

$\bar{K}N$ interaction, p-wave terms

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Abstract. The inclusion of new ingredients that are expected to be specially relevant at higher energies could reveal more information about the physics behind the NLO terms of the chiral Lagrangian. In the present work, we explore the relevance of including partial waves higher than the $L = 0$, which is usually the only component considered in the literature to study the $\bar{K}N$ scattering phenomenology. In particular, we focus on the p-wave contribution, the effect of which is expected to be non-negligible, as we aim at obtaining the K^-p scattering amplitudes at higher energies, necessary to describe the $\eta\Lambda$, $\eta\Sigma^0$, $K^0\Xi^0$ and $K^+\Xi^-$ production reactions. Extending the $\bar{K}N$ interaction to p-wave components is also relevant for studies of bound K^- mesons in nuclei since their local momentum can acquire sizable values.

1 Introduction

It is already more than two decades that Unitarized Chiral Perturbation Theory (ChPT) has been shown as a powerful approach to describe low-energy hadron interaction. This non-perturbative treatment of chiral perturbation theory (ChPT) allows one to dynamically generate resonances and bound states. ChPT is characterized by employing hadrons as degrees of freedom, instead of quarks and gluons, and by respecting the symmetries of the underlying theory of the strong interaction, Quantum Chromodynamics (QCD), particularly its spontaneously broken chiral symmetry [1].

Already from the early stages of the $\bar{K}N$ interaction study, such a theoretical scheme provided clear evidences of the molecular nature of $\Lambda(1405)$ (see Refs [2–7]), whose mass was systematically overestimated by quark models. The interpretation of the $\Lambda(1405)$ as a $\bar{K}N$ quasi-bound state was already anticipated by the authors of Refs [8, 9], but the confirmation of the $\Lambda(1405)$ being essentially a meson-baryon bound state came after establishing its double-pole nature [4, 10] from comparing different experimental line shapes [11], which unambiguously indicates the different coupling strength to the meson-baryon components of the $\Lambda(1405)$ wave-function.

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More recently, the irruption of several experiments triggered a renewed interest in improving the chiral unitary theories [19–26] for a better description of the $\bar{K}N$ interaction and related phenomenology. On the one hand, the COSY [12] and HADES [13] collaborations employed pp reactions aiming at establishing the shape of the $\Lambda(1405)$. On the other hand, with same purpose, photo- or electro-production processes were carried out at LEPS [14] and CLAS [15–17]. Additionally, the precise determination of the energy shift and width of the $1s$ state in kaonic hydrogen was measured by SIDDHARTA [18].

In our previous works [24–26], the relevance of the terms next in the hierarchy after the lowest order Weinberg–Tomozawa (WT) term was studied, in connection with the inclusion of higher-energy experimental data. An especial attention was paid on the $K\Xi$ production reactions because they do not proceed directly via the lowest order WT contribution [24]. Our further studies [25, 26] indeed demonstrated that the so-called Born diagrams as well as the next-to-leading order (NLO) terms are far from being mere corrections when it comes to the reproduction of the $K^-p \rightarrow \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$ reaction cross sections. In particular, the inclusion of isospin filtering reactions in [26, 27] served to emphasize their relevance for avoiding potential ambiguities in the isospin components of the scattering amplitude. All this translates into stronger constraints on the models by means of which one can derive more reliable values of the low-energy constants of the chiral Lagrangian.

In Ref. [28], as we expected, the inclusion of partial waves higher than the $L = 0$ reveal more information about the physics behind the NLO terms of the chiral Lagrangian. In particular, the focus was put on p-wave contributions whose effect is proved to be non-negligible in order to describe the $\eta\Lambda, \eta\Sigma^0, K^0\Xi^0$ and $K^+\Xi^-$ production reactions. As additional ingredients, new NLO contributions were taken into account, such terms have usually been ignored for having a low impact at lower energies. We show in the present manuscript that the new terms of the NLO Lagrangian are relevant, being in fact strongly intertwined with the p-wave components of the $\bar{K}N$ interaction.

2 Formalism

As already mentioned, UChPT has been proved as a proper tool to treat the meson–baryon scattering at energies around resonances. The key point lies in the fact that the nonperturbative schemes prevent plain ChPT from non converging and, at the same time, the unitarity and analyticity of the scattering amplitude are ensured. In the present work, the Bethe–Salpeter (BS) equation solved in coupled channels guarantees unitarity. The method followed is the same employed in Refs. [3, 29] and allows factorizing the interaction kernel and the scattering amplitude out of the integral equation thereby transforming a complex system of coupled integral equations into a simple system of algebraic equations which, in matrix form, reads:

$$T_{ij} = (1 - V_{il}G_l)^{-1}V_{lj}, \quad (1)$$

where V_{ij} is the driving kernel derived from the chiral Lagrangian, T_{ij} is the corresponding scattering amplitude for the transition from an i channel to a j one, and G_l is the loop function



Figure 1. Diagrammatic representation of the meson–baryon interaction kernels: Weinberg–Tomozawa term (i), direct and crossed Born terms (ii) and (iii), and NLO terms (iv). Dashed (solid) lines represent the pseudoscalar octet mesons (octet baryons).

of the intermediate channel l , which reads:

$$G_l = \frac{2M_l}{(4\pi)^2} \left\{ a_l(\mu) + \ln \frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s} \ln \frac{m_l^2}{M_l^2} + \frac{q_{cm}}{\sqrt{s}} \ln \left[\frac{(s + 2\sqrt{s}q_{cm})^2 - (M_l^2 - m_l^2)^2}{(s - 2\sqrt{s}q_{cm})^2 - (M_l^2 - m_l^2)^2} \right] \right\}, \quad (2)$$

with M_l and m_l being the baryon and meson masses of the channel. The subtraction constants a_l replace the divergence for a given dimensional regularization scale $\mu = 1000$ MeV. These constants are unknown parameters to be fitted to the experimental data. There are ten channels in the $S = -1$ sector, therefore one expects to have the same number of subtraction constants, although they can be reduced to six by taking into account isospin symmetry arguments.

Roughly speaking, the $SU(3)$ effective chiral Lagrangian consist of an expansion in powers of momentum over a characterizing parameter of the system. These building blocks preserve the symmetries of the fundamental interaction and are arranged in the expansion by order of relevance following a power counting scheme (see for instance [30] for a more detailed explanation). At leading order (LO), the most general Lagrangian can be expressed as:

$$\mathcal{L}_{\phi B}^{(1)} = i\langle \bar{B}\gamma_\mu [D^\mu, B] \rangle - M_0 \langle \bar{B}B \rangle - \frac{1}{2} D \langle \bar{B}\gamma_\mu \gamma_5 \{u^\mu, B\} \rangle - \frac{1}{2} F \langle \bar{B}\gamma_\mu \gamma_5 [u^\mu, B] \rangle, \quad (3)$$

where M_0 is the common baryon octet mass in the chiral limit, the constants D, F denote the axial vector couplings of the baryons to the mesons, and the symbol $\langle \cdot \rangle$ stands for the trace in flavour space. The baryon octet field (N, Λ, Σ, Ξ) is denoted by B , while the pseudoscalar meson octet field (π, K, η) enters in $u_\mu = iu^\dagger \partial_\mu U u^\dagger$, where $U(\phi) = u^2(\phi) = \exp(\sqrt{2}i\phi/f)$ (see [24]) with f being the pseudoscalar decay constant that acts as typifying scale factor in the expansion in powers of momentum. Finally, $[D_\mu, B]$ contains the covariant derivative that accounts for the local character of the chiral transformation of u and it is defined as:

$$[D_\mu, B] = \partial_\mu B + [\Gamma_\mu, B] \quad (4)$$

with $\Gamma_\mu = [u^\dagger, \partial_\mu u]/2$ being the chiral connection. The next-to-leading order (NLO) contributions are given by:

$$\begin{aligned} \mathcal{L}_{\phi B}^{(2)} = & b_D \langle \bar{B}[\chi_+, B] \rangle + b_F \langle \bar{B}[\chi_+, B] \rangle + b_0 \langle \bar{B}B \rangle \langle \chi_+ \rangle + d_1 \langle \bar{B}[u_\mu, [u^\mu, B]] \rangle \\ & + d_2 \langle \bar{B}[u_\mu, [u^\mu, B]] \rangle + d_3 \langle \bar{B}u_\mu \rangle \langle u^\mu B \rangle + d_4 \langle \bar{B}B \rangle \langle u^\mu u_\mu \rangle \\ & - \frac{g_1}{8M_N^2} \langle \bar{B}[u_\mu, [u_\nu, \{D^\mu, D^\nu\}B]] \rangle - \frac{g_2}{8M_N^2} \langle \bar{B}[u_\mu, [u_\nu, \{D^\mu, D^\nu\}B]] \rangle \\ & - \frac{g_3}{8M_N^2} \langle \bar{B}u_\mu \rangle \langle u_\nu, \{D^\mu, D^\nu\}B \rangle - \frac{g_4}{8M_N^2} \langle \bar{B}\{D^\mu, D^\nu\}B \rangle \langle u_\mu u_\nu \rangle \\ & - \frac{h_1}{4} \langle \bar{B}[\gamma^\mu, \gamma^\nu] B u_\mu u_\nu \rangle - \frac{h_2}{4} \langle \bar{B}[\gamma^\mu, \gamma^\nu] u_\mu [u_\nu, B] \rangle - \frac{h_3}{4} \langle \bar{B}[\gamma^\mu, \gamma^\nu] u_\mu \{u_\nu, B\} \rangle \\ & - \frac{h_4}{4} \langle \bar{B}[\gamma^\mu, \gamma^\nu] u_\mu \rangle \langle u_\nu, B \rangle + \text{h.c.}, \end{aligned} \quad (5)$$

where M_N stands for the nucleon mass, while the coefficients b_D, b_F, b_0, d_i ($i = 1, \dots, 4$), g_i ($i = 1, \dots, 4$) and h_i ($i = 1, \dots, 4$) are the low-energy constants (LECs) at this order. The LECs are unknown parameters that contain all the physics of the problem. Despite the symmetries of the underlying theory cannot fix these constants, some of them can be constrained by other observables, namely: the mass splitting of baryons, the pion-Nucleon sigma term or the strangeness content of the proton. In this case, we treat the LECs as free parameters in the fitting procedure as it is usually done in the literature. The quantity $\chi_+ = 2B_0(u^\dagger \mathcal{M} u^\dagger + u \mathcal{M} u)$ explicitly breaks chiral symmetry via the quark mass matrix $\mathcal{M} =$

$\text{diag}(m_u, m_d, m_s)$, while $B_0 = -\langle 0|\bar{q}q|0\rangle/f^2$ relates to the order parameter of spontaneously broken chiral symmetry.

The NLO Lagrangian of Eq. (5), taken from [31], differs from the one we employed in our previous works [24–26] in the terms accompanied by g_i ($i = 1, \dots, 4$) and h_i ($i = 1, \dots, 4$) coefficients (from now on g -terms and h -terms). The non-significant role of these terms at low energies has been the main reason to discard them in the study of the $S = -1$ meson–baryon scattering. By contrary, since our models explore higher energies, it is reasonable to consider these additional contributions. Actually, as we show in [28] and in the present manuscript, they play a relevant role mainly in the $\bar{K}N$ transitions to the $\eta\Lambda$, $\eta\Sigma^0$, and $K\Xi$ channels. It should be mentioned that there is a reduction of the number of LECs since the g_3 monomial cancels after the hermitian conjugation (h.c.) transformation in Eq. (5) as well as the appearance of new structures coming from h.c. of the g -terms.

All the interaction kernels for the $\phi_i B_i^s \rightarrow \phi_j B_j^{s'}$ processes (with the incoming and outgoing spins s, s' , respectively) can be derived from the previous Lagrangians. They can be found schematically represented in Fig. 1. Diagram (i) corresponds to the Weinberg–Tomozawa (V_{ij}^{WT}), while the Born contributions come in diagrams (ii) (direct Born term V_{ij}^{BD}) and (iii) (crossed Born term V_{ij}^{BC}), the vertices of which are obtained from the D and F terms of Eq. (3), and, finally, diagram (iv) accounts for the tree-level contributions at NLO (V_{ij}^{NLO}). The analytical form of all the contributions is given by Eqs. (7), (8), (9) and (10) in Ref. [28].

Since the total interaction kernel ($V_{ij} = V_{ij}^{WT} + V_{ij}^{BD} + V_{ij}^{BC} + V_{ij}^{NLO}$) is composed of a mixture of contributions with different angular momenta, it is not possible to incorporate V_{ij} into Eq. (1) directly. Thus, it is convenient to express the T -matrix in terms of the spin-nonflip and spin-flip parts.

$$T_{ij}(s, s') = \chi_j^{\dagger s'} [f(\sqrt{s}, \theta) - i(\vec{\sigma} \cdot \hat{n})g(\sqrt{s}, \theta)]\chi_i^s, \quad (6)$$

where θ is the CM angle between the initial and final meson momenta and $\hat{n} = \vec{q}_j \times \vec{q}_i / |\vec{q}_j \times \vec{q}_i|$ is the normal vector to the scattering plane (being \vec{q}_j and \vec{q}_i the outgoing and incoming three-momentum in the CM, respectively). The functions $f(\sqrt{s}, \theta)$ and $g(\sqrt{s}, \theta)$ can be expanded in Legendre polynomials as:

$$\begin{aligned} f(\sqrt{s}, \theta) &= \sum_{l=0}^{\infty} f_l(\sqrt{s}) P_l(\cos \theta), \\ g(\sqrt{s}, \theta) &= \sum_{l=1}^{\infty} g_l(\sqrt{s}) \sin \theta \frac{dP_l(\cos \theta)}{d \cos \theta}, \end{aligned} \quad (7)$$

with f_l and g_l being the projections of the sum of all the above kernels onto $P_l(\cos \theta)$ and $\sin \theta \frac{dP_l(\cos \theta)}{d \cos \theta}$, respectively.

The amplitudes can be redefined as:

$$\begin{aligned} f_{l+}(\sqrt{s}) &= \frac{1}{2l+1} (f_l(\sqrt{s}) + l g_l(\sqrt{s})) \text{ for } J = l + \frac{1}{2}, \\ f_{l-}(\sqrt{s}) &= \frac{1}{2l+1} (f_l(\sqrt{s}) - (l+1) g_l(\sqrt{s})) \text{ for } J = l - \frac{1}{2}, \end{aligned} \quad (8)$$

to ensure that the quantum numbers of spin ($\frac{1}{2}$), orbital angular momentum (l), and total angular momentum (J) are preserved in the unitarization procedure implemented by the BS equations. Following the notation of Ref. [32], each unitarized J -scattering amplitude should be calculated by a new version of Eq. (1), which in matrix form reads:

$$f_{l\pm} = [1 - f_{l\pm}^{\text{tree}} G]^{-1} f_{l\pm}^{\text{tree}}, \quad (9)$$

where the amplitudes f_{\pm}^{tree} are obtained from equations (6)–(8).

The aim of this work is the study of the effects caused by the inclusion of higher partial waves on the physical observables in this sector, particularly on the K^-p cross sections. The most general expression of the differential cross sections for a given $\phi_i B_i^{\chi_z} \rightarrow \phi_j B_j^{\chi'_z}$ process is

$$\frac{d\sigma_{ij}}{d\Omega} = \frac{M_i M_j q_j}{16\pi^2 s q_i} S_{ij}, \quad (10)$$

with

$$S_{ij} = \frac{1}{2\chi + 1} \sum_{\chi_z, \chi'_z} |T_{ij}(\chi_z, \chi'_z)|^2 = |f(\sqrt{s}, \theta)|^2 + |g(\sqrt{s}, \theta)|^2,$$

where the first factor averages over the initial baryon spin projections, giving 1/2 for this particular case, and where we have also summed over all possible final baryon spin projections.

Since the scope of the present study does not go beyond d-wave contributions, focusing mostly on p-wave effects, the expression for the total cross section incorporating such partial waves can be written as:

$$\sigma_{ij} = \frac{M_i M_j q_j}{4\pi s q_i} \left[|f_{0+}|^2 + 2|f_{1+}|^2 + |f_{1-}|^2 + 3|f_{2+}|^2 + 2|f_{2-}|^2 \right]. \quad (11)$$

3 Results and discussion

The $S = -1$ sector is an interesting benchmark to test meson-baryon Effective Field Theories given the existence of large sets of scattering data. For this very reason, it cannot only serve for checking their predictive power but also to extract information about the LECs, especially those beyond LO. This collection of experimental data has been employed to constrain the models developed in the study, the technical details of the data treatment as well as the fitting procedure can be found in Ref. [28]. Actually, three models were introduced in [28] to discuss the effects of the new elements, two new and a third one already developed in [26]. Being the two new models the natural extension of the old one in two steps.

- **s-wave (old)**: This first model corresponds to the fit called **WT+Born+NLO** carried out in [26]. It was constructed by adding the interaction kernels derived from the Lagrangian up to NLO and neglecting the h - and g -terms. We limited this model to the s-wave contribution.
- **s-wave**: The second model improves upon the first one by incorporating the novel h - and g -terms that come from the NLO interaction kernel and, as in the first model, only the s-wave contribution is taken into account.
- **s+p-waves**: This model employs the same Lagrangian as the **s-wave** fit, but it also incorporates the p-wave contributions.

As first comment on the values of the parameters present in the models, the most eye-catching feature is the difference observed when comparing these two new models (see Table 2 in Ref. [28]), being the set of the **s+p-waves** model closer to that of our earlier **s-wave (old)** model. This makes us believe that the **s+p-waves** model developed in the present work constitutes a very good starting point for subsequent implementations of higher partial waves. Both new models, **s-wave** and **s+p-waves**, reproduces experimental data very well and achieve very low $\chi_{d.o.f.}^2$ (0.77 and 0.86 respectively). This fact is clearly appreciated in Fig. 2 of [28], where total cross sections of K^-p scattering to all channels of the $S = -1$ sector are compared to experimental data. It should be commented that one can only notice larger differences among the models in the cross sections of the $K^-p \rightarrow \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$

Table 1. Threshold observables obtained from our fits (branching ratios and the energy shift and width of the 1s state of kaonic hydrogen). Experimental data is taken from [18, 33, 34].

	γ	R_n	R_c	ΔE_{1s}	Γ_{1s}
s+p-waves	2.36	0.188	0.662	297	532
s-wave	2.40	0.179	0.665	280	560
s-wave (old)	$2.36^{+0.03}_{-0.03}$	$0.188^{+0.010}_{-0.011}$	$0.659^{+0.005}_{-0.002}$	288^{+23}_{-8}	588^{+9}_{-40}
Exp.	2.36 ± 0.04	0.189 ± 0.015	0.664 ± 0.011	283 ± 36	541 ± 92

processes, specially at energies above 1900 MeV (see discussion in [28]). The results of the threshold observables for the previous models are collected in Table 1 as well as the corresponding experimental values. No substantial change in the reproduction of the experimental values can be appreciated when comparing the new models to the **s-wave (old)** one, which is completely aligned with the previous comment.

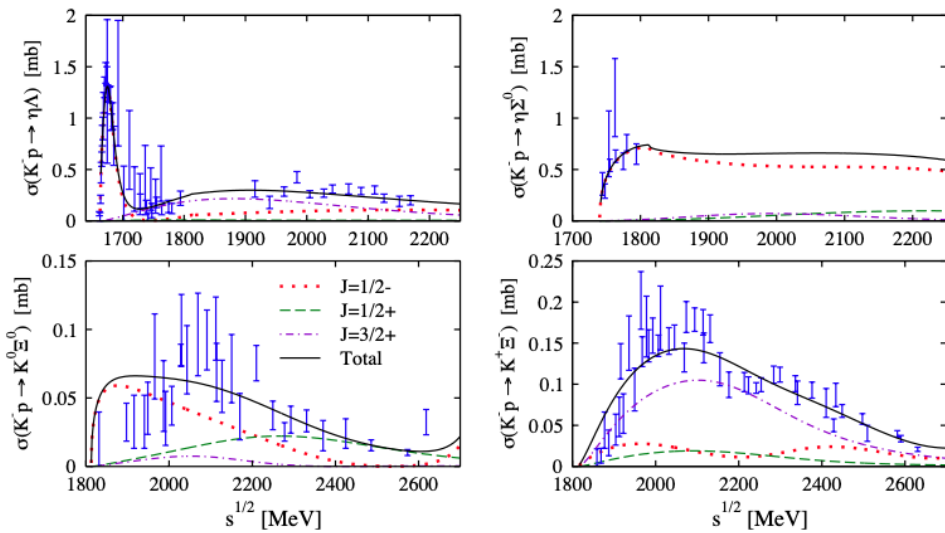


Figure 2. Total cross sections of the $K^-p \rightarrow \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$ reactions obtained for **s+p-waves** (solid black line), and the corresponding contributions for $J^P = \frac{1}{2}^-$ (dotted red line), $J^P = \frac{1}{2}^+$ (dashed green line) and $J^P = \frac{3}{2}^+$ (dash-dotted violet line). Experimental data has been taken from [35–50].

In order to aid better understanding of the role of the p-waves, in Fig. 2 we present the individual partial-wave contributions to the $K^-p \rightarrow \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$ processes included in the **s+p-waves** model, namely the $J^P = \frac{1}{2}^-$ (s-wave), $\frac{1}{2}^+$ and $\frac{3}{2}^+$ (p-wave) channels. We directly focus on these reactions since p-wave effects are barely noticeable in the classical processes.

Focusing first on the $K^-p \rightarrow \eta\Lambda$ cross section, it can be seen that the s-wave $J^P = \frac{1}{2}^-$ contribution dominates just above threshold due to the $\Lambda(1670)$ resonance, which is generated dynamically in this partial wave. Then, as the energy increases, its role is moderately losing relevance against the $J^P = \frac{3}{2}^+$ component which is the main contribution in the region ranging from 1750 to 2100 MeV. Now, we examine the $K^-p \rightarrow \eta\Sigma^0$ reaction. The s-wave $J^P = \frac{1}{2}^-$ contribution is clearly the dominant one over the whole range of energies explored. The

p-wave contributions start being noticeable from 1850 MeV, however, one should be very cautious about drawing conclusions because of the lack of experimental data in this region.

The partial-wave decomposition of the $K\Xi$ total cross section provides very valuable information about the relevance of higher partial waves in our model. On the one hand, the $K^0\Xi^0$ production discloses the fundamental role played by the $J^P = \frac{1}{2}^+$ contribution above 2200 MeV and how the low-energy regime is dominated by the $J^P = \frac{1}{2}^-$ wave. The $J^P = \frac{3}{2}^+$ component provides a non-negligible strength around 2100 MeV, which is however not enough to reproduce the sizable experimental structure. On the other hand, in the $K^-p \rightarrow K^+\Xi^-$ process, the $J^P = \frac{1}{2}^-$ and $J^P = \frac{1}{2}^+$ contributions are moderate and roughly constant, being the $J^P = \frac{3}{2}^+$ component the one that governs the description of the experimental data.

The main conclusion of this study is the need of higher partial-wave contributions to properly describe the K^-p inelastic scattering amplitudes opening up at higher energies. This is in contrast to our previous works, where the experimental data of such processes was reasonably reproduced employing pure s-wave scattering amplitudes. In other words, our previous models have effectively overestimated the lowest partial-wave contributions, thereby masking the physics of higher partial waves behind the values of the NLO parameters. As for the relevance of the new g - and h -terms considered in this work, compared to the rest of NLO contributions, we cannot say anything conclusive, as we have obtained very different values of the parameters in the two new fits presented in [28]. Given the fact that these terms provide a non-negligible contribution to the scattering amplitudes, especially far enough from thresholds, it is natural to consider the g - and h -terms when implementing higher partial waves in the scattering amplitudes for their strong dependence on momenta.

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