Title: Regulatory capital for credit risk: Is there a loophole in the system?

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“The content of this document is the sole responsibility of the author, who declares that he/she has not incurred plagiarism and that all references to other authors have been expressed in the text”.
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Abstract

Given the Spanish housing and mortgage market, there is an important incentive for young property buyers to exceed the debt-to-income (DTI) ratio, using a loophole in the database, CIRBE. The objective of this paper is to analyze whether there could be a possibility of a loophole in the system and therefore how it could affect the calculation of regulatory capital for credit risk requirements. The main hypothesis is that if this practice is commonly used by the borrowers, then there could be a significant difference between the VaR calculated by the banks and the actual VaR needed to cover the regulatory capital.

Keywords: CIRBE, Value-at-risk, loophole, credit risk, regulatory capital
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1. Introduction

In terms of banking, there is always an inconsistency with the time horizon when it comes to its assets and liabilities transactions. In simple terms, let’s say that a bank provides an interest rate for deposits and collects money from the market. This interest rate usually promised to be effective in the short term is lower than the interest rate that the bank will apply for its loans and then make benefit from the difference. The thing is that these loans are usually provided for a long-term period and there could be a considerable risk that they will be repaid on time.

The probability of insolvency is always present in the portfolio of a bank. Regulators try to imply their rulings so that the financial institutions do not make big mistakes to get benefits in a short time. There comes the need to regulate the capital needed to support any kind of insolvency in the assets generated by the financial institutions.

The main objective of this paper is to find out, whether there could be the possibility of a loophole in the information shared with the central bank, and how the existence of this loophole could affect the regulatory capital calculation, required by the banks. A brief study of the current norms in terms of credit risk capital requirements established to regulate the banking industry is also carried out. In the first part of the paper, we analyze the housing market and the income received by the households to consider that there is an important incentive for property buyers to exceed their debt-to-income ratio, for being able to buy a property.

In the second part of the paper, the objective is to understand the regulatory capital model needed to cover insolvency or credit default. In this part, a more detailed study has been done to calculate the regulatory capital needed at each point in time, using the Merton model.

1.1. CIRBE

CIRBE stands for “Central de Información de Riesgos del Banco de España”, which is a public database of the Spanish Central Bank. The banks are obliged to send information to the Central Bank informing about the risk provided to enterprises and households.
There is a limit of 1.000€ of the debt to form part of this public database. It is confidential information, and the client must give his/her permission to find information about their debt.

CIRBE is revised every month but when a loan is given to a company or a household, sometimes it could take almost 45 days to appear in the database due to the processes established between the financial institution and the Central Bank. We assume that this lag can be used by the banks’ clients to get more than one loan at a time, to cover the part of their own capital needed to buy a property.

If we bring this to the Spanish mortgage market, banks usually finance up to 80% of the house value or the buying price (the least of both values), and the other 20% has to be given by the client (buyer). Moreover, we have to add another 7% - 12%, depending on the age and income of the client to pay the taxes and other charges (registry, notary, etc). The total sum of the own capital needed usually ends up being between 27%-32% of the buying price.

Given the housing market in Spain, the prices in big cities are very high and this percentage marks an important figure for the households which do not let them have access to a mortgage. Connecting it to our main objective of this paper, many economic agents could use this time lag of CIRBE as a loophole to get this part of capital left from another bank as a personal loan.

The economic risk behind these operations is very high as the bank which is providing the personal loan has the debt data previous to the mortgage, and the debt-to-income ratio is lower. This leads to the approval of the personal loan and hence a wrong calculation of the regulatory capital needed to cover the risk. On the other hand, the same is true with the bank providing the mortgage, as the solvency of the client is not the same.

1.2. PD, LGD, EAD

The three main instruments used to measure the credit risk of a loan are:

1) EAD: Exposure at Default
2) PD: Probability of Default
3) LGD: Loss Given Default
With the assumptions we have taken above, all three elements are vulnerable to these kinds of financial operations. In the case of insolvency, for example, the firm that is providing the personal loan, won't be able to recover the sum provided. So, if we assume that the banks are using the IRB approach for calculating the regulatory capital for credit risk, the reports could have certain errors which even the financial institution is not considering and is, therefore, more vulnerable in situations of a market downturn.

Given this brief introduction and taking the assumption that these kinds of operations could exist due to the time lag of the risk database of the Central Bank, this paper will focus on how the regulatory capital needed for the credit risk could be wrongly calculated in some cases.
2. Spanish housing and mortgage market

This part of the paper is based on a brief study of the Spanish housing and mortgage market. According to the National Institute of Statistics (INE), in August of 2022, 36,721 new mortgages were signed, and the average capital provided is of 145,287€.

Usually, most of the Spanish banks provide up to 80% of the property price or the guarantee provided (value of the house), where the least of both values is considered. Even if there are banks that finance more than 80% of the price, considering certain parameters of the solvency of their clients, the usual practice is to finance up to 80%.

Clients on their side must have the other 20% plus the taxes and other expenses of notary, registry, management charges, etc. Assuming the least price of a 100,000€ house, usually, the own capital needed will be between 30% to 32% of the buying price if 10% of the property tax is paid. The same sum will be between 25% to 27% if a 5% property tax is paid.

There are cases where, if the house is bought by 2 holders, then there could be the case where the property tax paid is 7.5% and therefore the own capital needed is between 27% to 29.50% (always assuming that the least buying price is 100,000€).\(^1\)

The property tax exemption is based on 2 factors that are age and annual income. So, if the buyer is 32 years old or less and had the last tax declaration for an income of less than 30,000€, then he/she must pay only a 5% property tax.

Once we have seen different percentages of own capital needed, the least amount will still be between 25% to 27%. As this paper is not about the mortgage market, this part is reduced by taking some assumptions in order not to digress. A whole different study is needed for the mortgage market itself.

If we take the data of INE, the average capital provided in August of this year has been 145,287€. Assuming that it represents 80% of the buying price, the property price must have been around 181,609€. The own capital needed for buying this house must have been between 29% to 31% that resulting in between 52,000€ to 56,000€.

\(^1\) This calculation is done for the property tax in Catalonia for simplification purposes.
If we take other data on the Spanish population from INE, almost 40% of the population aged between 16 and 29 lived in a rented property on average between the years 2004 and 2021. This percentage is 20.8% if we consider the population between the age group of 30 to 44 years.

68% of the population aged between 30 to 44 years owns their property on an average basis between the years 2004 and 2021 (which is reducing annually).

Graph 1: Spanish average annual wages (in euros)

And if we do the same analysis by the age group, we can see that the average net income per household between the age of 16 to 29 years old was 20.534€ (considering a period between 2008-2020), and for the age group of 30 to 44 years, the average net income for the same period was of 27.749€. Another similar data could be seen in Graph 1, where the evolution of annual wages in Spain from 2000 to 2021 is shown. We can see the peak being around the period of the financial crisis.

If we take the data of Catalonia, the average gross salary for the age group of 25 to 34 years was 22.338,55€, and for the age group of 35-44 years, was 27.340,77€ in the year 2020. Both Spanish and Catalan average annual wages could be seen in the table 1 below.
If we take the initial data of the average mortgage given by the bank this year, which is 145,287€, an average person between the age of 16 to 29 years, won't be able to afford the monthly installment. The bank, in this case, will not approve the mortgage. Usually, the debt-to-income ratio is set at 35% as the maximum to provide a loan. In simple terms, this means that if a person is earning 1,000€ per month, as per the debt-to-income ratio, the mortgage will be approved only if the monthly installment is at a maximum of 350€.
As we can see in Graph 2 and 3, almost 40% of the income goes to pay the rent on an average basis in Spain. This percentage is around 56% in the cities like Catalonia and Madrid.
With the data presented above in Graph 4, if we assume that approximately the savings rate is around 10% of the gross income, in places like Catalonia, for the age group of 25 to 34 years, it represents around 2.238€ per year and for the age group of 35 to 44 years, the amount of savings would be around 2.734€ on an annual basis.

Now, if we state that the average mortgage provided in the year is 145.287€, it means that the own capital needed for buying the property would be between 52.000€ to 56.000€. Obviously, in the cities like Barcelona or Madrid, this amount will be even higher.

To reach the savings of 52.000€ will take a typical household of 25 to 34 years old, an estimated time of 23,23 years. To reach the savings of 56.000€ it will take almost 20,48 years for the age group of 25 to 44 years.

Based on the explanation above, even though many people use their parents’ savings for the acquisition of their first house, there is a strong motivation behind using the loophole in CIRBE, explained in the introduction.
3. Merton model

With the help of Merton formula, one can try to calculate the probability of default of a particular retail client or corporation. To understand the Merton model, let’s digress a bit to see what a Call option is. A call option is a financial contract that gives its holder the right to buy a specific underlying asset at a set price. It is a kind of contract where the buyer does have the option to buy at a set price but not the obligation to exercise the option. The option buyer will have a profit if the underlying asset is more expensive than before. There is a premium that you pay to get the call option and your maximum loss will be limited to this premium. The Merton model models the equity of a company as a call option on its assets.

Merton does the following mention:

The value of a firm’s equity is E, the total assets are A the liabilities are L. The total assets of the company will be equal to the sum of its Equity and Liabilities: 

\[ A = E + L \]

So, the firm's value to its shareholders at the end of period T will be \( A - L \), if the assets are greater than the liabilities. If the Liabilities are greater than the assets, then the shareholders get nothing.

\[ E_t = \max (A_t - L, 0) \]

The similarity of the model with a Call Option comes because shareholders would choose not to repay the liabilities in case the firm’s value is less than the debt. If the value is greater, they will exercise the option and choose to repay the debt and keep the difference.
Therefore, the default in the Merton model occurs when the assets fall below the point representing liabilities.

Assumptions of Merton model:

1) The asset value of a firm follows a stochastic process
2) The two main classes of securities are equity and debt
3) The equity doesn’t receive any dividends
4) The firm cannot issue new debt
5) The company’s debt is a zero-coupon bond which will become due at time T

If the value of a firm is higher than the debt, then no default occurs, the company pays the dues of the bond (debt), and the shareholders keep the residual. If the value of a firm is lower than the debt, then the company defaults.

In a European call option:

The valuation of a Call Option (using the Black-Scholes option pricing formula) on a stock price with $S$ at time $t$, can be denoted in the following manner:
\[ C^{BS}(t, S; r, \sigma, K, T) = S\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2) \]

Where \( \Phi \) denotes the standard normal distribution function

\[ r \] represents the risk-free interest rate

\[ \sigma \] denotes the volatility of the underlying stock

\[
d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}
\]

\[
d_2 = d_1 - \sigma\sqrt{T-t}
\]

Similarly, in the Merton model, the market value of the firm’s equity at \( t \leq T \) can be determined as the price of a European call option on the asset value \( V_t \) with the exercise price \( B \) and maturity \( T \). If we see the option value above, it is pretty much the same.

\[
S_t = V_t \Phi(d_{t,1}) - Be^{-r(T-t)}\Phi(d_{t,2})
\]

\[
d_{t,1} = \frac{\ln\left(\frac{V_t}{B}\right) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}
\]

\[
d_{t,2} = d_{t,1} - \sigma\sqrt{T-t}
\]

4. Regulatory Capital

The research carried out in this paper does not only is important for financial institutions but also for the regulatory authorities. In this section, a brief explanation of the Basel accords is provided, which shows us that it is important for the entire banking sector to fulfill its capital requirements.

The Basel Committee on Banking Supervision is in charge of the prudential regulation of banks and their solvency. The committee’s regulation standards are not mandatory but recommendations for the members to adopt them for the correct functioning of their financial institutions. The main objective of the committee is to enforce the regulation,
supervision, and banking practices at an international level to improve solvency and risk management in the financial sector.

The committee follows different activities to set and promote banking regulation standards, exchange information on the sector and its associated risks, and exchange information among other supervisors and the central banks.

The first Basel Accord was set in 1988, continued with a new version in 2004 (Basel II) and the most recent version was completed in 2017 with a framework known as Basel III.

Furtherly, we can see the parameters for the risk capital calculations provided by Basel III.

At a more general level, 3 main requirements must be always met:

(1) Common Equity Tier 1 must be at least 4.5% of risk-weighted assets (RWA).
(2) Tier 1 capital must be at least 6% of RWA.
(3) Total capital must be at least 8.0% of RWA.

The calculation of RWA could be the higher of:

1) The sum of RWA for credit risk, market risk, and operational risk.
2) If the bank is using an internal ratings-based approach for credit risk or advanced measurement approaches for operational risk, then this sum of credit risk, market risk, and operational risk will be adjusted as per the requirements of the capital floor.

As this paper is focused on credit risk, we will just consider that element, from the 2 options stated above. There are 2 main methods to calculate the credit RWA for banking book exposures:

1) The standardized approach
2) The IRB approach

For the IRB approach, the bank needs the approval of regulators, to use their methods while calculating risk parameters. In case a bank uses the standardized approach, a VaR model could be used to calculate the capital requirements, the previous business day's VaR number will be used and the new exposure is calculated using the following formula:

\[ E^* = \max\{0,(\Sigma E - \Sigma C) + \text{VaR output from an internal model}\} \]
As stated above, banks have some margin to present their regulatory methods, taking into account the general guidelines provided by the regulator. In case of this paper, a simple approach is considered by using the Merton model and transforming its parameters to adjust with a retail customer of a bank.

5. Simulations with 2 different loans and the calculation of VaR

Once we have seen the basics of the Merton model above, we can proceed to do a simulation. Initially, the setup of the Merton model is given as:

\[ V_t \text{is the asset value as an underlying which under the real-world probability measure } \mathcal{P}, \text{ follows a geometric Brownian motion} \]

\[ dV_t = \mu V_t dt + \sigma V_t dW_t, \quad 0 \leq t \leq T, \]

\[ V_T = V_0 e^{(\mu - \frac{1}{2} \sigma^2)T + \sigma W_T} \]

And since \( W_T \sim N(0, T) \), it follows that:

\[ \ln V_T \sim N(\ln V_0 + (\mu - \frac{1}{2} \sigma^2)T, \sigma^2 T) \]

In this case, the probability that the shareholders will not exercise their call option to buy the assets of the company at time \( T \) is

\[ P (V_T \leq B) = P (\ln V_T \leq \ln B) = \Phi \left( \frac{\ln \frac{B}{V_0} - (\mu - \frac{1}{2} \sigma^2)T}{\sigma \sqrt{T}} \right) \]

In this case, this model initially meant for corporations' default probability, is then transferred to a retail customer, that signs a mortgage to buy their property.

There are some important assumptions to be considered:

1) The mortgage approved by the bank is the total amount that is due by the client (the sum of capital and interests). This has been done to simplify the computation for every single loan as per the French amortization table. The amount of the loans
considered for this study has been generated randomly within the range of 50.000€ to 500.000€. A typical bank's mortgage portfolio is a bit different. This has been done to simplify the calculations and not take any further assumptions for the mortgage portfolio.

2) The time period considered is of 30 years.

3) The total amount due is returned at the end of the period (30 years). This assumption is very far from reality, as the mortgages are usually repaid in monthly installments.

4) The debt-to-income (DTI) ratio is considered between 25% to 35%. The barrier of 35% is the recommended DTI that banks take as a variable to approve the risk. The lower barrier of 25% is an assumption, even if it makes economic sense for the population having lower incomes.

5) Our underlying, which is $V_t$, is calculated using the random set of DTI. The reason behind that is very simple. The loan value during the period of 30 years must be between 25% to 35% of the total income accumulated during the same time. It can’t be higher than the 35% barrier because otherwise the loan would not be conceded, and it wouldn’t make part of this portfolio.

6) The $V_0$ is calculated as the discounted value to $V_t$ by assuming a 2% interest rate. With a 2% interest rate and 30 years of total time considered, the discount factor is calculated with the following formula: $e^{-rT}$

Once we got the discount factor, the $V_0$ is calculated as $V_t * e^{-rT}$

7) For the variables needed for the Merton model, the following assumptions have been considered:

a. $\mu_V \cdot T = E \left[ \frac{V(T) - V(0)}{V(0)} \right]$  
   $\mu_V = \left( \frac{1 - e^{-rT}}{e^{-rT}} \right) \left( \frac{1}{T} \right)$

b. Variance is assumed to be 0.01

c. Therefore, the standard deviation is 0.10

Once we had all the parameters for the Merton model, we get the default probabilities with the following formula:
\[ P (V_T \leq B) = P (\ln V_T \leq \ln B) = \Phi \left( \frac{\ln \frac{B}{V_0} - (\mu - \frac{1}{2} \sigma_V^2)T}{\sigma_V \sqrt{T}} \right) \]

5.1. Calculation of VaR

Value-at-risk (VaR) is a risk measure used to quantify the magnitude of possible financial losses within a portfolio (given a confidence level) over a specific period. It is very commonly used in risk management and therefore included in the models used for computing regulatory capital.

For the regulatory capital calculation, the one-factor framework could be used, assuming that the portfolios have their idiosyncratic risk and is diversified completely.

5.1.1. The asymptotic single risk factor model

The formulas for risk weights used for the further calculation of regulatory capital are based on the asymptotic single risk factor model developed by the Basel Committee.

Any capital charges computed under the assumption of an asymptotically fine-grained portfolio can underestimate the required capital for a real finite portfolio (Ortiz-Gracia, 2021)

By adapting Merton's model and assuming a correlation factor \( \rho_n \), we can furtherly calculate the VaR to get our regulatory capital. This can be done by using the formula ahead:

\[
VaR^A = \sum_{n=1}^{N} S_n \cdot L_n \cdot \Phi \left( \frac{t_n + \sqrt{\rho_n} \cdot \Phi^{-1}(\alpha)}{\sqrt{1 - \rho_n}} \right)
\]

Where,

\( S_n \) is our loan value.

\( L_n \) is assumed to be 100% to simplify the computation.
\( t_n \) is \( \Phi^{-1}(P_n) \) where \( P_n \) is the default probability of default of obligor \( n \).

Asset value correlation \( \rho_n \) is calculated using the following regulatory formula:

\[
\rho_n = 0.12 \cdot \frac{1 - e^{-50 \cdot P_n}}{1 - e^{-50}} + 0.24 \cdot \left( 1 - \frac{1 - e^{-50 \cdot P_n}}{1 - e^{-50}} \right)
\]

Once we get the VaR of our portfolio with a 99% confidence level and given the fact that we don’t have real data, a sensitivity analysis has been carried out. Ceteris paribus, values of VaR are calculated by increasing the loan values in every 10% of our mortgage portfolio. The objective is to see the difference between the VaR calculated by the bank (which will be the normal VaR) and the actual VaR needed for regulatory purposes. We keep the values of total income constant and assume that clients are taking another loan to pay a part of their down payment while purchasing a property.

The results for the values of VaR with a 5% increase in the loan values in every 10% of our portfolio can be seen in table 2, along with the normal VaR that is summarized in the same table. The default probabilities increase when the mortgage holders have a higher DTI due to increased loan value. We can see that the normal VaR is 7,36% which could be considered a genuine value, even if it is a bit higher. A gradual increase of VaR in every 10% of our portfolio can be noticed. If we see the difference between the normal VaR calculation and the one with the presence of increased DTI, significant values can be noticed. we can see that if in 100% of our portfolio, there is a presence of increased DTI, then there would be a difference of about 15.80% in the VaR calculated by the bank and the actual VaR needed for regulatory purposes. Graph 5 summarizes very well, the relationship between the VaR and the increase in loan value. The normal value, with no double loans in the mortgage portfolio, results in 7,36% as stated above, whereas it could be noticed that this value keeps on increasing if there is a presence of 2 loans of the same client, in every 10% of the portfolio.
Table 2: VaR calculation with sigma 0.01 and increments of 5% in loan values in every 10% of our portfolio

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>VaR</th>
<th>Difference with normal VaR</th>
<th>% difference with normal VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.0735952</td>
<td>0.00000000</td>
<td>0.00%</td>
</tr>
<tr>
<td>10%</td>
<td>0.0748051</td>
<td>0.00120984</td>
<td>1.64%</td>
</tr>
<tr>
<td>20%</td>
<td>0.0759863</td>
<td>0.00239105</td>
<td>3.25%</td>
</tr>
<tr>
<td>30%</td>
<td>0.0771585</td>
<td>0.00356322</td>
<td>4.84%</td>
</tr>
<tr>
<td>40%</td>
<td>0.0783306</td>
<td>0.00473538</td>
<td>6.43%</td>
</tr>
<tr>
<td>50%</td>
<td>0.0795105</td>
<td>0.00591523</td>
<td>8.04%</td>
</tr>
<tr>
<td>60%</td>
<td>0.0807086</td>
<td>0.00711341</td>
<td>9.67%</td>
</tr>
<tr>
<td>70%</td>
<td>0.0818658</td>
<td>0.00827052</td>
<td>11.24%</td>
</tr>
<tr>
<td>80%</td>
<td>0.0829806</td>
<td>0.00938535</td>
<td>12.75%</td>
</tr>
<tr>
<td>90%</td>
<td>0.0841141</td>
<td>0.01051885</td>
<td>14.29%</td>
</tr>
<tr>
<td>100%</td>
<td>0.085225</td>
<td>0.01162976</td>
<td>15.80%</td>
</tr>
</tbody>
</table>

Source: Own elaboration

Table 3: VaR calculation with sigma 0.007 and increments of 5% in loan values in every 10% of our portfolio

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>VaR</th>
<th>Difference with normal VaR</th>
<th>% difference with normal VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.02387585</td>
<td>0.00000000</td>
<td>0.00%</td>
</tr>
<tr>
<td>10%</td>
<td>0.02458704</td>
<td>0.00071119</td>
<td>2.98%</td>
</tr>
<tr>
<td>20%</td>
<td>0.02528164</td>
<td>0.00140579</td>
<td>5.89%</td>
</tr>
<tr>
<td>30%</td>
<td>0.02597313</td>
<td>0.00209728</td>
<td>8.78%</td>
</tr>
<tr>
<td>40%</td>
<td>0.02666166</td>
<td>0.00288632</td>
<td>11.67%</td>
</tr>
<tr>
<td>50%</td>
<td>0.02735537</td>
<td>0.00356322</td>
<td>14.57%</td>
</tr>
<tr>
<td>60%</td>
<td>0.02806578</td>
<td>0.00473538</td>
<td>17.55%</td>
</tr>
<tr>
<td>70%</td>
<td>0.02873672</td>
<td>0.00591523</td>
<td>20.36%</td>
</tr>
<tr>
<td>80%</td>
<td>0.02939402</td>
<td>0.00711341</td>
<td>23.11%</td>
</tr>
<tr>
<td>90%</td>
<td>0.03006743</td>
<td>0.00827052</td>
<td>25.93%</td>
</tr>
<tr>
<td>100%</td>
<td>0.03072597</td>
<td>0.01051885</td>
<td>28.69%</td>
</tr>
</tbody>
</table>

Source: Own elaboration
To get even more realistic results of our VaR, a lower value of sigma is considered. By assuming a value of 0.007, the resulting VaR is 2.39% which could be considered a more genuine value. We can notice in Graph 6 and table 3 that with a lower variance, the value of normal VaR (with no extra loans in the portfolio), reduces but the presence of an increased value of 5% in the loans, results in a higher difference from the normal VaR. As we can notice in table 3, there is a difference of 28.69% when there is a presence of 5% extra in the loan values among the 100% of our portfolio. A lower variance could make sense because the total income won’t vary too much on average.

If we increase the loan values up to 10%, the results could be seen in table 4 and graph 7 below. The differences from the normal VaR, are much higher in this case because a 10% increase in the loan values results in a higher probability of default and therefore a higher VaR. In this case, we can notice that just 10% of our portfolio having a 10% increase in its loan values, resulting in a difference of 6.62%. If our entire portfolio has an increased 10% in the loan values, there would be a difference of 61.18% between the VaR calculated by the bank and the regulatory VaR needed.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>VaR</th>
<th>Difference with normal VaR</th>
<th>% difference with normal VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.02387585</td>
<td>0.00000000</td>
<td>0.00%</td>
</tr>
<tr>
<td>10%</td>
<td>0.02545719</td>
<td>0.00158134</td>
<td>6.62%</td>
</tr>
<tr>
<td>20%</td>
<td>0.02698637</td>
<td>0.00311052</td>
<td>13.03%</td>
</tr>
<tr>
<td>30%</td>
<td>0.02849329</td>
<td>0.00461744</td>
<td>19.34%</td>
</tr>
<tr>
<td>40%</td>
<td>0.02998095</td>
<td>0.00610510</td>
<td>25.57%</td>
</tr>
<tr>
<td>50%</td>
<td>0.03146573</td>
<td>0.00758988</td>
<td>31.79%</td>
</tr>
<tr>
<td>60%</td>
<td>0.03297022</td>
<td>0.00909437</td>
<td>38.09%</td>
</tr>
<tr>
<td>70%</td>
<td>0.03438067</td>
<td>0.01050482</td>
<td>44.00%</td>
</tr>
<tr>
<td>80%</td>
<td>0.03574826</td>
<td>0.01187241</td>
<td>49.73%</td>
</tr>
<tr>
<td>90%</td>
<td>0.03713625</td>
<td>0.01326040</td>
<td>55.54%</td>
</tr>
<tr>
<td>100%</td>
<td>0.03848216</td>
<td>0.01460631</td>
<td>61.18%</td>
</tr>
</tbody>
</table>

Source: Own elaboration
As per the recommendations of the regulators, the total debt (long and short-term loans) should not exceed 40% of the net income. Following these guidelines, another experiment has been carried out where the loan values are set at 40% of the total income at the end of the 30 years period ($V_t$). Once again, maintaining the $V_t$ constant, an increase in the loan value is made in every 10% of our mortgage portfolio. The loan values are left with a 40% of $V_t$. Small financial institutions, consider this value while providing a loan for the part of the down payment needed while purchasing a property. The results are shown in graph 8 and table 5. As we can notice, just with 10% of our entire portfolio having a 40% of DTI, the difference between the VaR calculated by the bank and the regulatory VaR needed is 32.03%. If we see our entire portfolio, there could be an error of 245.69%.
The consequences of a legal loophole are clear. There is a difference between the regulatory capital calculated by the banks and the capital that is needed in case of a presence of fraud in the portfolio. We have seen that by this loophole in the information system, the error in the calculation of regulatory capital can be significant. As shown in table 5, just considering 50% of our portfolio with up to 40% of DTI, results in an error of 140.55% which in absolute terms, could be a significant amount for the financial institution.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>VaR</th>
<th>Difference with normal VaR</th>
<th>% difference with normal VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>0.02387585</td>
<td>0.00000000</td>
<td>0.00%</td>
</tr>
<tr>
<td>10%</td>
<td>0.03152355</td>
<td>0.00764770</td>
<td>32.03%</td>
</tr>
<tr>
<td>20%</td>
<td>0.03855986</td>
<td>0.01468401</td>
<td>61.50%</td>
</tr>
<tr>
<td>30%</td>
<td>0.04513147</td>
<td>0.02125562</td>
<td>89.03%</td>
</tr>
<tr>
<td>40%</td>
<td>0.05143151</td>
<td>0.02755566</td>
<td>115.41%</td>
</tr>
<tr>
<td>50%</td>
<td>0.05743315</td>
<td>0.03355730</td>
<td>140.55%</td>
</tr>
<tr>
<td>60%</td>
<td>0.06311389</td>
<td>0.03923804</td>
<td>164.34%</td>
</tr>
<tr>
<td>70%</td>
<td>0.06859705</td>
<td>0.04472120</td>
<td>187.31%</td>
</tr>
<tr>
<td>80%</td>
<td>0.07347295</td>
<td>0.04959710</td>
<td>207.73%</td>
</tr>
<tr>
<td>90%</td>
<td>0.07813887</td>
<td>0.05426302</td>
<td>227.27%</td>
</tr>
<tr>
<td>100%</td>
<td>0.08253745</td>
<td>0.05866160</td>
<td>245.69%</td>
</tr>
</tbody>
</table>

Table 5: VaR calculation with sigma 0.007 and 40% DTI in every 10% of our portfolio
Source: Own elaboration

6. Conclusions

In this paper, we have seen that it could be possible the existence of a loophole in the system that could be used by retail customers to get a mortgage and buy a property. The hypothesis of the existence of a loophole is justified due to high property prices and lower incomes of first property buyers in Spain, which afterward is contrasted with our simulation.

It has been concluded that the regulatory capital needed for the credit risk could be wrongly calculated up to a certain percentage. Due to this loophole, many of these clients
could face a problem of insolvency in periods of recession, and the banks could have serious issues in recovering back their debt. Another consequence that could be produced is that there is an important mismatch between the regulatory capital needed and the one that is calculated by the banks. If an extreme case is assumed, with the presence of a maximum amount of DTI recommended by regulators (40%), in the 100% of our portfolio, there could be a difference of 245.69% between the capital calculated by the bank and the real capital needed for the regulatory purposes.

The simulation was carried out with a total number of 10,000 mortgages ranging between 50,000€ to 500,000€ randomly, making a total sum of 2,731,239,053€. The results of the simulation show that with 5% increase in the initial loan value, in every 10% of the portfolio, the value of VaR increases and therefore, also the amount of regulatory capital needed increases. If we take into account that the total portfolio considered for the simulation is not even close to the total sum of mortgages from all the banks, then it is quite firm to say that a small percentage of the wrongly calculated capital could be significant in absolute terms for the entire sector. Some banks ask for a justification of the capital needed by the client before approving the mortgage but there are others, that don't. These latter are the ones possibly being deceived by the clients.

We are aware of the assumptions that have been considered for the model and the simulation but adjusting this study exactly to the reality could be a bigger challenge. New papers could be studied under the same topic, under different assumptions. An interesting following paper could be about the wrong calculation of the regulatory capital due to fraud in the registry values of the property. This practice is in another way, used by some agents to receive higher than 80% of the buying price, in their mortgage and buy a property. Obviously, in that case, we will be directly considering fraud with the banks.
7. Bibliography


Basel Committee on Banking Supervision. (2019c). *Basel Committee on Banking Supervision CRE Calculation of RWA for credit risk CRE30 IRB approach: Overview and asset class definitions*.


Basel Committee on Banking Supervision. (2019e). *Basel Committee on Banking Supervision CRE Calculation of RWA for credit risk CRE32 IRB approach: Risk components for each asset class*. 

Basel Committee on Banking Supervision. (2019g). Basel Committee on Banking Supervision CRE Calculation of RWA for credit risk CRE34 IRB approach: RWA for purchased receivables.


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https://apps.fomento.gob.es/CVP/


8. Annexes

VAR2

Abhishek Teji

20/12/2022

Temporal<-30
interes<-0.02
sig2<-0.01
crec<-0.05
intervalo<-0.99
set.seed(1)
Bt<-c(runif(10000, min = 50000, max = 50000))
DTI<-c(runif(10000, min = 0.25, max = 0.35))

DTI<-round(DTI,2)
Vt<-Bt/DTI

################## NOrmal ####################

exp_temp<-exp(-Temporal*interes)
V0<-Vt*exp_temp
Logbt_V0<-log(Bt/V0)

Mu<-(1-exp(-interes*Temporal))/exp(-interes*Temporal))*(1/Temporal)
sig<-sqrt(sig2)
bracket<-(Logbt_V0 - (Mu*(0.5*sig2))*Temporal)/(sig*sqrt(Temporal))

Pn<-pnorm(bracket)

# Temporal - (Mu*(0.5*sig2))*Temporal) + 0.24*
# (1-(1-exp(-50*Pn)) / 1-exp(-50))

invalpha<-qnorm(intervalo)

bracket2<-(-tn+sqrt(pn2)*invalpha)/(sqrt(1-pn2))
bracket2<-pnorm(bracket2)

VAR_aux<-sum(Bt*bracket2)

VAR_aux/sum(Bt)

## [1] 0.07359523

porcet = format(VAR_aux/sum(Bt),scientific = FALSE)

################# SERIE 1 ####################

Bt2 = Bt[1:1000]
Bt2<-Bt2*(1+crec)
Bt_aux<-Bt
Bt_aux[1:1000]<-Bt2
DTI<-round(DTI,2)
length(DTI)*0.1

## [1] 1000

exp_temp<-exp(-Temporal*interes)
V0<-Vt*exp_temp
Logbt_V0<-log(Bt_aux/V0)
Mu<-(1-exp(-interes*Temporal))/exp(-interes*Temporal))*(1/Temporal)
sig<-sqrt(sig2)
bracket<- (Logbt_V0 - (Mu-(0.5*sig2)*Temporal)/(sig*sqrt(Temporal)))
Pn<-pnorm(bracket)
tn<-qnorm(Pn)
pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50)) + 0.24*(1-((1-exp(-50*Pn))/1-exp(-50)))
invalpha<-qnorm(intervalo)
bracket2<-tn+sqrt(pn2)*invalpha/(sqrt(1-pn2))
bracket2<-pnorm(bracket2)
VAR_aux<-sum(Bt_aux*bracket2)
VAR_aux/sum(Bt_aux)

## [1] 0.07480507

porcet_1 = format(VAR_aux/sum(Bt_aux),scientific = FALSE)

################# SERIE 2 ####################


Bt2 = Bt[1:2000]
Bt2<-Bt2*(1+crec)
Bt_aux<-Bt
Bt_aux[1:2000]<-Bt2
DTI<-round(DTI,2)
length(DTI)*0.1

## [1] 1000

exp_temp<-exp(-Temporal*interes)
V0<-Vt*exp_temp
Logbt_V0<-log(Bt_aux/V0)
Mu<-(1-exp(-interes*Temporal))/exp(-interes*Temporal))*(1/Temporal)
sig<-sqrt(sig2)
bracket<-(Logbt_V0 - (Mu-(0.5*sig2))*Temporal)/(sig*sqrt(Temporal))
Pn<-pnorm(bracket)
tn<-qnorm(Pn)

pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50)) + 0.24*(1-((1-exp(-50*Pn))/1-exp(-50)))
intvalpha<-qnorm(intervalo)
bracket2<-(-tn+sqrt(pn2)*intvalpha)/(sqrt(1-pn2))

VAR_aux<-sum(Bt_aux*bracket2)
VAR_aux/sum(Bt_aux)

## [1] 0.07598628

porcet_2 = format(VAR_aux/sum(Bt_aux),scientific = FALSE)

#################################### SERIE 3 ####################################

Bt2 = Bt[1:3000]
Bt2<-Bt2*(1+crec)
Bt_aux<-Bt
Bt_aux[1:3000]<-Bt2
DTI<-round(DTI,2)
length(DTI)*0.1

## [1] 1000

exp_temp<-exp(-Temporal*interes)
V0<-Vt*exp_temp
Logbt_V0<-log(Bt_aux/V0)
Mu<-(1-exp(-interes*Temporal))/exp(-interes*Temporal))*(1/Temporal)
sig<-sqrt(sig2)
bracket<-(Logbt_V0 - (Mu-(0.5*sig2))*Temporal)/(sig*sqrt(Temporal))

Pn<-pnorm(bracket)
tn<-qnorm(Pn)

pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50))+0.24*(1-((1-exp(-50*Pn))/1-exp(-50)))

invalpha<-qnorm(intervalo)
bracket2<-(tn+sqrt(pn2)*invalpha)/(sqrt(1-pn2))

VAR_aux<-sum(Bt_aux*bracket2)
VAR_aux/sum(Bt_aux)

## [1] 0.07715845

porcet_3 = format(VAR_aux/sum(Bt_aux),scientific = FALSE)

############################ SERIE 4 #############################

Bt2 = Bt[1:4000]
Bt2<-Bt2*(1+cerc)
Bt_aux<-Bt

Bt_aux[1:4000]<-Bt2

DTI<-round(DTI,2)

length(DTI)*0.1

## [1] 1000

ex_temp<-exp(-Temporal*interes)

V0<-Vt*exp_temp

Logbt_V0<-log(Bt_aux/V0)

Mu<-((1-exp(-interes*Temporal))/exp(-interes*Temporal))*(1/Temporal)

sig<-sqrt(sig2)

bracket<-(Logbt_V0 - (Mu-(0.5*sig2))*Temporal)/(sig*sqrt(Temporal))

Pn<-pnorm(bracket)
tn<-qnorm(Pn)

pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50))+0.24*(1-((1-exp(-50*Pn))/1-exp(-50)))

invalpha<-qnorm(intervalo)
bracket2<-(tn+sqrt(pn2)*invalpha)/(sqrt(1-pn2))

VAR_aux<-sum(Bt_aux*bracket2)

VAR_aux/sum(Bt_aux)

## [1] 0.07833061
porcet_4 = format(VAR_aux/sum(Bt_aux),scientific = FALSE)

######################################################################## SERIE 5 ###################################################################

Bt2 = Bt[1:5000]
Bt2<-Bt2*(1+crec)
Bt_aux<-Bt
Bt_aux[1:5000]<-Bt2
DTI<-round(DTI,2)
length(DTI)*0.1
## [1] 1000

exp_temp<-exp(-Temporal*interes)
V0<-Vt*exp_temp
Logbt_V0<-log(Bt_aux/V0)
Mu<-(1-exp(-interes*Temporal))/exp(-interes*Temporal))*(1/Temporal)
sig<-sqrt(sig2)
bracket<-(Logbt_V0 - (Mu-(0.5*sig2))*Temporal)/(sig*sqrt(Temporal))
Pn<-pnorm(bracket)
tn<-qnorm(Pn)

pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50))+0.24*(1-(1-exp(-50*Pn))/1-exp(-50)))
intvalpha<-qnorm(intervalo)
bracket2<-(tn+sqrt(pn2)*intvalpha)/(sqrt(1-pn2))
bracket2<-pnorm(bracket2)
VAR_aux<-sum(Bt_aux*bracket2)
VAR_aux/sum(Bt_aux)
## [1] 0.07951046

porcet_5 = format(VAR_aux/sum(Bt_aux),scientific = FALSE)

######################################################################## SERIE 6 ####################################################################

Bt2 = Bt[1:6000]
Bt2<-Bt2*(1+crec)
Bt_aux<-Bt
Bt_aux[1:6000]<-Bt2
DTI<-round(DTI,2)
length(DTI)*0.1
## [1] 1000

exp_temp<-exp(-Temporal*interes)
V0<-Vt*exp_temp
Logbt_V0<-log(Bt_aux/V0)
Mu<-(1-exp(-interes*Temporal))/exp(-interes*Temporal)*(1/Temporal)
sig<-sqrt(sig2)
bracket<-((Logbt_V0 - (Mu-(0.5*sig2))*Temporal))/(sig*sqrt(Temporal))
Pn<-pnorm(bracket)

tn<-qnorm(Pn)

pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50))+0.24*(1-((1-exp(-50*Pn))/1-exp(-50)))
invalpha<-qnorm(intervalo)
bracket2<-((tn+sqrt(pn2)*invalpha)/(sqrt(1-pn2))

VAR_aux<-sum(Bt_aux*bracket2)

## [1] 0.08070864

porcet_6 = format(VAR_aux/sum(Bt_aux),scientific = FALSE)

############################### SERIE 7 ###############################

Bt2 = Bt[1:7000]
Bt2<-Bt2*(1+crec)
Bt_aux<-Bt
Bt_aux[1:7000]<-Bt2

DTI<-round(DTI,2)

length(DTI)*0.1

## [1] 1000

exp_temp<-exp(-Temporal*interes)
V0<-Vt*exp_temp
Logbt_V0<-log(Bt_aux/V0)
Mu<-(1-exp(-interes*Temporal))/exp(-interes*Temporal)*(1/Temporal)
sig<-sqrt(sig2)
bracket<-((Logbt_V0 - (Mu-(0.5*sig2))*Temporal))/(sig*sqrt(Temporal))
Pn<-pnorm(bracket)

tn<-qnorm(Pn)

pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50))+0.24*(1-((1-exp(-50*Pn))/1-exp(-50)))
\[1 - \exp(-50)\]

\[
inalpha <- \text{qnorm} (\text{intervalo)}
\]

\[
bracket2 <- (tn + \text{sqrt} (\text{pn2}) \ast invalpha) / (\text{sqrt}(1 - \text{pn2}))
\]

\[
bracket2 <- \text{pnorm} (\text{bracket2})
\]

\[
\text{VAR}_\text{aux} <- \text{sum}(Bt\_aux \ast \text{bracket2})
\]

\[
\text{VAR}_\text{aux} / \text{sum}(Bt\_aux)
\]

## [1] 0.08186575

\[
\text{porcet}_7 = \text{format} (\text{VAR}_\text{aux} / \text{sum}(Bt\_aux), \text{scientific} = \text{FALSE})
\]

### SERIE 8

\[
Bt2 = Bt[1:8000]
\]

\[
Bt2 <- Bt2 \ast (1 + \text{crec})
\]

\[
Bt\_aux <- Bt
\]

\[
Bt\_aux[1:8000] <- Bt2
\]

\[
\text{DTI} <- \text{round} (\text{DTI}, 2)
\]

\[
\text{length}(\text{DTI}) \ast 0.1
\]

## [1] 1000

\[
\exp_{\text{temp}} <- \exp(-\text{Temporal} \ast \text{interes})
\]

\[
V0 <- Vt \ast \exp_{\text{temp}}
\]

\[
\text{Logbt}_V0 <- \text{log} (\text{Bt}\_aux / V0)
\]

\[
\text{Mu} <- ((1 - \exp(-\text{interes} \ast \text{Temporal})) / \exp(-\text{interes} \ast \text{Temporal})) \ast (1 / \text{Temporal})
\]

\[
\text{sig} <- \text{sqrt} (\text{sig2})
\]

\[
\text{bracket} <- (\text{Logbt}_V0 - (\text{Mu} - (0.5 \ast \text{sig2}) \ast \text{Temporal}) / (\text{sig} \ast \text{sqrt} (\text{Temporal})))
\]

\[
\text{Pn} <- \text{pnorm} (\text{bracket})
\]

\[
\text{tn} <- \text{qnorm} (\text{Pn})
\]

\[
\text{pn2} <- 0.12 \ast ((1 - \exp(-50 \ast \text{Pn}) / 1 - \exp(-50)) + 0.24 \ast (1 - ((1 - \exp(-50 \ast \text{Pn}) / 1 - \exp(-50))))
\]

\[
inalpha <- \text{qnorm} (\text{intervalo})
\]

\[
bracket2 <- (tn + \text{sqrt} (\text{pn2}) \ast \text{invalpha}) / (\text{sqrt}(1 - \text{pn2}))
\]

\[
bracket2 <- \text{pnorm} (\text{bracket2})
\]

\[
\text{VAR}_\text{aux} <- \text{sum}(Bt\_aux \ast \text{bracket2})
\]

\[
\text{VAR}_\text{aux} / \text{sum}(Bt\_aux)
\]

## [1] 0.08298058

\[
\text{porcet}_8 = \text{format} (\text{VAR}_\text{aux} / \text{sum}(Bt\_aux), \text{scientific} = \text{FALSE})
\]

### SERIE 9
Bt2 = Bt[1:9000]
Bt2 <- Bt2 * (1 + crec)
Bt_aux <- Bt
Bt_aux[1:9000] <- Bt2
DTI <- round(DTI, 2)
length(DTI) * 0.1

## [1] 1000

exp_temp <- exp(-Temporal * interes)
V0 <- Vt * exp_temp
Logbt_V0 <- log(Bt_aux / V0)
Mu <- ((1 - exp(-interes * Temporal)) / exp(-interes * Temporal)) * (1 / Temporal)
sig <- sqrt(sig2)
bracket <- (Logbt_V0 - (Mu - (0.5 * sig2)) * Temporal) / (sig * sqrt(Temporal))
Pn <- pnorm(bracket)
tn <- qnorm(Pn)

pn2 <- 0.12 *((1 - exp(-50 * Pn)) / 1 - exp(-50)) + 0.24 * (1 - ((1 - exp(-50 * Pn)) / 1 - exp(-50)))
invalpha <- qnorm(intervalo)
bracket2 <- (tn + sqrt(pn2) * invalpha) / (sqrt(1 - pn2))

VAR_aux <- sum(Bt_aux * bracket2)
VAR_aux / sum(Bt_aux)

## [1] 0.08411408

porcet_9 = format(VAR_aux / sum(Bt_aux), scientific = FALSE)

#################################################### SERIE 10 ####################################################

Bt2 = Bt[1:10000]
Bt2 <- Bt2 * (1 + crec)
Bt_aux <- Bt
Bt_aux[1:10000] <- Bt2
DTI <- round(DTI, 2)
length(DTI) * 0.1

## [1] 1000

exp_temp <- exp(-Temporal * interes)
V0 <- Vt * exp_temp
Logbt_V0 <- log(Bt_aux / V0)
Mu <- ((1 - exp(-interes * Temporal)) / exp(-interes * Temporal)) * (1 / Temporal)
sig <- sqrt(sig2)
bracket<-(Logbt_V0 - (Mu-(0.5*sig2))*Temporal)/(sig*sqrt(Temporal))
Pn<-pnorm(bracket)
tn<-qnorm(Pn)

pn2<-0.12*((1-exp(-50*Pn))/1-exp(-50))+0.24*(1-((1-exp(-50*Pn))/1-exp(-50)))
intvalpha<-qnorm(intervalo)

bracket2<-(tn+sqrt(pn2)*intvalpha)/(sqrt(1-pn2))
bracket2<-pnorm(bracket2)

VAR_aux<-sum(Bt_aux*bracket2)
VAR_aux/sum(Bt_aux)

## [1] 0.08522499

porcet_10 = format(VAR_aux/sum(Bt_aux), scientific = FALSE)

################################## Resultados ##################################

porcet
## [1] "0.07359523"

porcet_1
## [1] "0.07480507"

porcet_2
## [1] "0.07598628"

porcet_3
## [1] "0.07715845"

porcet_4
## [1] "0.07833061"

porcet_5
## [1] "0.07951046"

porcet_6
## [1] "0.08070864"

porcet_7
## [1] "0.08186575"

porcet_8
```r
# [1] "0.08298058"
porcet_9
# [1] "0.08411408"
porcet_10
# [1] "0.08522499"
porcettot<-c(porcet,
             porcet_1,
             porcet_2,
             porcet_3,
             porcet_4,
             porcet_5,
             porcet_6,
             porcet_7,
             porcet_8,
             porcet_9,
             porcet_10)
porcettot<-as.numeric(porcettot)
x<-c(0:10)
plot(x,porcettot,type = "b", pch=19, col= "red", xlab="serie creciente", ylab="VaR%")
```
```r
resume <- data.frame(x, porcettotal)
resume$diff <- resume$porcettotal - resume$porcettotal[1]
resume$percentage <- round((resume$diff / resume$porcettotal[1]) * 100, 2)
resume
```

```
##     x porcettotal       diff percentage
## 1    0  0.07359523 0.000000000       0.00
## 2    1  0.07480507 0.001209840       1.64
## 3    2  0.07598628 0.002391054       3.25
## 4    3  0.07715845 0.003563222       4.84
## 5    4  0.07833061 0.004735379       6.43
## 6    5  0.07951046 0.005915233       8.04
## 7    6  0.08070864 0.007113409       9.67
## 8    7  0.08186575 0.008270520      11.24
## 9    8  0.08298058 0.009385349      12.75
## 10   9  0.08411408 0.010518850      14.29
## 11  10 0.08522499 0.011629760      15.80
```