

# Fluctuations in 2D fire-ant collectives

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**Abstract:** Active matter resides far from equilibrium since its constituents are constantly dissipating energy by converting ambient or internal energy into work. Previous studies showed that active matter exhibit giant number fluctuations(GNF). By studying fire-ant systems, we find that in the course of time, ants exhibit two distinctive phases: 'active', where essentially all ants move, and 'inactive', where a large fraction of ants are stationary. In both states, we find giant number fluctuations. However, the underlying origin for this result is different. In the 'active' case, the results are genuinely the same resulting in GNF like in other active matter systems. In the 'inactive' case, the GNF we see are due to the inhomogeneous nature of the system in this phase.

## I. INTRODUCTION

For the past centuries, physicists have had many concerns regarding biological organization and how could it be understood mathematically. An example of interest has been active matter, which consists of large number of self-propelling particles that can convert biochemical energy into work, often resulting in collective motion. Some living species such as birds, fish, insects or even bacteria can be described as active matter and under certain situations, they illustrate the collective motion state in the form of bird flocks, fish schools or bacterial colonies. Since particles are constantly injecting and dissipating energy into the system, active matter is far from equilibrium and therefore cannot be described using physics in equilibrium.

For a system in thermodynamic equilibrium which can interchange particles and energy with a reservoir, fluctuations in particle number scale linearly with the volume of the system and thus,  $\langle(\Delta N)^2\rangle \sim V \sim \langle N\rangle$ , where in the last relation it is used that the number density  $\rho = \langle N\rangle/V = cte$ . In contrast, for active systems, it is often observed that fluctuations no longer scale linearly with the number of particles, but instead, they scale as:  $\langle(\Delta N)^2\rangle \sim \langle N\rangle^\alpha$  with  $\alpha > 1$ . [1] This fact is referred to as giant number fluctuations(GNF). To understand their origin recall that if the particles in the system are spatially distributed randomly, the hypothesis of the central limit theorem(CLT) is fulfilled and we find that  $\langle(\Delta N)^2\rangle \sim \langle N\rangle$ . Conversely, if there is some spatial correlation between the particles in the system, for example, due to neighbouring alignment or any other sophisticated interactions, the system starts to deviate from the CLT expectation and fluctuations scale with  $\langle N\rangle$  to an exponent greater than 1. [2] This effect was first studied with thin rods made out of cuts of copper wire, confined in a quasi-2-dimensional cell that vibrated perpendicularly to the plane, and that resulted in the

in-plane motion of the rods with anomalously large fluctuations in density. It is the collective motion of the rods, resembling to a ferromagnetic state, that allows changes in the coarse grained  $\vec{v}$  of the collective of the rods, what results in more pronounced fluctuations relative to what is seen for disordered isotropic systems, or systems in equilibrium. [3]

My work focuses on exploring number fluctuations in fire-ant collectives. *Solenopsis invictae* collectives, are known to form interesting structures in nature [4]. They can form living rafts in order survive floods and even arrange themselves to form towers to climb to higher surfaces.

Recently, the group I am part of has been studying intensively the mechanical properties of these type of ant collectives as well as their social interactions, that have been modeled in terms of motility-induced phase separation(MIPS) [5]. Taking advantage of the data collected in their previous work, I analysed the number fluctuations on dense ant collectives. From the early data, it is observed that for long periods of time, many ants are stationary and aggregate into clusters with few other ants moving freely, but at some point clusters spontaneously break and the ants all become active and move. This behaviour occurred as cycles, with the collective being either 'inactive' or 'active'. Studying the fluctuations separately, we find that for 'active' periods  $\alpha > 1$  due to GNF, and for 'inactive' periods, we also find that  $\alpha > 1$ , but in this case due to the spatial heterogeneities in the system. To prove this last fact, we generated images simulating heterogeneous systems and determined how the exponent changed when varying the density distribution in the system.

## II. SET UP AND EXPERIMENTAL PROCEDURE

The experiment consists of a quasi-2-dimensional cylindrical system of diameter  $D_d = (4.50 \pm 0.02)$  cm, made by acrylic walls and an upper glass plate to cover the cell and confine the ants. The height of the cylinder is a

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bit larger than the mean height of the ants. The system of ants is thus 2D. The size and mass of the ants were computed by calculating the standard deviation from a sample of 1000 ants; these are  $L_{ant} = (3.4 \pm 0.7)$  mm and  $m_{ant} = (0.8 \pm 0.1)$  mg, respectively.

When ants were placed inside the experimental cell, a CCD camera recorded the system of ants at a frame rate of 3.75 fps. Eight different experiments were performed with ant masses of  $m_{exp} = [0.35, 0.36, 0.45, 0.45, 0.50, 0.57, 0.68, 0.81] \pm 0.01$  g, corresponding to  $N_{ants} \approx [438, 450, 563, 563, 625, 713, 850, 1013]$ . 40000 images were recorded for each experiment, meaning that the recording time for each experiment was  $t_{rec} \approx 3h$ .

### III. IMAGE PROCESSING

Each experiment consisted of roughly 40000 images in an 8-bit *jpeg* format in grey scale. Despite the resolution of the images was not the same in all experiments, on average, the image size was 760x756 pixels. Instead of calculating the number of ants within the image, which is hard at the experimental high densities, we counted the pixels corresponding to ants. To do so, we proceeded to simply subtract the  $i$ -th frame with a background image and selected a threshold in order to binarize the subtracted image. The processed image was then a matrix whose entries are ones and zeros corresponding ants and background, respectively. The image processing sequence can be seen in FIG. 1.

We used randomly selected images and compared the processed image with the original one to check the validity of the thresholding.

Once all images were binarized, we computed the fluctuations generating windows of radius  $R_j$  that explored the whole image, and counted the number of white pixels within the window. By repeating this count in the same window for all frames and then averaging for all windows we were able to compute the mean particle number  $\langle N_j \rangle$  and its fluctuation  $\langle (\Delta N_j)^2 \rangle = \langle (N_j)^2 \rangle - (\langle N_j \rangle)^2$ . This can be thought as if each window was an independent microstate and each frame was a replica of the system.

Repeating this process for different  $R_j$ , we were able to establish a relationship between  $\langle N \rangle$  and  $\langle (\Delta N)^2 \rangle$  which in logarithmic scale, we obtained a linear relationship, the slope of the corresponding line was  $\alpha$ .

## IV. RESULTS AND DISCUSSION

### A. Preliminary results

Before getting into the ant systems, we proceeded to study the fluctuations of two different point pattern systems: a disordered and a crystalline arrangement of points; see FIG. 2(a,b). The point distribution in the first case is spatially random; the system is homogeneous and

isotropic. However, for the crystalline case, the point pattern is homogeneous but anisotropic. Note that, since we are dealing with 2D homogeneous systems,  $\langle N \rangle = \rho \pi R^2$  with  $\langle N \rangle$  the average number of points inside a window of radius  $R_j$ , and  $\rho$  is the number density. This is indeed what we find, as shown in FIG. 2(c,d). We then compute  $\langle (\Delta N)^2 \rangle$  and plot the results as a function of  $\langle N \rangle$ . We find  $\langle (\Delta N)^2 \rangle \sim \langle N \rangle^\alpha$ , with  $\alpha = 0.9985 \pm 0.0015$  and  $\alpha = 0.54 \pm 0.03$  for the disordered and crystalline point pattern systems, respectively.

It is important to point that fluctuations deviate for large window sizes since the statistics diminish in those dimensions. The fits to obtain  $\alpha$  are computed for the  $\langle N \rangle$  values where  $\langle (\Delta N)^2 \rangle$  scale in power-law fashion.

We find that  $\alpha \approx 1$  for the disordered point pattern. That means that the particle distribution complies with the CLT and that no GNF are present for this system. For the crystalline point pattern, we obtain  $\alpha \approx 0.5$  since in this case, the only variation in particle number is given by the particles that lay near the perimeter of the circular window which is 1D, and not 2D. The points contributing to  $\langle (\Delta N)^2 \rangle$  are no longer the ones located inside the window, since the number of inner particles is the same for every window with radius  $R_j$  irrespective of its position. Therefore,  $\langle (\Delta N)^2 \rangle \sim R \sim \sqrt{\langle N \rangle}$ .

### B. Analysis of ant experiments

From the data, one can clearly notice that ants experience activity cycles. As an example, FIG. 3 shows two frames corresponding to both 'active' and 'inactive' states. Motivated by this transition, we analysed separately both the 'active' and 'inactive' periods and studied the differences in number fluctuations between the two.

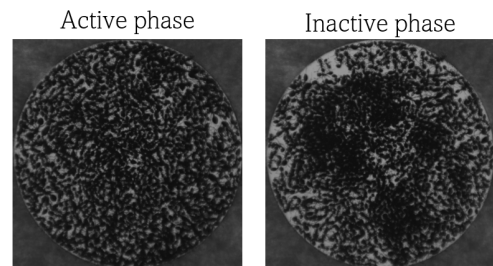


FIG. 3: Left image corresponds to a frame from an active cycle whereas the right image corresponds to a frame from an inactive cycle which can clearly be observed the cluster formation.

Several methods were proposed to classify the activity of the system such as subtracting consecutive images, computing the density of the system as a function of time or even fluctuations to parameterize the homogeneity of the system, though eyesight ended up being essential to bin the data by activity. After generating movies with

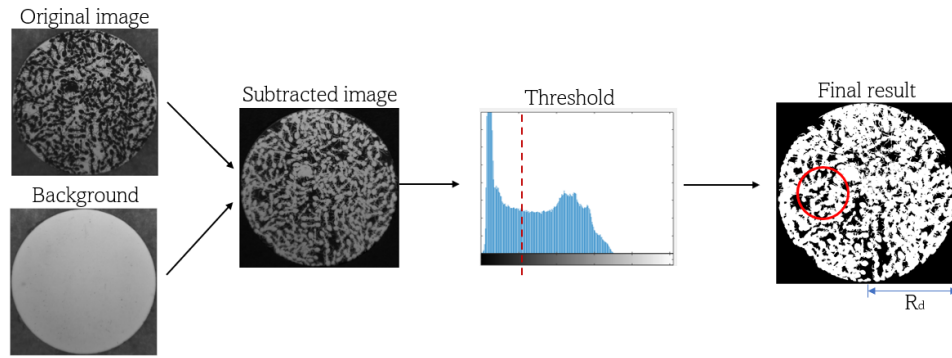


FIG. 1: Processing all frames undergo. The original image is subtracted to the background image, and finally, by selecting a threshold, the image is then binarized so ants correspond to ones and the rest are zeros. For the final result, it has been displayed in red, a window of radius  $R_j$ , as well as the radius of the dish,  $R_d$ .

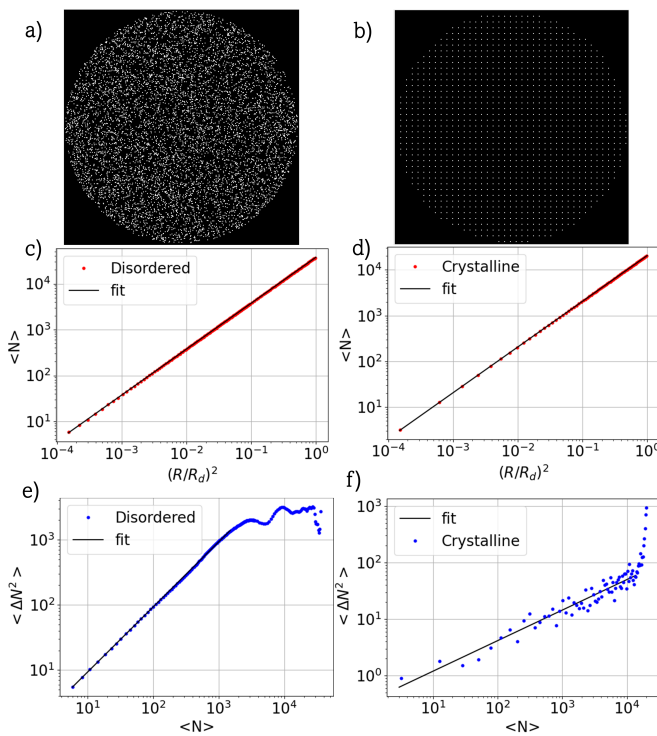


FIG. 2: Left column correspond to the results obtained for the disordered point pattern whilst the right column corresponds to the crystalline point pattern. Both c) and d) plots are represented with the window radius normalized to the dish radius  $R_d$

the frames, we could clearly observe, and then classify, periods where ants remained mostly inactive and periods where they were fully active.

After the classification, we proceeded to compute the fluctuations by averaging over the corresponding frames following the procedure explained in section III. *Image Processing*.

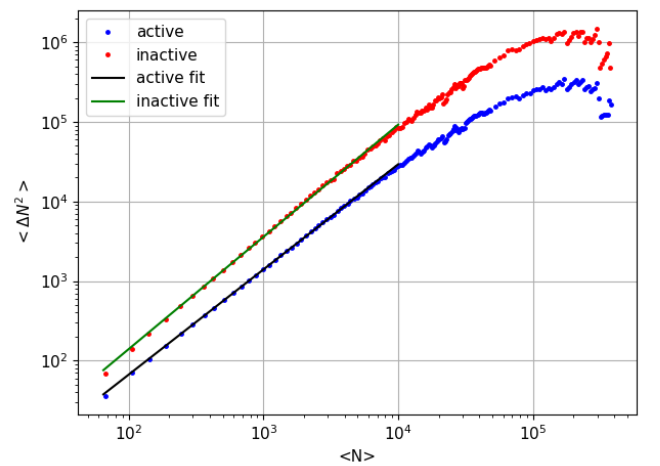


FIG. 4: Number fluctuations as a function  $\langle N \rangle$  and linear fits for both 'active' and 'inactive' states in a log-log scale for the  $0.81 \pm 0.01$  g experiment, the densest one.

Ant mass (g)	$\alpha$	
	'Active'	'Inactive'
$0.35 \pm 0.01$	$1.605 \pm 0.003$	$1.540 \pm 0.004$
$0.36 \pm 0.01$	$1.598 \pm 0.005$	$1.549 \pm 0.009$
$0.45 \pm 0.01$	$1.544 \pm 0.008$	$1.491 \pm 0.009$
$0.45 \pm 0.01$	$1.642 \pm 0.003$	$1.588 \pm 0.005$
$0.50 \pm 0.01$	$1.611 \pm 0.003$	$1.532 \pm 0.005$
$0.57 \pm 0.01$	$1.499 \pm 0.005$	$1.519 \pm 0.006$
$0.68 \pm 0.01$	$1.388 \pm 0.007$	$1.487 \pm 0.006$
$0.81 \pm 0.01$	$1.321 \pm 0.004$	$1.411 \pm 0.007$

TABLE I:  $\alpha$  results for all systems divided in two columns by the 'active' and the 'inactive' cycles.

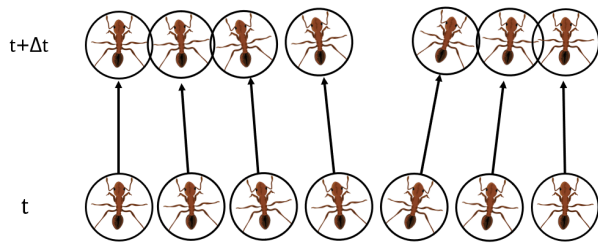


FIG. 5: An illustration of density fluctuation caused by local currents. The black arrow corresponds to the velocity vector of each ant.

### 1. 'Active' phase

The active phase was considered to be the period of time where all ants moved and no clustering was observed. The result for  $\langle(\Delta N)^2\rangle$  vs  $\langle N\rangle$  for the experiment with  $m = 0.81 \pm 0.01$  g is shown in FIG. 4 as blue dots. We find  $\alpha > 1$ . This is true for all experiments; see TABLE I. For the active periods, dense ant collectives do not fulfill the CLT and exhibit GNF. By locally analysing a subset of ants moving collectively, they can be thought of as tiny self-propelling regions with velocity  $\vec{v}$ . Given a certain configuration of these regions at time  $t$ , after a time  $\Delta t$ , the motion of a region may fluctuate as this would not locally change the ant alignment; see FIG. 5. As a consequence, ants experience local currents which generate fluctuations in the direction perpendicular to the direction of motion that enhance fluctuations overall and that explains why we obtain  $\alpha > 1$ . Note this implies that the 'active' phase corresponds to a collective motion state.

### 2. 'Inactive' phase

In the 'inactive' period most of the ants form large clusters that remain static for long periods of time. The 'inactive' phase is then characterized by large accumulations of ants in clusters with a few, usually located in the boundaries of the experimental cell, that move freely through the open areas left by the rest of the ants.

This motility phase coexistence can be explained by means of an effective potential, that depends strongly on the mean speed of the ants,  $U(\vec{r}) \sim \ln(\langle v(\vec{r}) \rangle / v_0)$ , where  $\langle v(\vec{r}) \rangle$  is the mean speed at  $\vec{r}$  and  $v_0$  is the mean speed of the ants at large separations. The decreasing speed, due to ants slowing down when approaching each other, lead to local particle accumulation, and hence further velocity reduction, generating a positive feedback loop which manifests as large-sized clusters that fill the system [5]. Though the presence of aligning interactions would disrupt the effective attraction, the fact that particles barely move when located within a cluster allows the aggregation to last for long periods of time. Furthermore, due to the large accumulations of ants, there are some regions

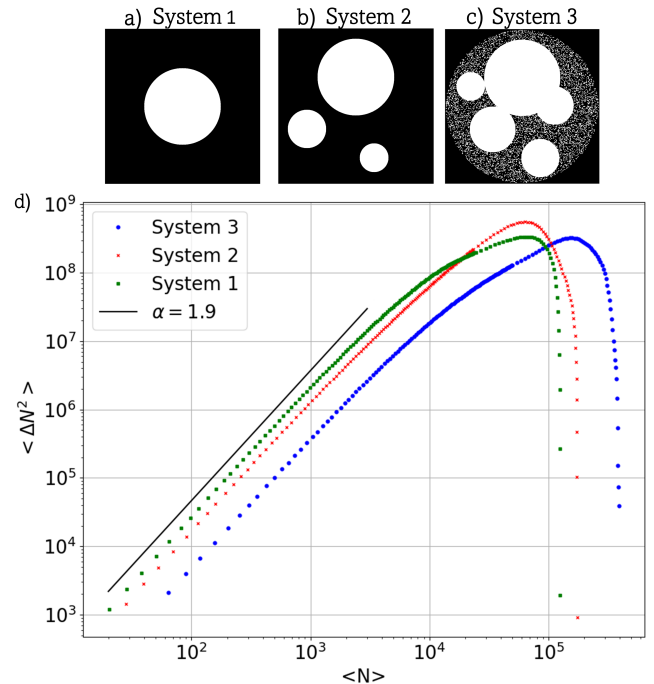


FIG. 6: Three proposed examples for heterogeneous systems. Mid row corresponds to number fluctuations vs. mean particle number. Bottom row corresponds to the dependency of the mean particle number with  $R^2$  normalised to the dish radius  $R_d$ .

System	$\alpha$
System 1	$1.913 \pm 0.003$
System 2	$1.887 \pm 0.004$
System 3	$1.857 \pm 0.003$

TABLE II:  $\alpha$  obtained for the three heterogeneous systems.

where density decreases considerably and the remaining ants that are not bounded to any clusters can move leading to a motility phase coexistence.

For these 'inactive' states, we find that  $\alpha > 1$  for all experiments; see FIG. 4 for the experiment with  $m = 0.81 \pm 0.01$  g where it is shown with red dots and TABLE I. Note that in this case there are no local currents that generate density fluctuations since inactive periods are not characterized by any collective motion of the ants. Instead, we believe the origin of this result is due to the heterogeneity of the system, with  $\rho$  being no longer constant but rather dependant on its position  $\rho = \rho(\vec{r})$ . To prove this, we generate images with distinctive but heterogeneous distributions of pixels; see FIG. 6(a-c).

Simulations of heterogeneous systems lead to  $\alpha > 1$  and it is observed that the more distributed the particles are, the lesser the exponent is; see FIG. 6(d) and TABLE II. These results illustrate that heterogeneity intensifies

the fluctuations of the system in accordance with the results obtained for the ant experiments in the 'inactive' phase.

## V. CONCLUSIONS

The main objective of my work was to examine the fluctuations in fire-ant collectives. We began by studying two point pattern systems which we knew what to obtain. We found  $\langle(\Delta N)^2\rangle \sim \langle N\rangle$  for the disordered arrangement of points and  $\langle(\Delta N)^2\rangle \sim \sqrt{\langle N\rangle}$  for the crystalline point pattern; the latter is due to the fact that only points near the border of the window contribute to the fluctuations. We then introduced the activity cycles ants experience in the course of time and, separately studied fluctuations for both 'active' and 'inactive' periods. We ended up obtaining an exponent  $\alpha > 1$  and discussed its origins for both cases by showing that the ant interactions in the 'active' periods lead to GNF similar to other active matter systems, whereas in the 'inactive' periods, we blamed heterogeneity as the cause of enhanced fluctuations as

seen in simulations of heterogeneous systems in FIG. 6.

It is worth mentioning that we discussed and brought up ideas to quantify the activity cycles so that we would be able to distinguish the 'active' and 'inactive' periods together with eyesight observations, albeit we only partially succeeded depending on the experiment. This is the subject of future work.

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