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Cross-sectional quantile regression for estimating conditional VaR of returns during periods of high volatility

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A B S T R A C T

Evaluating value at risk (VaR) for a firm's returns during periods of financial turmoil is a challenging task because of the high volatility in the market. We propose estimating conditional VaR and expected shortfall (ES) for a given firm's returns using quantile regression with cross-sectional (CSQR) data about other firms operating in the same market. An evaluation using US market data between 2000 and 2020 shows that our approach has certain advantages over a CAViaR model. Identification of low-risk firms and a reduction in computing times are additional advantages of the new method described.

1. Introduction

When evaluating a firm's stock returns, tails are essential for investors and regulators to manage investment decisions and evaluate capital allocation. More specifically, negative tails provide information about the possibility of future losses associated with a market asset or a portfolio. However, the movements in a firm's returns respond differently to periods of high market volatility.

A univariate time-series analysis of a single firm relies solely on past information for that firm and does not consider other firms in the same market. An example is the approach taken by the conditional autoregressive value at risk (CAViaR) model (Engle & Manganelli, 2004). However, to evaluate a firm's returns conditional on a specific characteristic, a cross-sectional sample of firms in the same market is required. Here, we propose using a cross-sectional quantile regression model to study tail returns for a single firm and we compare this approach with the CAViaR model.

Value at risk (VaR) is a relatively simple tool for summarizing risks (Bodnar, Hayt, & Marston, 1998) and, subsequently, several studies have successfully developed enhanced VaR estimation methods: including, the freedom to choose the probability distribution (Hull & White, 1998); and innovative approaches, such as ARCH and GARCH, which model heteroskedasticity (Engle, 2001); CoVaR, which measures systemic risk for institutions under adverse situations (Adrian & Brunnermeier, 2011); and, the CAViaR model, which estimates the tail with autoregressive processes (Engle & Manganelli, 2004). Additionally, risk evaluation has moved to calculate together with the VaR the Expected Shortfall (ES), mainly encouraged by the Basel III agreement (Basel Committee on Banking Supervision, 2016), which approximates the expected loss derived of the happening of an extreme event.

In recent years, many new methods for estimating VaR aimed at improving risk strategies involving asset evaluation have been proposed (e.g. Wang, Du, & Hsu, 2018; Sahamkhadam, Stephan, & Östermark, 2018; Lin, Sun, & Yu, 2018; Kwon, 2019; Gribisch & Eckernkemper, 2019; Cai & Stander, 2020; Pei, Wang, Xu, & Yue, 2021; Bodnar, Lindholm, Thorsén, & Tyrcha, 2021). Here, researchers highlight the importance of studying periods of high market volatility, which can be devastating in terms of losses and

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lack of liquidity (e.g. Dias, 2016; Alexandridis & Hasan, 2020; Babalos, Caporale, & Spagnolo, 2021; Belaid, Ben Amar, Goutte, & Guesmi, 2021); yet, in spite of the difficulties encountered in capturing variability in such periods, VaR and ES estimation remain the standard practice.

2. Methodology

2.1. Quantile regression

Quantile regression (QR) aims to fit the quantile of the response variable given a set of covariates Koenker (2017) and Koenker and Bassett Jr (1978). Here, we refer to cross-sectional quantile regression (CSQR) to emphasize the fixed period nature of our analysis. Moreover, CSQR is a useful method for understanding what influences the possibility of extreme returns being observed (see also, Uribe & Guillen, 2020 for related applications and an overview of time series).

Let y_i be a random variable with a probability distribution function F_i that depends on covariates $X_i' = \{X_{1i}, X_{2i}, \dots, X_{ki}\}$ for $i = 1, \dots, N$, where N is the number of observations, so $0 \le F_i(y|X_i) \le 1$. We specify the *α-th conditional quantile* $(0 \le \alpha \le 1)$, or simply the α-level conditional VaR, as:

$$Q_{Y_i|X_i}(\alpha) = \beta_{(\alpha)0} + \beta_{(\alpha)1} X_{1i} + \beta_{(\alpha)2} X_{2i} + \cdots + \beta_{(\alpha)k} X_{ki} = X_i' \beta_{(\alpha)} \ , \tag{1}$$

with parameter estimates $\hat{\beta}_{(\alpha)} = \arg\min_{\beta} \mathrm{E}[\rho_\alpha(Y_i - X_i'\beta)]$ where $\rho_\alpha(u) = [\alpha - \mathbb{1}_{\{u<0\}}]u$, and $\mathbb{1}_{\{u<0\}}$ is the identity function, with a value equal to 1 when the subscript is true and 0 otherwise.

To evaluate the performance of a CSQR model, a scoring function is defined to measure the discrepancy between a predicted and an observed value. Let $\tilde{y}_i(\alpha) := X_i' \hat{\beta}_{(\alpha)}$ be the fitted quantile for observation i at level α, and y_i the observed value for observation i, then we will use a summarization of the loss functions evaluated across a whole period as the scoring function like in Acerbi and Szekely (2014):

$$Q_\alpha^0 = \frac{1}{N} \sum_{i=1}^{N} \left[\alpha - \mathbb{1}_{\{y_i \le \tilde{y}_i(\alpha)\}}\right] \left(y_i - \tilde{y}_i(\alpha)\right) \ . \tag{2}$$

Note that Q_α^0 is a weighted average of the absolute distance between the observed value and the fitted quantile, where the weights balance the quantile level α. The lower the value of Q_α^0, the better the approximation.

The expected shortfall (ES) for a level α is defined as:

$$ES_\alpha(Y_i|X_i) = E[Y_i | Y_i \le VaR_\alpha(Y_i|X_i)] \ . \tag{3}$$

The ES, known alternatively as tail conditional expectation (TCE), conditional tail expectation (CTE) or tail value at risk (TVaR), is a risk measure that approximates the expected loss conditioned on the loss exceeding the VaR. When applied to returns, ES is usually negative given that returns may be negative, but on occasions it is expressed as an absolute value. Here, we opt not to change the sign.

For backtest the Expected Shortfall that will be calculated empirically after the VaR estimation as average value of Hits, we take use of the scoring function to evaluate jointly VaR and ES proposed by Fissler and Ziegel (2016), which renders the following equation:

$$\begin{aligned} S_\alpha(Q_i(\alpha), ES_{\alpha,i}, y_i) =& (\mathbb{1}_{\{y_i \le Q_i(\alpha)\}} - \alpha) G_1(Q_i(\alpha)) - \\ & \mathbb{1}_{\{y_i \le Q_i(\alpha)\}} G_1(y_i) + \\ & G_2(ES_{\alpha,i})(ES_{\alpha,i} - Q_i(\alpha)) + \\ & \mathbb{1}_{\{y_i \le Q_i(\alpha)\}} \frac{Q_i(\alpha) - y_i}{\alpha} - \\ & \mathcal{G}_2(ES_{\alpha,i}) + \\ & a(y_i) \end{aligned} \tag{4}$$

being G_1 an increasing function, $\mathcal{G}_2$ an increasing and convex function, and $\mathcal{G}_2' = G_2$. $\mathbb{1}$ represents the identity function, with a value of a 1 if the subscript is met and 0 otherwise. We will take the model specification from Acerbi and Szekely (2014), which uses $G_1(x) = -1/2Wx^2$, $\mathcal{G}_2 = 1/2\alpha x^2$ and $a = 0$. Other possible specifications used in literature can be found in Fissler and Ziegel (2016), Gneiting (2011) and Nolde and Ziegel (2017), and a comparison of different specifications is tested in Taylor (2020), without showing significant discrepancies between them.

2.2. CAViaR model

We use a CAViaR model to analyze a univariate time series of a firm's returns. Let y_t denote the return in time t, but, note, as we analyze only one firm, we have no need for the subscript i. This model was first proposed by Engle and Manganelli (2004) and, for a given α level, its general model specification is:

$$f_t(\beta) = \beta_0 + \sum_{i=1}^{q} \beta_i f_{t-i}(\beta) + \sum_{j=q+1}^{r} \beta_j l(y_{t-j}) \ , \tag{5}$$

where an α level is fixed but omitted here, y_{t-j} is the observed return in time $t-j$. $f_t(\beta)$ is an abbreviation for $f_t(y_{t-1}, \beta_\alpha)$ denoting the quantile α at t of the distribution of returns, which depends on the observed returns from previous periods. $\beta_\alpha = (\beta_0, \ldots, \beta_r)$ is the vector of parameters to be estimated using regression quantiles. Engle and Manganelli (2004) incorporated a lag function $l(\cdot)$ in order to link observed values to the information set.

The CAViaR model that we use to conduct our analysis, the indirect GARCH(1,1), defines their specification as:

$$f_t(\beta) = (\beta_1 + \beta_2 f_{t-1}^2(y_{t-2}, \beta_\alpha) + \beta_3 y_{t-1}^2)^{1/2} \; . \tag{6}$$

which using Eq. (2) to solve β, serves as a quantile regression model.

2.3. CSQR vs. CAViaR for the conditional VaR estimation of returns

In CSQR, we fix a point in time t, $1 \leq t \leq T$, and we adjust a QR model for the returns using the characteristics of all available firms in the market in that specific time t. In so doing, we have T QR models where each one characterizes each moment the state of the market.

By predicting with the T QR models a firm with its specific characteristics, we can compare the results of the CSQR model and the corresponding CAViaR model for the same firm. However, note that the two approaches are essentially different. CSQR uses all firms observed at a given point in time t and assumes that quantiles depend on all the firms' characteristics. CAViaR assumes that the quantiles of a single firm's returns depend on its own past returns. By implementing the CSQR and CAViaR approaches, we obtain two alternative estimates of VaR for each firm at each point in time t.

Implementing CSQR has certain advantages: (1) The computational requirements to calculate a quantile regression model at each t and extract an individual firm i are lower than fitting a CAViaR model (Eq. (6)) for the same firm i; (2) the CSQR model uses covariates, which allows us to include exogenous characteristics and to predict return quantiles for external firms that are not initially in our dataset; and (3) CAViaR needs a minimum observational time window prior to estimating the tails, while our approach does not require any previous observations, only contemporary t.

3. Data and characteristics in the cross-sectional quantile regression model

Our information database contains 204 characteristics for 26,298 different firms in the US market between 1990 and 2020. The data have a monthly frequency. We combined these data with the firms' returns, obtained from the Center for Research in Security Prices (CRSP). Our baseline quantile regression includes seven covariates: firm size (*MC*), book-to-market ratio (*BM*), operating profitability (*OP*), growth rate of investment (*INV*), 12-month momentum (*MOM*), liquidity of the firm (*LIQ*) and market beta (*beta*). Firm size (*MC*) was constructed using CRSP data, as in Uribe Gil, Guillén, and Vidal-Llana (2021), the other factors were retrieved from Chen and Zimmermann (2020) dataset. The variables chosen correspond to the standard magnitudes used to price average returns using cross-sectional factor firm characteristics. *MC*, *BM*, *OP* and *INV* are recommended by Fama and French (2020) while *MOM*, *LIQ* and *beta* were added in keeping with the discussions in Campbell (2017) and Malkiel (2019). We are aware of other factors that can be relevant for predicting VaR during high volatility periods, like the total volatility and idiosyncratic volatility of a firm (see Ang, Hodrick, Xing, & Zhang, 2006, 2009, Chen, Wang, Lin, & Huang, 2022), but we wanted to keep our model with the classic asset pricing specification. Moreover, with the addition of more factors, we would be able to improve even more our contribution and make the comparison less fair for the non cross-sectional model. Because of the specific requirements of the CAViaR model, we were obliged to restrict our comparative analysis to firms whose returns information cover the whole observation period (438 firms); CSQR, in contrast, can evaluate all the firms (26,298 firms).

We would like to add that generating synthetically more disaggregated data, e.g., daily information instead of monthly, does not contribute to answer our research question. We could achieve this by following the inverse steps of Kwon (2019), who transforms daily data into monthly using the square root method, while assuming normality. Indeed, our main research interest is to study quantiles of the distribution, and we assume absence of normality. Moreover, see results for the Variance–Covariance method in Section 4.1 which assumes normality and underperforms against any other tested model.

4. CSQR model for calculating VaR: a comparison with CAViaR

We compare the respective performances of the CSQR and CAViaR models for estimating VaR. Our dataset includes two crisis periods: the Great Recession (2007–2011) and the Covid-19 pandemic (2020). Note, we only present results from January 2000 to December 2020, given that we use data for the first 10 years to capture the autoregressive momentum required to estimate the CAViaR model. To study losers in terms of returns, we analyze the 0.05 α-level, a methodology that can be implemented for any quantile level. Code and results are available on request from the authors.

Fig. 1 shows the average predictions of both the CSQR (purple) and CAViaR models (green) for 438 firms over time. The CSQR model provides a more volatile series of predictions due to its use of other returns and covariates; this offers a richer perspective when calculating VaR. The CAViaR model presents a delay on the fitted VaR when compared with the CSQR model: for example, in the case of the Great Recession, the decrease in the predicted VaR is delayed by between four and seven months. The CAViaR model also appears to provide less extreme VaR estimates than those provided by the CSQR model. During the Great Recession, the CAViaR model produces a higher VaR at the 0.05 level – i.e. less extreme negative returns – than that produced by the CSQR

Fig. 1. Average VaR predicted for the 0.05 quantile for the CSQR (purple) and CAViaR models (green) for 438 firms between 2000 and 2020.

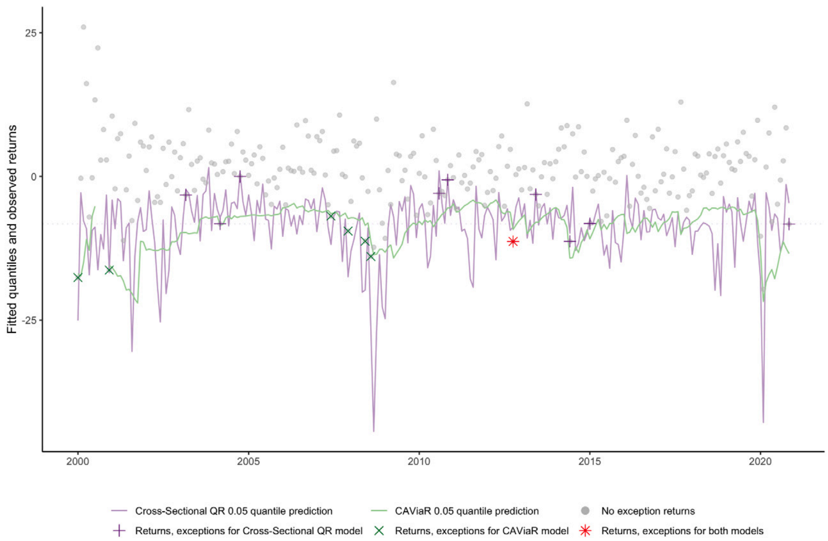

Fig. 2. Comparison of the evolution in returns and predictions for the 0.05 quantile for both the CSQR (purple) and CAViaR models (green) for firm *nº* 24010 between 2000 and 2020 (monthly data).

model. This indicates that the CSQR model detects more risk and, so, an increase in reserves is needed during the Great Recession. During the Covid-19 pandemic, it is unclear to decide which model predicts more risk.

By way of example, Fig. 2 shows the returns observed (gray dots) for an individual firm (firm *nº* 24010). The lines again show the predictions for the 0.05 quantile using the CSQR (purple) and CAViaR models (green). We consider a return lower than a 0.05 fitted quantile to be an Hit and that this should occur just 5% of the time. Hits are calculated with the Hit function (Engle & Manganelli, 2004) that renders a 1 when the return surpasses its calculated VaR and 0 otherwise. Hits – shown here with the same color as the corresponding model – refer to returns that are lower than the fitted VaR for that model but not for the other model: that is, purple crosses indicate returns that we consider to be Hits under the CSQR model but not under the CAViaR model, while green crosses indicate returns that are Hits under the CAViaR model but not under the CSQR model. Red asterisks are considered Hits for both models. The figure shows that during the Great Recession (2007–2011) and the Covid-19 pandemic (2020), the volatility of the returns increased for this particular firm. If we focus solely on the quantile predictions during the Great Recession, it is evident that the returns for this period are higher than the quantiles fitted for the CSQR model. In contrast, the CAViaR model identifies four points as Hits during this period (green crosses). These differences are an example of a localized potentially biased estimation of extreme returns during periods of high volatility.

Table 1
Eq. (2) scores at the 0.05 quantile for each model and the number of months that the model presented a lower (better) score than the other for 438 firms, for three time windows.

$Q^0_{0.05}$	All (2000–2020)	Great recession (2007–2011)	Covid-19 pandemic (2020)
Historical	1.14 (25,996)	1.39 (6,221)	1.89 (1,352)
Variance–Covariance	1.19 (2,051)	1.36 (573)	1.82 (149)
CSQR	0.97 (45,984)	**1.02 (10,008)**	**1.13 (2,546)**
CAViaR	**0.88 (48,536)**	1.05 (7,457)	1.31 (1,488)

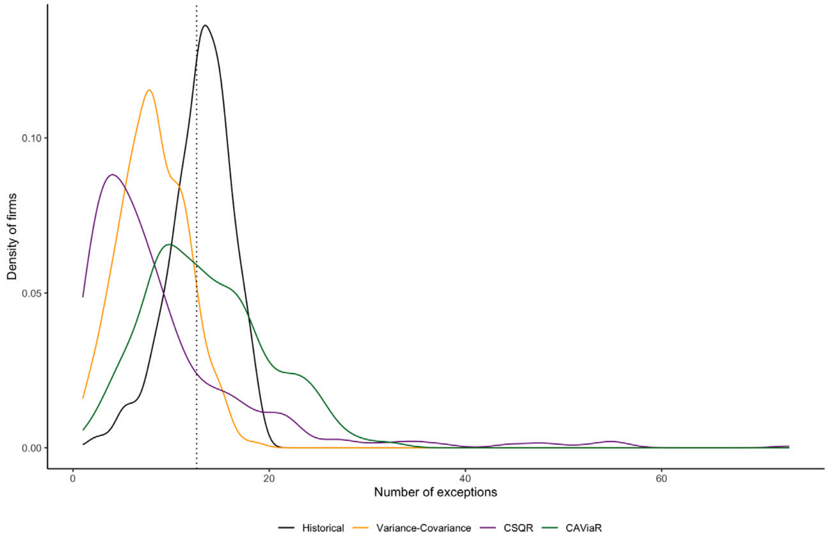

Fig. 3. Kernel density of the number of Hits by firm for the CSQR (purple) and CAViaR models (green) for the 0.05 quantile between 2000 and 2020 (438 firms). The dotted line indicates the benchmark score of 12.6 (i.e. expected number of Hits = 21 years × 12 months each × 0.05 quantile).

The example presented in Fig. 2 indicates that the method developed herein is robust to periods of turmoil. Indeed, this holds true for almost all the firms in our sample (see Fig. A.7 in the Appendix for further examples of this comparison conducted for other individual firms).

4.1. Scoring the models and distribution of hits

Our contribution focuses on the comparison between the Cross-Sectional Quantile Regression Model (CSQR) and the CAViaR model. This section is mainly based on scorings, so we are adding two other baseline models to show the improvement that a change to a more complex model adds. The complementary baseline models for calculating the VaR are the Historical method and the Variance–Covariance method.

In Table 1 we present the score in Eq. (2) for all months and for the two crisis periods, for the Historical method, Variance–Covariance method, for the CSQR model and the CAViaR model, for 438 firms. In parenthesis, we present the total number of months when a model (given in the corresponding row) has had a lower score, i.e. scores better, than the other models.

Table 1 shows that the CSQR model provides a better approximation to the 0.05 quantile during periods of high volatility, presenting a higher number of better scores (10,008 during the Great Recession and 2546 during the Covid-19 outbreak). However, over the whole period, the CAViaR model scores better more months than the other approximations. It appears that the CSQR model outperforms the CAViaR model in times of crisis due to the use of market data, suggesting that the former adapts to periods of high volatility more readily than is the case of the CAViaR model. It is interesting to note how the Variance–Covariance model does not show a good result in comparison of the other three models and that, while for the whole period the CAViaR model outperforms significantly the Historical method, over turmoil periods the number of months that CAViaR model outperforms the Historical method is very similar (25,996 vs 48,536 over the whole period but 6221 vs 7457 and 1352 vs 1488 during high volatility periods). This shows how our proposed model is as reliable during calm periods as it is reliable during turmoil events.

Fig. 3 shows the kernel density of the number of Hits for all firms (438) for the Historical method, the Variance–Covariance method, the CAViaR and CSQR models. In the case of the CAViaR and Historical models, we would expect a very high density of Hits overlapping the dotted line – corresponding here to a benchmark equal to 12.6 (that is, 21 years × 12 months each × 0.05 quantile) – because these models adjust quantiles from a time-series perspective, leaving approximately α% of observed values as Hist. In the case of the Variance–Covariance and the CSQR model, the density is less concentrated and presents fewer Hits than the

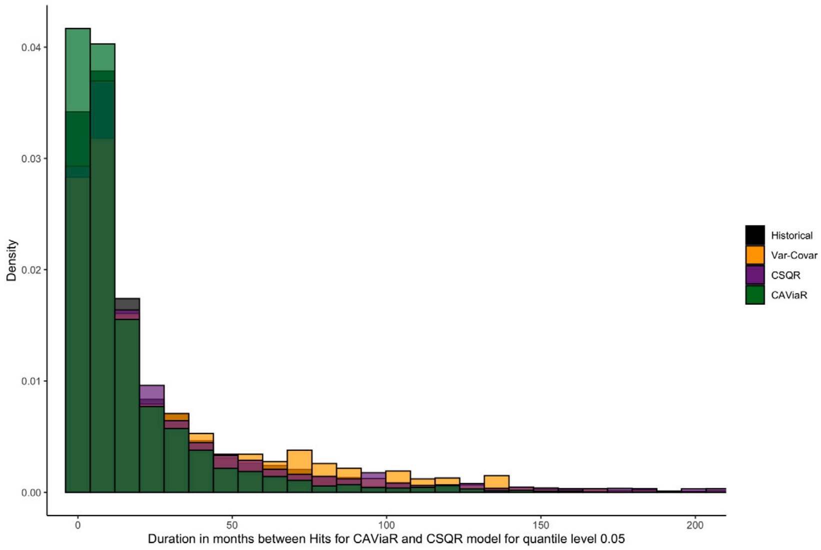

Fig. 4. Histogram of the duration between hits for Historical, Variance–Covariance, CSQR and CAViaR models for quantile level 0.05.

Table 2
Average of expected shortfall scores by firm present at Eq. (4) at the 0.05 quantile for each model for three time windows.

$\hat{S}_{0.05}$	All (2000–2020)	Great recession (2007–2011)	Covid-19 pandemic (2020)
Historical	−6,130.5	−5,853.3	−16,387.2
Variance–Covariance	−5,631.0	−5,728.2	−15,543.8
CSQR	−6,615.3	**−7,562.8**	**−20,434.8**
CAViaR	**−7,795.2**	−6,653.1	−11,534.7

Lower score indicates better prediction, marked in bold.

benchmark value. By locating individual firms in Fig. 3, we can identify those that present more Hits than expected and, as such, they can be considered underperforming.

Fig. 4 shows the histogram of the duration in months between two hits using the same procedure than Christoffersen and Pelletier (2004) and Engle and Manganelli (2004). We observe that for the CSQR model, and even more for the Variance–Covariance method, the distribution shows a bigger span between hits, while the CAViaR presents a distribution more truncated to the left side, with lower periods between two hits. The historical method does not present any notable information gain in comparison to other models. We performed a Kolmogorov–Smirnov test for the duration between Hits for both the CSQR and CAViaR models, where the null hypothesis tests the equal distribution of durations with an exponential duration and the alternative hypothesis assumes a different distribution. For the great majority of firms, the null hypothesis was not rejected.

In Table 2 we see the average scoring evaluated for the four models as specified in Eq. (4) for the quantile level 0.05. The results are similar than those present in Table 1, for the whole period the CAViaR model shows a better approximation to the Expected Shortfall, but for both turmoil periods, the CSQR model surpasses the other models tested. It is of note how the CAViaR model is the most underperforming model during the Covid-19 pandemic, which can be due to the necessity of the time-series based models of a long span of data, making them unfeasible to predict short terms, for example, at the start of a crisis.

4.2. Comparison of VaR and expected shortfall estimation

In Fig. 5, we present the density of the average estimated VaR 0.05 over time, calculated using the CSQR and CAViaR models, for all 438 firms and for three time windows. The density of the average VaRs for the 0.05 quantile calculated with the CSQR model is more strongly negative than that of the average VaRs calculated using the CAViaR model for the whole period (left subfigure). For periods of high volatility (center and right subfigures), the CAViaR model seems to include a group of firms with a fitted VaR lower than that fitted with the CSQR model, but the density of average CAViaR-fitted VaRs stays shifted to the right with respect to the density of the average CSQR-fitted VaRs.

Fig. 6 shows the difference between the empirical ES for the CSQR and CAViaR models, separately by time periods. In general, for the whole period (left subfigure), the average ES levels for the CAViaR model are greater than those for the CSQR model, meaning that, on average, capital requirements should be increased when using the CSQR model as opposed to the time-series perspective provided by the CAViaR model. During the Great Recession (center subfigure), the CSQR model predicts average ES levels similar

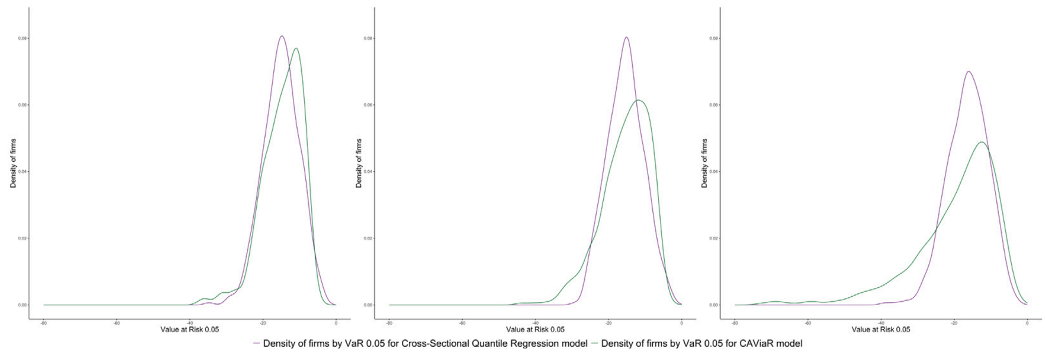

Fig. 5. Kernel density of estimated VaR using the CSQR (purple) and CAViaR models (green) for the 0.05 quantile, for the whole period (2000–2020, left), the Great Recession (2007–2012, middle) and Covid-19 pandemic (2020, right), for 438 firms.

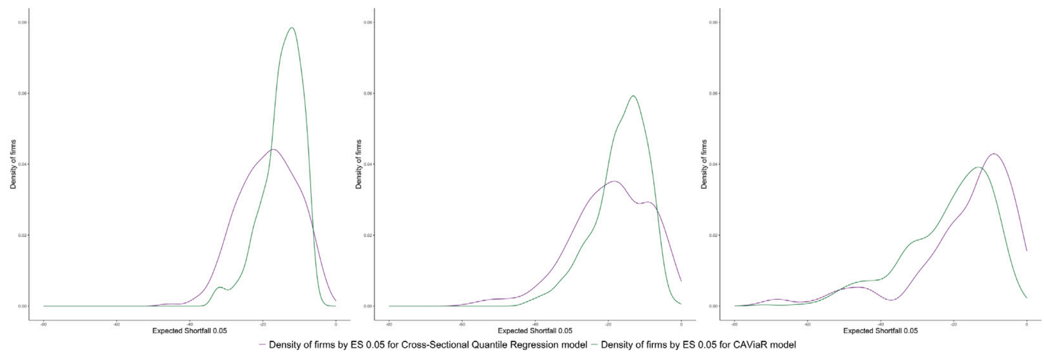

Fig. 6. Kernel density of estimated empirical ES using the CSQR (purple) and CAViaR models (green) for the 0.05 quantile, for the whole period (2000–2020, left), the Great Recession (2007–2012, middle) and Covid-19 pandemic (2020, right), for 438 firms.

to those of the CAViaR model. In the Covid-19 period (right subfigure), capital requirements are lower when employing the CSQR model (vs. the CAViaR model), because the density of the average estimated ES levels of the CSQR model has shifted to the right. This indicates that the choice of model does not have the same consequences for investment decisions.

5. Conclusions

We develop a cross-sectional quantile regression (CSQR) model using seven firm characteristics to evaluate the tail behavior of returns. For each firm, we compared the predicted VaR and empirical ES obtained using this CSQR model with the VaR and empirical ES estimated using the CAViaR model.

Our study included two periods of high volatility: the Great Recession (2007–2011) and the Covid-19 outbreak (2020). The CSQR model shows a lower estimated 0.05 quantile during these high volatility periods than that obtained using the CAViaR approach. Our results show the CSQR model performs better for the 0.05 quantile during both periods of turmoil and that this same CSQR model presents fewer Hits than are presented by the CAViaR model.

With the CSQR model, the VaR and empirical ES are, on average, lower than the corresponding values calculated using CAViaR; however, in periods of high volatility, the ES estimates using CSQR increase, exceeding those estimated with the CAViaR model. This is especially true for the period coinciding with the Covid-19 pandemic. From the perspective of risk management, if the ES is closer to zero in the CSQR model, this means that reserves should be reduced.

A natural step forward in this line of research would be to implement a mixture of both CAViaR model and CSQR model, for example by weighting both predictions depending of the volatility of the market, or by calculating the CSQR model's parameters as a stochastic process adjusted with a CAViaR.

In short, using the CSQR model should serve to enhance the evaluation of company returns, and can provide an improvement in the calculation of reserves during periods of turmoil. Moreover, the use of the CSQR model facilitates quantile approximation for out-of-sample firms.

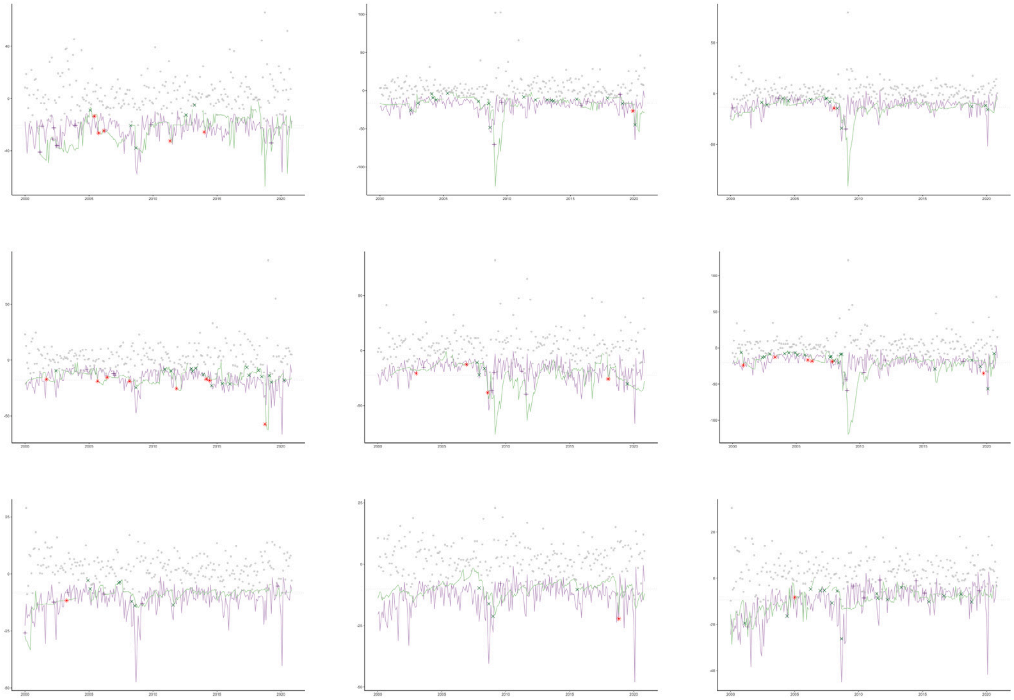

Fig. A.7. Monthly comparison between the evolution of returns and predictions for the 0.05 quantile for both the CSQR (purple) and CAViaR models (green) for firm *nº*s 10550, 17137, 21573, 45728, 51263, 54704, 57568, 61313 and 62092 between 2000 and 2020.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

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Appendix

See Fig. A.7.

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