Master in Actuarial and Financial Sciences

MASTER THESIS

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Abstract

The aim of this paper is to study the disparities in terms of internal rate of return (IRR) between the Spanish pay-as-you-go system and the Swedish notional accounts system focusing on variables such as wage growth, gender and retirement age. To achieve this, a program has been developed in RStudio to simulate the IRR of both systems for different profiles. The results obtained provide an in-depth understanding of the relationship between the IRR and intergenerational inequalities within the pure PAYG systems.

Key words: IRR, pay-as-you go system, notional accounts system, sustainability

Abstract

Este trabajo tiene como objetivo analizar las disparidades en términos de tasa interna de rentabilidad (TIR) entre el sistema de reparto español y el sistema de cuentas nocionales sueco haciendo énfasis en variables como el crecimiento salarial, género y edad de jubilación. Para lograrlo, se ha desarrollado un programa en RStudio que permite simular la TIR de ambos sitemas para diferentes perfiles. Los resultados obtenidos proporcionan una comprensión profunda de la relación existente, en los sistemas puros de reparto, entre la TIR e inequidades intergeneracionales que se producen dentro del sistema.

Palabras clave: TIR, sistema de reparto, sistema de cuentas nocionales, sostenibilidad

Contents

1 Introduction

According to the RAE definition, a pension is defined as a "periodic, temporary or lifelong amount paid by the social security system on account of retirement, widowhood, orphanhood or disability". Specifically, retirement pensions refer to the income that a worker will receive once he or she decides to exercise his or her right to retire after having contributed a series of income to the Social Security System during his or her time as an active worker.

Pension systems can vary significantly between countries around the world. There are currently three main types of pension systems:

1. **Pay-as-you-go systems:** a kind of system in which active workers finance the pensions of current retirees through their contributions to the social security system. In this system, individual funds are not accumulated, but resources are redistributed between generations. France and Spain are good examples of pay-as-you-go systems in which pensions are financed by active workers' contributions.

2. **Funded systems:** is a system in which each worker contributes for himself, this means that benefits are directly related to the contributions made and how they have evolved through time. The contributions made by the worker are invested in a range of financial instruments that are managed by specialised financial institutions that seek to maximise capital returns. At retirement, the worker will receive a pension based on the amount accumulated in his or her individual account and the return on investments. The country that stands out for its funded pension system is Chile, which in 1981 introduced a system of individually funded pension funds.

3. **Mixed systems:** a system that combines characteristics of both pay-as-you-go and funded systems. In this system, part of the contributions goes to finance current pensions (pay-as-you-go), while another part goes to individual accounts for each worker (funded). This kind of system seeks to combine intergenerational solidarity with the generation of individual savings. One of the most popular mixed pension systems is that of Sweden whose pension is determined by a mix of a guaranteed pension based on a pay-as-you-go system and a pension derived from a funded system.

The debate on which type of pension is better is complex and depends on various factors such as demographic, economic and social circumstances in each country.

One of the main advantages of the pay-as-you-go system is that it promotes intergenerational equity; however, it is exposed to demographic changes which, in a context of increasing life expectancy and a low birth rate, jeopardise the financial sustainability of the system.

The funded system generates individual decision-making capacity that allows free choice as to how to invest the accumulated capital. One of the major disadvantages of this system is that it increases pension inequalities, as those with higher salaries will be more protected than those with lower salaries.

The mixed system combines the benefits and strengths of the previously explained pay-as-you-go and funded systems. It is important to mention that such systems can be complex to manage due to the interaction between the public and private components of the system.

This paper first reviews the existing and most relevant literature on the impact of the internal rate of return on the equilibrium of pension systems. Subsequently, the functioning of the Spanish and Swedish pension systems is explained, with emphasis on aspects such as regulations and contributions. After understanding the behaviour of these systems, simulations of the internal rate of return are carried out in order to assess sustainability under some initial parameters.

2 Analytical framework

The concept of the internal rate of return (IRR from now on), is a term that can lead to confusion in a pension system context as it uses the term *"return"* when no asset class that provides an explicit return is actually invested. The most appropriate definition is the one used by Samuelson (1958) in which he defines IRR as "an indicator that will measure over the whole life cycle of a generation or cohort of individuals, what is the relationship between the contributions made (or reasonably expected to be made) and the benefits received (or reasonably expected to be received)".

The computation of the IRR represents a fundamental element in assessing the financial sustainability of a pay-as-you-go system since it helps to determine whether current contributions are sufficient to cover present and future pension obligations. According to Samuelson (1958), a pay-as-you-go pension system will only be viable in the long run if the IRR provided by the system does not exceed the growth rate of wages plus the steady growth rate of the contributing population. Samuelson's paper also demonstrated a significant finding: in a stationary economy [1] that exclusively produces consumer goods, the IRR of a pay-as-you-go system is equivalent to the population growth rate [2]. Consequently, it becomes evident that this type of system, characterised by the transfer of resources from younger to older individuals within the population pyramid, derives advantages from population growth. It is important to

[1] In macroeconomic terms, a stationary economy refers to a situation in which main economic variables (GDP, employment, inflation, interest rates, ...) remain stable and do not experience significant changes over time.

[2] Furthermore, in economies experiencing expansion, the IRR within the system rises proportionally with increases in productivity.

note that the above observations are not applicable to mixed or funded pension systems.

Potential factors directly influence the IRR offered by the pay-as-you-go systems. According to Bravo (1996), these systems often experience financial imbalances that affect their IRR. Bravo distinguishes between two pay-as-you-go systems: incipient and mature. An incipient system tends to exhibit surpluses due to its low liability/asset ratio, while a mature system typically shows deficits due to its high liability/asset ratio. Another potential element to take into account is demographic instability; research conducted by Lapkoff (1985) and Keyfitz (1985) in the United States reveals that evidence of demographic instability has a significant effect on the IRR of different generations.

The main objective of this paper is to analyse the internal rate of return of retirement pensions in the Spanish pay-as-you-go system and the Swedish notional accounts system by means of a comparison. This comparison will be carried out by establishing different pensioner profiles according to variables such as age at which the right to retire is exercised, contribution periods, the evolution of nominal income throughout the period of working activity, gender and year of birth.

In order to calculate the internal rate of return, it is essential to know an approximation of the pension that the contributor will receive once he decides to exercise his right to retire. For this approximation, it has been developed an application using the RStudio calculation tool that makes it possible to approximate, depending on the characteristics of each pension system (which will be explained in detail in the following section).

3 Pay-as-you-go systems vs. notional account systems

3.1 Spain: What is the pay-as-you-go system?

The pay-as-you-go (PAYG) system is a social security model in which the contributions of active workers are used to finance the benefits of current retirees. It is a system based on the principle of intergenerational equity, whereby when current contributors to the system retire, future generations of active workers finance their retirement benefits. At retirement, the worker stops contributing to the system and starts receiving a retirement pension, the amount of which depends on his or her contribution record and other variables involved.

One of the current challenging problems facing pay-as-you-go systems is the constant increase in life expectancy joined with low birth rates (Ayuso, Bravo, and Holzmann 2021) (Ayuso, Bravo, Holzmann, and E.Palmer 2021-b) (Ayuso, Bravo, Holzmann, and E.Palmer 2021-a). This causes the dependency ratio of the system to skyrocket, leading to financial imbalances that make the system unsustainable. The above point clearly reflects the importance of constant population growth developed by Samuelson. To solve these sustainability problems, the different systems have opted for measures such as: increases in the legal retirement ages, increases in contributions and adjustments in the calculation of pensions. (Ayuso, Bravo, Holzmann, and E.Palmer 2021-c)

According to Diamond & Barr (2008), in a PAYG system, unsatisfactory outcomes, such as longevity risk, are borne by contributors, i.e. the entire current working generation. Such risks can be transferred between generations through adjustments in benefits and contributions, or by including partial funding or borrowing in the system.

The Spanish pay-as-you-go system is a defined-benefit[3] system administered exclusively by the Social Security, which is governed by the General Social Security Act (Ley General de la Seguridad Social).

To qualify for retirement, it is essential to have been contributing for a minimum of 15 years, of which 2 years must be in the last 15 years prior to retirement. The Spanish system does not distinguish age retirement between gender. In the current year 2023, the system has set the ordinary retirement age at:

- **65 years** provided that contributions have been credited for **37 years and 9 months**.
- **66 years and 4 months** in the case of accumulating contributions of **less** than **37 years and 9 months**.

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The calculation of the Spanish contributory pension takes into account two main factors:

- **The regulatory base:** is calculated taking into account the worker's contribution bases. The contribution bases are defined as the gross monthly income on which the social security contributions have to be taken by employers and employees for each kind of contingency.[4]. Once the contribution bases have been defined, the contributions made by the employer and employee to the Social Security system are as follows (see Table 1):

  The formula for the regulatory base would then be denoted by:

$$B_r = \frac{\sum_{i=1}^{24} BC_i + \sum_{i=25}^{300} BC_i \frac{CPI_{25}}{CPI_i}}{350} \tag{1}$$

[3]A defined-benefit PAYG system is a type of pension system in which the amount of pension an individual will receive upon retirement is established in advance.

[4]The law establishes minimum and maximum contribution bases that are updated according to the country's economic situation. The monthly contributions per contingency are determined on the basis of this range.

[5]Fondo de Garantía Salarial

Contingency	**Employer (%)**	**Employee (%)**	**Total (%)**
Commons	23.60	4.70	28.30
Unemployment	5.50	1.55	7.05
FOGASA[5]	0.20	0.00	0.20
Professional formation	0.60	0.10	0.70

Table 1: Percentage of contributions on the contribution base by contingencies.
Source: Own elaboration

Where the numerator denotes the sum of the contribution bases (BC_i) in nominal value from month 1 to month 24 prior to retirement and the sum of the contribution bases in real value from month 25 to month 300 prior to retirement weighted by an updating factor that uploads them according to the CPI[6] shown at Spanish level for that month taking as a reference the CPI for month 25 prior to retirement. The amount of the sum in the numerator is divided by 350 months.

- **Reduction rate:** is a term that refers, as the name indicates, to the percentage reduction that is applied to the regulatory base for contributions years. In case of willing to retire with the minimum number of years of contributions required by law (15 years, 2 of which must be within the 15 years prior to retirement), the reduction rate is set at 50%. According to the transition of the percentages in the reduction rate published in the *Boletín Oficial del Estado (2022b)* for the year 2023, it is proposed that for each additional month of contribution over 180 months, the reduction rate will increase by 0.21% for the months 181 to 229, and finally increase the reduction rate by 0.19% for the months 230 to 438 (where the unit is reached with 36 years and 6 months of contributions).

  The reduction rate can be defined then as a function in steps:

  Reduction rate ($RR(t)$) as a function of contributed months (t):

$$\begin{cases} 0 & \text{for } 0 \leq t < 180 \\ 0.5 + 0.0021 \times (t - 180) & \text{for } 180 \leq t < 229 \\ 0.6029 + 0.0019 \times (t - 229) & \text{for } 229 \leq t < 438 \\ 1 & \text{for } t \geq 438 \end{cases}$$

In addition, the amount of the contributory pension defined above may be reduced or increased depending on whether the pensioner decides to take early or late retirement.

The Spanish system contemplates two types of early retirement:

[6]Consumer Price Index

- **Early retirement due to non-voluntary termination of employment (forced retirement)**. This is a type of early retirement that can be accessed when the worker leaves the company for objective reasons or when it is part of a collective dismissal derived from economic, technical, organisational or productive reasons. To access this type of retirement, the following requirements must be met:

  – To be registered in the employment offices as a job demander for a minimum of 6 months immediately prior to the date of application for retirement.

  – Be at least 4 years younger than the required retirement age. That is, a maximum of 61 years for those who have contributed a minimum of 37 years and 9 months, and 62 years and 4 months otherwise.

  – A minimum of 33 years of contributions, of which at least 2 years must be within the 15 years immediately prior to the time of termination.

  The amount of the reduction coefficients will vary according to the number of months of early retirement and the number of contribution periods. Obviously, the more months of early retirement, the higher the amount of the reduction coefficients, while the longer the contribution periods, the lower the reduction[7].

- **Early retirement by voluntary will of the worker (voluntary retirement)**. In order to be eligible for this type of pension, it is necessary to meet the following requirements:

  – To have an age that is less than 2 years younger than the required age. In other words, 63 years for people who have contributed a minimum of 37 years and 9 months, and 64 years and 4 months otherwise.

  – To demonstrate a minimum of 35 years, of which at least 2 have to be included in the 15 years previous to the moment of the ceasing of the contributions.

  As in the case of early retirement due to involuntary termination of employment, the reduction coefficients increase as the number of months before retirement increases and decrease as the number of contribution periods increases[8].

In the case of **delayed retirement**, for each year of delayed retirement, the pension is increased by 4%. However, the amount of the monthly retirement pension obtained after the increases resulting from the delay in retirement must in no case exceed the maximum monthly pension for 2023 set by the Government at 3,059.24€ (14 payments).

[7] The table of the reduction coefficients for compulsory retirement is attached in Appendix A

[8] The table of the reduction coefficients for voluntary retirement is attached in Appendix B

3.1.1 The Escrivá reform and its main implications

In the Real Decreto-Ley 2/2023 of 16 March[9], which came into effect the past 1 April 2023, special emphasis is placed on the long-term financial sustainability of the Spanish pension system taking into account an exceptional event that compromises its sustainability: the retirement of the baby boom generation.

In order to strengthen the long-term sustainability of the system, three types of fundamental actions are implemented:

1. **Gradual increase of the maximum contribution base**. It is proposed to increase the maximum contribution bases at an annual frequency marked by the increase of the CPI plus a fixed amount of 1.2 percentage points from 2024 to 2050.

2. **Introduction of the so-called Intergenerational Equity Mechanism (MEI)**. It consists of a contribution applicable to all regimes that will not be computable for benefit purposes and will feed the new Social Security Reserve Fund. The MEI contribution will be 0.6%, with 0.5 points paid by employers and the rest (0.1 points) paid by the employees. The MEI contribution is expected to double to 1.2% in 2029, distributed in 1 and 0.2 points for the employer and the employee respectively.

3. **Introduction of the solidarity quota.** It mainly affects salaries that exceed the maximum contribution base (from January 2023 fixed at 4.4495.50€). The solidarity quota will apply the following rates:

    - 5.5% to the part comprised between the maximum contribution base and 10 percent above the amount of the maximum base.
    - 6% if the remuneration over the maximum base is between 10 and 50 percent higher.
    - 7% to the part of remuneration that exceeds the previous point.

In addition, according to the mentioned law, the regulatory base used for the calculation of the retirement pension also experiences modifications and is the following one (equation (2)):

$$B_r = \frac{\sum_{i=1}^{24} BC_i + \sum_{i=25}^{348} BC_i \frac{CPI_{25}}{CPI_i}}{378} \qquad (2)$$

[9] Real Decreto-Ley 2/2023: https://www.boe.es/buscar/pdf/2023/BOE-A-2023-6967-consolidado.pdf

The numerator is the sum in nominal value of the first 24 contribution bases prior to retirement and the sum of the contribution bases in real value from month 25 to month 348 prior to retirement, taking as a reference the CPI of month 25. The amount obtained in the numerator is divided by 378 months. It is important to mention that from the 348 contribution bases used for the calculation in the new version of the regulatory base, the 324 contribution bases with the highest amount will be chosen.

3.2 Sweden: What is the notional account system?

According to the World Bank Pension Reform Primer[10], a notional account scheme is a kind of system "designed to mimic a defined-benefit PAYG plan, where the pension depends on contributions and investment returns".The returns that contributions earn is a notional one, meaning that are set by the government and not by the product of investment in financial markets.

The current Swedish system is characterised for being an example of a successful regime change transition that began to be implemented successfully in November 2001 in the form of a notional defined contribution system. The ordinary retirement age for the year 2023 is calculated at 66, with the possibility of advancing or postponing the retirement age by a wide range of ages from 61 to 70.

This system is distinguished for offering its retirees a contributory pension consisting of two pillars combining a pay-as-you-go and an individually notional funded approach:

1. **First tier**: is purely administrated by the Swedish Pensions Agency, which is responsible for providing a guaranteed basic pension (known as the "garantipension") for all Swedish citizens who do not have sufficient retirement income. In order to qualify for the "garantipension" in 2023, a number of conditions must be met, as detailed below:

    - Minimum age of 65 or be a pensioner who has had to take early retirement due to special circumstances.
    - Residence in the country for at least three consecutive years. To receive the full amount, the pensioner must have lived in the country for 40 years from the time he/she reached the age of 16. In case of a shorter period of residence, the guaranteed pension is reduced by 1/40th of a percentage point for each year.

[10]"Notional accounts: Notional defined contribution plans as a pension reform strategy", 2023

It is crucial to mention that the guaranteed pension quantity is determined by the "prisbasbelopp" which translates to "basic amount" in English. This basic amount serves as a benchmark for calculating allowances and social benefits, and it is updated annually based on the country's CPI evolution. According to Statistics Sweden (SCB), the "prisbasbelopp" for the year 2023 is set at 52,500 SEK[11], which is equivalent to 4,375 SEK per month.

The guaranteed pension sum depends on the individual's second pillar income ("inkomstpension") and marital status[12]. For single pensioners, the second pillar income threshold above which they are ineligible for the guaranteed pension is 16,177 SEK per month. The maximum monthly guaranteed pension for single individuals is 10,631 SEK. Thus, the guaranteed pension amount can be defined by the following piecewise function.[13]

Garantipension for single individuals according the earned *inkomstpension* :

$$\begin{cases} 2.43 \times PB - INK & \textit{if INK} \leq 1.26 \times PB \\ 1.17 \times PB - 0.48 \times (INK - 1.26 \times PB) & \textit{if INK} > 1.26 \times PB \end{cases}$$

In the case of married pensioners, the threshold for entitlement to the guaranteed pension is set at 14,649 SEK per month. The maximum monthly amount of the guaranteed pension for this group is 9,625 SEK. Similar to the previous scenario, the variation of the guaranteed pension with the second pillar income can be observed using a piecewise function.

Garantipension for married individuals according the earned *inkomstpension*:

$$\begin{cases} 2.20 \times PB - \textit{INK} & \textit{if INK} \leq 1.14 \times PB \\ 1,06 \times PB - 0.48 \times (\textit{INK} - 1.14 \times PB) & \textit{if INK} > 1.14 \times PB \end{cases}$$

Graphically (see Figure 1) it can be seen that the Swedish system generally provides guaranteed pensions that protect single pensioners (see green line) more than married pensioners (see light blue line). This disparity is explained by the fact that single people do not have a spouse with whom to share household expenses.

[11] SCB - prisbasbelopp

[12] Pensions Myndigheten

[13] The "prisbasbelopp" will be referenced using the term PB whereas the "inkomstpension" will be INK

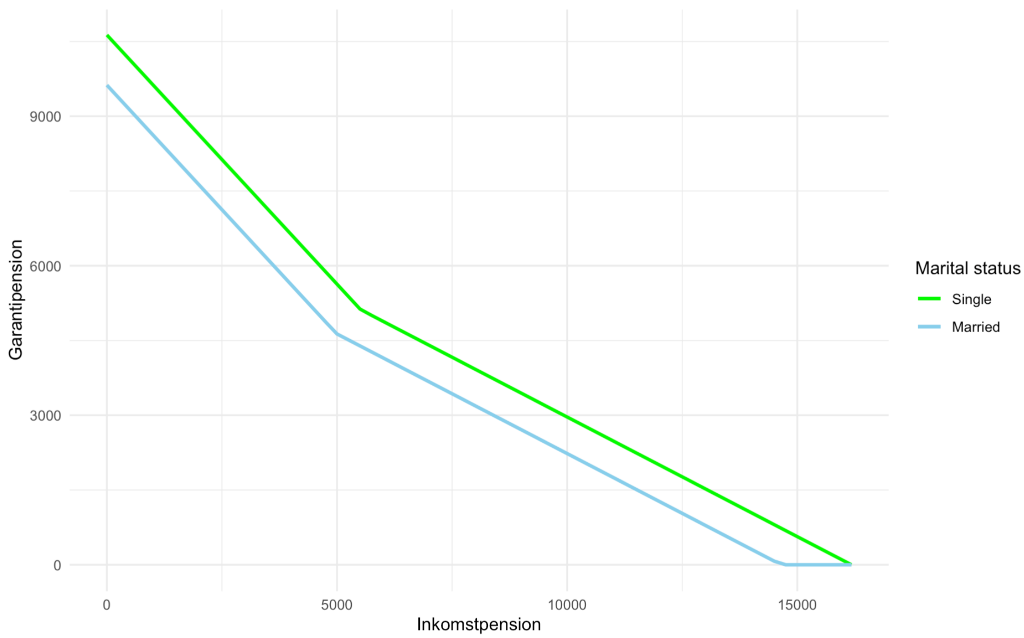

Figure 1: Guaranteed pension according to marital status
Source: Own elaboration

2. **Second tier**: is mandatory and collects 18.5% of workers' pensionable earnings[14] every month and distributes them into two large accumulation funds that generate two different types of pensions:

   - **Inkomstpension**: is the income derived from the accumulation of funds in a notional accounts scheme in which a total of 16% of the pensionable earnings are allocated. This accumulation takes place in a notional manner following a notional interest rate whose evolution takes into account the economic and employment situation of the country. Subsequently, the capital accumulation is reduced by the administration costs of the system and then multiplied by a growth factor that accumulates the funds of all members who have died and belong to the same cohort (inheritance gains). This accumulation factor ($AF_{i,t}$) has the following formula for retirement ages (where i=61,..,70) - (see formula 3):

[14]Pensionable earnings refer to the portion of an individual's income that is considered eligible for inclusion in the calculation of their pension benefits.

$$AF_{i,t} = \frac{L_{i-1,t} + L_{i,t}}{L_{i,t}^{a} + L_{i+1,t}} \tag{3}$$

[a] $L_{i,t}$ =number of survivors in year t in age group i out of 100,000 born.

The notional interest rate is indexed on two factors: the "inkomstindex" and the "balansindex". The inkomstindex is used to adjust workers' notional accounts in line with changes in the average income of the population with the aim of adapting the pension system to the economic situation in the country.

On the other hand, the "balansindex" is a factor that is used to stabilise the actuarial balance[15] of the pension system in the event that the number of liabilities (pensioners) is greater than the number of assets (contributors). The "balansindex" is activated when the ratio of assets to liabilities is less than one.

Since the implementation of the notional accounts system in 2001, the evolution of the "inkomstindex" and "balansindex" over the years has been as follows[16] (see Table 2):

Year	Inkomstindex/balansindex %	Year	Inkomstindex/balansindex %
2020	2.2	2010	-2.7
2019	3.8	2009	-1.4
2018	3.1	2008	6.2
2017	2.6	2007	4.5
2016	4.4	2006	3.2
2015	5.9	2005	2.7
2014	2.5	2004	2.4
2013	-1.1	2003	3.4
2012	5.8	2002	5.3
2011	5.2	2001	2.9

Table 2: Inkomstindex/balansindex evolution
Source: Own elaboration

[15] The actuarial balance refers to the long-term financial equilibrium of a pension system. It is calculated by comparing the projected revenues and expenditures of the pension system over time. If the actuarial balance is positive, indicates financial sustainability, while a negative balance indicates an insufficiency in resources to meet future benefits.

[16] Orange Report 2020 - page 57

Finally, the last term to be taken into account in the calculation of the *inkomstpension* is the annuity. This annuity acts as a divisor once the final notional accumulated capital has been calculated to calculate the monthly annuity that the pensioner will receive. The annuity calculation formula is as follows:

$$a_i = \frac{1}{12L_i} \sum_{k=1}^{R} \sum_{X=0}^{11} (L_k + \frac{(L_{k+1} - L_k)X}{12})(1.016)^{-(k-i)}(1.016)^{-\frac{X}{12}} \tag{4}$$

Where:

- a_i is the cohort-related annuity for age group i..
- L_i is number of survivors in age group i per 100,000 born.
- X is a variable comprised between values of 0 and 11 that refers to months of the year.

Individuals who retired before the age of 65 will see their income vary from year to year until they reach the age of 65 due to the recalculation of the annuity divisor. After reaching the age of 65, there is no further recalculation of the annuity divisors and therefore the amount of pension to be received remains fixed.

These annuity divisors are reviewed periodically due to constant increases in life expectancy which may put the sustainability of the pension system at risk. According to the latest Orange Report available on the web dating from 2020[17] the annuity divisors are the following (see Figure 2):

[17] Orange Report 2020 - page 117 - it is important to mention that the annuity divisors detailed in this report are the ones that will be used for the Swedish IRR simulation.

	61	62	63	64	65	66	67	68	69	70
1938	17.87	17.29	16.71	16.13	15.56	14.99	14.42	13.84	13.27	12.71
1939	17.94	17.36	16.78	16.19	15.62	15.04	14.47	13.89	13.32	12.76
1940	18.02	17.44	16.86	16.27	15.69	15.11	14.54	13.96	13.39	12.82
1941	18.14	17.56	16.98	16.39	15.81	15.23	14.65	14.08	13.50	12.94
1942	18.23	17.65	17.06	16.48	15.89	15.31	14.74	14.16	13.59	13.02
1943	18.33	17.75	17.16	16.58	15.99	15.41	14.84	14.26	13.68	13.11
1944	18.44	17.86	17.28	16.70	16.11	15.54	14.96	14.38	13.80	13.23
1945	18.55	17.96	17.38	16.80	16.22	15.64	15.07	14.48	13.91	13.33
1946	18.64	18.05	17.47	16.89	16.31	15.73	15.16	14.57	13.99	13.41
1947	18.73	18.15	17.56	16.98	16.40	15.83	15.24	14.66	14.07	13.49
1948	18.83	18.24	17.66	17.07	16.49	15.91	15.33	14.74	14.16	13.58
1949	18.89	18.31	17.72	17.13	16.55	15.97	15.38	14.79	14.21	13.63
1950	18.98	18.39	17.80	17.21	16.63	16.05	15.46	14.87	14.28	13.70
1951	19.06	18.48	17.89	17.30	16.71	16.13	15.54	14.95	14.37	13.78
1952	19.14	18.55	17.96	17.37	16.78	16.20	15.61	15.02	14.43	13.85
1953	19.20	18.62	18.03	17.44	16.85	16.26	15.68	15.09	14.50	13.91
1954	19.28	18.69	18.11	17.52	16.93	16.34	15.76	15.17	14.58	13.99
1955	19.34	18.75	18.16	17.58	16.99	16.40	15.81	15.22	14.63	14.04
1956	19.42	18.84	18.25	17.66	17.07	16.48	15.89	15.30	14.71	14.12

Figure 2: 2020 Annuity divisors
Source: Orange Report - 2020

- **Premiepension**: is income derived from the capital accumulation in an equity fund that is publicly managed and allocates 2.5% of the pensionable salary. The government automatically preselects the AP7 Såfa[18] capitalization fund, however, the worker is free to invest and change his portfolio in a maximum of five different funds. In turn, if the worker wishes to assume a higher risk, is free to choose equity funds instead of fixed income funds.

The Swedish system establishes that, as a general rule, employees must pay social security contributions of 7% of their gross salary[19]. Employers, on the other hand, are required to pay contributions of 31.42%[20], which is divided into the following elements (see Table 3[21]):

[18] AP7 Såfa
[19] The Swedish system does not impose any limits on employer and employee social security contributions. In the case of Spain, the contributions made to the system are limited by the maximum contribution base.
[20] The contributions made by the employers are calculated as a percentage of the total sum of salaries and benefits in a year.
[21] OCDE - Taxing Wages 2021: Sweden

Program	Employer (%)
Retirement pension	10.21
Survivor's pensions	0.60
Parental insurance	2.60
Health insurance	3.55
Labour market	2.64
Occupational health	0.20
General wage tax	11.60

Table 3: Employee social security contributions
Source: Own elaboration

4 The internal rate of return (IRR) and its relation with system sustainability

The work developed by Martinez and Soto (2021)[22] develops an indicator known as PM (proportionality measure - see equation (5)), which denotes actuarial fairness and assesses sustainability, fairness and intergenerational equity. This measure calculates lifetime benefits and contributions assuming a discount rate (r) and a GDP growth rate (g).

$$PM = \frac{\sum_{a=a_R}^{a_D} q_a \left(\frac{1+g}{1+r}\right)^{a-a_w}}{\sum_{a=a_w}^{a_{R-1}} \tau_a \left(\frac{1+g}{1+r}\right)^{a-a_w}} \quad (5)$$

From equation (5) a_W denotes the age at the entrance into working life, a_R denotes the age at retirement and a_D denotes the age at the end of the pension period. q_a and τ_a are the benefit and contribution rates as ratios of benefits and contributions to nominal GDP respectively.

The PM is consistent with the studies carried out by Aaron (1966) and Samuelson (1958) in which it can be concluded that a pure PAYG system will be fiscally sustainable when the IRR provided by the system is equal to GDP growth in the long run. Then when r=g (PM=1) actuarial fairness is achieved for a given cohort. A PM $>$ 1 is a sign that the pension system is in actuarial deficit due to demographic changes and will require intergenerational transfers.

[22] "Pension Reforms in Europe: How Far Have We Come and Gone?"

The choice of the discount rate (r) is crucial in the calculation of the PM since it is the rate at which benefits and contributions are evaluated and, at the same time, represents the return on alternative investments. The most relevant discount rate when evaluating public pension systems is the long-term government bonds yield since it represents the rate at which governments borrow and reflect the risk of long-term public promises, such as pensions (Queisser and Whitehouse 2006).

Therefore a system will be in actuarial fairness if the discount rate (r) is equal to the IRR provided by the system. An IRR $> r$ (PM >1) describes a pension system in which the relative burden of contributions during working life is offset by growth in retirement benefits. Conversely, an IRR $< r$ (PM <1) implies that the contributions made to the system would be higher than the benefits received.

In addition to the points mentioned above, empirical evidence supports the importance of population growth on the IRR of PAYG systems (Samuelson). Additionally, it has also been observed that imbalances in population growth can generate intergenerational inequities in terms of the IRR (Lapkoff & Keyfitz).

In this study, the calculation of the Internal Rate of Return (IRR) in pension systems is carried out considering demographic and economic variables. It is important to note that the calculation of the IRR in a pension system presents complexities due to the assumptions used regarding the financial equilibrium of the system.

According to Bravo, PAYG systems usually present financial imbalances. In this sense, Bravo distinguishes between two types of systems: incipient systems and mature systems. In incipient systems, a surplus is generally observed. This implies that the ratio of liabilities/assets is greater than 1. On the other hand, mature systems tend to have deficits, i.e., the ratio of liabilities/assets is less than 1.

To assess Spain's financial situation, the old-age dependency ratio[23] will be studied as a first approach. Based on the data available for 2021, Spain's old-age dependency ratio was 30%. This indicates that the Spanish pension system is in a deficit, as there are 3.33 active workers for every retired person.

It is important to note that previous analysis cannot be directly extrapolated to Sweden due to the unique characteristics of its pension system. Sweden incorporates notional accounts and utilises financial re-balancing tools such as balansindex and inkomstindex. These tools play an important role in maintaining the system's financial equilibrium.

[23] The old-age dependency ratio is a measure of the number of dependents aged above 65, compared with the total population aged 16 to 64 (labor force).

4.1 Calculation of IRR using J. Bravo's formula

Before explaining Bravo's[24] formula in detail, it is important to highlight the approach he takes when it comes to earnings growth and pension calculation.

Regarding earnings growth, it is assumed that the evolution of an individual's earnings over time t follows a constant annual rate of increase. Therefore, for any age x and at any time t, the individual's wage can be calculated using the following formula (where *sigma* is the rate of earnings growth):

$$y(x,t) = y(x,0) \times e^{\sigma t} \tag{6}$$

In terms of calculating pensions, Bravo sets contributions as a constant fraction (τ) of workers' earnings. In addition, it is assumed that the value of the pension (p) to be received arises from the product of the base wage[25] (s) and a replacement rate[26] (r) that is assumed to be constant.

Prior to continuing the IRR formula explanation used by Bravo (1996), it is important to understand the notation used in the formula. This will allow a better understanding of the calculation process.

- r: the replacement rate at the time of retirement
- τ: contribution rate on wages comprising employer and employee contributions to Social Security. It is assumed to be constant.
- E_j: life expectancy at retirement, i.e. the full life expectancy for those individuals who retire at age j where w term denotes actuarial infinity.

$$E_j = 0.5 + \sum_{n=1}^{w-j} \frac{l_{j+n}}{l_j} \tag{7}$$

[24]"La tasa de retorno de los sistemas de reparto" (1996)

[25]Bravo in his paper "La tasa de retorno de los sistemas de reparto" (1996) defines the base wage as the average of earnings (y) during the k years prior to retirement age.

[26]According to OECD definition, the replacement rate is defined as "the individual net pension entitlement divided by net pre-retirement earnings. It measures how effectively a pension system provides a retirement income to replace earnings, the main source of income before retirement."

- E_a: working-age life expectancy, i.e. the full life expectancy until retirement (j) for those individuals who have started working at age a.

$$E_a = 0.5 + \sum_{n=1}^{j-a} \frac{l_{a+n}}{l_a} \tag{8}$$

- A_X= is the average age of the contributors corresponding to the base wage calculation. It has the following formula (where k denotes the calculation period used in the base salary involved in the pension calculation):

$$A_X = j - \frac{k+1}{2} \tag{9}$$

- A_W= is the average age of contributors.

$$A_W = a + \frac{E_a - E_j \times_{j-a} P_a}{2} \tag{10}$$

- A_R= is the average age of pensioners.

$$A_R = j + \frac{E_j \times_{j-a} P_a}{2} \tag{11}$$

- $y(x, x)$ = is the earnings average received by the worker in the period k prior to retirement. Recall that k is the period of time considered for the calculation of the base salary involved in the pension calculation.
- $y(w, x)$ = is the earnings average received by the worker throughout his working history.
- σ = is the annual growth rate of the remunerations involved both in the base salary calculation and in the amount of the retirement pension to be received.
- i = represents the wage indexation and can take values in the range of 0 to 1. If the base salary used in the pension calculation is fully adjusted according to the general growth of earnings, i will have a value of 1.

Bravo (1996) defines the pension to be received with the following mathematical expression:

$$p = ry(x,x)e^{i(A_R - A_X)\sigma} \tag{12}$$

Then, the IRR (ρ) must satisfy the life-cycle constraint by equalising the discounted value of all contributions made to the system and the discounted value of all pensions received:

$$\int_a^j \tau y(x,x)e^{-\rho x}dx = \int_j^w ry(x,x)e^{i(A_R - A_X)\sigma}e^{-\rho x}dx \tag{13}$$

The general expression for the pension system's IRR (obtained by mathematical approximations of equation (13) and whose demonstration will not be emphasised), is as follows:

$$\rho = \frac{1}{(A_R - A_W)}\left[ln\left(\frac{r}{\tau}\right) + ln\left(\frac{E_j}{E_a}\right) + ln\left(\frac{y(x,x)}{y(w,x)}\right) + (A_X - A_W)\sigma + i(A_R - A_X)\sigma\right] \tag{14}$$

The general expression of the rate of return of the pension system must be adapted to the specific characteristics of each system studied in this paper.

In the case of the Spanish pension system, the regulatory base is used to calculate the pension, which takes into account the last k contribution bases (348 after the 2023 reform). To adapt the IRR (ρ) general expression to this characteristic, the value of $y(x,x)$ is replaced by the value obtained in each individual's regulatory base, as used in the approach proposed by Devesa and Vidal-Melia (2002). In this approach, the term $ln\left(\frac{y(x,x)}{y(w,x)}\right)$ is replaced by the logarithmic ratio between the regulatory base and the average wage of the period of active employment.

$$\rho_{Spain} = \frac{1}{(A_R - A_W)}\left[ln\left(\frac{r}{\tau}\right) + ln\left(\frac{E_j}{E_a}\right) + ln\left(\frac{BR_{348}}{y(w,x)}\right) + (A_X - A_W)\sigma + i(A_R - A_X)\sigma\right] \tag{15}$$

On the other hand, the Swedish pension system is mainly based on a notional accounts system in which the contributions made by an individual throughout his or her working history are notional capitalised, following a notional interest rate that is adapted according to the financial situation of the system. In this case, the base salary used in the calculation of the Swedish pension takes into account the average of all the salaries received during the employment history, so that the values of $y(x,x)$, $y(w,x)$ and A_X,A_W are equal. As a result, the IRR (ρ) expression for the Swedish system simplifies to:

$$\rho_{Sweden} = \frac{1}{(A_R - A_W)} \left[ln\left(\frac{r}{\tau}\right) + ln\left(\frac{E_j}{E_a}\right) + i(A_R - A_X)\sigma \right] \tag{16}$$

5 IRR simulation using RStudio

5.1 Assumptions in the simulation program

The program developed in RStudio is based on the following fundamental assumptions and adaptations of the Bravo (1996) formula:

- Given that the contributions made by an individual throughout his employment history are made on a monthly basis, it is crucial to focus the calculation of the IRR on this frequency for its subsequent annual transformation.
- Regarding the components of the internal rate of return formula that require actuarial calculation (E_j,E_a,A_X,A_W and A_R), mortality tables differentiated by gender will be used. In the case of Spain the PER 2020 tables are used, while for Sweden the mortality tables available in the Human Mortality Database[27] are used. As mentioned in the previous point, the calculation of the IRR requires a monthly approach, so these mortality tables require an adjustment to this frequency using the linear interpolation technique. The interpolation technique takes two known points in the mortality table and calculates the slope of the straight line connecting them; this slope is then used to estimate the intermediate value corresponding to a specific age in months.

[27] https://www.mortality.org/Country/Country?cntr=SWE

- The last contribution period corresponds to February 2023, which means that the retirement period starts in March 2023.
- No contribution gaps are assumed, which means that all individuals analysed have fulfilled their contribution obligations during their working life until retirement.
- Profiles are simulated for employed individuals who have lived their entire lives in each country of analysis. For Spain we consider workers affiliated to contribution group 1 of the general regime, which is composed of engineers and graduates[28]. In the case of Sweden, we simulate workers who reach retirement age with the marital status of married for the calculation of the *garantipension*.
- In order to carry out a more homogeneous analysis of these systems, the income derived from the *premiepension* has not been taken into account in the calculation of the Swedish pension.
- The income derived from the Swedish second pillar (inkomstpension) is capitalised at a notional interest rate determined by the average of the values set out in Table 2 (2.97% anually).
- The value of σ will be slightly varied in its calculation and will depend on the value a_1 which is the yearly constant increase in the initial gross annual salary. It is assumed that an individual's gross annual salary will gradually increase at this a_1 rate until five years before retirement age (j), at which point it begins to decrease at a rate equal to 0.15 times the previous percentage increase (a_2). See the following expression:

  Anual gross income evolution:

  $$\begin{cases} y(x,t) = y(x,0) \times e^{a_1 t} & \textit{for } t\textit{=0,1,2,3,...,x-5} \\ y(x,t) = \frac{y(x,x-5)}{e^{a_2 t-(x-5)}} & \textit{for } t\textit{=x-4,...,X} \end{cases}$$

  In the above function, t specifies all the annual periods contributed by the worker. Additionally, it is important to emphasise that the gross annual salary (as well as the retirement income) is received in 12 payments.

  The growth rate a_1 will be determined by the evolution of the CPI in each country, which also implies that there is no loss of purchasing power in wages. To determine the growth rate of remuneration (a_1), we first analyse the compound annual rate of change (CAGR) of growth in the CPI from the time the individual enters the labour market until the month prior to retirement. Once the amount of the CPI CAGR of the country has been determined, the parameter a_1 will be at least that amount.

[28]In 2023, the minimum and maximum contribution bases for this particular group are 1,759.50 and 4,495.50 euros/month.

Then, to determine the amount of σ, we will proceed to calculate the remuneration cumulative monthly rate growth:

- For the Spanish pension system case after the 2023 reform:

$$\sigma_{Spain} = \left(\frac{1}{144}\frac{y(x,x)}{y(x,x-29)}\right)^{\frac{1}{x\times 12}} - 1 \tag{17}$$

  The 29 refers to the number of years included in the calculation of the regulatory base (348 months).

- In the Swedish pension system case:

$$\sigma Sweden = \left(\frac{1}{144}\frac{y(x,x)}{y(x,0)}\right)^{\frac{1}{x\times 12}} - 1 \tag{18}$$

5.2 Economic features to consider

To calculate the value of i it is assumed that the overall wage growth for a given country follows the same evolution as its GDP per capita. Then, the value of i would be determined by the ratio between the monthly growth rate of the nominal wage (σ) and the cumulative annual growth rate of GDP per capita from the year in which the individual entered the labour market (or in the case of Spain, the year from which the contribution bases start to be considered in the regulatory base) and the year 2021 (this is set as the last year of the analysis due to limitations in the database extracted from the World Bank's website[29][30]).

$$i_{SPA} = \frac{\sigma}{\left(\frac{GDPcap_{2021}}{GDPcap_{2021-29}}\right)^{\frac{1}{29}} - 1} \tag{19}$$

$$i_{SWD} = \frac{\sigma}{\left(\frac{GDPcap_{2021}}{GDPcap_{2021-j-a}}\right)^{\frac{1}{j-a}} - 1} \tag{20}$$

[29] For equation (19): https://datos.bancomundial.org/indicator/NY.GDP.MKTP.CD?locations=ES
[30] For equation (20): https://datos.bancomundial.org/indicador/NY.GDP.MKTP.CD?locations=SE

6 Results obtained from the proposed simulation

6.1 Spain: How does the new reform affect the IRR?

As mentioned above, in April of this year 2023 a reform of the pension system was carried out which affected, among other things, the calculation of the regulatory base and the increase in contribution rates by 0.6 percentage points due to the introduction of the Intergenerational Equity Mechanism. In order to evaluate the effects of this reform on the IRR of the Spanish pension system, the following base profile is used as a starting point:

Condition	**Value**
$y(x,0)$	3,500 €
j	65 years
a	27 years and 3 months
Years contributed	37 years and 9 months

Table 4: Initial parameters
Source: Own elaboration

Table 4 presents the base profile used in the analysis, which corresponds to a contributor who decides to retire at age 65 after having made contributions for 37 years and 9 months. In addition, a starting salary ($y(x,0)$) of 3,500 € is established for this contributor at the beginning of his working career. From this starting amount, different annual wage growth rates (a_1) are simulated.

During the period analysed, the compounded annual CPI rate was 3.22%. Therefore, values of a_1 higher than this figure are used in the simulation to ensure that purchasing power is maintained (see Table 5). Table 5 shows the simulated IRR results for different values of a_1 differentiating between gender (IRR_M for men and IRR_F for women), replacement rates (r_j) and last gross annual wage received. The left-hand columns (BR_{300}) represent the calculations made with the old formula for the regulatory base, while the right-hand columns (BR_{348}) show the values obtained after the implementation of the new 2023 reform of the regulatory base.

		BR_{300}			BR_{348}		
$a_1\%$	$y(x,X)$	$IRR_M\%$	$IRR_F\%$	r_j	$IRR_M\%$	$IRR_F\%$	r_j
5.50	16,326.07	3.07	3.18	0.90	3.48	3.56	0.95
6.00	18,770,45	3.10	3.21	0.87	3.45	3.55	0.90
6.50	21,601.50	3.18	3.31	0.85	3.50	3.61	0.87
7.00	24,847.64	3.32	3.44	0.84	3.60	3.72	0.85
7.50	28,581.59	3.46	3.58	0.83	3.70	3.83	0.83
8.00	32,876.66	3.60	3.73	0.82	3.84	3.98	0.82
8.50	37,817.16	3.62	3.76	0.79	3.88	4.02	0.79
9.00	43,500.01	3.51	3.68	0.74	3.78	3.96	0.74
9.50	50,037.01	3.30	3.50	0.68	3.56	3.77	0.67
10.00	57,556.26	3.04	3.28	0.62	3.32	3.57	0.61
10.50	66,205.46	2.70	2.99	0.55	3.04	3.33	0.55
11.00	76,154.41	2.36	2.70	0.49	2.73	3.06	0.49
11.50	87.598,42	2.03	2.41	0.44	2.37	2.76	0.43
12.00	100,762.20	1.66	2.09	0.39	2.08	2.52	0.39
12.50	115,904.00	1.26	1.74	0.34	1.70	2.19	0.34
13.00	133,321.40	0.87	1.41	0.30	1.35	1.88	0.30

Table 5: IRR by gender with BR300 and BR348 computation
Source: Own elaboration

When analysing the Table 5, the following points can be highlighted:

- Regarding the replacement rate, it is observed that the 2023 new reform benefits individuals with less wage growth rate (a_1 with values of 5.50, 6.00 and 6.50). For the other profiles, there is no significant change in the replacement rate.
- The IRR increases on average by 0.33 percentage points with the new reform, independently of gender. However, gender differences are observed in both calculations of the regulatory base, with women obtaining on average 0.25 percentage points higher returns compared to men.

The following plot (Figure 3) shows the differences in the IRR values for four different groups according to gender and the type of calculation used for the regulatory base (BR_{348} for the new 2023 law and BR_{300} for the old law). The solid line represents the IRR values for men (green for the 2023 reform and blue for the old law), while the dashed line shows the IRR values for women (purple for the 2023 reform and red for the old law).

It is evident that the IRR values are higher when using the formula for the regulatory base that extends the calculation period to 348 months. The annual wage growth rate (a_1) that maximises the IRR, regardless of gender and the calculation of the regulatory base, is 8.5%. From this point on, the IRR decreases marginally as the wage growth rates (a_1) of the starting wage increase.

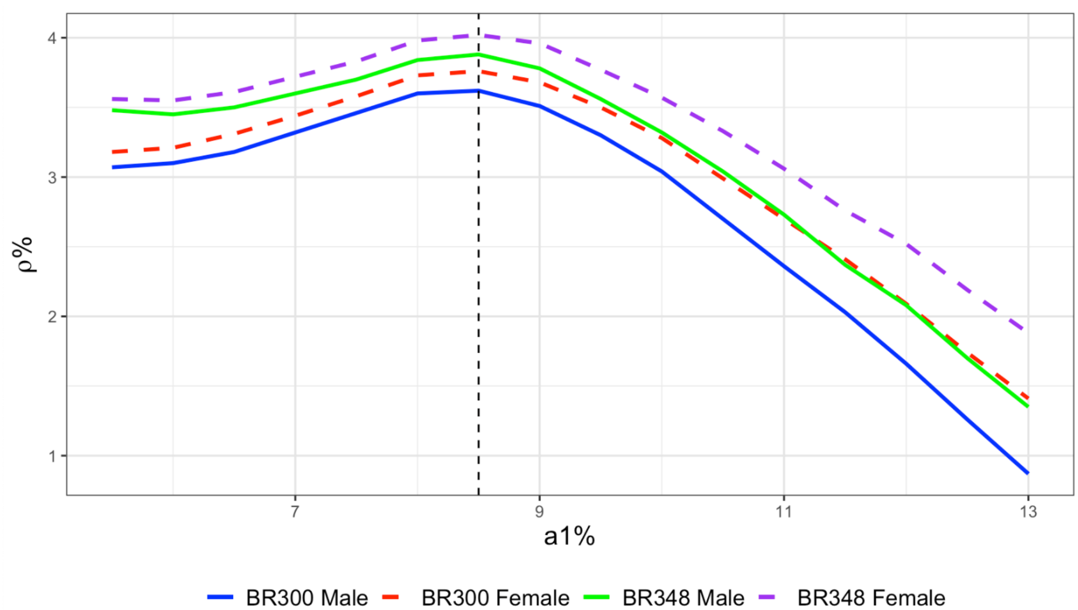

Figure 3: IRR comparison BR300 y BR348

6.2 Spain vs Sweden: a comparison of IRRs and sustainability

For the purpose of making a homogeneous analysis of both pension systems it will be assumed that the initial gross starting salary values are the same. In the case of Sweden the starting gross annual salary will be 100,000 SEK while in the case of Spain the starting value will be 8,800 € (equivalent amounts at the current exchange rate).[31]

A simulation of the IRR will be carried out for each country and by gender, considering three types of profiles that have contributed for 36, 37, 38, 39 and 40 years. These profiles are detailed in Table 6, where a_1 represents the annual salary increase and j indicates the retirement age.[32]

Profile	a_1%	j	Years contributed
Profile 1	5.00	65,66 and 68 years	36,37,38,39 and 40
Profile 2	6.00	65,66 and 68 years	36,37,38,39 and 40
Profile 3	7.00	65,66 and 68 years	36,37,38,39 and 40

Table 6: Initial parameters
Source: Own elaboration

The simulated results are available on pages 30-32. Tables 8 to 15 show the IRRs obtained by gender (IRR_M for men and IRR_F for women) for profiles 1, 2 and 3, together with the last gross annual salary received (remember that these salaries are equivalent to the exchange rate used to homogenise the analysis).

The analysis in this section will focus exclusively on examining the IRR differences derived from retirement age and wage growth under the Spanish and Swedish systems. The average of the values obtained in Tables 8 to 15 can then be taken to make an overall comparative analysis (see Table 7).

Looking at Table 7, we can see that the Spanish PAYG system seems to offer lower returns as the retirement age is postponed and the wage growth factor increases. For those who exercise retirement age at 65 and the pre-retirement wage was within the 30-40 thousand euros range (a_1=5%), the average IRR is 3.07 points, while for those who retire at 68 and the pre-retirement wage is on the 54-72 thousand euros rank (a_1=7%), the average IRR is about 2.2 points. On the other hand, the Swedish system shows higher returns as the

[31] Exchange rate: 0.088€/SEK

[32] It should be noted that in the case of Spain, the ordinary retirement age is 65 years if a minimum of 37 years and 9 months have been contributed, and 66 years and 4 months otherwise. Those profiles that retire at 65 and 66 with 36 and 37 years of contributions, respectively, would be opting for voluntary early retirement.

Annual wage growth	PAYG (Spain) $j=65$	PAYG (Spain) $j=66$	PAYG (Spain) $j=68$	Notional accounts (Sweden) $j=65$	Notional accounts (Sweden) $j=66$	Notional accounts (Sweden) $j=68$
$a_1 = 5\%$	3.07	3.05	2.99	3.36	3.70	4.52
$a_1 = 6\%$	2.94	2.91	2.82	3.17	3.54	4.41
$a_1 = 7\%$	2.40	2.35	2.20	3.13	3.51	4.39

Table 7: IRR mean comparison Spain-Sweden
Source: Own elaboration

retirement age is postponed and the wage growth factor increases. For the retirement age and salary rank mentioned previously, the average IRR stands at 3.36 and 4.39 points respectively.

Another important dynamic to note is that, in both systems, individuals retiring at the same age but with different wage trajectories obtain a higher return than those with less favourable earnings. This is partly due to the fact that both the Spanish and Swedish systems offer higher replacement rates for lower wages, through the implementation of a minimum pension in the case of Spain or a guaranteed pension ("garantipension") in the case of Sweden, the amount of which depends on the income from the "inkomstpension" of notional accounts.

Analysing the differences in simulated IRRs between the two systems, it is observed that the disparities are more favourable towards the Swedish pension system, especially as the retirement age is delayed. These disparities indicate that, in general, the Swedish system can provide a higher return compared to the Spanish system, particularly for those with lower wages and a longer delay in retirement.

The work of Martinez and Soto (2021) introduces the PM (proportionality measure) indicator, which assesses the sustainability, fairness and intergenerational equity of a PAYG system. This indicator is based on the use of the long-term growth rate of GDP (g) and the discount rate (r) as key elements. According to their findings, a pure PAYG system will be fiscally sustainable if it manages to generate an IRR (internal rate of return) comparable to long-term GDP growth (g). These results are consistent with previous research by Aaron (1966) and Samuelson(1958).

The PM establishes that for a pay-as-you-go system to be considered actuarially fair, the rates of r and g must be equal. For evaluating pension systems, the yield on long-term government bonds[33] has been found to be the best-fitting r value since it represents the rate at which governments borrow and reflect the risk of long-term public promises, such as pensions (Queisser and Whitehouse 2006). It is important to note that when the IRR is higher than the discount rate (PM>1), it indicates that the pension system provides lifetime benefits that exceed the contributions made during the labour life. On the other hand, when the IRR is lower than the discount rate (PM<1), it implies that the system provides lifetime benefits that are below the contributions made.

In the case of Spain, long-term GDP growth is in the 2.5-3 percentage point band. Furthermore, the yield provided by Spanish 10-year bonds has averaged 3 percentage points since the adoption of the euro in 2001 until early 2023[34].

Evaluating the simulated IRRs in this context, as shown in Figure 4, it is clear that for pensioners who have executed retirement age at 65 and have experienced low wage increments, the pension system offers returns above what is considered fiscally sustainable (see the red and blue lines with $a_1 = 5$) compared to the ones who have retired at 68 (see the green and purple dashed lines with $a_1 = 7$). Moreover, in the same case, the system provides a lifetime pension that exceeds the contributions made (IRR>r).

In conclusion, for individuals with a 65 retirement age and low wage evolution, the Spanish system provides returns that are not financially sustainable offering lifetime pensions above what is considered actuarially fair, leading to intergenerational inequities. On the other hand, for individuals with a postponed retirement age and high wage evolution, the system provides returns that are sustainable, but, with lifetime pensions below what is considered actuarially fair.

The above analysis cannot be extrapolated to Sweden since it does not have a pure pay-as-you-go system. However, it would be interesting to examine the relationship between the discount rate and the internal rates of return provided by the Swedish system. As shown in Figure 5, the IRR provided by the Swedish system is higher for those who have postponed retirement age and have experience high wage growth (represented by the green and purple dashed lines with $a_1 = 7$). If the Swedish system were a pure pay-as-you-go system, it is easy to observe that in most cases pensions would be higher compared to the contributions made ($IRR > r$) independently of wage increments.

[33]Government bonds with 10 years maturity are taken as a reference since they are the most common.
[34]Spain 10-year bond yield - historical data

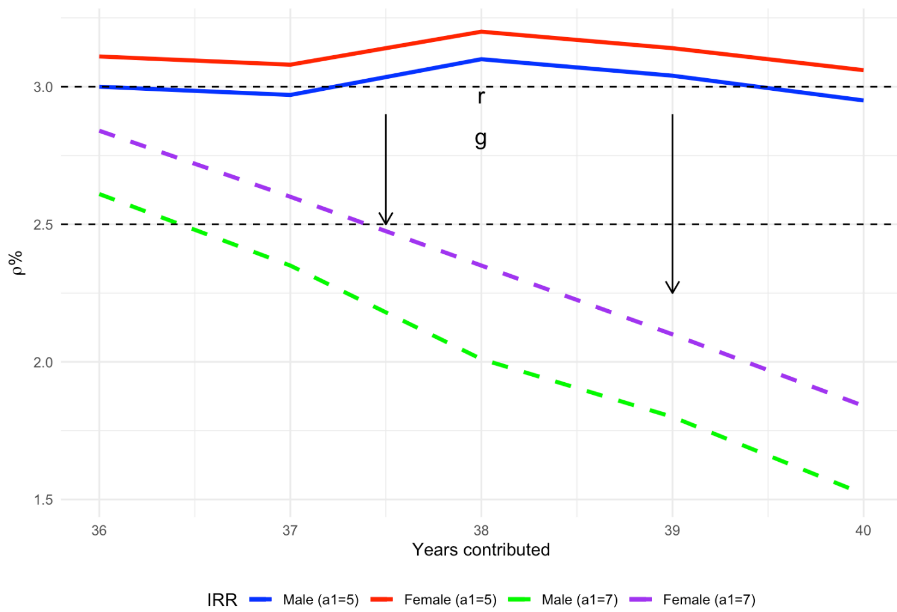

Figure 4: Spanish PAYG system and its actuarial fairness

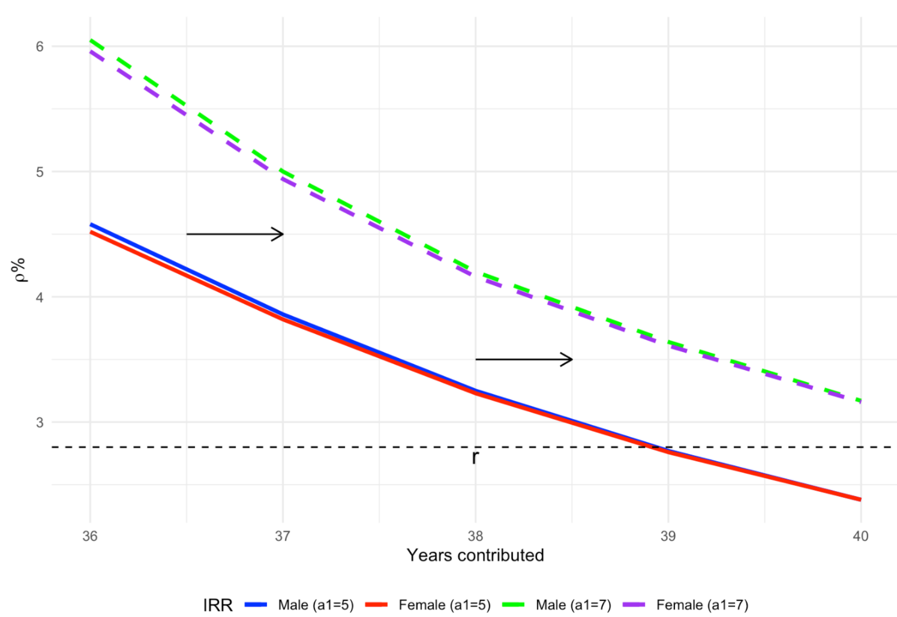

Figure 5: Swedish notional accounts system

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	$IRR_M\%$	$IRR_F\%$	$y(x,X)$	$IRR_M\%$	$IRR_F\%$	$y(x,X)$
36 years	3.00	3.11	32,289€	4.58	4.52	366,929SEK
37 years	2.97	3.08	33,945€	3.86	3.82	385,742SEK
38 years	3.10	3.20	35,685€	3.25	3.23	405,520SEK
39 years	3.04	3.14	37,515€	2.77	2.76	426,311SEK
40 years	2.95	3.06	39,438€	2.38	2.38	448,168SEK

Table 8: IRR comparison Spain-Sweden by gender with $a_1\%$=5 j=65
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	$IRR_M\%$	$IRR_F\%$	$y(x,X)$	$IRR_M\%$	$IRR_F\%$	$y(x,X)$
36 years	3.03	3.15	32,289€	5.06	4.99	366,929SEK
37 years	3.00	3.11	33,945€	4.25	4.20	385,742SEK
38 years	3.04	3.16	35,685€	3.58	3.54	405,520SEK
39 years	2.98	3.10	37,515€	3.04	3.03	426,311SEK
40 years	2.89	3.01	39,438€	2.66	2.66	448,168SEK

Table 9: IRR comparison Spain-Sweden by gender with $a_1\%$=5 j=66
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	$IRR_M\%$	$IRR_F\%$	$y(x,X)$	$IRR_M\%$	$IRR_F\%$	$y(x,X)$
36 years	3.02	3.15	32,289€	6.21	6.11	366,929SEK
37 years	2.98	3.12	33,945€	5.13	5.06	385,742SEK
38 years	2.95	3.08	35,685€	4.32	4.27	405,520SEK
39 years	2.89	3.02	37,515€	3.77	3.74	426,311SEK
40 years	2.79	2,93	39,438€	3.31	3.29	448,168SEK

Table 10: IRR comparison Spain-Sweden by gender with $a_1\%$=5 j=68
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	$IRR_M\%$	$IRR_F\%$	$y(x,X)$	$IRR_M\%$	$IRR_F\%$	$y(x,X)$
36 years	3.06	3.20	41,877€	4.31	4.26	475,882SEK
37 years	2.96	3.11	44,467€	3.59	3.57	505,309SEK
38 years	2.97	3.11	47,216€	3.05	3.03	536,555SEK
39 years	2.76	2.92	50,136€	2.64	2.64	569,734SEK
40 years	2.55	2.72	53,236€	2.28	2.29	604,964SEK

Table 11: IRR comparison Spain-Sweden by gender with $a_1\%=6$ and $j=65$
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	$IRR_M\%$	$IRR_F\%$	$y(x,X)$	$IRR_M\%$	$IRR_F\%$	$y(x,X)$
36 years	3.09	3.23	41,877€	4.79	4.73	475,882SEK
37 years	2.98	3.13	44,467€	4.02	3.98	505,309SEK
38 years	2.90	3.06	47,216€	3.41	3.39	536,555SEK
39 years	2.69	2.86	50,136€	2.96	2.95	569,734SEK
40 years	2.51	2.69	53,236€	2.57	2.57	604,964SEK

Table 12: IRR comparison Spain-Sweden by gender with $a_1\%=6$ $j=66$
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	$IRR_M\%$	$IRR_F\%$	$y(x,X)$	$IRR_M\%$	$IRR_F\%$	$y(x,X)$
36 years	3.07	3.23	41,877€	6.02	5.93	475,882SEK
37 years	2.93	3.10	44,467€	5.01	4.94	505,309SEK
38 years	2.75	2.93	47,216€	4.24	4.19	536,555SEK
39 years	2.56	2.76	50,136€	3.68	3.65	569,734SEK
40 years	2.34	2.55	53,236€	3.22	3.20	604,964SEK

Table 13: IRR comparison Spain-Sweden by gender with $a_1\%=6$ $j=68$
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	IRR_M%	IRR_F%	$y(x, X)$	IRR_M%	IRR_F%	$y(x, X)$
36 years	2.66	2.87	54,312€	4.27	4.22	617,185SEK
37 years	2.44	2.67	58,250€	3.59	3.56	661,936SEK
38 years	2.37	2.60	62,474€	3.01	3.00	709,932SEK
39 years	2.11	2.36	67,003€	2.60	2.60	761,408SEK
40 years	1.81	2.08	71,862€	2.24	2.25	816,617SEK

Table 14: IRR comparison Spain-Sweden by gender with a_1%=7 j=65
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	IRR_M%	IRR_F%	$y(x, X)$	IRR_M%	IRR_F%	$y(x, X)$
36 years	2.69	2.90	54,312€	4.79	4.73	617,185SEK
37 years	2.43	2.66	58,250€	4.02	3.97	661,936SEK
38 years	2.28	2.52	62,474€	3.37	3.35	709,932SEK
39 years	2.01	2.28	67,003€	2.91	2.91	761,408SEK
40 years	1.71	1.99	71,862€	2.52	2.53	816,617SEK

Table 15: IRR comparison Spain-Sweden by gender with a_1%=7 and j=66
Source: Own elaboration

	PAYG (Spain)			Notional accounts (Sweden)		
Years contributed	IRR_M%	IRR_F%	$y(x, X)$	IRR_M%	IRR_F%	$y(x, X)$
36 years	2.61	2.84	54,312€	6.05	5.96	617,185SEK
37 years	2.35	2.60	58,250€	5.00	4.94	661,936SEK
38 years	2.01	2.35	62,474€	4.20	4.16	709,932SEK
39 years	1.80	2.10	67,003€	3.64	3.61	761,408SEK
40 years	1.52	1.84	71,862€	3.17	3.16	816,617SEK

Table 16: IRR comparison Spain-Sweden by gender with a_1%=7 and j=68
Source: Own elaboration

7 Understanding the dynamics associated with the IRR

It is essential to understand the dynamics of the Spanish and Swedish pension systems by considering variables such as retirement age (j), wage growth (a_1) and gender, through an extensive number of simulations.

To do so, firstly, the IRR resulting from having contributed to these systems for 36,37,38,39 and 40 years has been simulated for profiles that decide to retire in the entire range between 63 and 70 years of age (j) and that have had annual salary increases (a_1) between 5 and 8% since their labour market enter (considering, as in section 6.2, a starting salary of 8,800€ and 100,000SEK for Spain and Sweden, respectively).

Once the simulations have been carried out, in order to understand the causal relationships of the variables specified above with the IRR of each system, different linear econometric models will be built in which the dependent variables are the underlying IRRs of having paid contributions for 36, 37, 38, 39 and 40 years. These models shown in Table 17 (Spain) and 18 (Sweden) are independent causal models.

7.1 Spanish IRR dynamics

In order to capture the dynamics of the Spanish PAYG system, Table 17 resumes the different estimated coefficients for each dependent variable ($IRR_{36:40}$)[35] in the Spanish system.

Coefficients interpretation:[36]

- a_1 (annual wage increase): in the Spanish PAYG system the increase in a_1 has a parabola effect on the IRR in the form of a "$\bigcap$". This means the impact on the IRR is initially positive, however, as a_1 increases the size of the overall effect on the dependent variable is gradually reduced due to the quadratic factor added. This quadratic factor is added to minimise the residuals of the model, an indispensable point in econometric modelling.
- $Male$: there are significant gender differences. Men on average have a lower IRR compared to women. Gender differences can be up to 0.26 percentage points in favour of women (see IRR_{40}).

[35] The IRR_{36} and IRR_{37} models assume early retirement by the voluntary will of the worker

[36] Remember that in an econometric model, the interpretation of the coefficients is done in mean values and assuming that the rest of the variables remain constant (ceteris paribus).

- j (retirement age): it can be seen that the system seems to give a lower return the later one decides to retire (negative coefficients) independently of the years contributed.

	Dependent variable:				
	IRR_{36}	IRR_{37}	IRR_{38}	IRR_{39}	IRR_{40}
	(1)	(2)	(3)	(4)	(5)
a_1	1.890***	1.812***	1.361***	1.041***	0.720***
	(0.135)	(0.152)	(0.178)	(0.161)	(0.161)
a_1^2	−0.173***	−0.174***	−0.145***	−0.126***	−0.108***
	(0.010)	(0.012)	(0.014)	(0.012)	(0.012)
$Male$	−0.219***	−0.232***	−0.224***	−0.240***	−0.258***
	(0.016)	(0.018)	(0.021)	(0.019)	(0.019)
j	−0.039***	−0.043***	−0.024***	−0.034***	−0.038***
	(0.005)	(0.005)	(0.005)	(0.004)	(0.004)
α	0.714	1.374**	1.568**	3.337***	4.643***
	(0.538)	(0.604)	(0.648)	(0.586)	(0.587)
Observations	156	156	208	208	208
R^2	0.938	0.946	0.920	0.949	0.959
Adjusted R^2	0.936	0.944	0.919	0.948	0.958
Note:			*p<0.1; **p<0.05; ***p<0.01		

Table 17: Spanish analysis
Source: Own elaboration

7.2 Swedish IRR dynamics

To understand the dynamics in the Swedish notional accounts system Table 18 shows the different coefficient estimates for each dependent variable ($IRR_{36:40}$).

Then, the coefficients interpretation is the following:

- a_1 (annual wage increase): in general terms, the effect of an increase in a_1 on the IRR of the Swedish notional account system takes the form of a "$\bigcup$". That is, it has a negative effect that is contrasted with the higher annual wage growth rates experienced.
- $Male$: In terms of gender, the differences are in favour of men, with the disparity being up to 0.07 percentage points (IRR_{36}).
- j (retirement age): the effect of an increase in this variable, as in the case of the wage increase, takes the form of a "$\bigcup$" in the IRR offered by the system. The effect is negative; however, the impact is reduced as retirement is taken later in life. The negative impact on the IRR of delaying the retirement age decreases markedly as the years of contribution increase. For example, for a person who has contributed 36 years (IRR_{36}) delaying retirement by 1 year means on average a decrease of 5.92 percentage points in the IRR; while for a person who has contributed 40 years (IRR_{40}) delaying retirement by 1 year means on average decreasing the IRR by 1.61 percentage points.

	Dependent variable:				
	IRR_{36}	IRR_{37}	IRR_{38}	IRR_{39}	IRR_{40}
	(1)	(2)	(3)	(4)	(5)
a_1	-0.822^{***}	-0.512^{***}	-0.406^{***}	-0.418^{***}	-0.340^{***}
	(0.144)	(0.092)	(0.030)	(0.075)	(0.017)
a_1^2	0.062^{***}	0.038^{***}	0.028^{***}	0.029^{***}	0.023^{***}
	(0.011)	(0.007)	(0.002)	(0.006)	(0.001)
$Male$	0.073^{***}	0.038^{***}	0.028^{***}	0.004	0.0003
	(0.017)	(0.011)	(0.004)	(0.009)	(0.002)
j	-5.966^{***}	-3.374^{***}	-2.533^{***}	-2.127^{***}	-1.623^{***}
	(0.249)	(0.160)	(0.052)	(0.129)	(0.029)
j^2	0.050^{***}	0.029^{***}	0.022^{***}	0.019^{***}	0.015^{***}
	(0.002)	(0.001)	(0.0004)	(0.001)	(0.0002)
α	185.234^{***}	101.954^{***}	75.859^{***}	63.613^{***}	47.498^{***}
	(8.269)	(5.318)	(1.719)	(4.306)	(0.961)
Observations	208	208	208	208	208
R^2	0.993	0.995	0.999	0.994	1.000
Adjusted R^2	0.993	0.995	0.999	0.994	1.000
Note:			$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$		

Table 18: Swedish analysis
Source: Own elaboration

8 Conclusions

Using the simulations developed and the hypotheses established, the points detailed in this section summarise and conclude the main findings.

The annual growth rate of earnings (a_1) has a positive effect on the IRR of the Spanish system, however, this positive effect is reduced for high wage growth values. This reduction can be explained by the low replacement rate that the system gives to high-income profiles.

In contrast, in the Swedish system, it is observed that annual wage growth has a negative effect on the IRR, however, this effect is mitigated for profiles with more favourable wage developments. This is due to the existence of the guaranteed pension in the Swedish system, which is inversely related to the growth of the "inkomstpension" of the second pillar. In the case of high salaries, the pension provided by the inkomstpension can compensate in terms of returns the loss of the guaranteed pension.

In terms of the convenience of choosing early or delayed retirement, the models suggest that in the Spanish pension system it is more advantageous to choose early retirement, due to the negative impact of the estimated coefficient (j) on the IRR. Therefore, it would be recommendable for the Spanish pension system to consider improving the rates of pension increase for each additional year contributed.

In the case of the Swedish pension system, delaying retirement has a negative effect that is reduced as the decision to delay retirement increases. This result is not surprising, since although contributions are notionally funded, the principles of capitalisation still apply. This implies an increase in the accumulation of capital returns over time, which in turn will increase the replacement rate and thus the IRR.

The life expectancy for men is usually lower compared to that of women, which can lead to significant differences in the calculation of the IRR by gender. In the case of Spain, it is observed that the IRR gender disparities are in favour of women and can be up to 0.22 percentage points higher than men. For Sweden, however, what is observed is that gender differences are in favourable to men in the largest case of up to 0.07 percentage points.

Regarding the sustainability of the Spanish system, the simulated results highlight that the system presents fiscally unsustainable IRRs in the long term for profiles with a low wage evolution during their working life and that opt to retire at 65 or 66 years of age. Moreover, the pension received by these profiles exceeds what is considered actuarially fair, i.e. lifetime benefits exceed contributions made during working life. These imbalances generate intergenerational inequities and affect the stability of the pay-as-you-go system.

The previous analysis applies exclusively to pure pay-as-you-go systems, such as the Spanish system, and cannot be extrapolated to the Swedish system. The Swedish system incorporates actuarial balancing mechanisms and seeks to ensure long-term sustainability by adjusting the notional interest rate ("inkomstindex") according to the country's economic and employment situation. These measures ensure greater balance and adaptability in the Swedish system compared to the Spanish system and results interesting to adapt

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A Appendix A

	Periodo cotizado: menos de 38 años y 6 meses	Periodo cotizado: igual o superior a 38 años y 6 meses e inferior a 41 años y 6 meses	Periodo cotizado: igual o superior a 41 años y 6 meses e inferior a 44 años y 6 meses	Periodo cotizado: igual o superior a 44 años y 6 meses
Meses que se adelanta la jubilación	**% reducción**	**% reducción**	**% reducción**	**% reducción**
48	30,00	28,00	26,00	24,00
47	29,38	27,42	25,46	23,50
46	28,75	26,83	24,92	23,00
45	28,13	26,25	24,38	22,50
44	27,50	25,67	23,83	22,00
43	26,88	25,08	23,29	21,50
42	26,25	24,50	22,75	21,00
41	25,63	23,92	22,21	20,50
40	25,00	23,33	21,67	20,00
39	24,38	22,75	21,13	19,50
38	23,75	22,17	20,58	19,00
37	23,13	21,58	20,04	18,50
36	22,50	21,00	19,50	18,00
35	21,88	20,42	18,96	17,50
34	21,25	19,83	18,42	17,00
33	20,63	19,25	17,88	16,50
32	20,00	18,67	17,33	16,00
31	19,38	18,08	16,79	15,50
30	18,75	17,50	16,25	15,00
29	18,13	16,92	15,71	14,50
28	17,50	16,33	15,17	14,00
27	16,88	15,75	14,63	13,50
26	16,25	15,17	14,08	13,00
25	15,63	14,58	13,54	12,50
24	15,00	14,00	13,00	12,00
23	14,38	13,42	12,46	11,50
22	13,75	12,83	11,92	11,00

	Periodo cotizado: menos de 38 años y 6 meses	Periodo cotizado: igual o superior a 38 años y 6 meses e inferior a 41 años y 6 meses	Periodo cotizado: igual o superior a 41 años y 6 meses e inferior a 44 años y 6 meses	Periodo cotizado: igual o superior a 44 años y 6 meses
Meses que se adelanta la jubilación	**% reducción**	**% reducción**	**% reducción**	**% reducción**
21	12,57	12,00	11,38	10,00
20	11,00	10,50	10,00	9,20
19	9,78	9,33	8,89	8,40
18	8,80	8,40	8,00	7,60
17	8,00	7,64	7,27	6,91
16	7,33	7,00	6,67	6,33
15	6,77	6,46	6,15	5,85
14	6,29	6,00	5,71	5,43
13	5,87	5,60	5,33	5,07
12	5,50	5,25	5,00	4,75
11	5,18	4,94	4,71	4,47
10	4,89	4,67	4,44	4,22
9	4,63	4,42	4,21	4,00
8	4,40	4,20	4,00	3,80
7	4,19	4,00	3,81	3,62
6	3,75	3,50	3,25	3,00
5	3,13	2,92	2,71	2,50
4	2,50	2,33	2,17	2,00
3	1,88	1,75	1,63	1,50
2	1,25	1,17	1,08	1,00
1	0,63	0,58	0,54	0,50

B Appendix B

	Periodo cotizado: menos de 38 años y 6 meses	Periodo cotizado: igual o superior a 38 años y 6 meses e inferior a 41 años y 6 meses	Periodo cotizado: igual o superior a 41 años y 6 meses e inferior a 44 años y 6 meses	Periodo cotizado: igual o superior a 44 años y 6 meses
Meses que se adelanta la jubilación	**% reducción**	**% reducción**	**% reducción**	**% reducción**
24	21,00	19,00	17,00	13,00
23	17,60	16,50	15,00	12,00
22	14,65	14,00	13,33	11,00
21	12,57	12,00	11,43	10,00
20	11,00	10,50	10,00	9,20
19	9,78	9,33	8,89	8,40
18	8,80	8,40	8,00	7,60
17	8,00	7,64	7,27	6,91
16	7,33	7,00	6,67	6,33
15	6,77	6,46	6,15	5,85
14	6,29	6,00	5,71	5,43
13	5,87	5,60	5,33	5,07
12	5,50	5,25	5,00	4,75
11	5,18	4,94	4,71	4,47
10	4,89	4,67	4,44	4,22
9	4,63	4,42	4,21	4,00
8	4,40	4,20	4,00	3,80
7	4,19	4,00	3,81	3,62
6	4,00	3,82	3,64	3,45
5	3,83	3,65	3,48	3,30
4	3,67	3,50	3,33	3,17
3	3,52	3,36	3,20	3,04
2	3,38	3,23	3,08	2,92
1	3,26	3,11	2,96	2,81

C Appendix C

```
install.packages("lubridate")
library(lubridate)
load("Desktop/TFM/PROGRAMA_SIMULACION_TIR/TABLAS_ESPA A/
   TAULES2122.RData")
calculos_pension<-function(){
  options(warn = -1)
  cat("C lculo de pensi n a recibir ","\n")
  cat("Introducir nombre del pa s para el c lculo de la
     pensi n" ,"\n")
  cat("ESPA A=ESP","\n")
  cat("SUECIA=SWD","\n")
  cat("Entrar opci n: ESP,SWD")
  pais<-readLines(n=1)

  if (pais=="ESP"){
    library(readxl)
    IPC_SPAIN <- read_excel("Desktop/TFM/PROGRAMA_SIMULACION_TIR/
       DATABASE_TFM_IRR.xlsx", col_types = c("text", "numeric","
       numeric"))
    IPC_SPAIN<-IPC_SPAIN[,2]
    cat("Recordemos que la edad de jubilaci n legal en Espa a
       es:" ,"\n")
    cat("65 a os si se han cotizado un m nimo de 37 a os y 9
       meses","\n")
    cat("66 a os y 4 meses en caso contrario","\n")

    cat("Introducir g nero del cotizado:")
    cat("M si es de g nero m sculino o F si es de g nero
       femenino")
    g nero<-readLines(n=1)
    g nero<-as.character(g nero)

    cat("Introducir edad de jubilaci n:","\n")
    cat("A os:","\n")
    a osesp<-readLines(n=1)
```

```
a osesp<-as.numeric(a osesp)
cat("y meses:","\n")
mesesp<-readLines(n=1)
mesesp<-as.numeric(mesesp)



cat("Introducir periodos cotizados","\n")
cat("A os:","\n")
a oscotesp<-readLines(n=1)
a oscotesp<-as.numeric(a oscotesp)
cat("y meses:","\n")
mescotesp<-readLines(n=1)
mescotesp<-as.numeric(mescotesp)

if ((a oscotesp+mescotesp/12)<15) {return(cat("Para acceder
   a la pensi n por jubilaci n son m nimo 15 a os
   cotizados","\n"))
  stop(cat("Fin","\n"))}

a oscot<-a oscotesp+mescotesp/12

cat("Introducir salario bruto anual inicial    :","\n")
saliniesp<-readLines(n=1)
saliniesp<-as.numeric(saliniesp)


fecha1 <- rep(0,dim(IPC_SPAIN)[1])
for (i in 1:dim(IPC_SPAIN)[1]) {
  fecha1[i] <- dmy(01032023)-months(i)
  fecha1 <- as_date(fecha1) }

mesescotizados<-a oscot*12
IPC_S<-data.frame(IPC_SPAIN[,1],row.names = fecha1)

alpha_mensual_IPC<-ifelse(dim(IPC_S)[1]>746,(IPC_S[1,1]/IPC_S
   [746,1])^(1/dim(IPC_S)[1])-1,(IPC_S[1,1]/IPC_S[
   mesescotizados,1])^(1/mesescotizados)-1)
```

```
alpha_anual_IPC<-(1+alpha_mensual_IPC)^12-1
cat("El tanto anual acumulativo (TAA) del IPC durante el
   periodo cotizado ha sido de:",round(alpha_anual_IPC*100,2)
   ,"%","\n")

cat("Introducir par metro a1 %:","\n")
incesp<-scan(n=13)
incesp<-as.numeric(incesp)


tesp<-1:(a oscot*12)
edad_insercion_laboral<-(a osesp+mesesp/12)-(a oscot)
edad<-floor(edad_insercion_laboral)
meses<-(edad_insercion_laboral-floor(edad_insercion_laboral))
   *12

##ev salario, asumimos que los 10 a os previos a la
   jubilaci n el salario decrece en un


s0 <- saliniesp  # salario inicial
e0 <- floor(edad_insercion_laboral)   # edad inicial
e1 <- floor(a osesp+mesesp/12)-5    # edad de inicio de
   disminuci n del salario
ej <- floor(a osesp+mesesp/12)  # edad de jubilaci n

a1<-rep(0,length(incesp))
a2<-rep(0,length(incesp))
for (i in 1:length(incesp)) {

a1[i] <- incesp[i]/100 # tasa de aumento del salario
a2[i] <- (incesp[i]/100)*0.15  # tasa de disminuci n del
   salario
}

salario_inicial<-matrix(0,nrow =(e1-e0)*12 ,ncol = length(
   incesp))
```

```
 for (i in 1:nrow(salario_inicial)) {
  for (j in 1:ncol(salario_inicial)) {
  salario_inicial[,j] <- s0 * exp(a1[j] * floor((tesp[1:nrow(
     salario_inicial)]/12)))
  }}

salario_inicial<-salario_inicial[dim(salario_inicial)[1]:1,]

salario_final<-matrix(0,nrow =(ej-e1)*12 ,ncol = length(
   incesp))
s0_1<-apply(salario_inicial, 2, function(x) x[1])

for (i in 1:nrow(salario_final)) {
  for (j in 1:ncol(salario_final)) {
    salario_final[,j] <-s0_1[j]* (1/exp(a2[j] * floor(tesp[1:
       nrow(salario_final)]/12)+1))
  }}

salario_final<-salario_inicial[dim(salario_final)[1]:1,]


salarios <- rbind(salario_final,salario_inicial)/12

fecha <- rep(0,dim(salarios)[1])
for (i in 1:length(fecha)) {
  fecha[i] <- dmy(01032023)-months(i)
  fecha <- as_date(fecha) }

Evolucion<-data.frame(IPC_SPAIN[1:nrow(salarios),],row.names
   = fecha[1:nrow(salarios)])

Evolucion<-cbind(Evolucion,salarios)

alpha_mensual_Salnom<-rep(0,length(incesp))
for (i in 1:length(a1)) {
  alpha_mensual_Salnom[i]<-(salarios[1,i]/salarios[348,i])^(1
     /348)-1
```

```
}

alpha_anual_Salnom<-rep(0,length(incesp))
for (i in 1:length(a1)){
alpha_anual_Salnom[i]<-(1+alpha_mensual_Salnom[i])^12-1}

salarios_finales<-apply(salarios*12, 2, function(x) x[1])
cat(" ltimo  salario anual bruto de:",round(salarios_finales
   ,2),"   ","\n")




basemaxima_esp <- c(rep(4495.50,6),rep(4139.40,12),rep
   (4070.10,12),rep(4070.10,12),rep(4070.10,12),rep
   (3803.70,4),rep(3751.20,20), rep(3642,12) ,rep(3606,12),
   rep(3597,12),rep(3425.70,12),rep(3262.50,12),rep
   (3230.10,12),rep(3198,12),rep(3166.2,12),rep(3074.10,12),
   rep(2996.10,12),rep(2897.70,12),rep(2813.4,12),rep
   (2731.5,12),rep(2652,12),rep(2574.9,12),rep(2499.91,12),
   rep(2450.87,12),rep(2402.73,12),rep(2360.17,12),rep
   (2311.67,12),rep(2253.07,12),rep(2176.81,12),rep
   (2103.24,12),rep(2032.20,12),rep(1931.77,12),rep
   (1839.82,12),rep(1752.19,12),rep(1657.71,12),rep
   (1609.39,12),rep(1562.51,12),rep(1488.05,12),rep
   (1377.88,12),rep(1287.73,12),rep(1129.60,12),rep
   (928.20,12))
basemaxima_esp <- basemaxima_esp[1:length(fecha)]
basemaxima_esp_table <- data.frame(basemaxima_esp,row.names =
    fecha)

baseminima_esp <- c(rep(1759.50,6),rep(1629.30,12),rep
   (1547,12),rep(1547,12),rep(1466.40,12),rep(1199.10,4+8),
   rep(1152.90,12), rep(1067.40,12), rep(1056,90,12),rep
   (1051.50,12),rep(1051.50,12),rep(1045.20,12),rep
   (1045.20,12),rep(1031.7,12),rep(1016.40,12),rep(977.40,12)
   ,rep(929.70,12),rep(881.10,12),rep(836.10,12),rep
   (799.80,12),rep(784.2,12),rep(768.90,12),rep(753.90,12),
```

```
   rep(739.06,12),rep(724.64,12),rep(711.84,12),rep
   (697.23,12),rep(679.56,12),rep(656.67,12),rep(634.49,12),
   rep(613.21,12),rep(589.59,12),rep(557.86,12),rep
   (523.78,12),rep(488.80,12),rep(474.56,12),rep(460.68,12),
   rep(438.68,12),rep(406.10,12),rep(379.54,12),rep
   (326.35,12),rep(288.85,12))
baseminima_esp <- baseminima_esp[1:length(fecha)]
baseminima_esp_table <- data.frame(baseminima_esp,row.names =
    fecha)

pension_minima <- 13521.71/12
pension_maxima <- 42829.29/12

base_cotizacion<-cbind(baseminima_esp_table,basemaxima_esp_
   table)

if (length(tesp)>=348){
  base_cotizacion<-base_cotizacion[1:348,]
} else {base_cotizacion<-base_cotizacion[1:length(tesp),]}

bases_cotizacion <- matrix(nrow =nrow(base_cotizacion) ,ncol
   = length(incesp))

salarios<-salarios[1:nrow(base_cotizacion),]

for (i in 1:nrow(base_cotizacion)) {

  # Recorremos cada salario de los individuos
  for (j in 1:ncol(salarios)) {

    # Calculamos el m nimo del m ximo entre el salario y
       las bases de cotizaci n
    bases_cotizacion[i, j] <- min(c(max(base_cotizacion$
       baseminima_esp[i],salarios[i,j]), base_cotizacion$
       basemaxima_esp[i]))
  }
}
```

```
inflacion_esp<-as.numeric(unlist(IPC_SPAIN))
fact_act<-inflacion_esp[25]/inflacion_esp[25:dim(base_
   cotizacion)[1]]

base_reguladora<-c(rep(0,length(incesp)))
for (i in 1:ncol(salarios)){
  base_reguladora[i]<-(sum(bases_cotizacion[1:24,i])+sum(
     bases_cotizacion[25:dim(base_cotizacion)[1],i]*fact_act)
     )/378
}


cat("Base reguladora de:", round(base_reguladora,2),"   ","\n
   ")

tasa_sustitucion <-c(rep(0,15*12-1),seq(0.5,by =0.0021,length
   .out = 49),seq((0.5+0.0021*49),by = 0.0019, to = 1))
mesestotales_cotizados <- a oscot*12
calculofin<-tasa_sustitucion[min(mesestotales_cotizados,
   length(tasa_sustitucion))]*base_reguladora
a ostotales_cotizados<-a oscot

if (a ostotales_cotizados >= 37+9/12) {
  edad_legal <- 65
} else {
  edad_legal <- 66+(4/12)
}

edad_retiro<-a osesp+mesesp/12

if (edad_retiro > edad_legal){
  cat("El jubilado est  optando a la jubilaci n demorada","
     \n")
  factor_incremento <- (0.04/12)*(edad_retiro-edad_legal)
```

```
  espa a_pension<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    espa a_pension[i]<-(1+factor_incremento)*calculofin[i]
  }
  espa a_pension


  cat("Este perfil de jubilado obtendr  una pensi n mensual
      de:", round(espa a_pension,2),"   ","\n")

  ts<-rep(0,length(incesp))
    for (i in 1:length(incesp)){
      ts[i]<-espa a_pension[i]/salarios[1,i]
    }

  cat("La tasa de sustituci n es de:",round(ts*100,2),"%","\
     n")

}




if(edad_retiro == edad_legal){

  espa a_pension<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    espa a_pension[i]<-min(max(calculofin[i],pension_minima)
       ,pension_maxima)
  }
  espa a_pension

  cat("Este perfil de jubilado obtendr  una pensi n de:",
     round(espa a_pension,2),"euros","\n")
  ts<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    ts[i]<-espa a_pension[i]/salarios[1,i]
  }
```

```
    cat("La tasa de sustituci n es de:",round(ts*100,2),"%","\
      n")
}


if(edad_retiro < edad_legal) {
  mesestotales_cotizados <- a oscot*12
  meses_anticipo <- (edad_legal/(1/12))-(a osesp*12+mesesp)
  cat("El jubilado puede optar a las siguientes opciones de
     jubilaci n anticipada: voluntaria o forzosa","\n")
  cat("Introducir VOL si se opta jubilaci n anticipada
     voluntaria o FRZ si se opta por jubilaci n anticipada
     forzosa","\n")
  opc<-readLines(n=1)

  if (opc=="VOL"){

    if (mesestotales_cotizados < 35*12 & meses_anticipo >24)
       {return(cat("No se puede optar a la jubilaci n
       voluntaria","\n"))
      stop(cat("Fin","\n"))}

    if (mesestotales_cotizados<= 38.5*12) {
      vector_descuento <- c
         (3.26,3.38,3.52,3.67,3.83,4,4.19,4.4,4.63,4.89,5.18
      ,5.5,5.87,6.29,6.77,7.33,8,8.8
      ,9.78,11,12.57,14.67,17.60,21)
      vector_descuento<-vector_descuento/100
      factor_descuento <- 1-vector_descuento[meses_anticipo]


      espa a_pension<-rep(0,length(incesp))
      for (i in 1:length(incesp)){
        espa a_pension[i]<-min(max(pension_minima,factor_
           descuento*calculofin[i]),factor_descuento*pension_
           maxima)
      }
```

```
  espa a_pension

  cat("Este perfil de jubilado obtendr  una pensi n de:
     ",round(espa a_pension,2),"euros","\n")

  ts<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    ts[i]<-espa a_pension[i]/salarios[1,i]
  }
  cat("La tasa de sustituci n es de:",round(ts*100,2),"%
     ","\n")
}

if (mesestotales_cotizados > 38.5*12 & mesestotales_
   cotizados <= 41.5*12) {
  vector_descuento <- c
     (3.11,3.23,3.36,3.5,3.65,3.82,4,4.2,4.42,4.67,4.94,
  5.25,5.6,6,6.46,7,7.64,8.4,9.33
  ,10.5,12,14,16.5,19)
  vector_descuento<-vector_descuento/100
  factor_descuento <- 1-vector_descuento[meses_anticipo]
  espa a_pension<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    espa a_pension[i]<-min(max(pension_minima,factor_
       descuento*calculofin[i]),factor_descuento*pension_
       maxima)
  }
  espa a_pension

  cat("Este perfil de jubilado obtendr  una pensi n de:
     ",round(espa a_pension,2),"euros","\n")

  ts<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    ts[i]<-espa a_pension[i]/salarios[1,i]
  }
  cat("La tasa de sustituci n es de:",round(ts*100,2),"%
     ","\n")
```

```
  }

if (mesestotales_cotizados > 41.5*12 & mesestotales_
   cotizados <= 44.5*12) {
  vector_descuento <-c
     (2.96,3.08,3.20,3.33,3.48,3.64,3.81,4,4.21,4.44,4.71
   ,5,5.33,5.71,6.15,6.67,7.27,8,8.89
   ,10,11.43,13.33,15,17)
  vector_descuento<-vector_descuento/100
  factor_descuento <- 1-vector_descuento[meses_anticipo]
  espa a_pension<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    espa a_pension[i]<-min(max(pension_minima,factor_
       descuento*calculofin[i]),factor_descuento*pension_
       maxima)
  }
  espa a_pension

  cat("Este perfil de jubilado obtendr  una pensi n de:
     ",round(espa a_pension,2),"euros","\n")

  ts<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    ts[i]<-espa a_pension[i]/salarios[1,i]
  }
  cat("La tasa de sustituci n es de:",round(ts*100,2),"%
     ","\n")



  }

if (mesestotales_cotizados > 44.5*12) {
  vector_descuento <-c
     (2.81,2.92,3.04,3.17,3.3,3.45,3.62,3.8,4,4.22,4.47
   ,4.75,5.07,5.43,5.85,6.33,6.91,7.6
```

```
      ,8.4,9.2,10,11,12,13)
      vector_descuento <-vector_descuento/100
      factor_descuento <- 1-vector_descuento[meses_anticipo]
      espa a_pension<-rep(0,length(incesp))
      for (i in 1:length(incesp)){
        espa a_pension[i]<-min(max(pension_minima,factor_
           descuento*calculofin[i]),factor_descuento*pension_
           maxima)
      }
      espa a_pension

      cat("Este perfil de jubilado obtendr  una pensi n de:
         ",round(espa a_pension,2),"euros","\n")

      ts<-rep(0,length(incesp))
      for (i in 1:length(incesp)){
        ts[i]<-espa a_pension[i]/salarios[1,i]
      }
      cat("La tasa de sustituci n es de:",round(ts*100,2),"%
         ","\n")

    }}

  if (opc=="FRZ"){

    if (mesestotales_cotizados<33*12 & meses_anticipo >48) {
       return(cat("No se puede optar a la jubilaci n forzosa
       ","\n"))
      stop(cat("Fin","\n"))}

    if (mesestotales_cotizados<= 38.5*12) {
      vector_descuento <- c
         (0.63,1.25,1.88,2.5,3.13,3.75,4.19,4.4,4.63,4.89,5.18

      ,5.5,5.87,6.29,6.77,7.33,8,8.8,9.78,11,12.57,13.75
      ,14.38,15,15.63,16.25,16.88,17.5,18.13,18.75
      ,19.38,20,20.63,21,.25,21.88,22.5,23.13,23.75,24.38,25
      ,25.63,26.25,26.88,27.5,28.13,28.75,29.38,30)
```

```
  vector_descuento<-vector_descuento/100
  factor_descuento <- 1-vector_descuento[meses_anticipo]
  espa a_pension<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    espa a_pension[i]<-min(max(pension_minima,factor_
       descuento*calculofin[i]),factor_descuento*pension_
       maxima)
  }
  espa a_pension

  cat("Este perfil de jubilado obtendr  una pensi n de:
     ",round(espa a_pension,2),"euros","\n")

  ts<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    ts[i]<-espa a_pension[i]/salarios[1,i]
  }
  cat("La tasa de sustituci n es de:",round(ts*100,2),"%
     ","\n")
}

if (mesestotales_cotizados > 38.5*12 & mesestotales_
   cotizados <= 41.5*12) {
  vector_descuento <- c
     (0.58,1.17,1.75,2.33,2.92,3.5,4,4.2,4.42,4.67,4.94
  ,5.25,5.60,6,6.46,7,7.64,8.4,9.33,10.5,12,12.83,13.42

  ,14,14.58,15.17,15.75,16.33,16.92,17.5,18.08,18.67,19.25

  ,19.83,20.42,21,21.58,22.17,22.75,23.33,23.92,24.5
  ,25.08,25.67,26.25,26.83,27.42,28)
  vector_descuento<-vector_descuento/100
  factor_descuento <- 1-vector_descuento[meses_anticipo]
  espa a_pension<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    espa a_pension[i]<-min(max(pension_minima,factor_
       descuento*calculofin[i]),factor_descuento*pension_
       maxima)
```

```
  }
  espa a_pension

  cat("Este perfil de jubilado obtendr  una pensi n de:
     ",round(espa a_pension,2),"euros","\n")

  ts<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    ts[i]<-espa a_pension[i]/salarios[1,i]
  }
  cat("La tasa de sustituci n es de:",round(ts*100,2),"%
     ","\n")

  }

if (mesestotales_cotizados > 41.5*12 & mesestotales_
   cotizados <= 44.5*12) {
  vector_descuento <-c
     (0.54,1.08,1.63,2.17,2.71,3.25,3.81,4,4.21,4.44,4.71
  ,5,5.33,5.71,6.15,6.67,7.27,8,8.89,10,11.38,11.92
  ,12.46,13,13.54,14.08,14.63,15.17,15.71,16.25,16.79
  ,17.33,17.88,18.42,18.96,19.5,20.04
  ,20.58,21.13,21.67,22.21
  ,22.75,23.29,23.83,24.38,24.92,25.46,26)
  vector_descuento<-vector_descuento/100
  factor_descuento <- 1-vector_descuento[meses_anticipo]
  espa a_pension<-rep(0,length(incesp))
  for (i in 1:length(incesp)){
    espa a_pension[i]<-min(max(pension_minima,factor_
       descuento*calculofin[i]),factor_descuento*pension_
       maxima)
  }
  espa a_pension

  cat("Este perfil de jubilado obtendr  una pensi n de:
     ",round(espa a_pension,2),"euros","\n")

  ts<-rep(0,length(incesp))
```

```
      for (i in 1:length(incesp)){
        ts[i]<-espa a_pension[i]/salarios[1,i]
      }
      cat("La tasa de sustituci n es de:",round(ts*100,2),"%
        ","\n")

      }

    if (mesestotales_cotizados > 44.5*12) {
      vector_descuento <-c
         (0.5,1,1.5,2,2.5,3,3.62,3.8,4,4.22,4.47,4.75,5.07
      ,5.43,5.85,6.33,6.91,7.6,8.4,9.2,10,11,11.5,12,12.5,13
      ,13.5,14,14.5,15,15.5,16,16.5,17,17.5,18
      ,18.5,19,19.5,20,20.5,21,21.5,22,22.5,23,23.5,24)
      vector_descuento <-vector_descuento/100
      factor_descuento <- 1-vector_descuento[meses_anticipo]
      espa a_pension<-rep(0,length(incesp))
      for (i in 1:length(incesp)){
        espa a_pension[i]<-min(max(pension_minima,factor_
           descuento*calculofin[i]),factor_descuento*pension_
           maxima)
      }
      espa a_pension

      cat("Este perfil de jubilado obtendr  una pensi n de:
        ",round(espa a_pension,2),"euros","\n")

      ts<-rep(0,length(incesp))
      for (i in 1:length(incesp)){
        ts[i]<-espa a_pension[i]/salarios[1,i]
      }
      cat("La tasa de sustituci n es de:",round(ts*100,2),"%
        ","\n")
    }}
}
```

```
##utilizaremos las tablas PER2020 haciendo distinci n entre
   sexo femenino y masculino

rj<-round(ts,2)
c<-0.2830+0.0705+0.0020+0.0070+0.006
##0.2830 son de contingencias comunes
##0.0705 son para contingencias de desempleo
##0.0020 son para FOGASA
##0.0070 son para FORMACION
##0.006 es mecanismo de equidad intergeneracional

edad_insercion_laboral<-(a osesp+mesesp/12)-(a oscotesp+
   mescotesp/12)
edad<-floor(edad_insercion_laboral)
meses<-(edad_insercion_laboral-floor(edad_insercion_laboral))
   *12
a<-edad_insercion_laboral
j<-a osesp+mesesp/12


##GDP PER CAPITA
GDPPC<- read_excel("Desktop/TFM/PROGRAMA_SIMULACION_TIR/
   DATABASE_TFM_IRR.xlsx",
                    sheet = "GDP PER CAPITA US DOL")
GDPPC<-GDPPC[,c(1,2,3)]
GDPPC<-as.data.frame(GDPPC)
colnames(GDPPC)[1]<-"Year"
GDPPC$Year<-as.Date(GDPPC$Year,format = "%Y")
GDPPC<-GDPPC[order(dim(GDPPC)[1]:1), ]



cat("El jubilado entr  al mercado laboral con:",edad,"a os
   y",meses,"meses","\n")
cohorte<-2023-floor(j)
cat("Perteneciendo a la cohorte del a o:",cohorte,"\n")
```

```
if (g nero=="M"){

  AP_M<-function(an,k) {
    LX<-conslm1(an)
    k<-as.numeric(k)
    vk<-c(0:(k-1))/k
    LY<-as.vector(t(outer(LX[-length(LX)],(1-vk),"*")+outer(
       LX[-1],vk,"*")))
    LY<-c(round(LY,4),0)
    return(LY)

  }

  lk12<-AP_M(cohorte,12)
  lim<-which(lk12==0)[1]
  Ej<-(sum(lk12[(j*12+2):lim])/lk12[(j*12+1)])+6
  Ea1<-(sum(lk12[(a*12+2):(j*12+1)])/lk12[(a*12+1)])+6
  pa<-lk12[(j*12+1)]/lk12[(a*12+1)]
  k<-348
  AB<-j*12-((k+1)/2)
  AR<-j*12+(pa*Ej)/2
  AC<-a*12+(Ea1-Ej*pa)/2
  sigma<-alpha_mensual_Salnom
  TAAGDP<-(GDPPC[1,2]/GDPPC[(28),2])^(1/(28))-1
  i<-sigma/TAAGDP

  TIR<-rep(0,length(incesp))
  for (l in 1:length(incesp)){
  media_salarios<-colMeans(salarios)
  TIR[l]<-(1/(AR-AC))*((log(rj[l]/c)+log(Ej/Ea1)+log(base_
     reguladora[l]/media_salarios[l])+(AB-AC)*sigma[l]+i[l]*(
     AR-AB)*sigma[l]))
  }
  TIR<-((1+TIR)^12)-1
  cat("Los valores de \u03C3 estimada para los contribuyentes
      son de:",round(((1+sigma)^12-1)*100,2),"%","\n")
  cat("Los valores de i estimada para los contribuyentes son
     de:",round(i,2),"\n")
```

```
  cat("La TIR aproximada para los contribuyentes son de:",
     round(TIR*100,2),"%","\n")

}

if (g nero=="F"){
  AP_F<-function(an,k) {
    LX<-conslf1(an)
    k<-as.numeric(k)
    vk<-c(0:(k-1))/k
    LY<-as.vector(t(outer(LX[-length(LX)],(1-vk),"*")+outer(
       LX[-1],vk,"*")))
    LY<-c(round(LY,4),0)
    return(LY)

  }

  lk12<-AP_F(cohorte,12)
  lim<-which(lk12==0)[1]
  Ej<-(sum(lk12[(j*12+2):lim])/lk12[(j*12+1)])+6
  Ea1<-(sum(lk12[(a*12+2):(j*12+1)])/lk12[(a*12+1)])+6
  pa<-lk12[(j*12+1)]/lk12[(a*12+1)]
  k<-348
  AB<-j*12-((k+12)/2)
  AR<-j*12+(pa*Ej)/2
  AC<-a*12+(Ea1-Ej*pa)/2
  sigma<-alpha_mensual_Salnom
  TAAGDP<-(GDPPC[1,2]/GDPPC[(28),2])^(1/(28))-1
  i<-sigma/TAAGDP
  TIR<-rep(0,length(incesp))
  for (l in 1:length(incesp)){
    media_salarios<-colMeans(salarios)
    TIR[l]<-(1/(AR-AC))*((log(rj[l]/c)+log(Ej/Ea1)+log(base_
       reguladora[l]/media_salarios[l])+(AB-AC)*sigma[l]+i[l]
       *(AR-AB)*sigma[l]))
  }

  TIR<-((1+TIR)^12)-1
```

```
    cat("Los valores de \u03C3 estimada para los contribuyentes
        son de:",round(((1+sigma)^12-1)*100,2),"%","\n")
    cat("Los valores de i estimada para los contribuyentes son
      de:",round(i,2),"\n")
    cat("La TIR aproximada para los contribuyentes son de:",
      round(TIR*100,2),"%","\n")

  }


}






if (pais=="SWD"){

  cat("El sistema sueco es un sistema de cuentas nocionales","\
    n")
  cat("Un 16% del salario pensionable se destina al fondo
    nocional ","  \n")
  cat("Un 2.5% del salario pensionable se destina a fondos de
    capitalizaci n","\n")

  IPC_SWD <- read_excel("Desktop/TFM/PROGRAMA_SIMULACION_TIR/
    DATABASE_TFM_IRR.xlsx",
                        sheet = "SWD")
  IPC_SWD<-IPC_SWD[,-1]
  c1<-IPC_SWD[1,c(1,2)]
  c1<-as.numeric(unlist(c1))
  c2<-IPC_SWD[2:dim(IPC_SWD)[1],]
  c2_1<-numeric(0)
  for (i in 1:nrow(c2)) {
    c2_1 <- c(c2_1, c2[i, ])
  }
```

```
c2_1<-as.numeric(unlist(c2_1))
IPC_SWD<-c(c1,c2_1)



cat("La edad de jubilaci n puede ejecutarse desde los 61 a
   70 a os","\n")

cat("Introducir g nero del cotizado:")
cat("M si es de g nero m sculino o F si es de g nero
   femenino")
g nero<-readLines(n=1)
g nero<-as.character(g nero)


cat("Introducir edad de jubilaci n:","\n")
cat("A os:","\n")
a os_swd<-readLines(n=1)
a os_swd<-as.numeric(a os_swd)
cat("y meses:","\n")
mes_swd<-readLines(n=1)
mes_swd<-as.numeric(mes_swd)
edad_ret_swd<-a os_swd+mes_swd/12
cat("Introducir periodos cotizados","\n")
cat("A os:","\n")
a oscot_swd<-readLines(n=1)
a oscot_swd<-as.numeric(a oscot_swd)
cat("y meses:","\n")
mescot_swd<-readLines(n=1)
mescot_swd<-as.numeric(mescot_swd)
a os_tot<-a oscot_swd+mescot_swd/12
cat("Introducir salario bruto anual inicial SEK:","\n")
saliniswd<-readLines(n=1)
saliniswd<-as.numeric(saliniswd)
fecha1 <- rep(0,length(IPC_SWD))
for (i in 1:length(IPC_SWD)) {
  fecha1[i] <- dmy(01032023)-months(i)
```

```
  fecha1 <- as_date(fecha1) }

mesescotizados<-a os_tot*12
IPC_SWD<-data.frame(IPC_SWD,row.names = fecha1)

alpha_mensual_IPC<-ifelse(dim(IPC_SWD)[1]>818,(IPC_SWD[1,1]/
   IPC_SWD[746,1])^(1/dim(IPC_SWD)[1])-1,(IPC_SWD[1,1]/IPC_
   SWD[mesescotizados,1])^(1/mesescotizados)-1)
alpha_anual_IPC<-(1+alpha_mensual_IPC)^12-1
cat("El tanto anual acumulativo (TAA) del IPC durante el
   periodo cotizado ha sido de:",round(alpha_anual_IPC*100,2)
   ,"%","\n")


cat("Introducir par metro a1 %:","\n")
incswd<-scan(n=13)
incswd<-as.numeric(incswd)


tswd<-1:(a os_tot*12)
edad_insercion_laboral<-(a os_swd+mes_swd/12)-(a oscot_swd+
   mescot_swd/12)
edad<-floor(edad_insercion_laboral)
meses<-(edad_insercion_laboral-floor(edad_insercion_laboral))
   *12
a<-edad_insercion_laboral



s0 <- saliniswd  # salario inicial
e0 <- floor(edad_insercion_laboral)   # edad inicial
e1 <- floor(a os_swd+mes_swd/12)-5    # edad de inicio de
   disminuci n del salario
ej <- floor(a os_swd+mes_swd/12)  # edad de jubilaci n

a1<-rep(0,length(incswd))
a2<-rep(0,length(incswd))
for (i in 1:length(incswd)) {
```

```
  a1[i] <- incswd[i]/100 # tasa de aumento del salario
  a2[i] <- (incswd[i]/100)*0.15  # tasa de disminuci n del
     salario
}

salario_inicial<-matrix(0,nrow =(e1-e0)*12 ,ncol = length(
   incswd))

for (i in 1:nrow(salario_inicial)) {
  for (j in 1:ncol(salario_inicial)) {
    salario_inicial[,j] <- s0 * exp(a1[j] * floor((tswd[1:
       nrow(salario_inicial)]/12)))
  }}

salario_inicial<-salario_inicial[dim(salario_inicial)[1]:1,]

salario_final<-matrix(0,nrow =(ej-e1)*12 ,ncol = length(
   incswd))
s0_1<-apply(salario_inicial, 2, function(x) x[1])

for (i in 1:nrow(salario_final)) {
  for (j in 1:ncol(salario_final)) {
    salario_final[,j] <-s0_1[j]* (1/exp(a2[j] * floor(tswd[1:
       nrow(salario_final)]/12)+1))
  }}

salario_final<-salario_inicial[dim(salario_final)[1]:1,]


salarios <- rbind(salario_final,salario_inicial)/12
##ev salario, asumimos que los 10 a os previos a la
   jubilaci n el salario decrece en un


fecha <- rep(0,dim(salarios)[1])
for (i in 1:length(fecha)) {
  fecha[i] <- dmy(01032023)-months(i)
```

```
  fecha <- as_date(fecha) }

Evolucion<-data.frame(IPC_SWD[1:nrow(salarios),],row.names =
   fecha[1:nrow(salarios)])
Evolucion<-cbind(Evolucion,salarios)

alpha_mensual_Salnom<-rep(0,length(incswd))
for (i in 1:length(a1)) {
  alpha_mensual_Salnom[i]<-((salarios[1,i]/salarios[dim(
     salarios)[1],i])^(1/dim(salarios)[1]))-1

}

alpha_anual_Salnom<-rep(0,length(incswd))
for (i in 1:length(a1)){
  alpha_anual_Salnom[i]<-(1+alpha_mensual_Salnom[i])^12-1}

salarios_finales<-apply(salarios*12, 2, function(x) x[1])
cat(" ltimo  salario anual bruto de:",round(salarios_finales
   ,2),"SEK","\n")




cat("Introducir tipo de cambio EUR/SEK actual:","\n")
EUR_SWD<-readLines(n=1)
EUR_SWD<-as.numeric(EUR_SWD)




##premiepension
##cap<-2.5/100
##salario_capitalizado<-salario_pensionable*cap
##int_cap<-0
##interes_capitalizacion<-(1+int_cap)^(1/12)-1
```

```
##premiepension<-sum(salario_capitalizado*(1+interes_
   capitalizacion/100)^(tswd/12))
##premiepension<-premiepension/EUR_SWD

##inkomstpensions
nocional<-0.16
salario_cuenta_nocional<-salarios*nocional

incomeindex<-c(2.2,3.8,3.1,2.6,4.4,5.9,2.5,-1.1,5.8,5.2
,-2.7,-1.4,6.2,4.5,3.2,2.7,2.4,3.4,5.3,2.9,1.4)
i_index<-(1+mean(incomeindex)/100)^(1/12)-1


salarios_capitalizado_nocional <- matrix(nrow =nrow(salario_
   cuenta_nocional) ,ncol = length(incswd))
rows<-1:nrow(salario_cuenta_nocional)
cols<-1:ncol(salario_cuenta_nocional)
salarios_capitalizado_nocional<-salario_cuenta_nocional[rows,
   cols]*(1+i_index)^(rows)


salarios_capitalizado_nocional<-colSums(salarios_capitalizado
   _nocional)


TABLASMORTALIDADSWD_UNISEX <- read.csv("~/Desktop/TFM/
   PROGRAMA_SIMULACION_TIR/TABLAS_SUECIA/TABLASMORTALIDADSWD_
   UNISEX.txt", sep="")
SWEDISHCOHORTS<-TABLASMORTALIDADSWD_UNISEX



##inheritance gains AF
inh_g<-floor(edad_ret_swd)
cohorte<-2023-floor(inh_g)
s<-which(SWEDISHCOHORTS$Year==cohorte)
cohort1<-SWEDISHCOHORTS[s,c(1,2,6)]
lk1_swd<-cohort1[,3]
```

```
lk1_swd<-as.numeric(unlist(lk1_swd))

AF<-(lk1_swd[inh_g-1+1]+lk1_swd[inh_g+1])/(lk1_swd[inh_g+1]+
   lk1_swd[inh_g+2])
AF

##annuity

##annuity
if (cohorte==1938 & floor(edad_ret_swd)==61) {annuity<-17.87}
if (cohorte==1938 & floor(edad_ret_swd)==62) {annuity<-17.29}
if (cohorte==1938 & floor(edad_ret_swd)==63) {annuity<-16.71}
if (cohorte==1938 & floor(edad_ret_swd)==64) {annuity<-16.13}
if (cohorte==1938 & floor(edad_ret_swd)==65) {annuity<-15.56}
if (cohorte==1938 & floor(edad_ret_swd)==66) {annuity<-14.99}
if (cohorte==1938 & floor(edad_ret_swd)==67) {annuity<-14.42}
if (cohorte==1938 & floor(edad_ret_swd)==68) {annuity<-13.84}
if (cohorte==1938 & floor(edad_ret_swd)==69) {annuity<-13.27}
if (cohorte==1938 & floor(edad_ret_swd)==70) {annuity<-12.71}

if (cohorte==1939 & floor(edad_ret_swd)==61) {annuity<-17.94}
if (cohorte==1939 & floor(edad_ret_swd)==62) {annuity<-17.36}
if (cohorte==1939 & floor(edad_ret_swd)==63) {annuity<-16.78}
if (cohorte==1939 & floor(edad_ret_swd)==64) {annuity<-16.19}
if (cohorte==1939 & floor(edad_ret_swd)==65) {annuity<-15.62}
if (cohorte==1939 & floor(edad_ret_swd)==66) {annuity<-15.04}
if (cohorte==1939 & floor(edad_ret_swd)==67) {annuity<-14.47}
if (cohorte==1939 & floor(edad_ret_swd)==68) {annuity<-13.89}
if (cohorte==1939 & floor(edad_ret_swd)==69) {annuity<-13.32}
if (cohorte==1939 & floor(edad_ret_swd)==70) {annuity<-12.76}

if (cohorte==1940 & floor(edad_ret_swd)==61) {annuity<-18.02}
if (cohorte==1940 & floor(edad_ret_swd)==62) {annuity<-17.44}
if (cohorte==1940 & floor(edad_ret_swd)==63) {annuity<-16.86}
if (cohorte==1940 & floor(edad_ret_swd)==64) {annuity<-16.27}
if (cohorte==1940 & floor(edad_ret_swd)==65) {annuity<-15.69}
if (cohorte==1940 & floor(edad_ret_swd)==66) {annuity<-15.11}
if (cohorte==1940 & floor(edad_ret_swd)==67) {annuity<-14.54}
```

```
if (cohorte==1940 & floor(edad_ret_swd)==68) {annuity<-13.96}
if (cohorte==1940 & floor(edad_ret_swd)==69) {annuity<-13.39}
if (cohorte==1940 & floor(edad_ret_swd)==70) {annuity<-12.82}

if (cohorte==1941 & floor(edad_ret_swd)==61) {annuity<-18.14}
if (cohorte==1941 & floor(edad_ret_swd)==62) {annuity<-17.56}
if (cohorte==1941 & floor(edad_ret_swd)==63) {annuity<-16.98}
if (cohorte==1941 & floor(edad_ret_swd)==64) {annuity<-16.39}
if (cohorte==1941 & floor(edad_ret_swd)==65) {annuity<-15.81}
if (cohorte==1941 & floor(edad_ret_swd)==66) {annuity<-15.23}
if (cohorte==1941 & floor(edad_ret_swd)==67) {annuity<-14.65}
if (cohorte==1941 & floor(edad_ret_swd)==68) {annuity<-14.08}
if (cohorte==1941 & floor(edad_ret_swd)==69) {annuity<-13.5}
if (cohorte==1941 & floor(edad_ret_swd)==70) {annuity<-12.94}

if (cohorte==1942 & floor(edad_ret_swd)==61) {annuity<-18.23}
if (cohorte==1942 & floor(edad_ret_swd)==62) {annuity<-17.65}
if (cohorte==1942 & floor(edad_ret_swd)==63) {annuity<-17.06}
if (cohorte==1942 & floor(edad_ret_swd)==64) {annuity<-16.48}
if (cohorte==1942 & floor(edad_ret_swd)==65) {annuity<-15.89}
if (cohorte==1942 & floor(edad_ret_swd)==66) {annuity<-15.31}
if (cohorte==1942 & floor(edad_ret_swd)==67) {annuity<-14.74}
if (cohorte==1942 & floor(edad_ret_swd)==68) {annuity<-14.16}
if (cohorte==1942 & floor(edad_ret_swd)==69) {annuity<-13.59}
if (cohorte==1942 & floor(edad_ret_swd)==70) {annuity<-13.02}

if (cohorte==1943 & floor(edad_ret_swd)==61) {annuity<-18.33}
if (cohorte==1943 & floor(edad_ret_swd)==62) {annuity<-17.75}
if (cohorte==1943 & floor(edad_ret_swd)==63) {annuity<-17.16}
if (cohorte==1943 & floor(edad_ret_swd)==64) {annuity<-16.58}
if (cohorte==1943 & floor(edad_ret_swd)==65) {annuity<-15.99}
if (cohorte==1943 & floor(edad_ret_swd)==66) {annuity<-15.41}
if (cohorte==1943 & floor(edad_ret_swd)==67) {annuity<-14.84}
if (cohorte==1943 & floor(edad_ret_swd)==68) {annuity<-14.26}
if (cohorte==1943 & floor(edad_ret_swd)==69) {annuity<-13.68}
if (cohorte==1943 & floor(edad_ret_swd)==70) {annuity<-13.11}

if (cohorte==1944 & floor(edad_ret_swd)==61) {annuity<-18.44}
```

```
if (cohorte==1944 & floor(edad_ret_swd)==62) {annuity<-17.86}
if (cohorte==1944 & floor(edad_ret_swd)==63) {annuity<-17.28}
if (cohorte==1944 & floor(edad_ret_swd)==64) {annuity<-16.7}
if (cohorte==1944 & floor(edad_ret_swd)==65) {annuity<-16.11}
if (cohorte==1944 & floor(edad_ret_swd)==66) {annuity<-15.54}
if (cohorte==1944 & floor(edad_ret_swd)==67) {annuity<-14.96}
if (cohorte==1944 & floor(edad_ret_swd)==68) {annuity<-14.38}
if (cohorte==1944 & floor(edad_ret_swd)==69) {annuity<-13.8}
if (cohorte==1944 & floor(edad_ret_swd)==70) {annuity<-13.23}

if (cohorte==1945 & floor(edad_ret_swd)==61) {annuity<-18.55}
if (cohorte==1945 & floor(edad_ret_swd)==62) {annuity<-17.96}
if (cohorte==1945 & floor(edad_ret_swd)==63) {annuity<-17.38}
if (cohorte==1945 & floor(edad_ret_swd)==64) {annuity<-16.8}
if (cohorte==1945 & floor(edad_ret_swd)==65) {annuity<-16.22}
if (cohorte==1945 & floor(edad_ret_swd)==66) {annuity<-15.64}
if (cohorte==1945 & floor(edad_ret_swd)==67) {annuity<-15.07}
if (cohorte==1945 & floor(edad_ret_swd)==68) {annuity<-14.48}
if (cohorte==1945 & floor(edad_ret_swd)==69) {annuity<-13.91}
if (cohorte==1945 & floor(edad_ret_swd)==70) {annuity<-13.33}

if (cohorte==1946 & floor(edad_ret_swd)==61) {annuity<-18.64}
if (cohorte==1946 & floor(edad_ret_swd)==62) {annuity<-18.05}
if (cohorte==1946 & floor(edad_ret_swd)==63) {annuity<-17.47}
if (cohorte==1946 & floor(edad_ret_swd)==64) {annuity<-16.89}
if (cohorte==1946 & floor(edad_ret_swd)==65) {annuity<-16.31}
if (cohorte==1946 & floor(edad_ret_swd)==66) {annuity<-15.73}
if (cohorte==1946 & floor(edad_ret_swd)==67) {annuity<-15.16}
if (cohorte==1946 & floor(edad_ret_swd)==68) {annuity<-14.57}
if (cohorte==1946 & floor(edad_ret_swd)==69) {annuity<-13.99}
if (cohorte==1946 & floor(edad_ret_swd)==70) {annuity<-13.41}

if (cohorte==1947 & floor(edad_ret_swd)==61) {annuity<-18.73}
if (cohorte==1947 & floor(edad_ret_swd)==62) {annuity<-18.15}
if (cohorte==1947 & floor(edad_ret_swd)==63) {annuity<-17.56}
if (cohorte==1947 & floor(edad_ret_swd)==64) {annuity<-16.98}
if (cohorte==1947 & floor(edad_ret_swd)==65) {annuity<-16.4}
if (cohorte==1947 & floor(edad_ret_swd)==66) {annuity<-15.83}
```

```
if (cohorte==1947 & floor(edad_ret_swd)==67) {annuity<-15.24}
if (cohorte==1947 & floor(edad_ret_swd)==68) {annuity<-14.66}
if (cohorte==1947 & floor(edad_ret_swd)==69) {annuity<-14.07}
if (cohorte==1947 & floor(edad_ret_swd)==70) {annuity<-13.49}

if (cohorte==1948 & floor(edad_ret_swd)==61) {annuity<-18.83}
if (cohorte==1948 & floor(edad_ret_swd)==62) {annuity<-18.24}
if (cohorte==1948 & floor(edad_ret_swd)==63) {annuity<-17.66}
if (cohorte==1948 & floor(edad_ret_swd)==64) {annuity<-17.07}
if (cohorte==1948 & floor(edad_ret_swd)==65) {annuity<-16.49}
if (cohorte==1948 & floor(edad_ret_swd)==66) {annuity<-15.91}
if (cohorte==1948 & floor(edad_ret_swd)==67) {annuity<-15.33}
if (cohorte==1948 & floor(edad_ret_swd)==68) {annuity<-14.74}
if (cohorte==1948 & floor(edad_ret_swd)==69) {annuity<-14.16}
if (cohorte==1948 & floor(edad_ret_swd)==70) {annuity<-13.58}

if (cohorte==1949 & floor(edad_ret_swd)==61) {annuity<-18.89}
if (cohorte==1949 & floor(edad_ret_swd)==62) {annuity<-18.31}
if (cohorte==1949 & floor(edad_ret_swd)==63) {annuity<-17.72}
if (cohorte==1949 & floor(edad_ret_swd)==64) {annuity<-17.13}
if (cohorte==1949 & floor(edad_ret_swd)==65) {annuity<-16.55}
if (cohorte==1949 & floor(edad_ret_swd)==66) {annuity<-15.97}
if (cohorte==1949 & floor(edad_ret_swd)==67) {annuity<-15.38}
if (cohorte==1949 & floor(edad_ret_swd)==68) {annuity<-14.79}
if (cohorte==1949 & floor(edad_ret_swd)==69) {annuity<-14.21}
if (cohorte==1949 & floor(edad_ret_swd)==70) {annuity<-13.63}

if (cohorte==1950 & floor(edad_ret_swd)==61) {annuity<-18.98}
if (cohorte==1950 & floor(edad_ret_swd)==62) {annuity<-18.39}
if (cohorte==1950 & floor(edad_ret_swd)==63) {annuity<-17.80}
if (cohorte==1950 & floor(edad_ret_swd)==64) {annuity<-17.21}
if (cohorte==1950 & floor(edad_ret_swd)==65) {annuity<-16.63}
if (cohorte==1950 & floor(edad_ret_swd)==66) {annuity<-16.05}
if (cohorte==1950 & floor(edad_ret_swd)==67) {annuity<-15.46}
if (cohorte==1950 & floor(edad_ret_swd)==68) {annuity<-14.87}
if (cohorte==1950 & floor(edad_ret_swd)==69) {annuity<-14.28}
if (cohorte==1950 & floor(edad_ret_swd)==70) {annuity<-13.7}
```

```
if (cohorte==1951 & floor(edad_ret_swd)==61) {annuity<-19.06}
if (cohorte==1951 & floor(edad_ret_swd)==62) {annuity<-18.48}
if (cohorte==1951 & floor(edad_ret_swd)==63) {annuity<-17.89}
if (cohorte==1951 & floor(edad_ret_swd)==64) {annuity<-17.3}
if (cohorte==1951 & floor(edad_ret_swd)==65) {annuity<-16.71}
if (cohorte==1951 & floor(edad_ret_swd)==66) {annuity<-16.13}
if (cohorte==1951 & floor(edad_ret_swd)==67) {annuity<-15.54}
if (cohorte==1951 & floor(edad_ret_swd)==68) {annuity<-14.95}
if (cohorte==1951 & floor(edad_ret_swd)==69) {annuity<-14.37}
if (cohorte==1951 & floor(edad_ret_swd)==70) {annuity<-13.78}

if (cohorte==1952 & floor(edad_ret_swd)==61) {annuity<-19.14}
if (cohorte==1952 & floor(edad_ret_swd)==62) {annuity<-18.55}
if (cohorte==1952 & floor(edad_ret_swd)==63) {annuity<-17.96}
if (cohorte==1952 & floor(edad_ret_swd)==64) {annuity<-17.37}
if (cohorte==1952 & floor(edad_ret_swd)==65) {annuity<-16.78}
if (cohorte==1952 & floor(edad_ret_swd)==66) {annuity<-16.2}
if (cohorte==1952 & floor(edad_ret_swd)==67) {annuity<-15.61}
if (cohorte==1952 & floor(edad_ret_swd)==68) {annuity<-15.02}
if (cohorte==1952 & floor(edad_ret_swd)==69) {annuity<-14.43}
if (cohorte==1952 & floor(edad_ret_swd)==70) {annuity<-13.85}

if (cohorte==1953 & floor(edad_ret_swd)==61) {annuity<-19.2}
if (cohorte==1953 & floor(edad_ret_swd)==62) {annuity<-18.62}
if (cohorte==1953 & floor(edad_ret_swd)==63) {annuity<-18.03}
if (cohorte==1953 & floor(edad_ret_swd)==64) {annuity<-17.44}
if (cohorte==1953 & floor(edad_ret_swd)==65) {annuity<-16.85}
if (cohorte==1953 & floor(edad_ret_swd)==66) {annuity<-16.26}
if (cohorte==1953 & floor(edad_ret_swd)==67) {annuity<-15.68}
if (cohorte==1953 & floor(edad_ret_swd)==68) {annuity<-15.09}
if (cohorte==1953 & floor(edad_ret_swd)==69) {annuity<-14.5}
if (cohorte==1953 & floor(edad_ret_swd)==70) {annuity<-13.91}

if (cohorte==1954 & floor(edad_ret_swd)==61) {annuity<-19.28}
if (cohorte==1954 & floor(edad_ret_swd)==62) {annuity<-18.69}
if (cohorte==1954 & floor(edad_ret_swd)==63) {annuity<-18.11}
if (cohorte==1954 & floor(edad_ret_swd)==64) {annuity<-17.52}
if (cohorte==1954 & floor(edad_ret_swd)==65) {annuity<-16.93}
```

```
if (cohorte==1954 & floor(edad_ret_swd)==66) {annuity<-16.34}
if (cohorte==1954 & floor(edad_ret_swd)==67) {annuity<-15.76}
if (cohorte==1954 & floor(edad_ret_swd)==68) {annuity<-15.17}
if (cohorte==1954 & floor(edad_ret_swd)==69) {annuity<-14.58}
if (cohorte==1954 & floor(edad_ret_swd)==70) {annuity<-13.99}

if (cohorte==1955 & floor(edad_ret_swd)==61) {annuity<-19.34}
if (cohorte==1955 & floor(edad_ret_swd)==62) {annuity<-18.75}
if (cohorte==1955 & floor(edad_ret_swd)==63) {annuity<-18.16}
if (cohorte==1955 & floor(edad_ret_swd)==64) {annuity<-17.58}
if (cohorte==1955 & floor(edad_ret_swd)==65) {annuity<-16.99}
if (cohorte==1955 & floor(edad_ret_swd)==66) {annuity<-16.4}
if (cohorte==1955 & floor(edad_ret_swd)==67) {annuity<-15.81}
if (cohorte==1955 & floor(edad_ret_swd)==68) {annuity<-15.22}
if (cohorte==1955 & floor(edad_ret_swd)==69) {annuity<-14.63}
if (cohorte==1955 & floor(edad_ret_swd)==70) {annuity<-14.04}

if (cohorte>=1956 & floor(edad_ret_swd)==61) {annuity<-19.42}
if (cohorte>=1956 & floor(edad_ret_swd)==62) {annuity<-18.84}
if (cohorte>=1956 & floor(edad_ret_swd)==63) {annuity<-18.25}
if (cohorte>=1956 & floor(edad_ret_swd)==64) {annuity<-17.66}
if (cohorte>=1956 & floor(edad_ret_swd)==65) {annuity<-17.07}
if (cohorte>=1956 & floor(edad_ret_swd)==66) {annuity<-16.48}
if (cohorte>=1956 & floor(edad_ret_swd)==67) {annuity<-15.89}
if (cohorte>=1956 & floor(edad_ret_swd)==68) {annuity<-15.3}
if (cohorte>=1956 & floor(edad_ret_swd)==69) {annuity<-14.71}
if (cohorte>=1956 & floor(edad_ret_swd)==70) {annuity<-14.12}


inkomstpension<-(((salarios_capitalizado_nocional*AF)/annuity
   ))/12

prisbasbelopp<-4375

garanti_pension <- function(inkomstpension, edad_ret_swd,
   prisbasbelopp) {

  garantipension <- rep(0, length(inkomstpension))
```

```
  for (i in 1:length(inkomstpension)) {

    if (round(inkomstpension[i], 2) >= 14649 & edad_ret_swd
       >= 65) {
      garantipension[i] <- 0
    } else if (edad_ret_swd< 65) {
      garantipension[i] <- 0
    } else if (round(inkomstpension[i], 2) <= 1.14 *
       prisbasbelopp) {
      garantipension[i] <- 2.2 * prisbasbelopp -
         inkomstpension[i]
    } else if (round(inkomstpension[i], 2) > 1.14 *
       prisbasbelopp & round(inkomstpension[i], 2) < 14649) {
      garantipension[i] <- 1.06 * prisbasbelopp - 0.48 * (
         inkomstpension[i] - 1.14 * prisbasbelopp)
    }
  }

  return(garantipension)
}

garantipension<-garanti_pension(inkomstpension,edad_ret_swd,
   prisbasbelopp)


cat("El jubilado entr  al mercado laboral con:",edad," a os
   y",meses,"meses","\n")
cat("Perteneciendo a la cohorte del  a o:",cohorte,"\n")
cat("Recibir  una pensi n derivada del primer pilar de:",
   garantipension,"SEK","\n")
cat("Recibir  una pensi n derivad del segundo pilar de:",
   inkomstpension,"SEK","\n")

pcontributiva<-rep(0,length(incswd))
for (i in 1:length(incswd)){
  pcontributiva[i]<-inkomstpension[i]+garantipension[i]
}
```

```
cat("Lo cu l suma una cantidad total de:", round(
   pcontributiva,2),"SEK","\n")
cat("Que se traduce en:",round(round(garantipension+
   inkomstpension,2)*EUR_SWD,2),"    ","\n")
last_salary<-apply(salarios, 2, function(x) x[1])
rj<-rep(0,length(incswd))
for (i in 1:length(incswd)){
  rj[i]<-(inkomstpension[i]+garantipension[i])/last_salary[i]
}
cat("La tasa de sustituci n es de:",round(rj*100,2),"%","\n"
   )

##GDP PER CAPITA
GDPPC<- read_excel("Desktop/TFM/PROGRAMA_SIMULACION_TIR/
   DATABASE_TFM_IRR.xlsx",
                    sheet = "GDP PER CAPITA US DOL")
GDPPC<-GDPPC[,c(1,2,3)]
GDPPC<-as.data.frame(GDPPC)
colnames(GDPPC)[1]<-"Year"
GDPPC$Year<-as.Date(GDPPC$Year,format = "%Y")
GDPPC<-GDPPC[order(dim(GDPPC)[1]:1), ]

if (g nero=="M"){

  last_salary<-apply(salarios, 2, function(x) x[1])
  rj<-rep(0,length(incswd))
  for (i in 1:length(incswd)){
    rj[i]<-(inkomstpension[i]+garantipension[i])/last_salary[
       i]
  }
  c<-0.3142+0.07

  TABLAS_M<- read.csv("~/Desktop/TFM/PROGRAMA_SIMULACION_TIR/
     TABLAS_SUECIA/TABLASHOMBRES.txt", sep="")
  am<-which(TABLAS_M$Year==cohorte)
  cohortm<-TABLAS_M[am,c(1,2,6)]
```

```
lkm_swd<-cohortm[,3]
lkm_swd<-as.numeric(unlist(lkm_swd))

AP_M<-function(an,k) {
  LX<-lkm_swd
  k<-as.numeric(k)
  vk<-c(0:(k-1))/k
  LY<-as.vector(t(outer(LX[-length(LX)],(1-vk),"*")+outer(
     LX[-1],vk,"*")))
  LY<-c(round(LY,4),0)
  return(LY)

}

lk12m<-AP_M(cohorte,12)
j<-a os_tot
lim<-which(lk12m==0)[1]
Ej<-(sum(lk12m[(j*12+1):lim])/lk12m[(j*12+1)])+6
Ea1<-(sum(lk12m[(a*12+1):(j*12+1)])/lk12m[(a*12+1)])+6
pa<-lk12m[(j*12+1)]/lk12m[(a*12+1)]
k<-a os_tot*12
AB<-j*12-((k+1)/2)
AR<-j*12+(pa*Ej)/2
AC<-a*12+(Ea1-Ej*pa)/2
sigma<-alpha_mensual_Salnom
TAAGDP<-(GDPPC[1,3]/GDPPC[(a oscot_swd-1),3])^(1/(a oscot
   _swd-1))-1
i<-sigma/TAAGDP

TIR<-rep(0,length(incswd))
for (l in 1:length(incswd)){
  TIR[l]<-(1/(AR-AC))*((log(rj[l]/c)+log(Ej/Ea1))+i[l]*(AR-
     AB)*sigma[l])
}
TIR<-((1+TIR)^12)-1

cat("Los valores de \u03C3 estimada para los contribuyentes
    son de:",round(((1+sigma)^12-1)*100,2),"%","\n")
```

```
    cat("Los valores de i estimada para los contribuyentes son
       de:",round(i,2),"\n")
    cat("La TIR aproximada para los contribuyentes son de:",
       round(TIR*100,2),"%","\n")

  }

  if (g nero=="F"){
    last_salary<-apply(salarios, 2, function(x) x[1])
    rj<-rep(0,length(incswd))
    for (i in 1:length(incswd)){
      rj[i]<-(inkomstpension[i]+garantipension[i])/last_salary[
         i]
    }
    c<-0.3142+0.07
    TABLAS_F<- read.csv("~/Desktop/TFM/PROGRAMA_SIMULACION_TIR/
       TABLAS_SUECIA/TABLASMUJERES.txt", sep="")
    af<-which(TABLAS_F$Year==cohorte)
    cohortf<-TABLAS_F[af,c(1,2,6)]
    lkf_swd<-cohort1[,3]
    lkf_swd<-as.numeric(unlist(lkf_swd))

    AP_F<-function(an,k) {
      LX<-lkf_swd
      k<-as.numeric(k)
      vk<-c(0:(k-1))/k
      LY<-as.vector(t(outer(LX[-length(LX)],(1-vk),"*")+outer(
         LX[-1],vk,"*")))
      LY<-c(round(LY,4),0)
      return(LY)

    }

    lk12f<-AP_F(cohorte,12)
    j<-a os_tot
    lim<-which(lk12f==0)[1]
    Ej<-(sum(lk12f[(j*12+1):lim])/lk12f[(j*12+1)])+6
    Ea1<-(sum(lk12f[(a*12+1):(j*12+1)])/lk12f[(a*12+1)])+6
```

```
      pa<-lk12f[(j*12+1)]/lk12f[(a*12+1)]
      k<-a os_tot*12
      AB<-j*12-((k+1)/2)
      AR<-j*12+(pa*Ej)/2
      AC<-a*12+(Ea1-Ej*pa)/2

      sigma<-alpha_mensual_Salnom
      TAAGDP<-(GDPPC[1,3]/GDPPC[(a oscot_swd-1),3])^(1/(a oscot
         _swd-1))-1
      i<-sigma/TAAGDP

      TIR<-rep(0,length(incswd))
      for (l in 1:length(incswd)){
        TIR[l]<-(1/(AR-AC))*((log(rj[l]/c)+log(Ej/Ea1))+i[l]*(AR-
           AB)*sigma[l])
      }

      TIR<-((1+TIR)^12)-1
      cat("Los valores de \u03C3 estimada para los contribuyentes
          son de:",round(((1+sigma)^12-1)*100,2),"%","\n")
      cat("Los valores de i estimada para los contribuyentes son
         de:",round(i,2),"\n")
      cat("La TIR aproximada para los contribuyentes son de:",
         round(TIR*100,2),"%","\n")

    }
  }
}
```