}.$$

Therefore, by computing the expectation of Θ knowing that the policyholder is in class i, r_i^* is also obtained and coincides with the optimal premium level deduced from the Norberg's formula. Hence, this expected value provides the optimal premium.

An important quality of r_i^* is that it meets the property of financial equilibrium, ensuring that the collection remains constant over time (Denuit & Charpentier, 2009). Thus, the pure premiums that the insurance company obtains from each individual change over time, but the total amount collected from the policyholders, in terms of pure premium, remains constant,

$$E[r_i^*] = \sum_{i=0}^{s} r_i^* \cdot Pr[I = i] = \sum_{i=0}^{s} E[\Theta|I = i] \cdot Pr[I = i] = E\big[E[\Theta|I = i]\big] = E[\Theta] = 1.$$

4. Construction of a segmented BMS

In Section 3 it has been assumed that the number of claims of a randomly taken insured is distributed as $MPoi(\lambda, \Theta)$. In a BMS without segmentation it is considered that λ is the same for all the individuals within a portfolio. However, when the *a priori* data suggest that the individuals could be classified in different risk groups, it might be convenient to correspond a different value of λ to each group. These are λ_k and collect the part of the risk that could be deduced from the *a priori* information of the k_th risk group. These parameters are noted as the claim frequency of the risk group *k* deduced from the *a priori* information.

Therefore, now it is considered a portfolio that has been partitioned in k different risk sets based on the *a priori* information. The number of claims of an insured in the k_th risk group is now distributed as $MPoi(\lambda_k, \Theta)$. Hence, if a random new policyholder is selected, his annual expected claim frequency deduced *a priori* is now represented by a discrete random variable Λ, since the class from which this insured comes from is not known. Given that the different risk groups may not have the same importance in the portfolio, w_k is defined as the weight of the k_th risk group. Therefore, the value of w_k could correspond to the relative number of individuals in the group.

4.1. Norberg method in a segmented case

As it happens with the not segmented case, Norberg (1976) provides a formula that could be applied when different risk groups are computed *a priori*. If I is the class occupied by a random insured in the steady state of the segmented portfolio, then the probability mass function of I is given by

$$Pr[I=i]=\sum_{\forall k} w_k \cdot \int_{\theta>0} \pi_i(\lambda_k \theta) \cdot u(\theta)\ d\theta.$$

Again, by computing the mean square deviation between Θ and r_i, the optimal RPP could be obtained. Firstly, the expression of Q must be developed,

$$\begin{aligned} Q = E[(\Theta - r_i)^2] &= \sum_{I=0}^{s} E[(\Theta - r_i)^2 | I = i]\ \cdot Pr[I = i] \\ &= \sum_{I=0}^{s} \int_{\theta>0} (\theta - r_i)^2 \cdot u(\theta|i)\ d\theta \cdot Pr[I=i]\ d\theta \\ &= \sum_{I=0}^{s} \int_{\theta>0} (\theta - r_i)^2 \cdot Pr[I = i | \Theta = \theta] \cdot u(\theta)\ d\theta \\ &= \sum_{\forall k} w_k \int_{\theta>0} (\theta - r_i)^2 \cdot \pi_i(\lambda_k\theta) \cdot u(\theta)\ d\theta. \end{aligned}$$

Then, $\frac{\partial Q}{\partial r_i} = 0$ must be calculated to find the minimum

$$0 = \sum_{\forall k} w_k \int_{\theta>0} (\theta - r_i) \cdot \pi_i(\lambda_k\theta) \cdot u(\theta)\ d\theta,$$

isolating r_i from this expression it is obtained

$$r_i^* = \frac{\sum_{\forall k} w_k \int_{\theta>0} \theta \cdot \pi_i(\lambda_k\theta) \cdot u(\theta)\ d\theta}{\sum_{\forall k} w_k \int_{\theta>0} \pi_i(\lambda_k\theta) \cdot u(\theta)\ d\theta}. \qquad (6)$$

As it has happened with the not segmented case it is straightforward to show that r_i^* and $E[\Theta | I = i]$ are equivalent.

5. Comparison of the segmented and not segmented BMS under different assumptions

In this section the performance of the segmented and not segmented version of different BMSs will be compared. Each of these BMSs considers different scenarios and assumptions. Hence, it will be detected if there are substantial differences in considering or not the risk groups obtained *a priori* for the construction of the systems under different situations. Before performing this analysis, some notes about the concept of pure premium should be done for the correct interpretation of the results in this section and the next one.

According to *equivalence principle* (Promislow, 2010) also known as the *pure premium principle*, the pure premium should coincide with the expected value of the risk. Since the risk under study is the number of claims, it should be considered the expected number of claims. As stated before, the coefficients that collect the risk of having a claim are λ_k and θ. Therefore, their values could be considered to determine the pure premium that correspond to the individuals of each risk group. When the insured enters to the portfolio and the *a priori* pure premiums must be computed, it is possible to determine the values λ_k but not those of θ since they are the realization of the random variable Θ whose values are *a priori* unknown. However, at this instant, could be assumed or inferred a discrete structure function, whose values are θ_l, from which the expected value of Θ could be calculated. Therefore, the *a priori* pure premium could be computed as the product of λ_k and $E[\Theta]$. Moreover, the discrete nature of Θ, under this assumption, leads to a finite number of values that could take this random variable denoted θ_l.

Despite the *equivalence principle* is justifiable considering the law of the large numbers it is not coherent according to the risk theory. This is due to the fact that this theory verifies that the use of premiums that are not larger than the pure premium calculated in this way lead in the long term to bankrupt with probability 1. To avoid this situation, insurance companies usually consider a security surcharge applied to the pure premium.

To address these aspects, it is going to be considered the following:

- The most reasonable scenario of θ_l is the one that assumes "normal drivers" *a posteriori*. This means that the *a posteriori* behavior of the drivers is unknown, but it is assumed that in general their future number of claims will not differ much in mean from its *a priori* claim frequency and there will be the same probabilities of being a good or bad driver for the company. Hence, the *a priori* pure premiums are going to coincide with the λ_k since $E[\Theta] = 1$. The *a posteriori* pure premium will be the product of RPP (r_i) assuming normal drivers and the BP. [8]

- If it is desired to consider that the insured will exhibit a bad driving behavior and, therefore, its future expected number of claims will be larger than λ_k, the *a priori* pure premium would be the product of λ_k and $E[\Theta] > 1$. The *a posteriori* one will be result of multiplying the r_i assuming "bad drivers" by BP. Therefore, it is considered that the relative surcharge applied to the pure premiums *a priori* coincide with the difference between the average of Θ when it is considered "bad drivers" and the expected value of Θ under the assumption of normal drivers. Whilst the r_i under the "bad drivers" assumption will contain the relative surcharge considered *a posteriori*.

  It could be also considered that *a posteriori* the behavior of the drivers will be advantageous for the company. In this case the *a priori* pure premium would be the product of λ_k and $E[\Theta] < 1$ and the *a posteriori* one the result of multiplying the r_i assuming good drivers and BP. This could be used to consider a discount in the premiums. However, if this is applied, it would be difficult to justifiable since, according to the risk theory, this would lead to insolvency.

In this section will be derived the r_i for the different BMSs under the three assumptions about the future driving behavior of the policyholders. Furthermore, in Section 6 it is

[8] Since the BMS under study only considers the number of claims this quantity must be multiplied by the expected value of the claim amount, E[X]. This issue will be discussed further on.

going to be assessed the effect of considering a surcharge in the premiums to pay by the insureds from two simulated samples about the number of claims.

Moreover, since under the Norberg framework the RPPs are always computed considering the discrete structure function of Θ, it is considered not necessary that the BP depends on θ_l. Thus, for the different risk groups the BP coincide with the λ_k.[9]

5.1. Study of the impact of segmentation in a BMS with scale -1/+1

Firstly, it is considered a BMS that does not imply many levels of penalization per claim. That is, each claim is going to be penalized with an increase of one class. This BMS will be called "Light BMS" and its specific features are:

- The number of classes are $s = 6$.

- The transition rules of this BMS can be described by the scale -1/+1. Thus, for each year without reporting a guilty claim the insured will be regarded with a fall of 1 class, while for each reported accident it will be punished with an increase of 1 class in the scale. These transition rules are also placed in Table 3.

Table 3: System table of the "Light BMS"

Class	Classes after n claims					
i	0	1	2	3	4	5+
6	5	6	6	6	6	6
5	4	6	6	6	6	6
4	3	5	6	6	6	6
3	2	4	5	6	6	6
2	1	3	4	5	6	6
1	1	2	3	4	5	6

Source: Own elaboration

- The entrance class, that is the class in which all the policyholders are placed when they get into the portfolio, is $e = 5$. Thus, the vector of initial probabilities is obviously

$$P^{(0)}(\lambda\Theta) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

- Given these transition rules the transition matrix, M, is:

[9] This BP will be used in section 6 for the obtention of the pure premium in units of claim amount or when the claim amount is 1.

$$M = \begin{pmatrix} PN_0 & PN_1 & PN_2 & PN_3 & PN_4 & 1-\Sigma_1 \\ PN_0 & 0 & PN_1 & PN_2 & PN_3 & 1-\Sigma_2 \\ 0 & PN_0 & 0 & PN_1 & PN_2 & 1-\Sigma_3 \\ 0 & 0 & PN_0 & 0 & PN_1 & 1-\Sigma_4 \\ 0 & 0 & 0 & PN_0 & 0 & 1-\Sigma_5 \\ 0 & 0 & 0 & 0 & PN_0 & 1-\Sigma_6 \end{pmatrix},$$

where PN_n, $n = 0, \dots, 4$ are the probabilities of reporting *n* claims during one period and $\Sigma_s = \sum_{n=0}^{N} PN_n$ the sum of the elements of the s row. Since it is assumed that $N \sim MPoi(\lambda\Theta)$, the PN_n are obtained through the pdf of the Poisson distribution.

$$PN_n = P[N = n|\lambda\Theta] = \frac{(\lambda\Theta)^n}{n!} \cdot e^{-(\lambda\Theta)} \; for \; n = 1, \dots, 4,$$

being the parameter λ the same for all the individuals in the portfolio in the not segmented case, and the realization of the discrete random variable Λ in the segmented. Whether segmentation is or not considered, it is assumed that λ was obtained based on the information *a priori* able in the databases about claims that the insurance companies had.

With the purpose of applying the methodology purposed by Norberg (1976) in a practical context, the author suggests considering Θ as a discrete random variable, distributed according to a specific structure function that approximates adequately the car policies under study. With comparative purposes three different assumptions about this distribution are made. Although the values that Θ take under all the scenarios are the same, the probability that each realization takes place change depending on the assumed distribution. [10]

Table 4: Discrete structure function

Values of θ_l for all the scenarios						
θ_l	0.25	0.50	0.75	1.25	1.5	1.75
Probabilities under the equiprobable case ("Normal drivers")						
$Pr[\Theta = \theta_l]$	1/6	1/6	1/6	1/6	1/6	1/6
$E[\Theta] = 1$						
Higher probabilities for the large θ ("Bad drivers")						
$Pr[\Theta = \theta_l]$	1/12	1/12	1/12	1/4	1/4	1/4
$E[\Theta] = 1.25$						
Higher probabilities for the lower θ ("Good drivers")						
$\Pr[\Theta = \theta_l]$	1/4	1/4	1/4	1/12	1/12	1/12
$E[\Theta] = 0.75$						

Source: Own elaboration

[10] Norberg presents a specific discrete structure function according to the characteristics of the Norwegian private car insurance policies.

As it has been stated, θ_l represents the part of the risk of having a claim for a random policyholder that cannot be detected with the *a priori* information. Therefore, for each of these three assumptions, the different values of θ_l in the discrete structure function of Table 4, could be considered as the *a priori* unknown part of the claim frequency for a policyholder with specific driving skills and habits. That is, the higher values of θ_l correspond to the "worst drivers" and the lower once to the "best drivers" [11]. Thus, without changing the values of this coefficient, it has been assumed that the portfolio has better or worse drivers *a posteriori* by only changing the probabilities of each realization.

With this data and assuming that $\lambda = 0.645$, it is possible to compute the r_i in the not segmented case through the Norberg's formula. Since, it has been assumed that the *a posteriori* risk parameter, θ_l, is the realization of a discrete random variable, (5) now could be rewritten as:

$$r_i^* = \frac{\sum_{i=1}^{6}\sum_{l=1}^{6}\theta_l \cdot \pi_i(\lambda\theta_l) \cdot \Pr[\Theta = \theta_l]}{\sum_{i=1}^{6}\sum_{l=1}^{6}\pi_i(\lambda\theta_l) \cdot \Pr[\Theta = \theta_l]}.$$

Moreover, the discrete nature of Θ allows the obtention of the same result also through the procedure explained in Denuit & Charpentier (2009). Indeed, both, the Norberg's formula and the framework proposed by Denuit & Charpentier, have been applied in this research. The calculations were performed through the software R, which code is able in Annex 1. Whatever has been the proceed followed it results in the r_i given in Table 5.

Table 5: "Light BMS" - RPPs in the not segmented case under the three scenarios of the discrete structure function

Class	Good drivers	Normal drivers	Bad drivers
6	1.39	1.48	1.52
5	1.20	1.37	1.45
4	0.96	1.18	1.34
3	0.75	0.92	1.14
2	0.58	0.67	0.84
1	0.44	0.47	0.54

Source: Own elaboration

As it could be seen in Table 5 the lowest r_i for all classes are obtained under the "Good drivers" assumption, while the higher are in derived in the "Bad drivers" case. Thus, the pure premium for an insured in the entry class 5 would be 120% of the BP if he is considered a good driver, 137% if he is considered normal driver and 145% if considered bad driver. [12] These results are due to the fact that the presumption of high quantity of

[11] This notation considers not only the driving skills but also to the other unknown risk factors that affect the probability of having a claim that are not observable *a priori*. Hence, a "Bad driver" refers to that one that is less convenient for the company.

[12] Even though this is the not segmented case the values of the premiums in units of average claim amount, that is, when it is assumed that the claim amount is 1, depend on the *a priori* premium associated to the risk group of the insured. Hence, when different lambdas are assumed, a policyholder that is classified in risk group 4 and is in class 5 must pay 1.2·0.6 = 0.72·E[X] monetary units in the BMS that consider good behaved drivers.

drivers with more propension to have an accident *a posteriori* require higher premiums, while the opposite happens when the individuals under study are assumed less likely to report a claim. Furthermore, whatever the assumption about the behavior of the insureds is made the premiums increase with the classes. Hence, it can be stated that this BMS satisfy the *equitable* principle since larger premiums are obtained when it is assumed a sample of riskier driver *a posteriori* and when the insureds are placed in the most penalized levels, that are supposed to contain the riskiest drivers.

For the obtention of the r_i under the segmented case it is required the computation of the corresponding λ_k of each risk group. This segmentation is produced *a priori*, hence, as it has been stated in the introduction, a GLM model could be applied to an old representative sample to identify the influence of the *a priori* observable variables and determine with it the different risk groups for the new policyholders. Then, the risk parameter λ_k associated with group k could be computed as the sample mean of the number of claims reported by policyholders in group k of the old portfolio, that present the same categories in the variables as the individuals in the set k of new portfolio. Moreover, the same assumptions about the data and the variables as in the not segmented case are considered, but in this case the existence of different *a priori* risk sets is considered.

Since w_k is the weight or importance of the k_th risk group within the portfolio, it could be interpreted as the proportion individuals in group k in comparison to the total portfolio. With comparative purposes, two assumptions about the distribution of λ_k have been done. The supposed values for λ_k are placed in Table 6. The first assumption considers that the propensity of the risk groups to have an accident is quite different while the second reflects lower differences between the groups. In both cases the average weighted mean of λ_k coincides with the overall value of the claim frequency in the not segmented case, that was $\lambda = 0.645$. The w_k are the same under both assumptions.

Table 6: Assumed distributions of λ_k

Assumption 1: λ_k for different risk groups						
λ_k	0.2	0.3	0.4	0.6	0.7	0.9
w_k	0.1	0.1	0.1	0.15	0.15	0.4
Assumption 2: λ_k for similar risk groups						
λ_k	0.35	0.55	0.57	0.62	0.7	0.75
w_k	0.1	0.1	0.1	0.15	0.15	0.4

Source: Own elaboration

As in the not segmented case, once the information required for the computation of the r_i has been collected, the results could be obtained via the segmented version of the Norberg's formula, or the methodology explained by Denuit & Charpentier (2009). The code used in both cases is also able in Annex 1.

The segmented version of the Norberg's formula (6) could be adapted for discrete Θ

$$r_i^* = \frac{\sum_{k=1}^{6} w_k \sum_{i=1}^{6} \sum_{l=1}^{6} \theta_l \cdot \pi_i(\lambda_k \theta_l) \cdot \Pr[\Theta = \theta_l]}{\sum_{k=1}^{6} w_k \sum_{i=1}^{6} \sum_{l=1}^{6} \pi_i(\lambda_k \theta_l) \cdot \Pr[\Theta = \theta_l]}$$

In the segmented case, since the values of λ_k are different under the two assumptions, two sets of RPP's will be obtained, one for each assumed distribution of λ_k.

Table 7: "Light BMS" - RPPs in the segmented case under the three scenarios of the discrete structure function

	Assumption 1: Different λ_k			**Assumption 2: Similar λ_k**		
Class	**Good drivers**	**Normal drivers**	**Bad drivers**	**Good drivers**	**Normal drivers**	**Bad drivers**
6	1.30	1.44	1.50	1.37	1.47	1.51
5	1.10	1.30	1.42	1.17	1.35	1.44
4	0.91	1.13	1.32	0.95	1.17	1.34
3	0.76	0.96	1.20	0.75	0.94	1.17
2	0.63	0.80	1.05	0.59	0.71	0.92
1	0.50	0.62	0.83	0.45	0.50	0.63

Source: Own elaboration

As it happens in the not segmented case, the BMS in Table 7 accomplish the equitable principle. It is shown how under the assumption of worst behaved drivers; the premiums are higher and there is a monotonic increase of the premiums with the classes for all the scenarios.

Furthermore, if these results are compared with the ones obtained in the not segmented case, it could be seen that the r_i are always closer to 1 in the normal drivers' scenario and closer to 0.75 and 1.25 under the assumption of good and "bad drivers" respectively (see Annex 2). These results ensure that the pure premium that the insured should pay when segmentation is considered is going to be always closer to the estimated claim frequency that describes him, $\lambda_k \cdot E[\Theta]$. This aspect is going to be further discussed in Section 6. Moreover, it is obtained that the difference between RPPs and the claim frequency of the risk groups are smaller when different λ_k are assumed. This happens because the adjustment that produces segmentation under the Norberg framework is more accurate when there are larger differences among the risk groups.

5.2. Study of the impact of segmentation in a scale -1/+3

It is also interesting to evaluate what is the effect of segmentation under a BMS that penalizes with more classes when a claim is reported. Thus, same scenarios as above will be analyzed but for a -1/+3 BMS. This system will be denoted "Strict BMS" and it is characterized by:

- The number of classes is $s = 6$.

- The transition rules of this BMS now are described by the scale -1/+3. Thus, for each year without reporting a claim the insured is regarded with a fall of 1 class. However, if the insured reports a claim a penalization of 3 classes is applied. These transition rules are displayed by Table 8.

Table 8: Scheme of the system table of the "Strict BMS"

Class	**Classes after n claims**		
i	**0**	**1**	**2**
6	5	6	6
5	4	6	6
4	3	6	6
3	2	6	6
2	1	5	6
1	1	4	6

Source: Own elaboration

- The entrance class is also $e = 5$. Hence, the obtentions of the vector of initial probabilities is straightforward

$$P^{(0)}(\lambda\Theta) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

- Given the defined transition rules the transition matrix, M, is:

$$M = \begin{pmatrix} PN_0 & 0 & 0 & PN_1 & 0 & 1-\Sigma_1 \\ PN_0 & 0 & 0 & 0 & PN_1 & 1-\Sigma_2 \\ 0 & PN_0 & 0 & 0 & 0 & 1-\Sigma_3 \\ 0 & 0 & PN_0 & 0 & 0 & 1-\Sigma_4 \\ 0 & 0 & 0 & PN_0 & 0 & 1-\Sigma_5 \\ 0 & 0 & 0 & 0 & PN_0 & 1-\Sigma_6 \end{pmatrix}$$

The assumptions in this example are the same as in Subsection 5.1:

- $PN_n = P[N = n|\lambda\Theta] = \frac{(\lambda\Theta)^j}{j!} \cdot e^{-(\lambda\Theta)}\ for\ n = 1, \ldots, 2$
- The three scenarios in Table 4 will be considered for Θ.
- The same two assumptions in Table 6 are made about the values of λ_k for each risk group in the segmented case.

Therefore, if the Norberg's formulas or the formulation proposed by Denuit & Charpentier are applied, the results for the segmented and not segmented case are derived.

Table 9: “Strict BMS” - RPPs under the three scenarios of the discrete structure function

	Not Segmented			Assumption 1: Different λ_k			Assumption 2: Similar λ_k		
Class	Good	Normal	Bad	Good	Normal	Bad	Good	Normal	Bad
6	1.07	1.30	1.43	1.05	1.28	1.42	1.07	1.29	1.43
5	0.90	1.14	1.33	0.88	1.12	1.32	0.89	1.13	1.33
4	0.68	0.87	1.12	0.69	0.91	1.16	0.68	0.88	1.13
3	0.59	0.73	0.96	0.63	0.81	1.06	0.60	0.75	0.99
2	0.52	0.61	0.80	0.58	0.73	0.98	0.54	0.64	0.85
1	0.39	0.41	0.47	0.46	0.54	0.73	0.40	0.44	0.53

Source: Own elaboration

The figures in Table 9 display how under all the scenarios and assumptions and for all classes, the premiums in the “Strict BMS” are smaller than those obtained in the “Light BMS”, Tables 5 and 7. This is due to the fact that a BMS should ensure that each insured pays the quantity that corresponds to its risk. Therefore, if for each claim the BMS is penalized with more levels in the scale, it does not require such as larger RPPs.

Likewise, in this case and for all scenarios, as in the “Light BMS”, there exist a monotonic increase in the premiums with classes and the premiums charged to the insureds increase if the BMS is calculated under the assumption of worse drivers. It is straightforward, that again the scope of corrections that the segmented BMS does with respect the not segmented ones are smaller for lower differences on λ_k between risks groups. Thus, as in the -1/+1, the r_i are always closer to the expected value of $\lambda_k \cdot \mathrm{E}[\Theta]$ when the differences of the claim frequencies of the risk groups are larger.

6. Analysis of the paths of premiums of the policyholders in the not segmented and segmented cases

In Section 5, the different r_i; obtained under the “strict” and “light” BMS considering different scenarios about the information known *a priori*, collected by λ_k, and *a posteriori*, represented by θ_l; are compared.

In this section, the different premiums derived from the segmented and not segmented case are going to be assessed for some individuals in two simulated portfolios, according to the annual number of claims that they declare during a period of 5 years. That is, it will be assessed how the premiums to pay, in units of E[X], by some randomly taken insureds from different risk group change over the periods under different assumptions. Firstly, the paths of posteriori pure premiums obtained from the four different combinations of assumptions are going to be used but only under the assumption of normal drivers. [13] Then the differences premiums derived from the “normal drivers” and “bad drivers” scenarios

[13] The four combinations of assumptions and its reduced denotation in brackets are: Strict BMS and Different λ_k [SandD], Strict BMS and Similar λ_k [SandS], Light BMS and Different λ_k [LandD], Light BMS and Similar λ_k [LandS].

about θ_l will be compared. This last, is analyzed to provide a purpose of surcharge in the pure premium through the Norberg's framework.

6.1. Simulation of the sample

The two simulated samples correspond to the two assumptions about λ_k. Both contain the annual number of claims per insured for a period of 5 years, being the sample size of 10,000 policyholders. Moreover, the random values were obtained under the assumption about the distribution of the number of claims and the parameters proposed by Norberg. That is, the data of each risk group is randomly generated applying a Poisson distribution of parameter λ_k. This could be done in the software R via the function *rpois*(*n,lambda*). In the case under analysis *n* is the number of individuals in each risk set and results from the product of w_k by the 10,000 insureds of the total portfolio. On the other hand, *lambda* is the λ_k. That is,

$$rpois(10000w_k, \lambda_k) \qquad for\ k = 1, \dots, 6$$

Table 10: Parameters required for the simulation of the number of claims

Risk group k	λ_k Assumption 1	λ_k Assumption 2	w_k
1	0.9	0.7	0.4
2	0.7	0.6	0.15
3	0.6	0.55	0.15
4	0.4	0.45	0.1
5	0.3	0.4	0.1
6	0.2	0.3	0.1

Source: Own elaboration

Table 10 presents the values to consider in the simulations. It could be observed that the only thing that changes are the values of the parameter, being the same the weights. The simulated values and the code used for its obtention are included in Annex 1.

6.2. Obtention of the premium in units of average claim severity

Let S be the total risk of a driver, considering the number of claims and the amount of each one. Then, considering the aforementioned equivalence principle, the pure premium that an insured must pay in a period for its total risk, *S*, is $E[S]$. The independence between *N* and *X* and the fact that $\{\mathbb{X}_i\}_{i=1}^{\infty}$ are iid ensures that the $E[S]$ results from the product of the expected value of the number of claims, *E*[*N*], and the expected value of the claim amount, *E*[*X*] (Denuit & Charpentier, 2004).

Moreover, the RPP (r_i) in the BMSs under study, provide the percentage of the BP of each risk group ($BP_k\ for\ k = 1, \dots, 6$) that the insureds of a specific risk set must pay in the different classes, considering only his number of claims. That is, $E[N] = r_i \cdot BP_k$ $for\ i = 1, \dots, 6$ is a vector that contains the part of the pure premiums that corresponds to the number of claims, that an insured of risk group k must pay when occupies the different classes.

Therefore, to evaluate how the pure premium evolve over the years it is going to assume a unitary expected claim severity which enables the obtention of the pure premium in units of $E[X]$. Moreover, taking into account that the BP_k could be considered equal to λ_k, the pure premium under unitary $E[X]$ for an insured in risk group k in each BMS class, are

$$\textit{Pure pre.} = E[S] = E[N]{\cdot}E[X] = r_i \cdot BP_k{\cdot}\, E[X] = \{E[X] = 1;\ BP_k = \lambda_k\} = r_i \cdot \lambda_k.$$

Twelve BMSs are derived from the different combinations of assumptions about λ_k, the classes penalized and the scenarios of θ_l. Thus, in all BMS are obtained six vectors, each of which associated with one risk group. All these vectors will have six components that correspond to the premium to pay in each class (see Annex 2).

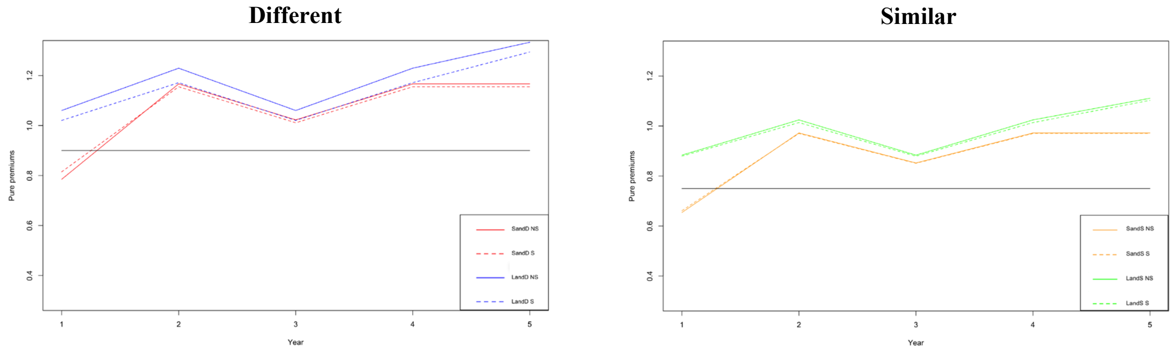
6.3. Comparison of the paths in the segmented and not segmented case

Once the simulation of the portfolio and the computation of the pure premiums in units of $E[X]$ has been done, the evolution of the premiums from some policyholders in the sample will be analyzed. Hence, the path of premiums described in the 5 simulated periods by policyholders from different risk groups, will be assessed to identify the different behaviors of the segmented and not segmented case.

Following the Markov theory, in which the BMS is based, the RPP that the policyholder must pay each year only depend on the number of claims of the current year and the class of the past year. Nevertheless, under the framework under study the pure premium that the insured finally pays depends on the risk group where he was initially allocated. Thus, the quantity that he finally pays in concept of pure premium under a specific BMS, is always taken from the vector of pure premiums in units of E[X] that corresponds to its risk group. Then, the class occupied by the insured in a specific year will determine the component of its associated vector that contains the premium to pay. For the moment, only the “normal drivers” scenario of θ_l is analyzed. Hence, the premiums under analysis are the pure premiums without surcharge.

Figure 1: Evolution of the premiums for a random insured of risk group 1(λ_k = 0.9 different or λ_k = 0.7 similar) when his claims are concordant with his *a priori* risk

	Year 1	Year 2	Year 3	Year 4	Year 5
Claims	0	1	0	1	2
Class strict	4	6	5	6	6
Class light	4	5	4	5	6



Source: Own elaboration

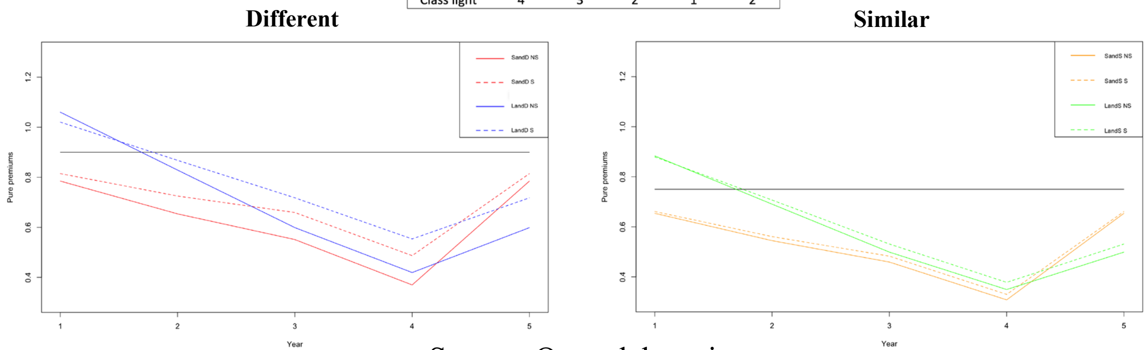
Figure 1 displays the premiums for unitary claim amount that an insured in risk group 1 must pay under the different assumptions about λ_k and the number of classes that are punished by each claim. Furthermore, the number of claims per year and the occupied class by this policyholder are placed above the graphs. The left-hand plot describes the evolution of the premiums when large differences in the values of λ_k are considered, whereas the graph on the right contains the premium to pay when similar λ_k are assumed. Since it has been obtained one random sample for each assumption of λ_k one insured of each simulated portfolio must be chosen. The individuals selected correspond with the observation 3,889 in the first sample and 3,972 in the second. Both have 4 claims in the five years period, that it is quite concordant with its *a priori* claim frequency. [14]What is more the claims are equally distributed over the years. However, the classes occupied by each policyholder differ in some periods depending on the BMS under analysis.

The graphs show that the *a posteriori* premiums are always closer to the ones *a priori* in the segmented case being the differences when segmentation is or not considered quite little, specially under similar *a priori* claim frequencies assumption. This also implies that the insureds in the higher levels of the scale, that are those who have more risk and for that a higher premium, pay less in the segmented case than in the not segmented. Furthermore, the premium to pay by the insureds in the less risky classes are lower in the segmented case. Therefore, this suggest that segmentation introduces some solidarity with respect the case in which segmentation is not considered.

Moreover, in this particular example the premiums are larger in the BMSs that punish with less levels. The differences between the "Strict" and the "Light" BMSs become smaller when the class of the policyholders decreases. Furthermore, the premiums are larger under the assumption about λ_k of more distinguishable risk groups, even in the segmented case.
The r_i in the not segmented case do not depend on the assumption about λ_k. However, when the premiums in units of $E[X]$ are derived, the BP that coincides with the λ_k of the group must be used. Thus, the premiums of this insured are lower when similar λ_k are considered, even in the segmented case, because the value of this coefficient associated with risk group 1 is lower under this assumption.

Figure 2: Evolution of the premiums for a random insured of risk group 1 ($\lambda_k = 0.9$ different $\lambda_k = 0.7$ similar) when his claims are not concordant with his *a priori* risk

	Year 1	Year 2	Year 3	Year 4	Year 5
Claims	0	0	0	0	1
Class strict	4	3	2	1	4
Class light	4	3	2	1	2



Source: Own elaboration

[14] It is straightforward that the mean number of claims in the period is 0.8, that is very close to the claim frequency of the risk group where the insured was allocated *a priori*. This were 0.9 and 0.75 under assumption 1 and 2 about λ_k respectively.

It is also relevant to analyze what occurs when the claims reported by an insured are not concordant with its *a priori* characteristics. Therefore, Figure 2 displays the paths with the *a posteriori* premiums that an insured in risk group 1 from each randomly obtained sample should pay when they have less claims than expected, according to the features observed *a priori*.[15] The observation number of the drivers taken in this case are 725 and 2,774 from sample 1 and 2 respectively.

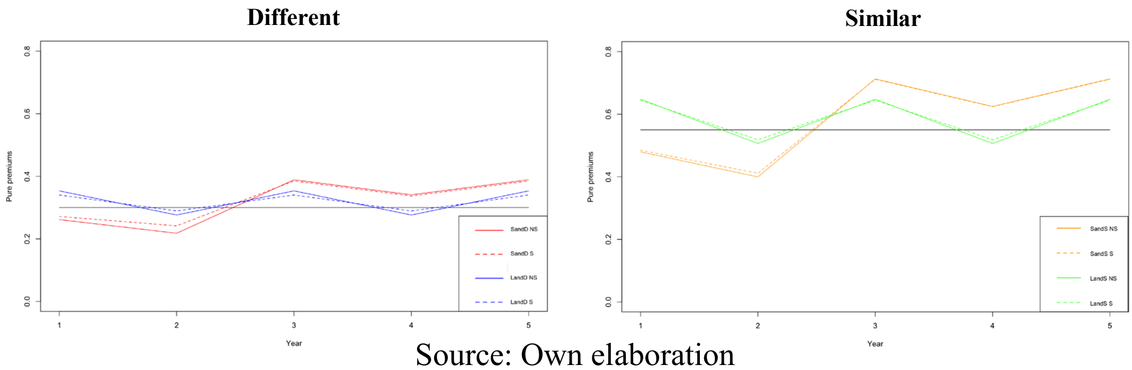
Since in this case the insured reports no claims in the first four years, all BMSs under study exhibits a decrease in the premiums as the levels occupied in the scale reduces. Then, the claim in year 5 make the premiums raise. Again, the property of computing closer premiums to the ones *a priori* is shown in this graph. However, now larger differences between the premiums obtained with and without segmentation arose. This happens when the classes in which the insured is allocated are not very concordant to its high claim frequency obtained *a priori.* This is due to the fact that the segmented case does not allow such a large decrease in the premium thanks to its capacity of still considering that the insured was classified with a high claim frequency *a priori*. For instance, in the BMS that considers larger punished classes and different λ_k (SandD), when the insured moves from class 4 to class 3 the bonification with respect the previous premium is 21.8% in the segmented case and 19.6% when segmentation is not considered.

This example shows how segmentation has a larger corrective effect when the insured is allocated in far lower classes than expected.
Moreover, this effect of segmentation is more evident under the assumption of different λ_k because the risk groups are more distinguishable. Despite the fact that in this case the premiums are not always larger under the "Light BMSs", this only occurs in year five as a consequence of the larger differences in the position of the scale occupied under both assumptions. Whilst in the "Strict BMSs" the insured is placed in class 4, in the BMSs that punish only one level per claim he occupies class 2.

Even though it is also important to evaluate the paths for the insureds from the different risk groups, this would require repeating the above analysis five more times. Instead of this, the path of premiums will be assessed for insureds in risk group 5. This kind of drivers in contrast with the one above, have a smaller claim frequency.

Figure 3: Evolution of the premiums for a random insured of risk group 5 (λ_k = 0.3 different or λ_k = 0.55 similar) when his claims are concordant with his *a priori* risk

	Year 1	Year 2	Year 3	Year 4	Year 5
Claims	0	0	1	0	1
Class strict	4	3	6	5	6
Class light	4	3	4	3	4



Source: Own elaboration

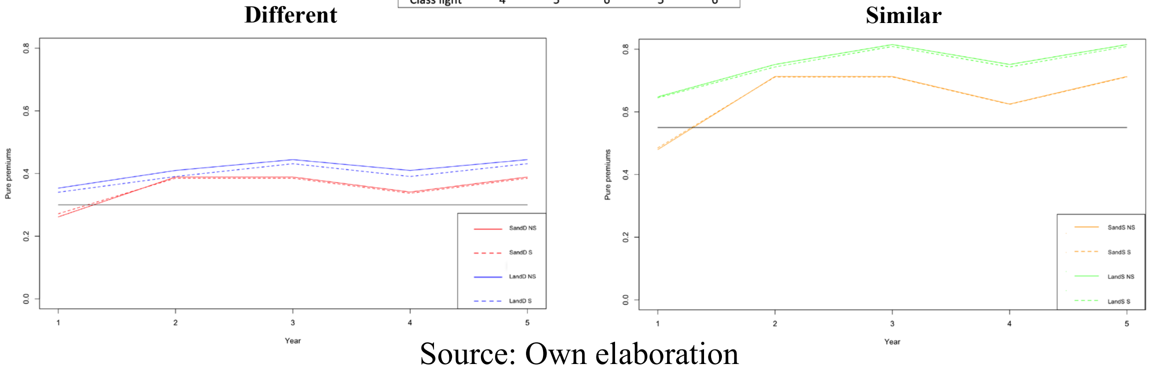
[15] The randomly taken insureds have a mean number of claims in the period of 0.2 that is far from the claim frequency of 0.9 computed for risk group 1.

As in the previous case, firstly, the premiums for a policyholder in each simulated sample when they have a reasonable number of claims in the period, are analyzed. With respect the premiums assessed in Figure 1, these ones seem to display changes of lower magnitude from one year to the next. This happens because the BP, that multiplies the r_i, are in this case smaller than for an insured in risk group 1. Again, the premiums derived from the segmented method are closer to the *a priori* computed claim frequency in all methods.

Another relevant aspect is that now the premiums are not always higher for the BMSs that punish with less classes. Moreover, during the first three years in which the classes coincide in both variants of BMSs the premiums are higher under the light versions. Nevertheless, when the drivers report claims, the higher penalization in terms of levels that implies the "Strict BMSs" leads to larger premiums than under the other assumption.

Figure 4: Evolution of the premiums for a random insured of risk group 5 (λ_k = 0.3 different or λ_k = 0.55 similar) when his claims are not concordant with his *a priori* risk

	Year 1	Year 2	Year 3	Year 4	Year 5
Claims	0	1	1	0	2
Class strict	4	6	6	5	6
Class light	4	5	6	5	6



Source: Own elaboration

Now, as it has been done for the first individuals, the paths of premiums described by an insured in risk group 5 from each sample which have an unexpected number of claims, are displayed in Figure 4. Since the claim frequency of this group is quite low, it is interesting to analyze when the mean number of claims in the sample period is larger than expected.

In contrast with the paths described for an insured in risk group 1, now, comparing Figures 3 and 4, the corrections in the premiums that implies segmentation are generally lower when the insured has an unexpected number of claims. This could seem strange since segmentation is expected to imply larger differences with the not segmented case when the insured is allocated in classes that are not expected for him according to its *a priori* claim frequency. Nevertheless, this stems from the structure of the Norberg's formula and specifically from the stationary distribution and the weights, that are the two components of the formula that change when segmentation is considered. Therefore, under the assumption of larger proportion of insureds in the riskiest classes and this stationary probabilities, larger differences between the segmented and not segmented case are obtained when the insured is in the lowest levels of the scale. In these particular examples assessed, this effect is concordant with the composition of the portfolio. Indeed, the large number of insureds in the riskiest groups suggest that is more serious if the insured is allocated in a lower class than the ones expected. Hence, larger corrections with the introduction of segmentation are required for the lower classes.

6.4. Impact of implementing a surcharge in the pure premium through the Norberg's framework

In Subsection 6.3 the different paths of pure premium that provide the segmented and not segmented case were evaluated and compared only for the "normal drivers" scenario and for some random insureds.

In this section, the oscillation in the pure premiums produced by the introduction of the surcharge through the "bad drivers" assumption will be assessed. The surcharged *a priori* premiums are computed with the product of λ_k and E[Θ] under the "bad drivers" assumption.

Table 11: *A priori* pure premium without and with surcharge

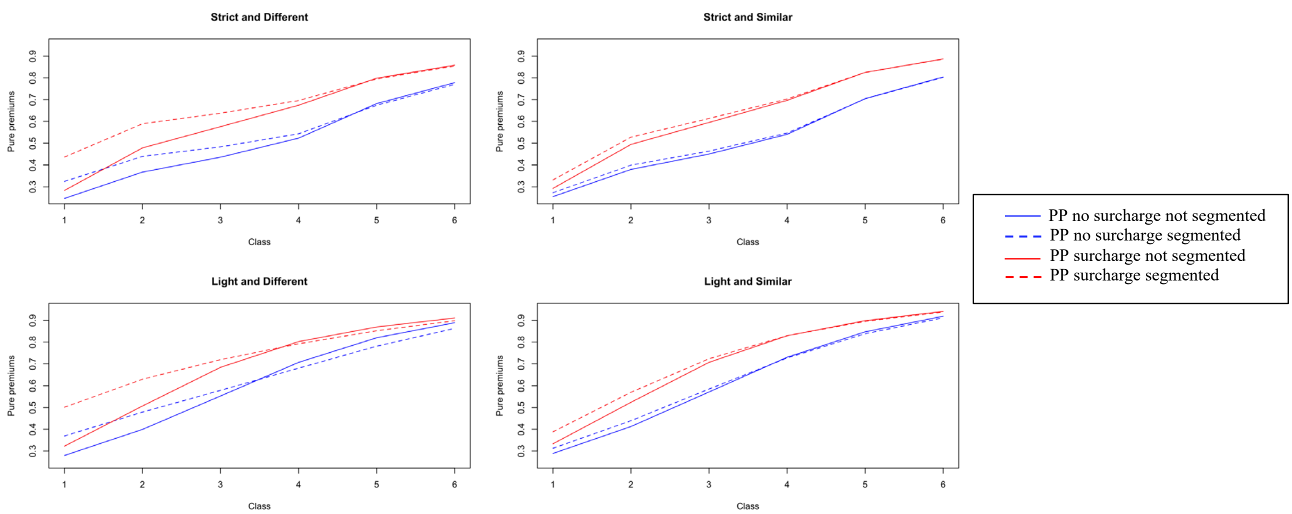
	Assumption 1: different λ_k		Assumption 1: similar λ_k	
	Without surcharge	With surcharge	Without surcharge	With surcharge
Risk group 1	0.9	1.125	0.75	0.9375
Risk group 2	0.7	0.875	0.7	0.875
Risk group 3	0.6	0.75	0.62	0.775
Risk group 4	0.4	0.5	0.57	0.7125
Risk group 5	0.3	0.375	0.55	0.6875
Risk group 6	0.2	0.25	0.35	0.4375

Source: Own elaboration

In Table 11 are provided the *a priori* pure premiums with and without the surcharge for the insureds of the different risk groups. Since the E[Θ] under the "bad drivers" assumptions is 1.25 the surcharge is of 25% of the *a priori* pure premium. This aspect leads to an increase in the amounts to pay by the insureds between 0.05 and 0.225 times the average claim amount.

To obtain the *a posteriori* pure premiums surcharged it must be taken the r_i under the assumption of bad drivers and multiply it by the BP that coincides with λ_k. This will be done for all the levels in the scale and for an individual of each risk group.

Figure 5: Pure premium and pure premiums surcharged for an insured in risk group 1 by classes



Source: Own elaboration

The premiums to pay in each one of the levels of the scale, under the four combinations of assumptions and only for the individual in risk group 1 are displayed in Figure 4. The graphs that correspond to the other risk groups are available in Annex 3. From Figure 4 could be noticed that the differences between the paths of premiums under the not segmented case tend to be reduce considerably for the extreme classes specially for the lowest. However, when segmentation is considered, this convergence between the premiums is not as sharp as when it is not considered. This suggests that the segmented case provide more stable surcharges and therefore more stability for the margin of security that the insurance company receive. These conclusions are also robust with the other five risk groups, since this behavior is present for all of them.

Moreover, the security margin increases with the claim frequency of the risk groups since larger BPs are assigned to riskiest sets. That is, being the surcharge r_i and the not surcharge r_i the same for all the risk groups, when they are multiplied by larger BP the differences between both premiums, and thus the surcharge, become bigger. Additionally, the graphs in Figure 4 and Annex 3 also display how the differences in the premiums between the segmented and not segmented case are wider for the lowest classes.

7. Conclusions

The BMSs treated as Markov Chains are widely applied in motor insurance contracts. That is because they provide a method to update the premiums from the insured when the specific information that describe them is *a posteriori* revealed. This methodology is widely applied because is relatively easy to understand. In contrast with other methods, as the ones under the credibility theory, the classical or not segmented BMS updates the premiums without considering the information of previous years to the recent one.

Insurers are normally large companies with long experience in the market who have enormous databases of useful past information about policyholders. Hence, the coefficients that rely on the *a priori* observable characteristics of the driver are calculated with large amount of data that gives statistical reliability to them. Therefore, forgetting this information in the *a posteriori* pricing might not be the best alternative. Even though a not segmented BMS in which is chosen as BP the *a priori* claim frequency, provides *a posteriori* premiums that depend indirectly of the initially obtained features of the driver, the relativities, r_i, do not consider this information *a priori*. However, segmentation enables considering the initial features of the drivers, directly for the obtention of the r_i in the construction of a BMS.

It may happen that the number of claims reported by the insured in a certain period do not reflect his real risk. While the classical BMS do not consider this aspect the segmented determines the premium considering this deviation from the risk stated *a priori*. For instance, it has been proved that the segmented BMS provide lower premiums than in the not segmented case, for the insureds that being in a class with an associated large premium they have *a priori* risk parameter λ_k that is relatively small. In the same way, when an insured is in a lower level than the expected according to its *a priori* risk, the premiums that he should pay are larger if segmentation is considered. Therefore, the segmented pure premiums will always be closer to the *a priori* determined risk of the driver reflected by $\lambda_k \cdot E[\Theta]$.

The analysis of the RPPs has shown that under the considered assumptions, the premiums increase with the levels of the system. Therefore, in accordance with the *equitable* principle, the riskiest insureds pay a larger quantity of money. However, to ensure that the premiums are always closer to the *a priori* obtained claim frequency, segmentation provides lower amounts to be paid by the drivers in the higher classes and larger premiums for those in the lower levels of the system compared with the classical BMS. Therefore, segmentation introduces in some way some *solidarity* to the system. Moreover, it has been assessed how the premiums change when the probabilities in the discrete structure function about Θ are modified. The parameter θ_l considers the part of the risk that remain *a priori* unknown. Thus, assuming different probabilities for each realization when no source of information to infer them is able, is not advisable. However, it has been seen that assuming higher probabilities for the larger realizations of Θ allows the insurance companies to surcharge the pure premiums, something necessary according to the risk theory.

It has been proved how a BMS that penalizes with more classes and could seem that provide worst premiums for the drivers, supply lower RPPs than a system that penalizes with less levels in the scale, as a way of compensation. The results also suggest that the degree of differentiation of the risk groups is important for an appropriate implementation of segmentation. Hence, the *a posteriori* calculated premiums will be closer to the obtained *a priori* risk of each group when the λ_k exhibits large differences.

The implementation of the Norberg's framework in practice with a simulated dataset, has allowed the analysis of what would be the real premiums to pay. This has been done under the assumption of unitary claim severity with the different scenarios studied before. Furthermore, it allowed a clear evaluation of the directly analyzed aspects performed with the RPPs and to identify additional particularities. For instance, in both segmented and not segmented cases, it was seen that the premiums are larger for the riskiest groups under the assumption of different λ_k. This happens because the BP depends on λ_k and the larger risk groups have a larger value in this parameter under this assumption. The opposite happens for the groups with lowest risk.

The analysis of the paths also has shown that for an insured classified in a highly risky group, the correction in the premiums that segmentation introduces are larger when its behavior does not reflect the expected *a priori* derived risk. The opposite happens with the less risky groups that exhibit larger differences between the segmented and not segmented case when the individual is allocated in a concordant class with its risk *a priori*. This is due to the weights and stationary probabilities considered in the segmented version of the Norberg's formula. Indeed, in the cases under study the formula allows considering that a larger quantity of drivers are allocated in the riskiest groups. Therefore, the corrections that imply the segmented case with respect to the not segmented are larger when the insureds exhibit lower number of claims than it is expected.

The paths also have shown that, even though the RPPs are larger for the "Light BMSs", the different levels in the scale in which an insured could be allocated, depending on how many classes are penalized for each claim, allow for higher premiums in the "Strict" case. The comparison of the premiums in the studied examples has also revealed that less differences appear in the unitary claim severity premiums, obtained under the different assumptions, when the risk of the individual is small. That happens because these insureds have a lower BP.

The implementation of surcharged pure premiums through the framework under study has also been studied. Indeed, this aspect was evaluated for the *a priori* premiums and the *a posteriori* ones under the different assumptions and for all the levels in the scale. It was identified that the surcharge derived from the implementation of the bad driver's assumption remains more stable when segmentation is considered. This is more concordant with the *solvency* principle than in the classical systems. This aspect could be relevant, especially if insurers are interested in more stability to improve the solvency of the company. It was also appreciated that this security surcharge is affected by the BP and therefore by the claim frequency of the risk groups. Furthermore, this evaluation has enabled an easier visualization of the higher corrections of segmentation for the lower classes mentioned above.

It is important to highlight some limitations of this research that could be assessed in future research lines. The principal one is that the Norberg's method is an asymptotic criterion which assumes that the premiums have been in the portfolio during infinite periods, what is not accomplished in practice. Thus, to compute the premiums of the insureds over the first years in the portfolio with the methodology under study it may not be the best choice.

In order to settle this issue, Borgan *et al*. (1981) proposed the definition of some weights that allows to consider the temporality of the analysis. Furthermore, the framework proposed by these three authors provide a way of computing an optimal entrance class in terms of efficiency, that could also be introduced in further analysis. Hence, in future studies the derivation of segmented BMS through this methodology should be studied.

Another aspect to consider is that the formula provided by Norberg and even the improvement suggested by Borgan *et. al.* (1981) often result in very irregular premiums that could not be desirable for commercial reasons. For instance, Gilde & Sundt (1989) provide a framework that allows the premiums to increase linearly. Therefore, in future research the implementation of this alternative framework for the obtention of BMS assuming segmentation should be analyzed.

It would also be desirable that extensive research that treated segmentation in BMSs, made use of measures as the coefficient of variation, the Loimaranta efficiency (Loimaranta, 1972) or the relative stationary average level to assess and compare different BMSs. It would be worthwhile to implement the methodology under study to an overdispersed dataset via the negative binomial or another distribution for count data. What is more, the distributions of λ_k and Θ can be set in a way that allows the consideration of overdispersion in the construction of a BMS.

This research bases its analysis in the number of claims, nevertheless, other authors consider a multivariate discrete distribution to include in the analysis the claim size (Gómez-Déniz & Calderín-Ojeda, 2018). Hence, analyzing the performance of the Norberg's framework considering claim severity as well, may be interesting. Moreover, comparing the methodology developed with other models that segmentate the data as the credibility models could provide relevant results. For instance, the premiums calculated in the segmented BMS might be compared with the ones obtained with Bülhmann-Straub calculated with the Empirical Bayes method or with premiums derived from the Bayesian Approach (Tse, 2009)

This research has been an excellent opportunity to go further in the knowledge about the widely used BMSs, assessing a way of considering segmentation and the implementation of an optimal condition in the construction of them. Moreover, segmentation has provided a framework to compute the premiums considering the past information beyond the last year without renouncing to the capacity of penalizing or regarding the insured with a simple system of classes.

It is worth mentioning that there exist other methodologies different than the one in this thesis that introduce an optimal condition in these systems, for example the aforementioned methodologies purposed by Borgan or Gilde & Sundt and also the so-called optimal BMS treated in Lemaire (1995). The implementation in practice of the methods under study in this research has been done through the software R (R Development Core Team, 2022), which code is included in Annex 1, and Excel.

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Annexes

Annex 1: R code

The R code used for the obtention of the different results obtained in this research are the following:

STRICT BMS:

<u>Calculation of the RPP</u>

Not Segmented case:

Note: The not segmented case is common under both assumptions of lambda of the risk groups because they do not affect in this part.

```
#Lambda of all the portfolio
nmed<-0.645

lambda<-nmed

# Values of thita
cita1<-0.25
cita2<-0.5
cita3<-0.75
cita4<-1.25
cita5<-1.5
cita6<-1.75


#Total claim frequency
cf1<-lambda*cita1;cf1
cf2<-lambda*cita2;cf2
cf3<-lambda*cita3;cf3
cf4<-lambda*cita4;cf4
cf5<-lambda*cita5;cf5
cf6<-lambda*cita6;cf6

#For cita1

#Transition matrix
p0<-dpois(0, cf1);p0
p1<-dpois(1, cf1);p1
P<-matrix(c(p0,p0,0,0,0,0,0,0,p0,0,0,0,0,0,0,p0,0,0,p1,0,0,0,p0,0,0,
        p1,0,0,0,p0,1-p0-p1,1-p0-p1,1-p0,1-p0,1-p0,1-p0),nrow=6,ncol=6);P

#Transpose the transition matrix
Pt<-t(P)

#Vector of initial probabilities
```

```
p0<-c(0,0,0,0,1,0)


#Obtention of the stationary probabilities

### 1) Limits

Plim <- P
for (k in 1:10) Plim <- Plim %*% Plim ## i.e., Mlim <- Mlim?(2?10)
Plim
pilim <- Plim[5,]; pilim

### 2) Rolki Formula
lim.distr =
  function(matrix) {
    et = matrix(nrow=1, ncol=dim(matrix)[2], data=1)
    E = matrix(nrow=dim(matrix)[1], ncol=dim(matrix)[2], data=1)
    mat = diag(dim(matrix)[1]) - matrix + E
    inverse.mat = solve(mat)
    p = et %*% inverse.mat
    return(p)}
pi = lim.distr(P) ; pi

### 3) Eigen values and eigen vectors

eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep


# 4) Simulation method
Next<-
matrix(c(1,4,6,6,6,6,1,5,6,6,6,6,2,6,6,6,6,6,3,6,6,6,6,6,4,6,6,6,6,6,5,6,6,6,6,6),nrow=6,nc
ol=6);Next
TMax <- 50; NSim <- 100000; FinalBM <- numeric(NSim)
for (n in 1:NSim)
{ cn1 <- rpois(TMax,cf1); cn1 <- pmin(cn1, 2) + 1
BM <- 5; for (i in 1:TMax) BM <- Next[cn1[i],BM]
FinalBM[n] <- BM
}
pi <-
c(sum(FinalBM==1)/NSim,sum(FinalBM==2)/NSim,sum(FinalBM==3)/NSim,sum(Fin
alBM==4)/NSim,sum(FinalBM==5)/NSim,sum(FinalBM==6)/NSim); pi


pst1<-pivep


#For cita2
```

```
#Transition matrix
p0<-dpois(0, cf2);p0
p1<-dpois(1, cf2);p1
#p2<-dpois(2, cf2);p2
P<-matrix(c(p0,p0,0,0,0,0,0,0,p0,0,0,0,0,0,0,p0,0,0,p1,0,0,0,p0,0,0,
        p1,0,0,0,p0,1-p0-p1,1-p0-p1,1-p0,1-p0,1-p0,1-p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst2<-pivep


#For cita3

#Transition matrix
p0<-dpois(0, cf3);p0
p1<-dpois(1, cf3);p1
#p2<-dpois(2, cf3);p2
P<-matrix(c(p0,p0,0,0,0,0,0,0,p0,0,0,0,0,0,0,p0,0,0,p1,0,0,0,p0,0,0,
        p1,0,0,0,p0,1-p0-p1,1-p0-p1,1-p0,1-p0,1-p0,1-p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst3<-pivep


#For cita4

#Transition matrix
p0<-dpois(0, cf4);p0
p1<-dpois(1, cf4);p1
#p2<-dpois(2, cf4);p2
P<-matrix(c(p0,p0,0,0,0,0,0,0,p0,0,0,0,0,0,0,p0,0,0,p1,0,0,0,p0,0,0,
        p1,0,0,0,p0,1-p0-p1,1-p0-p1,1-p0,1-p0,1-p0,1-p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst4<-pivep


#For cita5
```

```
#Transition matrix
p0<-dpois(0, cf5);p0
p1<-dpois(1, cf5);p1
#p2<-dpois(2, cf5);p2
P<-matrix(c(p0,p0,0,0,0,0,0,0,p0,0,0,0,0,0,0,p0,0,0,p1,0,0,0,p0,0,0,
        p1,0,0,0,p0,1-p0-p1,1-p0-p1,1-p0,1-p0,1-p0,1-p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst5<-pivep



#For cita6

#Transition matrix
p0<-dpois(0, cf6);p0
p1<-dpois(1, cf6);p1
#p2<-dpois(2, cf6);p2
P<-matrix(c(p0,p0,0,0,0,0,0,0,p0,0,0,0,0,0,0,p0,0,0,p1,0,0,0,p0,0,0,
        p1,0,0,0,p0,1-p0-p1,1-p0-p1,1-p0,1-p0,1-p0,1-p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst6<-pivep
```

<u>Deriving the Not Segmented RPP through Denuit & Charpentier formulation (cita normal drivers)</u>

```
# Probability of being a cita i driver if the policyholder is in class L=l

pst<-rbind(pst1,pst2,pst3,pst4,pst5,pst6)

Pc1l1<-
pst[1,1]*(1/6)/(pst[1,1]*(1/6)+pst[2,1]*(1/6)+pst[3,1]*(1/6)+pst[4,1]*(1/6)+pst[5,1]*(1/
6)+pst[6,1]*(1/6))



Pcl<-matrix(0,6,6)

for(i in 1:6){
```

```
  for(j in 1:6){
    Pcl[i,j]<-
pst[i,j]*(1/6)/(pst[1,j]*(1/6)+pst[2,j]*(1/6)+pst[3,j]*(1/6)+pst[4,j]*(1/6)+pst[5,j]*(1/6)+
pst[6,j]*(1/6))
  }
}

colnames(Pcl)<-c("l1","l2","l3","l4","l5","l6")
rownames(Pcl)<-c("cita 1","cita 2","cita 3","cita 4","cita 5","cita 6")


#Relative pure premium a posteriori not segmented case

# Pr[theta|L=1]

PP1<-
Pcl[1,1]*cita1+Pcl[2,1]*cita2+Pcl[3,1]*cita3+Pcl[4,1]*cita4+Pcl[5,1]*cita5+Pcl[6,1]*ci
ta6

# Pr[theta|L=2]

PP2<-
Pcl[1,2]*cita1+Pcl[2,2]*cita2+Pcl[3,2]*cita3+Pcl[4,2]*cita4+Pcl[5,2]*cita5+Pcl[6,2]*ci
ta6

# Pr[theta|L=3]

PP3<-
Pcl[1,3]*cita1+Pcl[2,3]*cita2+Pcl[3,3]*cita3+Pcl[4,3]*cita4+Pcl[5,3]*cita5+Pcl[6,3]*ci
ta6

# Pr[theta|L=4]

PP4<-
Pcl[1,4]*cita1+Pcl[2,4]*cita2+Pcl[3,4]*cita3+Pcl[4,4]*cita4+Pcl[5,4]*cita5+Pcl[6,4]*ci
ta6

# Pr[theta|L=5]

PP5<-
Pcl[1,5]*cita1+Pcl[2,5]*cita2+Pcl[3,5]*cita3+Pcl[4,5]*cita4+Pcl[5,5]*cita5+Pcl[6,5]*ci
ta6

# Pr[theta|L=6]

PP6<-
Pcl[1,6]*cita1+Pcl[2,6]*cita2+Pcl[3,6]*cita3+Pcl[4,6]*cita4+Pcl[5,6]*cita5+Pcl[6,6]*ci
ta6

RPPPNS<-c(PP1,PP2,PP3,PP4,PP5,PP6)
```

Obtention of the Not segmented RPP with the Normberg's formula

Cita normal drivers

```
RPPPNSnorb<-c(0,0,0,0,0,0)
for(i in 1:6){
RPPPNSnorb[i]<-
(cita1*pst[1,i]*(1/6)+cita2*pst[2,i]*(1/6)+cita3*pst[3,i]*(1/6)+cita4*pst[4,i]*(1/6)+cita
5*pst[5,i]*(1/6)+cita6*pst[6,i]*(1/6))/
(pst[1,i]*(1/6)+pst[2,i]*(1/6)+pst[3,i]*(1/6)+pst[4,i]*(1/6)+pst[5,i]*(1/6)+pst[6,i]*(1/6))
}

RPPPNSnorb
```

Cita bad drivers

```
RPPPNSbad<-c(0,0,0,0,0,0)
for(i in 1:6){
  RPPPNSbad[i]<-
(cita1*pst[1,i]*(1/12)+cita2*pst[2,i]*(1/12)+cita3*pst[3,i]*(1/12)+cita4*pst[4,i]*(1/4)+
cita5*pst[5,i]*(1/4)+cita6*pst[6,i]*(1/4))/
(pst[1,i]*(1/12)+pst[2,i]*(1/12)+pst[3,i]*(1/12)+pst[4,i]*(1/4)+pst[5,i]*(1/4)+pst[6,i]*(1
/4))
}
```

Cita good drivers

```
RPPPNSgood<-c(0,0,0,0,0,0)
for(i in 1:6){
  RPPPNSgood[i]<-
(cita1*pst[1,i]*(1/4)+cita2*pst[2,i]*(1/4)+cita3*pst[3,i]*(1/4)+cita4*pst[4,i]*(1/12)+cit
a5*pst[5,i]*(1/12)+cita6*pst[6,i]*(1/12))/

(pst[1,i]*(1/4)+pst[2,i]*(1/4)+pst[3,i]*(1/4)+pst[4,i]*(1/12)+pst[5,i]*(1/12)+pst[6,i]*(1/
12))
}

RPPPNSgood
```

Segmented case

Depending on the assumption about λ_k that is desired for the obtention of the BMS it must be executed one of the following chunks of code, before proceeding to run the script below.

Ass. 1: different λ_k	Ass. 2: simmilar λ_k
nmedA1<-0.9 nmedA2<-0.7 nmedA3<-0.6 nmedA4<-0.4 nmedA5<-0.3 nmedA6<-0.2	nmedA1<-0.75 nmedA2<-0.7 nmedA3<-0.62 nmedA4<-0.57 nmedA5<-0.55 nmedA6<-0.35

```
wA1<-0.4
wA2<-0.15
wA3<-0.15
wA4<-0.1
wA5<-0.1
wA6<-0.1

nmed<-
nmedA1*wA1+nmedA2*wA2+nmedA3*wA3+nmedA4*wA4+nmedA5*wA5+nmedA
6*wA6;nmed

w<-c(wA1,wA2,wA3,wA4,wA5,wA6)

nmed<-c(nmedA1,nmedA2,nmedA3,nmedA4,nmedA5,nmedA6)
cita<-c(0.25,0.5,0.75,1.25,1.5,1.75)

P<-matrix(0,6,6)
Pij<-matrix(0,6*6,6)

a<-1
for(i in 1:6){
  for(j in 1:6){
    p0<-dpois (0, nmed[i]*cita[j])
    p1<-dpois(1, nmed[i]*cita[j])
    p2<-dpois(2, nmed[i]*cita[j])
    P<-matrix(c(p0,p0,0,0,0,0,0,0,p0,0,0,0,0,0,0,p0,0,0,p1,0,0,0,p0,0,0,
              p1,0,0,0,p0,1-p0-p1,1-p0-p1,1-p0,1-p0,1-p0,1-p0),nrow=6,ncol=6)

    eigen(t(P))
    pi <- eigen(t(P))$vectors[,1]
    pi <- pi/sum(pi)
```

```
    pivep <- Re(pi)

    Pij[a,]<-pivep
    a=a+1
  }
}

Pij
wr<-c(rep(wA1,6),rep(wA2,6),rep(wA3,6),rep(wA4,6),rep(wA5,6),rep(wA6,6))

sum(w)
sum(wr)

#We change the notation of the formula above:

Pij
wr

pil<-
c((1/6)*sum(Pij[,1]*wr),(1/6)*sum(Pij[,2]*wr),(1/6)*sum(Pij[,3]*wr),(1/6)*sum(Pij[,4]
*wr),(1/6)*sum(Pij[,5]*wr),(1/6)*sum(Pij[,6]*wr));pil
sum(pil)
```

<u>Deriving the Segmented RPP through</u> <u>Denuit & Charpentier formulation (cita normal drivers)</u>

```
#Pr[Θ=0.25|L=l]

P025l<-matrix(0,6,6)
P025l[1,]<-(1/(6*pil))*(w[1]*Pij[1,])
for(i in 2:6){
  P025l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+1,])
}

P025l<-colSums(P025l)


#Pr[Θ=0.5|L=l]

P05l<-matrix(0,6,6)
P05l[1,]<-(1/(6*pil))*(w[1]*Pij[2,])
for(i in 2:6){
  P05l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+2,])
}

P05l<-colSums(P05l)


#Pr[Θ=0.75|L=l]
```

```
P075l<-matrix(0,6,6)
P075l[1,]<-(1/(6*pil))*(w[1]*Pij[3,])
for(i in 2:6){
  P075l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+3,])
}

P075l<-colSums(P075l)

#Pr[Θ=1.25|L=l]

P125l<-matrix(0,6,6)
P125l[1,]<-(1/(6*pil))*(w[1]*Pij[4,])
for(i in 2:6){
  P125l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+4,])
}

P125l<-colSums(P125l)

#Pr[Θ=1.5|L=l]

P15l<-matrix(0,6,6)
P15l[1,]<-(1/(6*pil))*(w[1]*Pij[5,])
for(i in 2:6){
  P15l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+5,])
}

P15l<-colSums(P15l)

#Pr[Θ=1.75|L=l]

P175l<-matrix(0,6,6)
P175l[1,]<-(1/(6*pil))*(w[1]*Pij[6,])
for(i in 2:6){
  P175l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+6,])
}

P175l<-colSums(P175l)


# Segmented RPP

#E[Θ|L=l]=0.25·Pr[Θ=0.25|L=l]+...+1.75·Pr[Θ=1.75|L=l]

RPPPS<-0.25*P025l+0.5*P05l+0.75*P075l+1.25*P125l+1.5*P15l+1.75*P175l
```

<u>Obtention of the Not segmented RPP with the Normberg's formula</u>

Cita normal drivers

```
RPN<-matrix(0,6,6)
RPD<-matrix(0,6,6)
for(i in 1:6){
  RPN[i,]<-(w[i]*(cita[1]*Pij[6*(i-1)+1,]*(1/6)+cita[2]*Pij[6*(i-1)+2,]*(1/6)+cita[3]*Pij[6*(i-1)+3,]*(1/6)+cita[4]*Pij[6*(i-1)+4,]*(1/6)+cita[5]*Pij[6*(i-1)+5,]*(1/6)+cita[6]*Pij[6*(i-1)+6,]*(1/6)))
  RPD[i,]<-(w[i]*(Pij[6*(i-1)+1,]*(1/6)+Pij[6*(i-1)+2,]*(1/6)+Pij[6*(i-1)+3,]*(1/6)+Pij[6*(i-1)+4,]*(1/6)+Pij[6*(i-1)+5,]*(1/6)+Pij[6*(i-1)+6,]*(1/6)))
}

PN<-colSums(RPN)
PD<-colSums(RPD)

RPPPSnorb<-PN/PD
```

Cita bad drivers

```
RPN<-matrix(0,6,6)
RPD<-matrix(0,6,6)
for(i in 1:6){
  RPN[i,]<-(w[i]*(cita[1]*Pij[6*(i-1)+1,]*(1/12)+cita[2]*Pij[6*(i-1)+2,]*(1/12)+cita[3]*Pij[6*(i-1)+3,]*(1/12)+cita[4]*Pij[6*(i-1)+4,]*(1/4)+cita[5]*Pij[6*(i-1)+5,]*(1/4)+cita[6]*Pij[6*(i-1)+6,]*(1/4)))
  RPD[i,]<-(w[i]*(Pij[6*(i-1)+1,]*(1/12)+Pij[6*(i-1)+2,]*(1/12)+Pij[6*(i-1)+3,]*(1/12)+Pij[6*(i-1)+4,]*(1/4)+Pij[6*(i-1)+5,]*(1/4)+Pij[6*(i-1)+6,]*(1/4)))
}

PN<-colSums(RPN)
PD<-colSums(RPD)

RPPPSbad<-PN/PD
```

Cita good drivers

```
RPN<-matrix(0,6,6)
RPD<-matrix(0,6,6)
for(i in 1:6){
  RPN[i,]<-(w[i]*(cita[1]*Pij[6*(i-1)+1,]*(1/4)+cita[2]*Pij[6*(i-1)+2,]*(1/4)+cita[3]*Pij[6*(i-1)+3,]*(1/4)+cita[4]*Pij[6*(i-1)+4,]*(1/12)+cita[5]*Pij[6*(i-1)+5,]*(1/12)+cita[6]*Pij[6*(i-1)+6,]*(1/12)))
  RPD[i,]<-(w[i]*(Pij[6*(i-1)+1,]*(1/4)+Pij[6*(i-1)+2,]*(1/4)+Pij[6*(i-1)+3,]*(1/4)+Pij[6*(i-1)+4,]*(1/12)+Pij[6*(i-1)+5,]*(1/12)+Pij[6*(i-1)+6,]*(1/12)))
}

PN<-colSums(RPN)
PD<-colSums(RPD)
```

```
RPPPSgood<-PN/PD
```

Note: If it has been executed the Ass .1 the RPPPS and RPPPSnor would be the segmented RPP for a strict BMS when the λ_k are assumed different otherwise it would be the RPP for a strict BMS but for λ_k that are considered similar.

<u>Pure premium for unitary claim amount</u>

Not segmented case

```
PBrg<-rep(0,6)
PPEXnormalNS<-matrix(0,6,6)
PPEXbadNS<-matrix(0,6,6)
PPEXgoodNS<-matrix(0,6,6)
for(i in 1:6){
  PBrg[i]<-nmed[i]

  PPEXnormalNS[i,]<-RPPPNS*PBrg[i]
  PPEXbadNS[i,]<-RPPPNSbad*PBrg[i]
  PPEXgoodNS[i,]<-RPPPNSgood*PBrg[i]
}

rownames(PPEXnormalNS)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXbadNS)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXgoodNS)<-c("RG1","RG2","RG3","RG4","RG5","RG6")

PPEXnormalNS
PPEXbadNS
PPEXgoodNS
```

Segmented case

```
PBrg<-rep(0,6)
PPEXnormal<-matrix(0,6,6)
PPEXbad<-matrix(0,6,6)
PPEXgood<-matrix(0,6,6)
for(i in 1:6){
  PBrg[i]<-nmed[i]

  PPEXnormal[i,]<-RPPPS*PBrg[i]
  PPEXbad[i,]<-RPPPSbad*PBrg[i]
  PPEXgood[i,]<-RPPPSgood*PBrg[i]
}

rownames(PPEXnormal)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXbad)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXgood)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
```

PPEXnormal
PPEXbad
PPEXgood

Note: If it has been executed the Ass .1 all the PPEX and PPEXNS would correspond with the Strict and different case while if the Ass. 2 was the once excited they would made reference to the Strict and similar case.

From the following chunk of code it must be executed the once that correspond with the assumption considered above:

Ass. 1: different λ_k

```
SDPPEXnormalNS<-PPEXnormalNS
SDPPEXnormal<-PPEXnormal
SDPPEXbadNS<-PPEXbadNS
SDPPEXbad<-PPEXbad
SDPPEXgoodNS<-PPEXgoodNS
SDPPEXgood<-PPEXgood
```

Ass. 2: simmilar λ_k

```
SSPPEXnormalNS<-PPEXnormalNS
SSPPEXnormal<-PPEXnormal
SSPPEXbadNS<-PPEXbadNS
SSPPEXbad<-PPEXbad
SSPPEXgoodNS<-PPEXgoodNS
SSPPEXgood<-PPEXgood
```

Light BMS:

Calculation of the RPP

Not Segmented case:

Note: The not segmented case is common under both assumptions of lambda of the risk groups because they do not affect in this part.

```
#Lambda of all the portfolio
nmed<-0.645

lambda<-nmed

# Values of thita
cita1<-0.25
cita2<-0.5
cita3<-0.75
cita4<-1.25
cita5<-1.5
cita6<-1.75


#Total claim frequency
cf1<-lambda*cita1;cf1
cf2<-lambda*cita2;cf2
cf3<-lambda*cita3;cf3
```

```
cf4<-lambda*cita4;cf4
cf5<-lambda*cita5;cf5
cf6<-lambda*cita6;cf6

#For cita1

#Transition matrix
p0<-dpois(0, cf1);p0
p1<-dpois(1, cf1);p1
p2<-dpois(2, cf1);p2
p3<-dpois(3, cf1);p3
p4<-dpois(4, cf1);p4
P<-matrix(c(p0,p0,0,0,0,0,p1,0,p0,0,0,0,p2,p1,0,p0,0,0,p3,p2,p1,0,p0,0,p4,
p3,p2,p1,0,p0,1-p0-p1-p2-p3-p4,1-p0-p1-p2-p3,1-p0-p1-p2,1-p0-p1,1-p0,1-
p0),nrow=6,ncol=6);P

#Transpose the transition matrix
Pt<-t(P)

#Vector of initial probabilities
p0<-c(0,0,0,0,1,0)


#Obtention of the stationary probabilities

### 1) Límites

Plim <- P
for (k in 1:10) Plim <- Plim %*% Plim ## i.e., Mlim <- Mlim?(2?10)
Plim
pilim <- Plim[5,]; pilim

### 2) Fórmula Rolski
lim.distr =
  function(matrix) {
    et = matrix(nrow=1, ncol=dim(matrix)[2], data=1)
    E = matrix(nrow=dim(matrix)[1], ncol=dim(matrix)[2], data=1)
    mat = diag(dim(matrix)[1]) - matrix + E
    inverse.mat = solve(mat)
    p = et %*% inverse.mat
    return(p)}
pi = lim.distr(P) ; pi

### 3) Valores y vectores propios

eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep
```

```
# 4) Método simulación
Next<-
matrix(c(1,2,3,4,5,6,1,3,4,5,6,6,2,4,5,6,6,6,3,5,6,6,6,6,4,6,6,6,6,6,5,6,6,6,6,6),nrow=6,nc
ol=6);Next
TMax <- 50; NSim <- 100000; FinalBM <- numeric(NSim)
for (n in 1:NSim)
{ cn1 <- rpois(TMax,cf1); cn1 <- pmin(cn1, 2) + 1
BM <- 5; for (i in 1:TMax) BM <- Next[cn1[i],BM]
FinalBM[n] <- BM
}
pi <-
c(sum(FinalBM==1)/NSim,sum(FinalBM==2)/NSim,sum(FinalBM==3)/NSim,sum(Fin
alBM==4)/NSim,sum(FinalBM==5)/NSim,sum(FinalBM==6)/NSim); pi


pst1<-pivep

#For cita2

#Transition matrix
p0<-dpois(0, cf1);p0
p1<-dpois(1, cf1);p1
p2<-dpois(2, cf1);p2
p3<-dpois(3, cf1);p3
p4<-dpois(4, cf1);p4
P<-matrix(c(p0,p0,0,0,0,0,p1,0,p0,0,0,0,p2,p1,0,p0,0,0,p3,p2,p1,0,p0,0,p4,
p3,p2,p1,0,p0,1-p0-p1-p2-p3-p4,1-p0-p1-p2-p3,1-p0-p1-p2,1-p0-p1,1-p0,1-
p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst2<-pivep


#For cita3

#Transition matrix
p0<-dpois(0, cf1);p0
p1<-dpois(1, cf1);p1
p2<-dpois(2, cf1);p2
p3<-dpois(3, cf1);p3
p4<-dpois(4, cf1);p4
P<-matrix(c(p0,p0,0,0,0,0,p1,0,p0,0,0,0,p2,p1,0,p0,0,0,p3,p2,p1,0,p0,0,p4,
p3,p2,p1,0,p0,1-p0-p1-p2-p3-p4,1-p0-p1-p2-p3,1-p0-p1-p2,1-p0-p1,1-p0,1-
p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
```

```
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst3<-pivep


#For cita4

#Transition matrix
p0<-dpois(0, cf1);p0
p1<-dpois(1, cf1);p1
p2<-dpois(2, cf1);p2
p3<-dpois(3, cf1);p3
p4<-dpois(4, cf1);p4
P<-matrix(c(p0,p0,0,0,0,0,p1,0,p0,0,0,0,p2,p1,0,p0,0,0,p3,p2,p1,0,p0,0,p4,
p3,p2,p1,0,p0,1-p0-p1-p2-p3-p4,1-p0-p1-p2-p3,1-p0-p1-p2,1-p0-p1,1-p0,1-
p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst4<-pivep


#For cita5

#Transition matrix
p0<-dpois(0, cf1);p0
p1<-dpois(1, cf1);p1
p2<-dpois(2, cf1);p2
p3<-dpois(3, cf1);p3
p4<-dpois(4, cf1);p4
P<-matrix(c(p0,p0,0,0,0,0,p1,0,p0,0,0,0,p2,p1,0,p0,0,0,p3,p2,p1,0,p0,0,p4,
p3,p2,p1,0,p0,1-p0-p1-p2-p3-p4,1-p0-p1-p2-p3,1-p0-p1-p2,1-p0-p1,1-p0,1-
p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst5<-pivep



#For cita6
```

```
#Transition matrix
p0<-dpois(0, cf1);p0
p1<-dpois(1, cf1);p1
p2<-dpois(2, cf1);p2
p3<-dpois(3, cf1);p3
p4<-dpois(4, cf1);p4
P<-matrix(c(p0,p0,0,0,0,0,p1,0,p0,0,0,0,p2,p1,0,p0,0,0,p3,p2,p1,0,p0,0,p4,
p3,p2,p1,0,p0,1-p0-p1-p2-p3-p4,1-p0-p1-p2-p3,1-p0-p1-p2,1-p0-p1,1-p0,1-
p0),nrow=6,ncol=6);P

#Obtention of the stationary probabilities
eigen(t(P))
pi <- eigen(t(P))$vectors[,1]; pi
pi <- pi/sum(pi); pivep <- Re(pi);pivep

pst6<-pivep
```

<u>Deriving the Not Segmented RPP through Denuit & Charpentier formulation (cita normal drivers)</u>

```
# Probability of being a cita i driver if the policyholder is in class L=l

pst<-rbind(pst1,pst2,pst3,pst4,pst5,pst6)

Pc1l1<-
pst[1,1]*(1/6)/(pst[1,1]*(1/6)+pst[2,1]*(1/6)+pst[3,1]*(1/6)+pst[4,1]*(1/6)+pst[5,1]*(1/
6)+pst[6,1]*(1/6))



Pcl<-matrix(0,6,6)

for(i in 1:6){
  for(j in 1:6){
    Pcl[i,j]<-
pst[i,j]*(1/6)/(pst[1,j]*(1/6)+pst[2,j]*(1/6)+pst[3,j]*(1/6)+pst[4,j]*(1/6)+pst[5,j]*(1/6)+
pst[6,j]*(1/6))
  }
}

colnames(Pcl)<-c("l1","l2","l3","l4","l5","l6")
rownames(Pcl)<-c("cita 1","cita 2","cita 3","cita 4","cita 5","cita 6")


#Relative pure premium a posteriori not segmented case

# Pr[theta|L=l]
```

```
PP1<-
Pcl[1,1]*cita1+Pcl[2,1]*cita2+Pcl[3,1]*cita3+Pcl[4,1]*cita4+Pcl[5,1]*cita5+Pcl[6,1]*ci
ta6

# Pr[theta|L=2]

PP2<-
Pcl[1,2]*cita1+Pcl[2,2]*cita2+Pcl[3,2]*cita3+Pcl[4,2]*cita4+Pcl[5,2]*cita5+Pcl[6,2]*ci
ta6

# Pr[theta|L=3]

PP3<-
Pcl[1,3]*cita1+Pcl[2,3]*cita2+Pcl[3,3]*cita3+Pcl[4,3]*cita4+Pcl[5,3]*cita5+Pcl[6,3]*ci
ta6

# Pr[theta|L=4]

PP4<-
Pcl[1,4]*cita1+Pcl[2,4]*cita2+Pcl[3,4]*cita3+Pcl[4,4]*cita4+Pcl[5,4]*cita5+Pcl[6,4]*ci
ta6

# Pr[theta|L=5]

PP5<-
Pcl[1,5]*cita1+Pcl[2,5]*cita2+Pcl[3,5]*cita3+Pcl[4,5]*cita4+Pcl[5,5]*cita5+Pcl[6,5]*ci
ta6

# Pr[theta|L=6]

PP6<-
Pcl[1,6]*cita1+Pcl[2,6]*cita2+Pcl[3,6]*cita3+Pcl[4,6]*cita4+Pcl[5,6]*cita5+Pcl[6,6]*ci
ta6

RPPPNS<-c(PP1,PP2,PP3,PP4,PP5,PP6)
```

<u>Obtention of the Not segmented RPP with the Normberg's formula</u>

<u>Cita normal drivers</u>

```
RPPPNSnorb<-c(0,0,0,0,0,0)
for(i in 1:6){
RPPPNSnorb[i]<-
(cita1*pst[1,i]*(1/6)+cita2*pst[2,i]*(1/6)+cita3*pst[3,i]*(1/6)+cita4*pst[4,i]*(1/6)+cita
5*pst[5,i]*(1/6)+cita6*pst[6,i]*(1/6))/
(pst[1,i]*(1/6)+pst[2,i]*(1/6)+pst[3,i]*(1/6)+pst[4,i]*(1/6)+pst[5,i]*(1/6)+pst[6,i]*(1/6))
}

RPPPNSnorb
```

Cita bad drivers

```
RPPPNSbad<-c(0,0,0,0,0,0)
for(i in 1:6){
  RPPPNSbad[i]<-
(cita1*pst[1,i]*(1/12)+cita2*pst[2,i]*(1/12)+cita3*pst[3,i]*(1/12)+cita4*pst[4,i]*(1/4)+
cita5*pst[5,i]*(1/4)+cita6*pst[6,i]*(1/4))/
(pst[1,i]*(1/12)+pst[2,i]*(1/12)+pst[3,i]*(1/12)+pst[4,i]*(1/4)+pst[5,i]*(1/4)+pst[6,i]*(1
/4))
}
```

Cita good drivers

```
RPPPNSgood<-c(0,0,0,0,0,0)
for(i in 1:6){
  RPPPNSgood[i]<-
(cita1*pst[1,i]*(1/4)+cita2*pst[2,i]*(1/4)+cita3*pst[3,i]*(1/4)+cita4*pst[4,i]*(1/12)+cit
a5*pst[5,i]*(1/12)+cita6*pst[6,i]*(1/12))/

(pst[1,i]*(1/4)+pst[2,i]*(1/4)+pst[3,i]*(1/4)+pst[4,i]*(1/12)+pst[5,i]*(1/12)+pst[6,i]*(1/
12))
}

RPPPNSgood
```

Segmented case

Depending on the assumption about λ_k that is desired for the obtention of the BMS it must be executed one of the following chunks of code, before proceeding to run the script below.

Ass. 1: different λ_k

```
nmedA1<-0.9
nmedA2<-0.7
nmedA3<-0.6
nmedA4<-0.4
nmedA5<-0.3
nmedA6<-0.2
```

Ass. 2: simmilar λ_k

```
nmedA1<-0.75
nmedA2<-0.7
nmedA3<-0.62
nmedA4<-0.57
nmedA5<-0.55
nmedA6<-0.35
```

```
wA1<-0.4
wA2<-0.15
wA3<-0.15
wA4<-0.1
wA5<-0.1
```

```
wA6<-0.1

nmed<-
nmedA1*wA1+nmedA2*wA2+nmedA3*wA3+nmedA4*wA4+nmedA5*wA5+nmedA
6*wA6;nmed

w<-c(wA1,wA2,wA3,wA4,wA5,wA6)

nmed<-c(nmedA1,nmedA2,nmedA3,nmedA4,nmedA5,nmedA6)
cita<-c(0.25,0.5,0.75,1.25,1.5,1.75)

P<-matrix(0,6,6)
Pij<-matrix(0,6*6,6)

a<-1
for(i in 1:6){
  for(j in 1:6){
    p0<-dpois(0, nmed[i]*cita[j]);p0
    p1<-dpois(1, nmed[i]*cita[j]);p1
    p2<-dpois(2, nmed[i]*cita[j]);p2
    p3<-dpois(3, nmed[i]*cita[j]);p3
    p4<-dpois(4, nmed[i]*cita[j]);p4

    P<-
matrix(c(p0,p0,0,0,0,0,p1,0,p0,0,0,0,p2,p1,0,p0,0,0,p3,p2,p1,0,p0,0,p4,p3,p2,p1,0,p0,1-
p0-p1-p2-p3-p4,1-p0-p1-p2-p3,1-p0-p1-p2,1-p0-p1,1-p0,1-p0),nrow=6,ncol=6);P

    eigen(t(P))
    pi <- eigen(t(P))$vectors[,1]
    pi <- pi/sum(pi)
    pivep <- Re(pi)

    Pij[a,]<-pivep
    a=a+1
  }
}

Pij
wr<-c(rep(wA1,6),rep(wA2,6),rep(wA3,6),rep(wA4,6),rep(wA5,6),rep(wA6,6))

sum(w)
sum(wr)

#We change the notation of the formula above:

Pij
wr
```

```
pil<-
c((1/6)*sum(Pij[,1]*wr),(1/6)*sum(Pij[,2]*wr),(1/6)*sum(Pij[,3]*wr),(1/6)*sum(Pij[,4]
*wr),(1/6)*sum(Pij[,5]*wr),(1/6)*sum(Pij[,6]*wr));pil
sum(pil)
```

<u>Deriving the Segmented RPP through Denuit & Charpentier formulation (cita normal drivers)</u>

```
#Pr[Θ=0.25|L=l]

P025l<-matrix(0,6,6)
P025l[1,]<-(1/(6*pil))*(w[1]*Pij[1,])
for(i in 2:6){
  P025l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+1,])
}

P025l<-colSums(P025l)


#Pr[Θ=0.5|L=l]

P05l<-matrix(0,6,6)
P05l[1,]<-(1/(6*pil))*(w[1]*Pij[2,])
for(i in 2:6){
  P05l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+2,])
}

P05l<-colSums(P05l)


#Pr[Θ=0.75|L=l]

P075l<-matrix(0,6,6)
P075l[1,]<-(1/(6*pil))*(w[1]*Pij[3,])
for(i in 2:6){
  P075l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+3,])
}

P075l<-colSums(P075l)

#Pr[Θ=1.25|L=l]

P125l<-matrix(0,6,6)
P125l[1,]<-(1/(6*pil))*(w[1]*Pij[4,])
for(i in 2:6){
  P125l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+4,])
}

P125l<-colSums(P125l)
```

```
#Pr[Θ=1.5|L=l]

P15l<-matrix(0,6,6)
P15l[1,]<-(1/(6*pil))*(w[1]*Pij[5,])
for(i in 2:6){
  P15l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+5,])
}

P15l<-colSums(P15l)

#Pr[Θ=1.75|L=l]

P175l<-matrix(0,6,6)
P175l[1,]<-(1/(6*pil))*(w[1]*Pij[6,])
for(i in 2:6){
  P175l[i,]<-(1/(6*pil))*(w[i]*Pij[6*(i-1)+6,])
}

P175l<-colSums(P175l)


# Segmented RPP

#E[Θ|L=l]=0.25·Pr[Θ=0.25|L=l]+...+1.75·Pr[Θ=1.75|L=l]

RPPPS<-0.25*P025l+0.5*P05l+0.75*P075l+1.25*P125l+1.5*P15l+1.75*P175l
```

Obtention of the Not segmented RPP with the Normberg's formula

Cita normal drivers

```
RPN<-matrix(0,6,6)
RPD<-matrix(0,6,6)
for(i in 1:6){
  RPN[i,]<-(w[i]*(cita[1]*Pij[6*(i-1)+1,]*(1/6)+cita[2]*Pij[6*(i-
1)+2,]*(1/6)+cita[3]*Pij[6*(i-1)+3,]*(1/6)+cita[4]*Pij[6*(i-
1)+4,]*(1/6)+cita[5]*Pij[6*(i-1)+5,]*(1/6)+cita[6]*Pij[6*(i-1)+6,]*(1/6)))
  RPD[i,]<-(w[i]*(Pij[6*(i-1)+1,]*(1/6)+Pij[6*(i-1)+2,]*(1/6)+Pij[6*(i-
1)+3,]*(1/6)+Pij[6*(i-1)+4,]*(1/6)+Pij[6*(i-1)+5,]*(1/6)+Pij[6*(i-1)+6,]*(1/6)))
}

PN<-colSums(RPN)
PD<-colSums(RPD)

RPPPSnorb<-PN/PD
```

Cita bad drivers

```
RPN<-matrix(0,6,6)
RPD<-matrix(0,6,6)
for(i in 1:6){
  RPN[i,]<-(w[i]*(cita[1]*Pij[6*(i-1)+1,]*(1/12)+cita[2]*Pij[6*(i-
1)+2,]*(1/12)+cita[3]*Pij[6*(i-1)+3,]*(1/12)+cita[4]*Pij[6*(i-
1)+4,]*(1/4)+cita[5]*Pij[6*(i-1)+5,]*(1/4)+cita[6]*Pij[6*(i-1)+6,]*(1/4)))
  RPD[i,]<-(w[i]*(Pij[6*(i-1)+1,]*(1/12)+Pij[6*(i-1)+2,]*(1/12)+Pij[6*(i-
1)+3,]*(1/12)+Pij[6*(i-1)+4,]*(1/4)+Pij[6*(i-1)+5,]*(1/4)+Pij[6*(i-1)+6,]*(1/4)))
}

PN<-colSums(RPN)
PD<-colSums(RPD)

RPPPSbad<-PN/PD
```

Cita good drivers

```
RPN<-matrix(0,6,6)
RPD<-matrix(0,6,6)
for(i in 1:6){
  RPN[i,]<-(w[i]*(cita[1]*Pij[6*(i-1)+1,]*(1/4)+cita[2]*Pij[6*(i-
1)+2,]*(1/4)+cita[3]*Pij[6*(i-1)+3,]*(1/4)+cita[4]*Pij[6*(i-
1)+4,]*(1/12)+cita[5]*Pij[6*(i-1)+5,]*(1/12)+cita[6]*Pij[6*(i-1)+6,]*(1/12)))
  RPD[i,]<-(w[i]*(Pij[6*(i-1)+1,]*(1/4)+Pij[6*(i-1)+2,]*(1/4)+Pij[6*(i-
1)+3,]*(1/4)+Pij[6*(i-1)+4,]*(1/12)+Pij[6*(i-1)+5,]*(1/12)+Pij[6*(i-1)+6,]*(1/12)))
}

PN<-colSums(RPN)
PD<-colSums(RPD)

RPPPSgood<-PN/PD
```

Note: If it has been executed the Ass .1 the RPPPS and RPPPSnor would be the segmented RPP for a light BMS when the λ_k are assumed different otherwise it would be the RPP for a light BMS but for λ_k that are considered similar.

Pure premium for unitary claim amount

Not segmented case

```
PBrg<-rep(0,6)
PPEXnormalNS<-matrix(0,6,6)
PPEXbadNS<-matrix(0,6,6)
PPEXgoodNS<-matrix(0,6,6)
for(i in 1:6){
  PBrg[i]<-nmed[i]
```

```
  PPEXnormalNS[i,]<-RPPPNS*PBrg[i]
  PPEXbadNS[i,]<-RPPPNSbad*PBrg[i]
  PPEXgoodNS[i,]<-RPPPNSgood*PBrg[i]
}

rownames(PPEXnormalNS)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXbadNS)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXgoodNS)<-c("RG1","RG2","RG3","RG4","RG5","RG6")

PPEXnormalNS
PPEXbadNS
PPEXgoodNS
```

Segmented case

```
PBrg<-rep(0,6)
PPEXnormal<-matrix(0,6,6)
PPEXbad<-matrix(0,6,6)
PPEXgood<-matrix(0,6,6)
for(i in 1:6){
  PBrg[i]<-nmed[i]

  PPEXnormal[i,]<-RPPPS*PBrg[i]
  PPEXbad[i,]<-RPPPSbad*PBrg[i]
  PPEXgood[i,]<-RPPPSgood*PBrg[i]
}

rownames(PPEXnormal)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXbad)<-c("RG1","RG2","RG3","RG4","RG5","RG6")
rownames(PPEXgood)<-c("RG1","RG2","RG3","RG4","RG5","RG6")

PPEXnormal
PPEXbad
PPEXgood
```

Note: If it has been executed the Ass .1 all the PPEX and PPEXNS would correspond with the light and different case while if the Ass. 2 was the once excited they would made reference to the light and similar case.

From the following chunk of code, it must be executed the once that correspond with the assumption considered above:

Ass. 1: different λ_k

```
LDPPEXnormalNS<-PPEXnormalNS
LDPPEXnormal<-PPEXnormal
LDPPEXbadNS<-PPEXbadNS
LDPPEXbad<-PPEXbad
LDPPEXgoodNS<-PPEXgoodNS
LDPPEXgood<-PPEXgood
```

Ass. 2: simmilar λ_k

```
LSPPEXnormalNS<-PPEXnormalNS
LPPEXnormal<-PPEXnormal
LSPPEXbadNS<-PPEXbadNS
LSPPEXbad<-PPEXbad
LSPPEXgoodNS<-PPEXgoodNS
LSPPEXgood<-PPEXgood
```

SIMULATION AND GRAPHIC OF THE PATHS FOR NORMAL DRIVERS

Note: It is important to first execute the above code for the obtention of the four combinations of assumptions of punishment per claim and values of λ_k: strict and different, strict and similar, light and different and light and similar.

Different λ_k case

```
#10,000 insureds
n<-10000


#Simulation for lambda different
sD<-matrix(0,10000,5)

for(i in 1:5){
   set.seed(i)
   s1<-rpois(n*0.4,0.9)
   s2<-rpois(n*0.15,0.7)
   s3<-rpois(n*0.15,0.6)
   s4<-rpois(n*0.1,0.4)
   s5<-rpois(n*0.1,0.3)
   s6<-rpois(n*0.1,0.2)

   sD[,i]<-c(s1,s2,s3,s4,s5,s6)
}

colnames(sD)<-c("Y1","Y2","Y3","Y4","Y5")

sD


#Simulation for lambda similar
sS<-matrix(0,10000,5)

for(i in 1:5){
   set.seed(i+10)
   s1<-rpois(n*0.4,0.75)
   s2<-rpois(n*0.15,0.7)
   s3<-rpois(n*0.15,0.62)
   s4<-rpois(n*0.1,0.57)
   s5<-rpois(n*0.1,0.55)
   s6<-rpois(n*0.1,0.35)

   sS[,i]<-c(s1,s2,s3,s4,s5,s6)
```

```
}

colnames(sS)<-c("Y1","Y2","Y3","Y4","Y5")

sS
```

Selection of random individual RG1 (lambda=0.9 (D) and 0.75 (S))) for the good driver's case

```
#case a: the individual presents a behavior concordant with its claim frequency

options(max.print = 99999)
sD[1:4000,]
SA<-sD[1:4000,]
which(SA[,1]==0 & SA[,2]==1 & SA[,3]==0 & SA[,4]==1 & SA[,5]==2)
sD[3889,]


SB<-sS[1:4000,]
which(SB[,1]==0 & SB[,2]==1 & SB[,3]==0 & SB[,4]==1 & SB[,5]==2)

sS[3972,]


#different lambdas
classS<-c(4,6,5,6,6)

#simmilar lambdas
classL<-c(4,5,4,5,6)



#Generation of the trajectories

PTNSSandD<-
c(SDPPEXnormalNS[1,classS[1]],SDPPEXnormalNS[1,classS[2]],SDPPEXnormalNS[
1,classS[3]],SDPPEXnormalNS[1,classS[4]],SDPPEXnormalNS[1,classS[5]]);PTNSSa
ndD

PTSSandD<-
c(SDPPEXnormal[1,classS[1]],SDPPEXnormal[1,classS[2]],SDPPEXnormal[1,classS[
3]], SDPPEXnormal[1,classS[4]],SDPPEXnormal[1,classS[5]]);PTSSandD

PTNSSandS<-
c(SSPPEXnormalNS[1,classS[1]],SSPPEXnormalNS[1,classS[2]],SSPPEXnormalNS[1
,classS[3]],SSPPEXnormalNS[1,classS[4]],SSPPEXnormalNS[1,classS[5]]);PTNSSa
ndS
```

```
PTSSandS<-
c(SSPPEXnormal[1,classS[1]],SSPPEXnormal[1,classS[2]],SSPPEXnormal[1,classS[3]
], SSPPEXnormal[1,classS[4]],SSPPEXnormal[1,classS[5]]);PTSSandS

PTNSLandD<-
c(LDPPEXnormalNS[1,classL[1]],LDPPEXnormalNS[1,classL[2]],LDPPEXnormalNS
[1,classL[3]],LDPPEXnormalNS[1,classL[4]],LDPPEXnormalNS[1,classL[5]]);PTNSL
andD

PTSLandD<-
c(LDPPEXnormal[1,classL[1]],LDPPEXnormal[1,classL[2]],LDPPEXnormal[1,classL[
3]], LDPPEXnormal[1,classL[4]],LDPPEXnormal[1,classL[5]]);PTSLandD

PTNSLandS<-
c(LSPPEXnormalNS[1,classL[1]],LSPPEXnormalNS[1,classL[2]],LSPPEXnormalNS[
1,classL[3]],LSPPEXnormalNS[1,classL[4]],LSPPEXnormalNS[1,classL[5]]);LSPTNS
LandS

PTSLandS<-
c(LSPPEXnormal[1,classL[1]],LSPPEXnormal[1,classL[2]],LSPPEXnormal[1,classL[3
]], LSPPEXnormal[1,classL[4]],LSPPEXnormal[1,classL[5]]);PTSLandS


#Plots for the individual in RG1

y<-c(0.9,0.9,0.9,0.9,0.9)

#SandD drivers
plot(PTNSSandD, type = "l", ylim=c(0.3,1.3), col="red", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandD, type = "l", col="red", lty=2)
lines(y, type="l")

#LandD drivers
lines(PTNSLandD, type = "l", col="blue")
lines(PTSLandD, type = "l", col="blue", lty=2)

legend(4.405, 0.643, legend=c("SandD NS", "SandD S","LandD NS","LandD S"),
    col=c("red", "red","blue","blue"), lty=1:2, cex=0.6)


y<-c(0.75,0.75,0.75,0.75,0.75)

#SandS drivers
plot(PTNSSandS, type = "l",ylim=c(0.3,1.3), col="orange", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandS, type = "l", col="orange", lty=2)
lines(y, type="l")

#LandS drivers
```

```
lines(PTNSLandS, type = "l", col="green")
lines(PTSLandS, type = "l", col="green", lty=2)

legend(4.405, 0.6431, legend=c("SandS NS", "SandS S","LandS NS","LandS S"),
    col=c("orange", "orange","green","green"), lty=1:2, cex=0.6)

#case b: the individual does not presents a behavior concordant with its claim frequency

options(max.print = 99999)
sD[1:4000,]
SA<-sD[1:4000,]
which(SA[,1]==0 & SA[,2]==0 & SA[,3]==0 & SA[,4]==0 & SA[,5]==1)
sD[725,]


SB<-sS[1:4000,]
which(SB[,1]==0 & SB[,2]==0 & SB[,3]==0 & SB[,4]==0 & SB[,5]==1)

sS[2774,]


#different lambdas

classS<-c(4,3,2,1,4)

#simmilar lambdas

classL<-c(4,3,2,1,2)

#Generation of the trajectories

PTNSSandD<-
c(SDPPEXnormalNS[1,classS[1]],SDPPEXnormalNS[1,classS[2]],SDPPEXnormalNS[
1,classS[3]],SDPPEXnormalNS[1,classS[4]],SDPPEXnormalNS[1,classS[5]]);PTNSSa
ndD

PTSSandD<-
c(SDPPEXnormal[1,classS[1]],SDPPEXnormal[1,classS[2]],SDPPEXnormal[1,classS[
3]], SDPPEXnormal[1,classS[4]],SDPPEXnormal[1,classS[5]]);PTSSandD

PTNSSandS<-
c(SSPPEXnormalNS[1,classS[1]],SSPPEXnormalNS[1,classS[2]],SSPPEXnormalNS[1
,classS[3]],SSPPEXnormalNS[1,classS[4]],SSPPEXnormalNS[1,classS[5]]);PTNSSand
S

PTSSandS<-
c(SSPPEXnormal[1,classS[1]],SSPPEXnormal[1,classS[2]],SSPPEXnormal[1,classS[3]
], SSPPEXnormal[1,classS[4]],SSPPEXnormal[1,classS[5]]);PTSSandS
```

```
PTNSLandD<-
c(LDPPEXnormalNS[1,classL[1]],LDPPEXnormalNS[1,classL[2]],LDPPEXnormalNS
[1,classL[3]],LDPPEXnormalNS[1,classL[4]],LDPPEXnormalNS[1,classL[5]]);PTNSL
andD

PTSLandD<-
c(LDPPEXnormal[1,classL[1]],LDPPEXnormal[1,classL[2]],LDPPEXnormal[1,classL[
3]], LDPPEXnormal[1,classL[4]],LDPPEXnormal[1,classL[5]]);PTSLandD

PTNSLandS<-
c(LSPPEXnormalNS[1,classL[1]],LSPPEXnormalNS[1,classL[2]],LSPPEXnormalNS[
1,classL[3]],

LSPPEXnormalNS[1,classL[4]],LSPPEXnormalNS[1,classL[5]]);LSPTNSLandS
PTSLandS<-
c(LSPPEXnormal[1,classL[1]],LSPPEXnormal[1,classL[2]],LSPPEXnormal[1,classL[3
]], LSPPEXnormal[1,classL[4]],LSPPEXnormal[1,classL[5]]);PTSLandS


#Plots for the individual in RG1

y<-c(0.9,0.9,0.9,0.9,0.9)

#SandD drivers
plot(PTNSSandD, type = "l", ylim=c(0.3,1.3), col="red", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandD, type = "l", col="red", lty=2)
lines(y, type="l")

#LandD drivers
lines(PTNSLandD, type = "l", col="blue")
lines(PTSLandD, type = "l", col="blue", lty=2)

legend(4.405, 1.34, legend=c("SandD NS", "SandD S","LandD NS","LandD S"),
      col=c("red", "red","blue","blue"), lty=1:2, cex=0.6)

y<-c(0.75,0.75,0.75,0.75,0.75)

#SandS drivers
plot(PTNSSandS, type = "l",ylim=c(0.3,1.3), col="orange", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandS, type = "l", col="orange", lty=2)
lines(y, type="l")

#LandS drivers
lines(PTNSLandS, type = "l", col="green")
lines(PTSLandS, type = "l", col="green", lty=2)

legend(4.41, 1.34, legend=c("SandS NS", "SandS S","LandS NS","LandS S"),
      col=c("orange", "orange","green","green"), lty=1:2, cex=0.6)
```

Selection of random individual RG5 (lambda=0.3 (D) and 0.55 (S)) for the good driver's case

```
#case a: the individual presents a behavior concordant with its claim frequency


options(max.print = 99999)
sD[8001:9000,]
SA<-sD[8001:9000,]
which(SA[,1]==0 & SA[,2]==0 & SA[,3]==1 & SA[,4]==0 & SA[,5]==1)
sD[8000+10,]


SB<-sS[8001:9000,]
which(SB[,1]==0 & SB[,2]==0 & SB[,3]==1 & SB[,4]==0 & SB[,5]==1)

sS[8000+819,]

#different lambdas
classS<-c(4,3,6,5,6)

#simmilar lambdas
classL<-c(4,3,4,3,4)


#Generation of the trajectories

PTNSSandD<-
c(SDPPEXnormalNS[5,classS[1]],SDPPEXnormalNS[5,classS[2]],SDPPEXnormalNS[
5,classS[3]],SDPPEXnormalNS[5,classS[4]],SDPPEXnormalNS[5,classS[5]]);PTNSSa
ndD

PTSSandD<-
c(SDPPEXnormal[5,classS[1]],SDPPEXnormal[5,classS[2]],SDPPEXnormal[5,classS[
3]], SDPPEXnormal[5,classS[4]],SDPPEXnormal[5,classS[5]]);PTSSandD

PTNSSandS<-
c(SSPPEXnormalNS[5,classS[1]],SSPPEXnormalNS[5,classS[2]],SSPPEXnormalNS[5
,classS[3]],SSPPEXnormalNS[5,classS[4]],SSPPEXnormalNS[5,classS[5]]);PTNSSand
S

PTSSandS<-
c(SSPPEXnormal[5,classS[1]],SSPPEXnormal[5,classS[2]],SSPPEXnormal[5,classS[3]
], SSPPEXnormal[5,classS[4]],SSPPEXnormal[5,classS[5]]);PTSSandS
```

```
PTNSLandD<-
c(LDPPEXnormalNS[5,classL[1]],LDPPEXnormalNS[5,classL[2]],LDPPEXnormalNS
[5,classL[3]],LDPPEXnormalNS[5,classL[4]],LDPPEXnormalNS[5,classL[5]]);PTNSL
andD
PTSLandD<-
c(LDPPEXnormal[5,classL[1]],LDPPEXnormal[5,classL[2]],LDPPEXnormal[5,classL[
3]], LDPPEXnormal[5,classL[4]],LDPPEXnormal[5,classL[5]]);PTSLandD

PTNSLandS<-
c(LSPPEXnormalNS[5,classL[1]],LSPPEXnormalNS[5,classL[2]],LSPPEXnormalNS[
5,classL[3]],LSPPEXnormalNS[5,classL[4]],LSPPEXnormalNS[5,classL[5]]);LSPTNS
LandS

PTSLandS<-
c(LSPPEXnormal[5,classL[1]],LSPPEXnormal[5,classL[2]],LSPPEXnormal[5,classL[3
]], LSPPEXnormal[5,classL[4]],LSPPEXnormal[5,classL[5]]);PTSLandS


#Plots for the individual in RG5

y<-c(0.3,0.3,0.3,0.3,0.3)

#SandD drivers
plot(PTNSSandD, type = "l", ylim=c(0,0.8), col="red", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandD, type = "l", col="red", lty=2)
lines(y, type="l")

#LandD drivers
lines(PTNSLandD, type = "l", col="blue")
lines(PTSLandD, type = "l", col="blue", lty=2)

legend(4.405, 0.273, legend=c("SandD NS", "SandD S","LandD NS","LandD S"),
      col=c("red", "red","blue","blue"), lty=1:2, cex=0.6)


y<-c(0.55,0.55,0.55,0.55,0.55)

#SandS drivers
plot(PTNSSandS, type = "l",ylim=c(0,0.8), col="orange", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandS, type = "l", col="orange", lty=2)
lines(y, type="l")

#LandS drivers
lines(PTNSLandS, type = "l", col="green")
lines(PTSLandS, type = "l", col="green", lty=2)

legend(4.405, 0.2732, legend=c("SandS NS", "SandS S","LandS NS","LandS S"),
      col=c("orange", "orange","green","green"), lty=1:2, cex=0.6)
```

#case b: the individual does not presents a behavior concordant with its claim frequency

```
options(max.print = 99999)
sD[8001:9000,]
SA<-sD[8001:9000,]
which(SA[,1]==0 & SA[,2]==1 & SA[,3]==1 & SA[,4]==0 & SA[,5]==2)
sD[8000+399,]


SB<-sS[8001:9000,]
which(SB[,1]==0 & SB[,2]==1 & SB[,3]==1 & SB[,4]==0 & SB[,5]==2)

sS[8000+800,]

#different lambdas

classS<-c(4,6,6,5,6)

#simmilar lambdas

classL<-c(4,5,6,5,6)


#Generation of the trajectories

PTNSSandD<-
c(SDPPEXnormalNS[5,classS[1]],SDPPEXnormalNS[5,classS[2]],SDPPEXnormalNS[
5,classS[3]],SDPPEXnormalNS[5,classS[4]],SDPPEXnormalNS[5,classS[5]]);PTNSSa
ndD

PTSSandD<-
c(SDPPEXnormal[5,classS[1]],SDPPEXnormal[5,classS[2]],SDPPEXnormal[5,classS[
3]], SDPPEXnormal[5,classS[4]],SDPPEXnormal[5,classS[5]]);PTSSandD

PTNSSandS<-
c(SSPPEXnormalNS[5,classS[1]],SSPPEXnormalNS[5,classS[2]],SSPPEXnormalNS[5
,classS[3]],SSPPEXnormalNS[5,classS[4]],SSPPEXnormalNS[5,classS[5]]);PTNSSand
S

PTSSandS<-
c(SSPPEXnormal[5,classS[1]],SSPPEXnormal[5,classS[2]],SSPPEXnormal[5,classS[3]
], SSPPEXnormal[5,classS[4]],SSPPEXnormal[5,classS[5]]);PTSSandS

PTNSLandD<-
c(LDPPEXnormalNS[5,classL[1]],LDPPEXnormalNS[5,classL[2]],LDPPEXnormalNS
[5,classL[3]],LDPPEXnormalNS[5,classL[4]],LDPPEXnormalNS[5,classL[5]]);PTNSL
andD
```

```
PTSLandD<-
c(LDPPEXnormal[5,classL[1]],LDPPEXnormal[5,classL[2]],LDPPEXnormal[5,classL[
3]], LDPPEXnormal[5,classL[4]],LDPPEXnormal[5,classL[5]]);PTSLandD

PTNSLandS<-
c(LSPPEXnormalNS[5,classL[1]],LSPPEXnormalNS[5,classL[2]],LSPPEXnormalNS[
5,classL[3]],LSPPEXnormalNS[5,classL[4]],LSPPEXnormalNS[5,classL[5]]);LSPTNS
LandS

PTSLandS<-
c(LSPPEXnormal[5,classL[1]],LSPPEXnormal[5,classL[2]],LSPPEXnormal[5,classL[3
]], LSPPEXnormal[5,classL[4]],LSPPEXnormal[5,classL[5]]);PTSLandS



y<-c(0.3,0.3,0.3,0.3,0.3)

#SandD drivers
plot(PTNSSandD, type = "l", ylim=c(0,0.8), col="red", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandD, type = "l", col="red", lty=2)
lines(y, type="l")

#LandD drivers
lines(PTNSLandD, type = "l", col="blue")
lines(PTSLandD, type = "l", col="blue", lty=2)

legend(4.405, 0.2731, legend=c("SandD NS", "SandD S","LandD NS","LandD S"),
    col=c("red", "red","blue","blue"), lty=1:2, cex=0.6)

#Plots for the individual in RG5

y<-c(0.55,0.55,0.55,0.55,0.55)

#SandS drivers
plot(PTNSSandS, type = "l",ylim=c(0,0.8), col="orange", xlab = "Year", ylab = "Pure
premiums")
lines(PTSSandS, type = "l", col="orange", lty=2)
lines(y, type="l")

#LandS drivers
lines(PTNSLandS, type = "l", col="green")
lines(PTSLandS, type = "l", col="green", lty=2)

legend(4.405, 0.2731, legend=c("SandS NS", "SandS S","LandS NS","LandS S"),
    col=c("orange", "orange","green","green"), lty=1:2, cex=0.6)
```

SIMULATION AND GRAPHIC OF THE PATHS EFECT OF A SURCHARGE

```
for(i in 1:6){
  par(mfrow=c(2,2))

  #S and D
  #            Y1 Y2 Y3 Y4 Y5
  # claims       0  0  0  0  2
  # class strict   4  3  2  1  4

  y<-c(0.6,0.6,0.6,0.6,0.6)
  yb<-c(0.6*1.25,0.6*1.25,0.6*1.25,0.6*1.25,0.6*1.25)

  classS<-c(4,3,2,1,4)

  PTNSSandDn<-
c(SDPPEXnormalNS[i,1],SDPPEXnormalNS[i,2],SDPPEXnormalNS[i,3],

SDPPEXnormalNS[i,4],SDPPEXnormalNS[i,5],SDPPEXnormalNS[i,6]);PTNSSandDn
  PTSSandDn<-c(SDPPEXnormal[i,1],SDPPEXnormal[i,2],SDPPEXnormal[i,3],
         SDPPEXnormal[i,4],SDPPEXnormal[i,5],SDPPEXnormal[i,6]);PTSSandDn

  PTNSSandDb<-c(SDPPEXbadNS[i,1],SDPPEXbadNS[i,2],SDPPEXbadNS[i,3],

SDPPEXbadNS[i,4],SDPPEXbadNS[i,5],SDPPEXbadNS[i,6]);PTNSSandDb
  PTSSandDb<-c(SDPPEXbad[i,1],SDPPEXbad[i,2],SDPPEXbad[i,3],
         SDPPEXbad[i,4],SDPPEXbad[i,5],SDPPEXbad[i,6]);PTSSandDb

  plot(PTNSSandDn, type = "l", ylim=c(0.1,1.3), col="blue", xlab = "Class", ylab =
"Pure premiums", main = "Strict and Different")
  lines(PTSSandDn, type = "l", col="blue", lty=2)
  lines(PTNSSandDb, type = "l", col="red")
  lines(PTSSandDb, type = "l", col="red", lty=2)

  #S and S
  #            Y1 Y2 Y3 Y4 Y5
  # claims       0  0  0  0  1
  # class strict   4  3  2  1  2

  y<-c(0.62,0.62,0.62,0.62,0.62)
  yb<-c(0.62*1.25,0.62*1.25,0.62*1.25,0.62*1.25,0.62*1.25)

  classS<-c(4,3,2,1,2)

  PTNSSandSn<-
c(SSPPEXnormalNS[i,1],SSPPEXnormalNS[i,2],SSPPEXnormalNS[i,3],

SSPPEXnormalNS[i,4],SSPPEXnormalNS[i,5],SSPPEXnormalNS[i,6]);PTNSSandDn
  PTSSandSn<-c(SSPPEXnormal[i,1],SSPPEXnormal[i,2],SSPPEXnormal[i,3],
         SSPPEXnormal[i,4],SSPPEXnormal[i,5],SSPPEXnormal[i,6]);PTSSandDn
```

```
PTNSSandSb<-c(SSPPEXbadNS[i,1],SSPPEXbadNS[i,2],SSPPEXbadNS[i,3],
          SSPPEXbadNS[i,4],SSPPEXbadNS[i,5],SSPPEXbadNS[i,6]);PTNSSandDb
PTSSandSb<-c(SSPPEXbad[i,1],SSPPEXbad[i,2],SSPPEXbad[i,3],
          SSPPEXbad[i,4],SSPPEXbad[i,5],SSPPEXbad[i,6]);PTSSandDb


plot(PTNSSandSn, type = "l", ylim=c(0.1,1.3), col="blue", xlab = "Class", ylab =
"Pure premiums", main = "Strict and Similar")
lines(PTSSandSn, type = "l", col="blue", lty=2)
lines(PTNSSandSb, type = "l", col="red")
lines(PTSSandSb, type = "l", col="red", lty=2)

#L and D
#              Y1 Y2 Y3 Y4 Y5
# claims        1  0  0  0  0
# class strict  6  5  4  3  2

y<-c(0.6,0.6,0.6,0.6,0.6)#Hacer para todos!!!!!!!!
yb<-c(0.6*1.25,0.6*1.25,0.6*1.25,0.6*1.25,0.6*1.25)


classS<-c(6,5,4,3,2)

PTNSLandDn<-
c(LDPPEXnormalNS[i,1],LDPPEXnormalNS[i,2],LDPPEXnormalNS[i,3],

LDPPEXnormalNS[i,4],LDPPEXnormalNS[i,5],LDPPEXnormalNS[i,6]);PTNSSandD
n
PTSLandDn<-c(LDPPEXnormal[i,1],LDPPEXnormal[i,2],LDPPEXnormal[i,3],
          LDPPEXnormal[i,4],LDPPEXnormal[i,5],LDPPEXnormal[i,6]);PTSSandDn

PTNSLandDb<-c(LDPPEXbadNS[i,1],LDPPEXbadNS[i,2],LDPPEXbadNS[i,3],

LDPPEXbadNS[i,4],LDPPEXbadNS[i,5],LDPPEXbadNS[i,6]);PTNSSandDb
PTSLandDb<-c(LDPPEXbad[i,1],LDPPEXbad[i,2],LDPPEXbad[i,3],
          LDPPEXbad[i,4],LDPPEXbad[i,5],LDPPEXbad[i,6]);PTSSandDb


plot(PTNSLandDn, type = "l", ylim=c(0.1,1.3), col="blue", xlab = "Class", ylab =
"Pure premiums", main = "Light and Different")
lines(PTSLandDn, type = "l", col="blue", lty=2)
lines(PTNSLandDb, type = "l", col="red")
lines(PTSLandDb, type = "l", col="red", lty=2)
```

```
#L and S
#             Y1 Y2 Y3 Y4 Y5
# claims       1  0  0  0  0
# class strict  6  5  4  3  2

y<-c(0.62,0.62,0.62,0.62,0.62)
yb<-c(0.62*1.25,0.62*1.25,0.62*1.25,0.62*1.25,0.62*1.25)

#Buscar que exista un individuo así en la muestra y decir su número de
observación!!!

classS<-c(6,5,4,3,2)

PTNSLandSn<-
c(LSPPEXnormalNS[i,1],LSPPEXnormalNS[i,2],LSPPEXnormalNS[i,3],

LSPPEXnormalNS[i,4],LSPPEXnormalNS[i,5],LSPPEXnormalNS[i,6]);PTNSSandDn
PTSLandSn<-c(LSPPEXnormal[i,1],LSPPEXnormal[i,2],LSPPEXnormal[i,3],
         LSPPEXnormal[i,4],LSPPEXnormal[i,5],LSPPEXnormal[i,6]);PTSSandDn

PTNSLandSb<-c(LSPPEXbadNS[i,1],LSPPEXbadNS[i,2],LSPPEXbadNS[i,3],
          LSPPEXbadNS[i,4],LSPPEXbadNS[i,5],LSPPEXbadNS[i,6]);PTNSSandDb
PTSLandSb<-c(LSPPEXbad[i,1],LSPPEXbad[i,2],LSPPEXbad[i,3],
         LSPPEXbad[i,4],LSPPEXbad[i,5],LSPPEXbad[i,6]);PTSSandDb


plot(PTNSLandSn, type = "l", ylim=c(0.1,1.3), col="blue", xlab = "Class", ylab =
"Pure premiums", main = "Light and Similar")
lines(PTSLandSn, type = "l", col="blue", lty=2)
lines(PTNSLandSb, type = "l", col="red")
lines(PTSLandSb, type = "l", col="red", lty=2)
#lines(y, type="l")
#lines(yb, type="l", col="green")
}
```

First observations in the simulation:

```
> head(sD)
     Y1 Y2 Y3 Y4 Y5
[1,]  0  0  0  1  0
[2,]  0  1  2  0  1
[3,]  1  1  0  0  2
[4,]  2  0  0  0  0
[5,]  0  3  1  2  0
[6,]  2  3  1  0  1
```

```
> head(sS)
     Y1 Y2 Y3 Y4 Y5
[1,]  0  0  1  0  1
[2,]  0  1  0  1  0
[3,]  1  2  0  2  3
[4,]  0  0  0  1  1
[5,]  0  0  3  3  0
[6,]  2  0  0  1  3
```

First observations in the simulation:

```
> tail(sD)
         Y1 Y2 Y3 Y4 Y5
 [9995,]  0  0  2  0  1
 [9996,]  0  0  0  0  0
 [9997,]  0  1  0  0  0
 [9998,]  0  0  0  0  0
 [9999,]  0  0  0  0  0
[10000,]  0  1  0  0  0
```

```
> tail(sS)
         Y1 Y2 Y3 Y4 Y5
 [9995,]  0  0  0  1  0
 [9996,]  0  1  0  0  0
 [9997,]  0  0  0  1  0
 [9998,]  0  0  0  1  0
 [9999,]  1  1  2  0  0
[10000,]  1  0  2  0  0
```

Annex 2: Verification that the segmented case provides closer premiums to the claim frequency

For the RPP:

S and D

NOT SEGMENTED							SEGMENTED													
>	RPPPNS						>	RPPPS						1	2	3	4	5	6	
[1]	0.411	0.612	0.726	0.872	1.136	1.296	[1]	0.541	0.732	0.806	0.905	1.123	1.283	Seg	Seg	Seg	Seg	Seg	Seg	0
>	RPPPNSbad						>	RPPPSbad												
[1]	0.472	0.798	0.960	1.124	1.330	1.430	[1]	0.727	0.982	1.063	1.160	1.324	1.424	Seg	Seg	Seg	Seg	Seg	Seg	0
>	RPPPNSgood						>	RPPPSgood												
[1]	0.389	0.523	0.588	0.677	0.895	1.072	[1]	0.455	0.579	0.626	0.694	0.880	1.054	Seg	Seg	Seg	Seg	Seg	Seg	0

S and S

NOT SEGMENTED							SEGMENTED													
>	RPPPNS						>	RPPPS						1	2	3				
[1]	0.411	0.612	0.726	0.872	1.136	1.296	[1]	0.440	0.644	0.748	0.882	1.135	1.294	Seg	Seg	Seg	Seg	Seg	Seg	0
>	RPPPNSbad						>	RPPPSbad												
[1]	0.472	0.798	0.960	1.124	1.330	1.430	[1]	0.535	0.851	0.990	1.134	1.330	1.429	Seg	Seg	Seg	Seg	Seg	Seg	0
>	RPPPNSgood						>	RPPPSgood												
[1]	0.389	0.523	0.588	0.677	0.895	1.072	[1]	0.404	0.537	0.598	0.682	0.893	1.068	Seg	Seg	Seg	Seg	Seg	Seg	0
														el indiv						

L and D

NOT SEGMENTED							SEGMENTED													
>	RPPPNS						>	RPPPS												
[1]	0.466	0.665	0.921	1.178	1.366	1.481	[1]	0.615	0.798	0.964	1.134	1.302	1.438	Seg	Seg	Seg	Seg	Seg	Seg	0
>	RPPPNSbad						>	RPPPSbad												
[1]	0.537	0.845	1.140	1.337	1.449	1.518	[1]	0.835	1.049	1.199	1.320	1.420	1.497	Seg	Seg	Seg	Seg	Seg	Seg	0
>	RPPPNSgood						>	RPPPSgood												
[1]	0.439	0.576	0.745	0.963	1.199	1.390	[1]	0.502	0.629	0.758	0.909	1.099	1.304	Seg	Seg	NO Seg	Seg	Seg	Seg	1
															NS	0.005				
															S	-0.01				
															DIF	0.013				

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NOT SEGMENTED							SEGMENTED													
>	RPPPNS						>	RPPPS												
[1]	0.466	0.665	0.921	1.178	1.366	1.481	[1]	0.504	0.708	0.943	1.172	1.352	1.470	Seg	Seg	Seg	Seg	Seg	Seg	0
>	RPPPNSbad						>	RPPPSbad												
[1]	0.537	0.845	1.140	1.337	1.449	1.518	[1]	0.626	0.919	1.167	1.337	1.443	1.512	Seg	Seg	Seg	NO Seg	Seg	Seg	1
>	RPPPNSgood						>	RPPPSgood												
[1]	0.439	0.576	0.745	0.963	1.199	1.390	[1]	0.455	0.593	0.755	0.953	1.174	1.367	Seg	Seg	Seg	Seg	Seg	Seg	0
															NS	-0.09				
															S	-0.09				
															DIF	6E-04				

Formula:

>	RPPPNS						>	RPPPS						1	2	3	4	5	
[1]	0.411	0.612	0.726	0.872	1.136	1.296	[1]	0.541	0.732	0.806	0.905	1.123	1.283	Seg	Seg	Seg	Seg	Seg	=SI(ABS(1-H6)>ABS(1-P6);"Seg"; "NO Seg")

For the premiums in units of E[X]:

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NOT SEGMENTED

> PPEXnormalNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.370	0.551	0.654	0.785	1.023	1.167
RG2	0.288	0.429	0.508	0.610	0.795	0.907
RG3	0.247	0.367	0.436	0.523	0.682	0.778
RG4	0.164	0.245	0.291	0.349	0.454	0.519
RG5	0.123	0.184	0.218	0.262	0.341	0.389
RG6	0.082	0.122	0.145	0.174	0.227	0.259

>PPEXbadNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.425	0.718	0.864	1.011	1.197	1.287
RG2	0.330	0.558	0.672	0.787	0.931	1.001
RG3	0.283	0.479	0.576	0.674	0.798	0.858
RG4	0.189	0.319	0.384	0.449	0.532	0.572
RG5	0.142	0.239	0.288	0.337	0.399	0.429
RG6	0.094	0.160	0.192	0.225	0.266	0.286

>PPEXgoodNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.350	0.471	0.529	0.609	0.806	0.965
RG2	0.272	0.366	0.411	0.474	0.627	0.750
RG3	0.233	0.314	0.353	0.406	0.537	0.643
RG4	0.155	0.209	0.235	0.271	0.358	0.429
RG5	0.117	0.157	0.176	0.203	0.269	0.322
RG6	0.078	0.105	0.118	0.135	0.179	0.214

SEGMENTED

> PPEXnormal

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.487	0.659	0.725	0.815	1.011	1.155
RG2	0.379	0.513	0.564	0.634	0.786	0.898
RG3	0.325	0.439	0.483	0.543	0.674	0.770
RG4	0.216	0.293	0.322	0.362	0.449	0.513
RG5	0.162	0.220	0.242	0.272	0.337	0.385
RG6	0.108	0.146	0.161	0.181	0.225	0.257

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

> PPEXbad

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.655	0.884	0.957	1.044	1.192	1.281
RG2	0.509	0.687	0.744	0.812	0.927	0.997
RG3	0.436	0.589	0.638	0.696	0.794	0.854
RG4	0.291	0.393	0.425	0.464	0.530	0.569
RG5	0.218	0.295	0.319	0.348	0.397	0.427
RG6	0.145	0.196	0.213	0.232	0.265	0.285

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

>PPEXgood

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.410	0.521	0.564	0.625	0.792	0.948
RG2	0.319	0.405	0.438	0.486	0.616	0.737
RG3	0.273	0.347	0.376	0.417	0.528	0.632
RG4	0.182	0.231	0.251	0.278	0.352	0.421
RG5	0.137	0.174	0.188	0.208	0.264	0.316
RG6	0.091	0.116	0.125	0.139	0.176	0.211

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

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> PPEXnormalNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.308	0.459	0.545	0.654	0.852	0.972
RG2	0.288	0.429	0.508	0.610	0.795	0.907
RG3	0.255	0.380	0.450	0.541	0.704	0.804
RG4	0.234	0.349	0.414	0.497	0.648	0.739
RG5	0.226	0.337	0.400	0.480	0.625	0.713
RG6	0.144	0.214	0.254	0.305	0.398	0.454

> PPEXbadNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.354	0.598	0.720	0.843	0.998	1.072
RG2	0.330	0.558	0.672	0.787	0.931	1.001
RG3	0.293	0.494	0.595	0.697	0.825	0.887
RG4	0.269	0.455	0.547	0.640	0.758	0.815
RG5	0.260	0.439	0.528	0.618	0.732	0.786
RG6	0.165	0.279	0.336	0.393	0.466	0.500

> PPEXgoodNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.292	0.392	0.441	0.508	0.671	0.804
RG2	0.272	0.366	0.411	0.474	0.627	0.750
RG3	0.241	0.324	0.364	0.420	0.555	0.664
RG4	0.222	0.298	0.335	0.386	0.510	0.611
RG5	0.214	0.288	0.323	0.372	0.492	0.589
RG6	0.136	0.183	0.206	0.237	0.313	0.375

SEGMENTED

> PPEXnormal

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.330	0.483	0.561	0.661	0.851	0.970
RG2	0.308	0.451	0.524	0.617	0.794	0.906
RG3	0.273	0.399	0.464	0.547	0.703	0.802
RG4	0.251	0.367	0.426	0.503	0.647	0.737
RG5	0.242	0.354	0.411	0.485	0.624	0.712
RG6	0.154	0.225	0.262	0.309	0.397	0.453

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

> PPEXbad

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.401	0.638	0.742	0.851	0.997	1.072
RG2	0.374	0.596	0.693	0.794	0.931	1.000
RG3	0.331	0.528	0.614	0.703	0.824	0.886
RG4	0.305	0.485	0.564	0.646	0.758	0.814
RG5	0.294	0.468	0.544	0.624	0.731	0.786
RG6	0.187	0.298	0.346	0.397	0.465	0.500

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

> PPEXgood

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.303	0.403	0.448	0.511	0.669	0.801
RG2	0.283	0.376	0.419	0.477	0.625	0.747
RG3	0.250	0.333	0.371	0.423	0.553	0.662
RG4	0.230	0.306	0.341	0.389	0.509	0.609
RG5	0.222	0.295	0.329	0.375	0.491	0.587
RG6	0.141	0.188	0.209	0.239	0.312	0.374

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

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> PPEXnormalNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.419	0.599	0.829	1.060	1.229	1.333
RG2	0.326	0.466	0.645	0.825	0.956	1.037
RG3	0.280	0.399	0.553	0.707	0.820	0.889
RG4	0.186	0.266	0.368	0.471	0.546	0.593
RG5	0.140	0.200	0.276	0.353	0.410	0.444
RG6	0.093	0.133	0.184	0.236	0.273	0.296

> PPEXbadNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.484	0.760	1.026	1.203	1.304	1.366
RG2	0.376	0.591	0.798	0.936	1.014	1.062
RG3	0.322	0.507	0.684	0.802	0.869	0.911
RG4	0.215	0.338	0.456	0.535	0.579	0.607
RG5	0.161	0.253	0.342	0.401	0.435	0.455
RG6	0.107	0.169	0.228	0.267	0.290	0.304

> PPEXgoodNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.395	0.519	0.671	0.866	1.080	1.251
RG2	0.308	0.403	0.522	0.674	0.840	0.973
RG3	0.264	0.346	0.447	0.578	0.720	0.834
RG4	0.176	0.231	0.298	0.385	0.480	0.556
RG5	0.132	0.173	0.224	0.289	0.360	0.417
RG6	0.088	0.115	0.149	0.193	0.240	0.278

SEGMENTED

> PPEXnormal

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.554	0.718	0.868	1.020	1.171	1.294
RG2	0.431	0.558	0.675	0.794	0.911	1.007
RG3	0.369	0.479	0.578	0.680	0.781	0.863
RG4	0.246	0.319	0.386	0.454	0.521	0.575
RG5	0.185	0.239	0.289	0.340	0.390	0.431
RG6	0.123	0.160	0.193	0.227	0.260	0.288

> PPEXbad

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.751	0.944	1.079	1.188	1.278	1.347
RG2	0.584	0.734	0.839	0.924	0.994	1.048
RG3	0.501	0.629	0.719	0.792	0.852	0.898
RG4	0.334	0.420	0.479	0.528	0.568	0.599
RG5	0.250	0.315	0.360	0.396	0.426	0.449
RG6	0.167	0.210	0.240	0.264	0.284	0.299

> PPEXgood

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.452	0.566	0.683	0.818	0.989	1.174
RG2	0.352	0.440	0.531	0.636	0.769	0.913
RG3	0.301	0.378	0.455	0.545	0.659	0.783
RG4	0.201	0.252	0.303	0.364	0.440	0.522
RG5	0.151	0.189	0.228	0.273	0.330	0.391
RG6	0.100	0.126	0.152	0.182	0.220	0.261

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	NO Seg	Seg	Seg	Seg
RG2	Seg	Seg	NO Seg	Seg	Seg	Seg
RG3	Seg	Seg	NO Seg	Seg	Seg	Seg
RG4	Seg	Seg	NO Seg	Seg	Seg	Seg
RG5	Seg	Seg	NO Seg	Seg	Seg	Seg
RG6	Seg	Seg	NO Seg	Seg	Seg	Seg

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NOT SEGMENTED

> PPEXnormalNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.349	0.499	0.691	0.883	1.025	1.111
RG2	0.326	0.466	0.645	0.825	0.956	1.037
RG3	0.289	0.413	0.571	0.730	0.847	0.918
RG4	0.266	0.379	0.525	0.671	0.779	0.844
RG5	0.256	0.366	0.506	0.648	0.751	0.815
RG6	0.163	0.233	0.322	0.412	0.478	0.519

> PPEXbadNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.403	0.633	0.855	1.003	1.087	1.138
RG2	0.376	0.591	0.798	0.936	1.014	1.062
RG3	0.333	0.524	0.707	0.829	0.898	0.941
RG4	0.306	0.481	0.650	0.762	0.826	0.865
RG5	0.295	0.464	0.627	0.735	0.797	0.835
RG6	0.188	0.296	0.399	0.468	0.507	0.531

> PPEXgoodNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.330	0.432	0.559	0.722	0.900	1.043
RG2	0.308	0.403	0.522	0.674	0.840	0.973
RG3	0.272	0.357	0.462	0.597	0.744	0.862
RG4	0.250	0.328	0.425	0.549	0.684	0.792
RG5	0.242	0.317	0.410	0.530	0.660	0.765
RG6	0.154	0.202	0.261	0.337	0.420	0.487

SEGMENTED

> PPEXnormal

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.378	0.531	0.707	0.879	1.014	1.103
RG2	0.353	0.496	0.660	0.821	0.946	1.029
RG3	0.313	0.439	0.584	0.727	0.838	0.912
RG4	0.287	0.404	0.537	0.668	0.770	0.838
RG5	0.277	0.390	0.518	0.645	0.743	0.809
RG6	0.176	0.248	0.330	0.410	0.473	0.515

> PPEXbad

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.470	0.689	0.875	1.003	1.082	1.134
RG2	0.438	0.643	0.817	0.936	1.010	1.059
RG3	0.388	0.570	0.723	0.829	0.895	0.938
RG4	0.357	0.524	0.665	0.762	0.823	0.862
RG5	0.344	0.505	0.642	0.736	0.794	0.832
RG6	0.219	0.322	0.408	0.468	0.505	0.529

> PPEXgood

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.341	0.444	0.566	0.715	0.881	1.025
RG2	0.318	0.415	0.528	0.667	0.822	0.957
RG3	0.282	0.367	0.468	0.591	0.728	0.847
RG4	0.259	0.338	0.430	0.543	0.669	0.779
RG5	0.250	0.326	0.415	0.524	0.646	0.752
RG6	0.159	0.207	0.264	0.334	0.411	0.478

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	NO Seg	Seg	Seg
RG2	Seg	Seg	Seg	NO Seg	Seg	Seg
RG3	Seg	Seg	Seg	NO Seg	Seg	Seg
RG4	Seg	Seg	Seg	NO Seg	Seg	Seg
RG5	Seg	Seg	Seg	NO Seg	Seg	Seg
RG6	Seg	Seg	Seg	NO Seg	Seg	Seg

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	Seg
RG2	Seg	Seg	Seg	Seg	Seg	Seg
RG3	Seg	Seg	Seg	Seg	Seg	Seg
RG4	Seg	Seg	Seg	Seg	Seg	Seg
RG5	Seg	Seg	Seg	Seg	Seg	Seg
RG6	Seg	Seg	Seg	Seg	Seg	Seg

Formula:

> PPEXnormalNS

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.370	0.551	0.654	0.785	1.023	1.167

> PPEXnormal

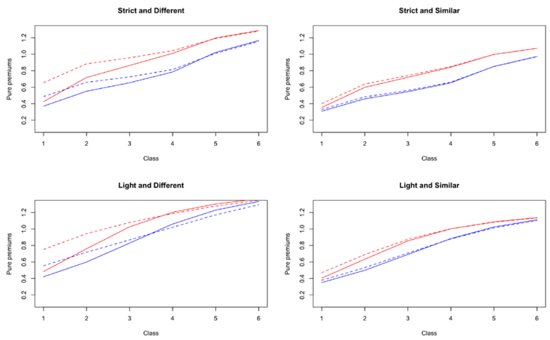
	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	0.487	0.659	0.725	0.815	1.011	1.155

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]
RG1	Seg	Seg	Seg	Seg	Seg	=SI(ABS(0.9-H7)>ABS(0.9-P7);"Seg"; "NO Seg")

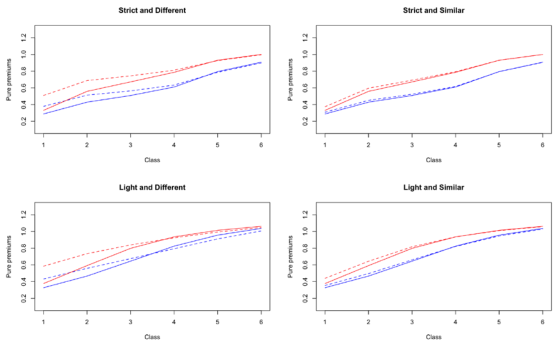
Despite the fact that the not segmented cases exhibit closer premiums to the claim frequency of the insureds in the highlighted cases, the differences in this class for both methods are very small. This suggest that this estrange behavior is due to the decimals that use internally the R and Excel software's. Moreover, in both cases where that occurs is when the paths of premiums cross each other, supporting this reason.

Annex 3: Pure Premiums and Pure Premium surcharged in units of E[X] for all Risk Groups

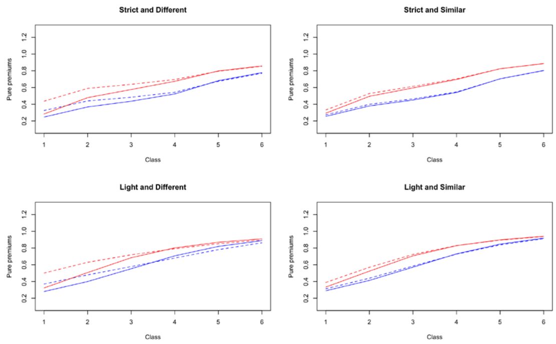
Risk group 1 ($\lambda_k = 0.9(D)$ or 0.75 (S))



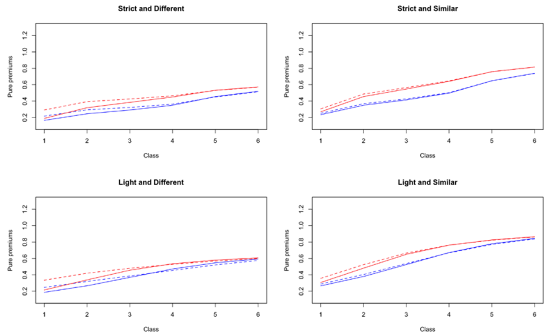
Risk group 2 ($\lambda_k = 0.7(D)$ or 0.7 (S))



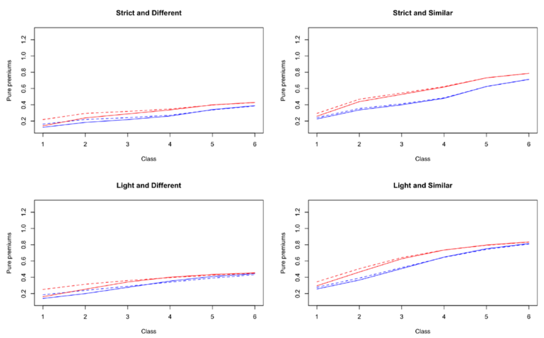
Risk group 3 ($\lambda_k = 0.6(D)$ or 0.62 (S))



Risk group 4 ($\lambda_k = 0.4(D)$ or 0.57 (S))



Risk group 5 ($\lambda_k = 0.3(D)$ or 0.55 (S))



Risk group 1 ($\lambda_k = 0.2(D)$ or 0.35 (S))