Return-point memory of the random field Ising model at zero temperature in the mean-field approximation

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The random field Ising model at zero temperature (RFIM T=0) is a paradigmatic model of hysteresis and memory of driven extended disordered systems. Under quasistatic driving these systems display interesting properties such as return-point memory (RPM) of partial cycles. Here we show how to solve the T=0 RFIM in the mean-field approximation, and prove that in this simple approximation the model still displays hysteresis, though only for sufficiently narrow distributions of random fields. We simulate partial hysteresis cycles and first-order reversal curves (FORC) for this case, and show that the T=0 RFIM in the mean-field approximation verifies also the RPM.

I. INTRODUCTION

Hysteresis is a highly recurrent phenomenon in the fields of condensed matter physics, materials and meta-materials, hydrology, biology, engineering and robotics, as described in the 3-volume book series *The Science of Hysteresis* [1].

Hysteresis, from the greek "lagging behind", is the dependence of the state of the system on its history. Broadly speaking, hysteresis can be classified into rate-dependent and rate-independent. The rate-dependent hysteresis applies when the response of the system and a periodic external forcing have similar time scales. The rate-independent hysteresis, also known as quasistatic hysteresis, applies when the response of the system is much faster than the time scale defined by the rate of change of the forcing. This work will focus on quasistatic driven systems, where hysteresis survives even in the limit of zero driving rate due to disorder, impurities or the formation of domains. In hysteretic systems, we apply an external input and the system shows an output (response), which depends on the input history. The simplest examples are bistable systems, i.e. systems that have two possible equilibrium states and adopt one or another depending on the past values of the external input. In this paper we will adopt the formalism of magnetic systems: the input is the external magnetic field, H, and the output is the magnetization of the system, M [2].

Hysteresis loops, see Fig. 1,show the possible magnetization values for the system. For the extreme values of H, positive and negative, the magnetization saturates to its limiting values. In between, there are all the possible values of M. Partial loops are inner cycles attached to the main loop, and occur when the external field is reversed before the system reaches one saturated magnetization value. Then, before the system reaches the other saturated value of the magnetization, the field is reversed again. Eventually, the system returns to the same state in the main loop. This is known as return-point memory (RPM).

Memory is the faculty by which a system stores and remembers information. We can assume that a system

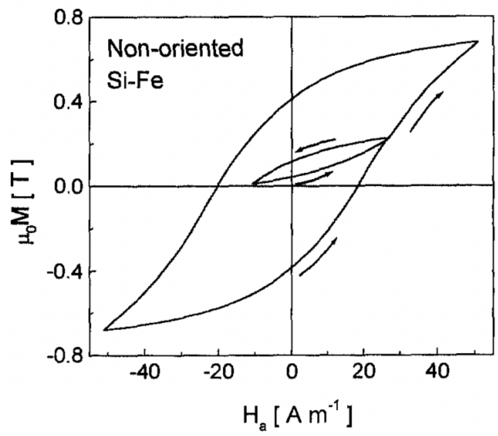

FIG. 1: Magnetization vs applied magnetic field of a typical uniaxial ferromagnet. Shown are the main hysteresis loop and a partial loop. Figure from Ref. [2].

which is not fully relaxed to equilibrium may retain information of its previous states. On the contrary, a system in equilibrium has no memory of its past. Accordingly, hysteresis cycles have memory while they are not on a saturated state and they will erase all the information once they are saturated, relaxed.

Hysteresis emerges in all kinds of dissipative problems where the underlying microscopic processes are irreversible. At the microscopic scale the free energy landscape of the system dictates the minima where the system can be found. For a bistable system the free energy landscape has only two minima because the response of the system only takes two different values. If a bistable system is not externally forced, and thermal fluctuations can be neglected ($T = 0$), it will remain frozen in its actual equilibrium state. However, by varying the external field, see Fig. 2, the system is forced and the free energy landscape is distorted. The system loses stability and jumps from one equilibrium state to the other. During the jump, part of the energy is dissipated, making the process irreversible. Hysteresis is the macroscopic man-

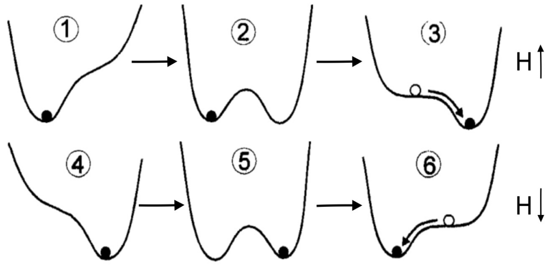

FIG. 2: Sequence of free energy landscapes of a bistable system, for varying H. The first sequence is for increasing external field. The system evolves from a state with negative magnetization to a state of positive magnetization. The second sequence is for decreasing external field. The magnetization jumps back from positive to negative. The presence of the energy barrier makes the two jumps to occur at different values of the magnetic field, giving rise to a hysteresis cycle. Figure adapted from Ref. [2]

ifestation of energy dissipation. In extended disordered systems the free energy landscape is rugged, with many metastable minima of varying height separated by large energy barriers. When the system is externally driven with H, it goes irreversibly from one metastable equilibrium to another, eventually through an avalanche of nonequilibrium transitions known as Barkhausen jumps.

There are two noteworthy family models that approach this phenomenon. On the one hand, the Preisach model, introduced by F. Preisach in 1935 [3], which is a black box model based on hysterons. A hysteron is an operator that gives a rectangular response depending on the field applied. It is considered that the switching value of the external field of the hysterons from down to up is α and the value for the switching from up to down is β, therefore it is assumed that $\alpha \geq \beta$ [4]. The output of the elementary operators (hysterons) can only take two values ± 1, they can be interpreted as two-position operator, corresponding to $\hat{\gamma}_{\alpha\beta} H = \pm 1$. The global response of the system can be obtained as the sum of the response of the hysterons of the system, with an arbitrary weight function, $\mu(\alpha, \beta)$, that represents the population of operators with switching values α and β. In this model the magnetization is given by:

$$M(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \hat{\gamma}_{\alpha\beta} H(t) d\alpha d\beta. \tag{1}$$

The distribution $\mu(\alpha, \beta)$ may be found from the first order reversal curves (FORC), Ref. [5]. These curves are attached to a branch of the main loop. They are formed when a monotonic increase of $H(t)$ is followed by a subsequent monotonic decrease down to saturation, see Fig. 4. Equivalentlly, a FORC could depart from the decreasing magnetization branch all the way up to the positive magnetization saturation value. Although the Preisach model reproduces characteristics of hysteresis quite well, it is phenomenological, and it is hard to link it with the physics of the hysteretic systems.

On the other hand, there is the zero-temperature Random Field Ising Model ($T = 0$ RFIM) introduced in this context by Sethna et al [6]. It is based on the Ising model, hence it is a reticular model and it is fluctuationless since it is studied at 0 temperature. The hamiltonian reads:

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} s_i s_j - H \sum_i s_i - \sum_i b_i s_i. \tag{2}$$

Each spin, s_i, interacts with its nearest neighbours, with an external field, H, and a random local field, b_i. J is the magnitude of the exchange interaction. The flipping value of a spin depends on its interactions. Depending on the variation of the external field and its nearest neighbours, the spins of the lattice will be set in a way that minimizes the energy of the system. At zero temperature the hamiltonian coincides with the free energy of the system, which can be written as:

$$\mathcal{H} = U - HM, \tag{3}$$

where U is the internal energy of the system and HM is the product of the two conjugate variables through which we vary the energy of the system. In a multispin system with frozen disorder, the free energy landscape becomes a rugged free energy landscape with multiple minima. Thus, a variation of the external field or a change in the magnetization will distort the free energy landscape and may induce a spin flip. In turn, the free energy landscape will be, again, modified and might induce the flip of another spin and so forth, causing an avalanche of spin flipping. An avalanche could be described as the transition of the system through non-equilibrium states until it reaches again a metastable state.

An alternative model of hysteresis is the mean-field model of Lim and Saloma, inspired in the mean field solution of a reticular model with delay, but written in continuous instead of discrete variables [7].

The aim of this research is applying the mean-field approximation to the T=0 RFIM and prove that it has return-point memory. The mean-field approximation is a theory where the interactions among the neighbours of the lattice are treated approximately. They are replaced by an interaction with an average field. Therefore, the correlations of the fluctuations (the spin flips) are disregarded. The expression of the average field, or mean effective field, can be easily found, just replacing the value of each local magnitude for the mean value. Then, each spin sees the same effective field and the many-body problem becomes a single-body problem. On the whole, the theory does not take into account the positions in the lattice and relies on the self-consistency condition

$$\langle s_i \rangle = \langle s_j \rangle. \tag{4}$$

This report is organized as follows: first the mean-field approximation of the T=0 RFIM is exposed in Sec.

II. The results obtained with mean field theory for the simulation of the RPM are presented in Sec. III. Finally in Sec. IV we draw the conclusions of the investigation.

II. MEAN-FIELD APPROXIMATION OF THE T=0 RFIM

The random field Ising model is a model that describes the state of the system by means of the orientation of the spins in the lattice. The orientations can only be up (+1) or down (-1), $s_i = \pm 1$. The hamiltonian of the model, Eq. (2), is built by the three interactions of the spins contained in $\mathcal{H}$. The random field is specific of each lattice site and follows a statistical distribution, which adds disorder to the system. This research will take the gaussian distribution with mean value $\langle b_i \rangle = 0$, and the variance of the distribution, σ, will be subject of study.

Firstly, note that a new variable, the local field F_i acting on the spin i can be defined by factoring out the spin s_i:

$$\mathcal{H} = -\sum_i F_i s_i, \tag{5}$$

where F_i accounts for the interaction of the spin with its neighbours, the external field and with its local random field:

$$F_i = J \sum_{\langle j|i \rangle} s_j + H + b_i. \tag{6}$$

In mean-field approximation, by reason of Eq. (4), the sum over j can be interpreted as the magnetization per spin multiplied by the number of nearest neighbours of a lattice site (the coordination number, z). Then, in the thermodynamic limit, according to Ref.[8], the state equation of the system becomes:

$$m = \tanh(\beta z J m + \beta H + \beta b_i), \tag{7}$$

where β stands for $1/kT$ and k is the Boltzmann constant. For $T = 0$, $\beta \to \infty$ and the tanh becomes a step function between $m = -1$ and $m = +1$. Then, the condition for a spin to flip corresponds to the argument of the tanh equal to zero. The values of the external field that satisfy this condition are given by:

$$H = -zJm - b_i. \tag{8}$$

These are the only values that will cause a change of the magnetization. Starting at the equilibrium point, $m = -1, H = -\infty$, all spins are pointing downwards, the condition for the first spin to flip (the spin with largest random field, b_1) will therefore be:

$$H_1 = -zJ(-1) - b_1. \tag{9}$$

The corresponding magnetization after this spin flip is:

$$m = 1 - 2p(b_1), \tag{10}$$

where the factor 2 arises from the change of the spin 1 from -1 to +1. Iterating this procedure to the k spin with random field b_k, we obtain that the corresponding spin flip takes place at:

$$H_k = -zJ\left(1 - 2\sum_{i=1}^{k-1} p(b_i)\right) - b_k, \tag{11}$$

where the term in brackets is the magnetization of the system before this spin k flips. Once the new spin flip has taken place, the new magnetization is:

$$m = 1 - 2\sum_{i=1}^{k} p(b_i). \tag{12}$$

Bringing it to the continuum limit, this expression becomes:

$$M = 1 - 2\int_{-\infty}^{-zJM-H} p(b)db, \tag{13}$$

where we have used that the upper limit is given by Eq. (8), i.e. $b = -zJM - H$. We have thus obtained a mean-field self-consistency condition for the metastable equilibrium values $M(H)$.

It is remarkable that the result of the mean-field approximation depends on the variance σ of the distribution (through $p(b)$) but not on the spatial dimensionality of the system. In this approximation the interactions among the neighbours are replaced by the interaction with an effective field, for which the dimension of the system becomes irrelevant but the size of the system and the variance of the distribution do not. The variance is a measure of dispersion that takes into account the spread of all local random fields; for large variance the local random fields will be very spread in magnitude and the flip of a spin will, typically, not induce another spin flip. On the contrary, for smaller variances the random local fields will be similar and a single spin flip may induce an infinite avalanche of all the spins. On the other hand, for a given variance, the size of the system will make the distribution more or less populated. Therefore the size of the system will have an opposite effect to the variance σ. The critical variance, σ_C, defined as the value of the variance for which a single point of the loop has infinite slope, dictates the change of regime. According to Ref. [9], $\sigma_C = 0.798$. However, in small systems and for $\sigma = \sigma_C$, hysteresis appears before a point of the loop has infinite slope. For that reason, the value $\sigma_C = 0.798$ is only valid in the thermodynamic limit, where $N \to \infty$. It is worth noting that in the mean-field approximation the system shows hysteresis only for $\sigma < \sigma_C$, as shown in Fig. 3. In this regime, the coupling between neighbouring spins is more important than the amount of disorder of the system. In this condition, the values of the random

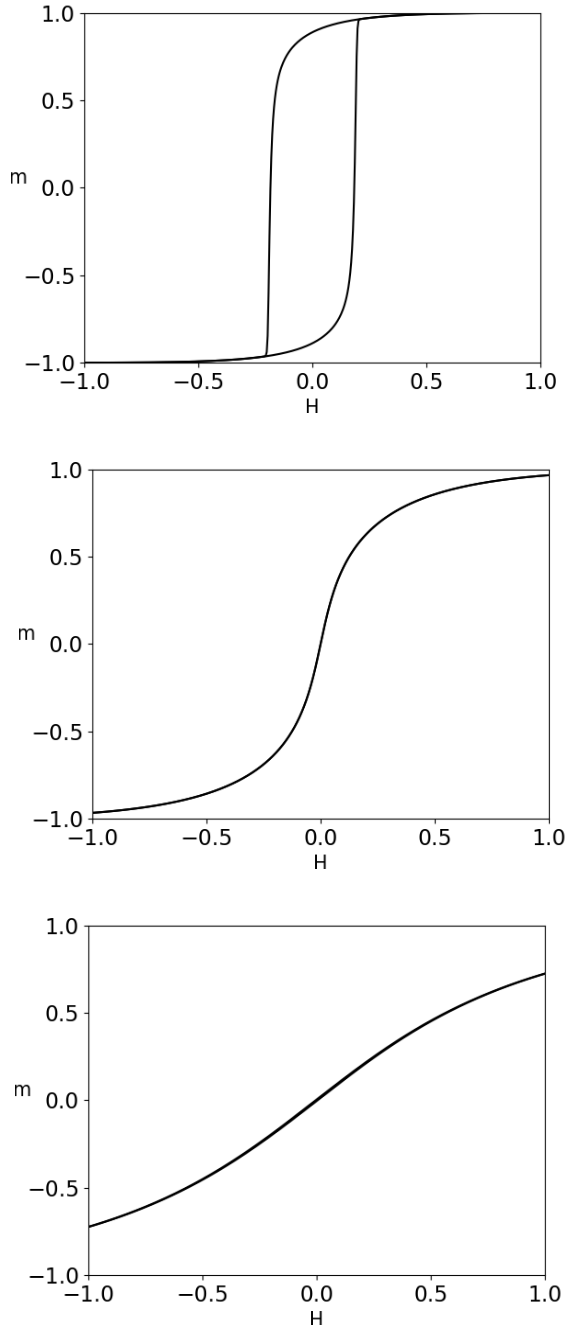

FIG. 3: Magnetization vs applied magnetic field for a system with 10^5 spins in mean-field approximation for the three regimes. From top to bottom: variance $\sigma < \sigma_C$, variance $\sigma = \sigma_C$ and variance $\sigma > \sigma_C$, with $\sigma_C = 0.798$ [9].

fields will be more similar and big avalanches are more likely. Avalanches will cause energy dissipation and irreversibility, therefore, hysteresis cycles. This effect is more important in the mean-field approximation since in this approximation the position of the spins is not relevant but the distribution of the local random fields is.

III. MEAN FIELD RETURN-POINT MEMORY

One of the exceptionalities of the T=0 RFIM is that it was the first model to prove an interesting memory property such as the RPM from physical arguments. This property is observed in many hysteretic systems in the quasiestatic limit. It is closely related with the wiping-out property. Systems with RPM are able to store memory of its past states and are able to go back to those exact states under proper conditions. On the other hand, the wiping-out property ensures that when the system returns to a state (H, M) where it has already been, the memory of the past states in this inner loop is erased.

In the original work of Sethna et al [6], the origin of this RPM was proved to be associated with a partial ordering of the microscopic configurations, which was preserved by the quasistatic dynamics. Under a monotonous excursion of the driving field, one configuration ahead of another remains always ahead (the no passing rule introduced by A. Middleton in the context of charge density waves (CDW) [10]). This property together with the adiabatic character of the dynamics makes that a non-monotonic excursion of the driving field can always be bounded by two monotonic excursions, and all of them take the system to the same final state.

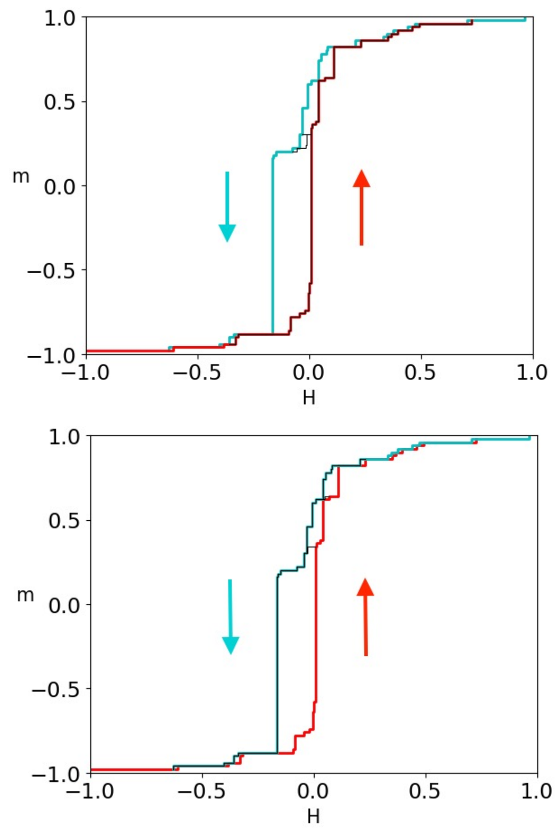

FIG. 4: Magnetization vs applied magnetic field for a system with 100 spins and variance $\sigma = 0.7$. Top: a FORC for increasing field. Bottom: a FORC for decreasing field.

We can prove that our mean-field solution of the T=0 RFIM displays also the RPM property. In the mean-field approximation, this property can be shown only for $\sigma < \sigma_C$ because it is there where hysteresis appears. We take a system with $\sigma = 0.7$ and N=100, where N is the size of the system. The FORCs of the system are shown in Fig. 4. In either the increasing or decreasing field cases, there are no anomalous crossings of trajectories. This is related to the fact that the area of the hysteresis cycle in the suitable variables (the conjugated variables H and M) corresponds to the energy irreversibly dissipated. The properties of the trajectories in the hysteresis cycles reflect the restrictions imposed by the first and second laws of thermodynamics.

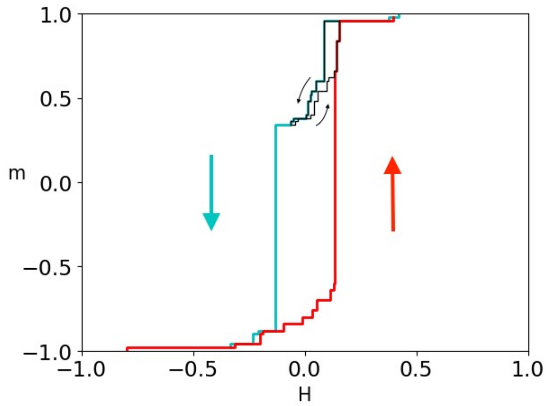

FIG. 5: Magnetization vs applied magnetic field for a system with 100 spins and variance $\sigma = 0.7$. The ascending branch is drawn in red, the descending branch in cian, and the partial loop in black.

Figure 5 shows the results for a system with the same characteristics as in Fig. 4 ($N = 100, \sigma = 0.7$) but for another realization of values of the random fields. It presents a partial loop initiated from the decreasing branch. The descending trajectory after the partial cycle rejoins exactly the original descending trajectory at the return point. The system returns to the exact same state once the internal cycle is completed. The memory of the return point is stored exactly for a particular realization of the local random fields.

IV. CONCLUSIONS

Hysteresis is a wide-interest phenomenon which can be described with various models. We have applied the mean-field approximation on the T=0 RFIM, and shown that the result becomes independent of the spatial dimensionality. In this approximation we have found a recurrency relation for the magnetization vs the applied magnetic field in the discrete formalism, Eq. (12), and the corresponding expression for the continuous limit, Eq. (13). The latter reproduces the results of Dahmen and Sethna [9]. With this approximation, the system only exhibits hysteresis for $\sigma < \sigma_C$. We have proved for the first time that in this regime the hysteresis cycle exhibits also the return-point memory property, by which the system recovers its original configuration after a closed cycle in the $M - H$ parameter space.

Acknowledgments

I would like to thank my advisor for the guidance, support and hope in the dark times of the research. I would also like to acknowledge my family for the precious support during all this time and my college friends for their help and advice.

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