Simulating the ocean dynamics of the midlatitudes circulation. A question of scale ?

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Abstract: Today, numerical simulations are the most common tool for understanding ocean dynamics. However, due to the non-linearity of the system, numerical simulations only provide a discretized approximation to particular solutions. Are these approximations unique, or do they depend on the choices made during discretization? In this work we have reviewed the issue of horizontal resolution and illustrated the importance of allowing the development of mesoscale features to ensure a more realistic description of the ocean dynamics.

I. INTRODUCTION

The ocean. The greatest paradox of our world. Such a chaotic fluid yet repeatedly perceived as transmitting peace. Early explorers and navigators demonstrated bravery as they embarked into unknown territories, discovering diverse cultures and expanding our understanding of the world. The study of oceanography as a science started at the end of the 19^{th} Century when first data on ocean currents, winds, and other oceanographic phenomena were collected. During the 20^{th} Century some idealized analytical solutions that could explain the ocean dynamics were proposed by Ekman, Svedrup, Stommel or Munk [1].

The study of the oceans changed significantly with the arrival of satellites in the 1970s. For the first time, quasi-synoptic, quasi-global and quasi-real-time data were available, a new way of studying oceanography became possible. It became evident that the idealized analytical solutions of Ekman, Svedrup, Stommel or Munk, although good at describing some isolated phenomenons that occur in the ocean, were inadequate in providing realistic descriptions of the ocean dynamics. Thanks to the technological developments, computers became powerful enough to handle the data and calculations required to model the ocean. Since then, many numerical simulations, that combine quasi-real-time data and ever-improving numerical methods, have enhanced the understanding of the oceans.

In this paper, we simulate the three-dimensional ocean circulation in the mid-latitudes. This region is of particular interest due to the crucial role of the Gulf Stream in transporting warm waters from the tropics to the northern areas, which has a substantial impact on regional climate patterns and plays a crucial role in regulating the temperature of the North Atlantic Ocean. We will investigate the impact of the horizontal resolution for a realistic description of the ocean dynamics by executing a set of five experiments with the MITgcm, a state-of-the-art model for both the ocean and the atmosphere [2].

II. METHODOLOGY

The underlying physical equations used to describe the ocean dynamics derive from Navier-Stokes equations for a Newtonian fluid:

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}\right)=-\nabla P+\mu\nabla^2\vec{v}+\vec{F}_{vol} \qquad (1)$$

where ρ is the density of the fluid, the term between parenthesis is the Lagrangian derivative $\left(\frac{D}{Dt}\right)$ of the velocity. On the right hand-side, the first term is the pressure gradient, the second is the viscous term where μ is the dynamic viscosity coefficient and the third term corresponds to any external volume force. The 'grad' (∇) in spherical coordinates:

$$\nabla\equiv\left(\frac{1}{r\cos\varphi}\frac{\partial}{\partial\lambda},\frac{1}{r}\frac{\partial}{\partial\varphi},\frac{\partial}{\partial r}\right)$$

The Hydrostatic Primitive Equations (HPEs) [3] used in our experiments are an approximation of the Navier-Stokes. These equations, commonly employed in oceanography to study the large-scale circulation, incorporate several key approximations [4]. The thin-shell approximation that assumes small vertical variations compared with horizontal ones, the quasi-hidrostatic approximation balancing the vertical gradient pressure and the buoyancy forces, the Bousinesq approximation neglecting density variations except when calculating the buoyancy term, and the assumption of water incompressibility. Additionally, a turbulent closure hypothesis is made by expressing the turbulent fluxes in terms of large-scale features. To introduce the turbulent mixing and diffusion of momentum caused by subgrid-scale processes, an eddy-viscosity coefficient, A_h, is introduced in HPEs.

It is important to note that the HPEs are thus simplifications of the true fluid dynamics, as discussed in [3], and there are limitations to their applicability. In some cases, such as studying fine-scale turbulence, coastal dynamics, or certain boundary layer processes, some features may be missing out. As this system of equations is non-linear, there has not been derived a general solution for it, requiring numerical discretization for its solution.

A. The model

As stated before, we will use the MITgcm code that calculates solutions of the HPEs with the help of finite differences and by describing the vertical structure of the ocean as a set of stacked layers of specified thickness. The experiment set-up derives from the MITgcm Baroclinic Ocean Gyre Tutorial [2]. The domain is a rectangular basin that lies in a sphere and spans from the tropics to the mid-latitudes, with 15 vertical layers that vary from $\Delta z = 50$m deep in the surface layer, to 190m deep in the bottom layer. The lateral grid is discretized into gridcells that are uniformly spaced in longitude, $\Delta\lambda$, and latitude, $\Delta\varphi$, as

$$\Delta x = a\cos(\varphi)\Delta\lambda \qquad \Delta y = a\Delta\varphi \tag{2}$$

where $a = 6.37\cdot 10^{-6}$m is the Earth's radius. In the upcoming section II B we will define the different resolutions $(\Delta\lambda, \Delta\varphi)$ employed for each experiment.

The experiment set-up corresponds to a wind- and buoyancy-forced stratified ocean. The Coriolis parameter, f, varies with latitude, φ, as

$$f(\varphi) = 2\Omega\sin\varphi \tag{3}$$

where the angular velocity of the Earth's rotation, Ω, is set to $\frac{2\pi}{86400}s^{-1}$). The equation of state used is linear

$$\rho = \rho_0(1 - \alpha_\theta(\theta - \theta_0)) \tag{4}$$

where $\rho_0 = 999.8$ kg m^{-3}, $\alpha_\theta = 2\times 10^{-4}$ K^{-1}, and θ is the potential temperature initially stratified that varies from $\theta_0 = 30$°C at the surface to $\theta_0 = 2$°C at the bottom.

Two forcing terms are applied at the surface: a meridional wind-stress that varies sinusoidally with latitude and a linear profile restoration for temperature to keep a tropical-to-pole temperature gradient at the surface.

$$\tau_\lambda(\varphi) = -\tau_0\cos 2\pi\frac{\varphi - \varphi_0}{L_\varphi} \tag{5}$$

$$F_\theta = -\frac{1}{\tau_\theta}(\theta - \theta^*) \tag{6}$$

where $L_\varphi = 60°$ is the lateral grid extent, $\varphi_0 = 15°$N and $\tau_0 = 0.1$ Nm^{-2}. In equation (5) the relaxation timescale $\tau_\theta = 30$ days and $\theta^* = \frac{\theta_{max} - \theta_{min}}{L_\varphi}(\varphi - \varphi_0)$ with $\theta_{max} = 30$°C and $\theta_{min} = 0$°C.

B. Experimental procedure

The study comprises a set of 5 experiments. The first three experiments are considered coarse resolution experiments, $\Delta\lambda = \Delta\varphi = 2°, 1°, 1/2°$. We will show that these represent a non-eddying ocean, i.e., an ocean where the structure is just the double-gyre solution described by the analytical models of Stommel and Munk [1]. The fourth experiment is an eddy-permitting one, $\Delta\lambda = \Delta\varphi = 1/4°$, where some eddies start to appear. And the last experiment corresponds to an eddy-resolving one, $\Delta\lambda = \Delta\varphi = 1/8°$, where the dynamics of mesoscale eddies in the ocean are solved.

For the finite differences model to remain numerically stable one criterion must be fulfilled, the Courant–Friedrichs–Lewy condition (CFL) [5] that makes sure information from a cell only propagates, at most, to its neighbors. This condition limits the time step, Δt that can be used for each resolution.

$$\Delta t < \frac{1}{4|c_{max}|}\Delta x_{min} \tag{7}$$

where the maximum horizontal advection is set to $|c_{max}| = 2$ms^{-1} and Δx_{min} is the shortest gridsize.

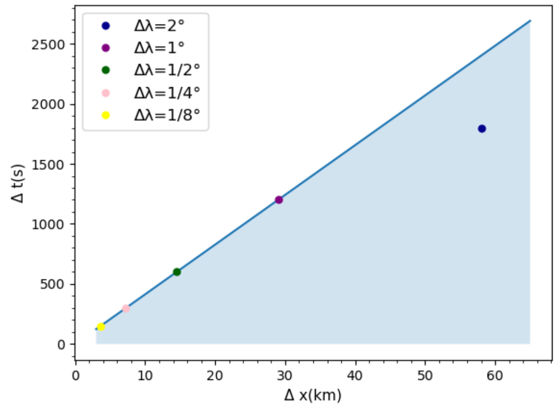

Figure 1: Required time interval for stability, in seconds, as a function of the lateral grid spacing, in kilometers. The colored dots show the chosen time step, Δt for each resolution specified in column 3 of table I.

Fig.1 illustrates the fact that smaller time-steps are needed when the horizontal resolution increases. This means that finer resolutions come at the cost of, not only larger computational grids but also longer simulation times.

Additionally, the ability of our set-up to properly simulate a western boundary current (WBC) is related to the Munk boundary width, which makes sure the western boundary layer is perceived by the model.

$$M = \frac{2\pi}{\sqrt{3}}\left(\frac{A_h}{\beta}\right)^{\frac{1}{3}} \tag{8}$$

where A_h is the horizontal eddy viscosity and the gradient of the Coriolis parameter, $\beta = 10^{-11}$ s^{-1} m^{-1}. In our experiments, we set this Munk width to 3 gridcells so that the simulation has at least 3 points to describe the WBC.

$$A_h = \beta \left(\frac{3\sqrt{3}\Delta x_{max}}{2\pi} \right)^3 \quad (9)$$

where Δx_{max} is the longest gridsize.

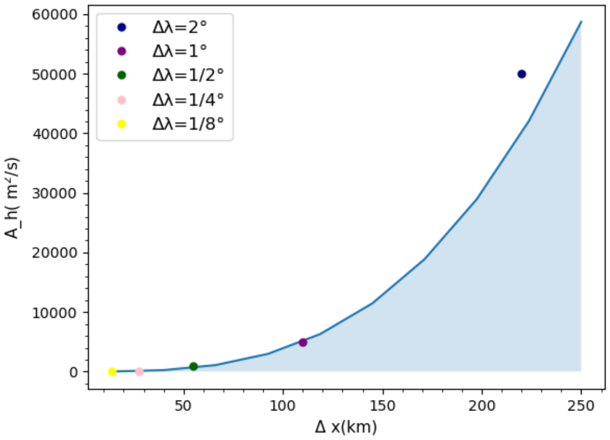

Figure 2: Eddy viscosity coefficient, in square meters per second, as a function of the lateral grid spacing, in kilometers. The colored dots show the chosen eddy viscosity coefficient for each resolution specified in column 3 of table I.

Fig.2 illustrates the fact that, as the horizontal resolution increases, a smaller horiozontal viscosity coefficient is allowed. This means that non-linear terms of HPEs start to become relevant, compared with the dissipation term, and a more energetic and less viscous ocean is simulated.

$\Delta\lambda$ ($^\circ$)	Δx_{min} (km)	Grid size (# × #)	Δt (s)	A_h (m^{2}s^{-1})	t_{sim} (year)	t_{cpu}/t_{sim} (h/year)
2	48	32x32	1800	50000	100	0.14
1	29	62x62	1200	5000	25	0.64
1/2	14.5	122x122	600	1000	5	3.1
1/4	7.25	242x242	300	80	2	18.75
1/8	3.625	482x482	150	10	0.25	240*

Table I: Running time per simulated time(t_{cpu}/t_{sim}), timestep (Δt), minimum lateral grid spacing, Δx_{min}, and eddy viscosity coefficient (A_h) used for the experiments done for each resolution. *The model grid has been subdivided into four separate tiles, and the model equations have been solved in parallel on four separate processes.

The experiment was first run for 100 years, so that the main circulation and the thermocline were stabilized. We started with a resolution of $\Delta\lambda = 2^\circ$ and, after interpolating the result to the corresponding grid of $\Delta\lambda = 1^\circ$, we ran it for 25 years with the $\Delta\lambda = 1/2^\circ$. We followed the same methodology while we kept increasing the resolution untill an eddy-permitting one ($\Delta\lambda = 1/8^\circ$) was obtained.

III. RESULTS AND DISCUSSION

To study how the resolution affects the results, we will compare different magnitudes obtained from the coarse resolution ($\Delta\lambda = 2^\circ$) to the eddy resolving one ($\Delta\lambda = 1/8^\circ$).

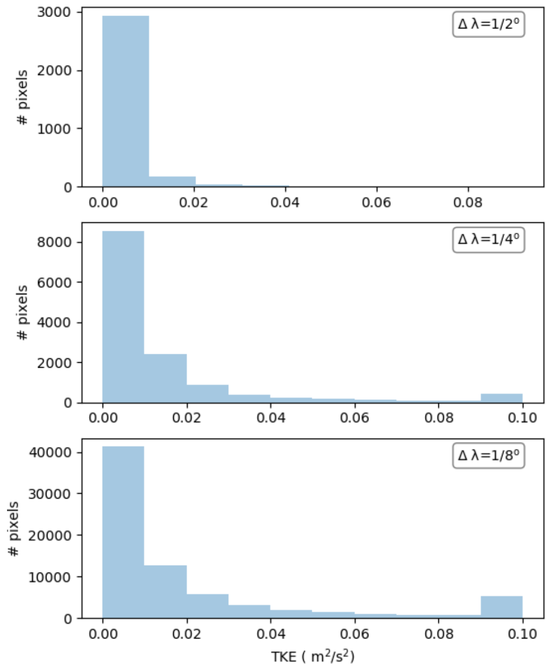

Figure 3: Histograms that represent the number of pixels with the corresponding values of the surface total kinetic energy, TKE, for $\Delta\lambda = 1/2^\circ, 1/4^\circ, 1/8^\circ$. Values of TKE> 0.1 m^2 s^{-2} have been saturated to TKE= 0.1 m^2 s^{-2} to highlight the differences between the experiments. The data was restricted to the most energetic region for the same reason.

First, we want to see if the ocean becomes more energetic as we keep increasing the resolution. As mentioned before, a finer resolution affects the balance of the terms in eq. (9). We have calculated the surface total kinetic energy, TKE:

$$\text{TKE} = \frac{1}{2}(u^2 + v^2) \quad (10)$$

where u and v are the zonal and meridional flows respectively at each cellgrid.

The histograms of TKE for the experiments $\Delta\lambda = 1/2°, 1/4°, 1/8°$ shown in fig. 3 illustrate that, as the resolution of the experiment increases, the surface total kinetic energy becomes larger and the energetic region expands, having more pixels with TKE> 0.1 m^2 s^{-2}.

Furthermore, the maximum of TKE, shown in table II, illustrates how the ocean becomes more energetic when we perform the eddy-permitting simulations, actually, MKE in $\Delta\lambda = 1/8°$ simulation is two orders of magnitude larger than in $\Delta\lambda = 1°$. This result reveals the importance of increasing the resolution so that we can use smaller A_h and start considering the non-linear terms in HPEs which let us obtain a more realistic description of the ocean.

	2°	1°	1/2°	1/4°	1/8°
Δx_{max} (km)	220	110	55	22.5	11.25
MKE(m^{2}s^{-2})	0.027	0.032	0.092	0.614	3.449

Table II: Data that represents the results for the different resolution experiments. MKE is the domain-mean surface maximum kinetic energy and Δx_{max} is the maximun lateral grid spacing (southern part of the domain).

Fig.4 shows the TKE in the studied domain for $\Delta\lambda = 1°$ and $\Delta\lambda = 1/8°$. We can observe how the western boundary current, WBC, detaches from the coast and penetrates the ocean domain between the latitudes, $\Delta\varphi$ = 40-50°. Moreover, the jet strongly intensifies and some mesoscale eddies start to form. For $\Delta\lambda = 1°$, non-linear effects are completely suppressed and the WBC, seems to be weak and vanish. In contrast, for $\Delta\lambda = 1/8°$, the WBC is stronger, starts to expand zonally and penetrates further. This last result shows a more energetic ocean that starts to match the observations.

Additionally, in the eddy-resolving experiment ($\Delta\lambda = 1/8°$) the free surface height begins to exhibit patterns associated with mesoscale eddies. This is due to the fact that the Rossby radius of deformation [7] is λ_m = 30-40km in our domain, which means that to solve the mesoscale eddies properly we need $\Delta x_{max} \approx$ 10km. Table II shows that to obtain this lateral grid spacing, resolutions of $\Delta\lambda = 1/8°$ are needed.

So far, the importance of the choice made during discretization has been highlighted. The results change drastically as we increase the resolution and get to an eddy-resolving one. The mesoscale physics have a clear impact on the large-scale, and it could be interesting to see if the submesoscale physics affect the ocean dynamics, as discussed in [6].

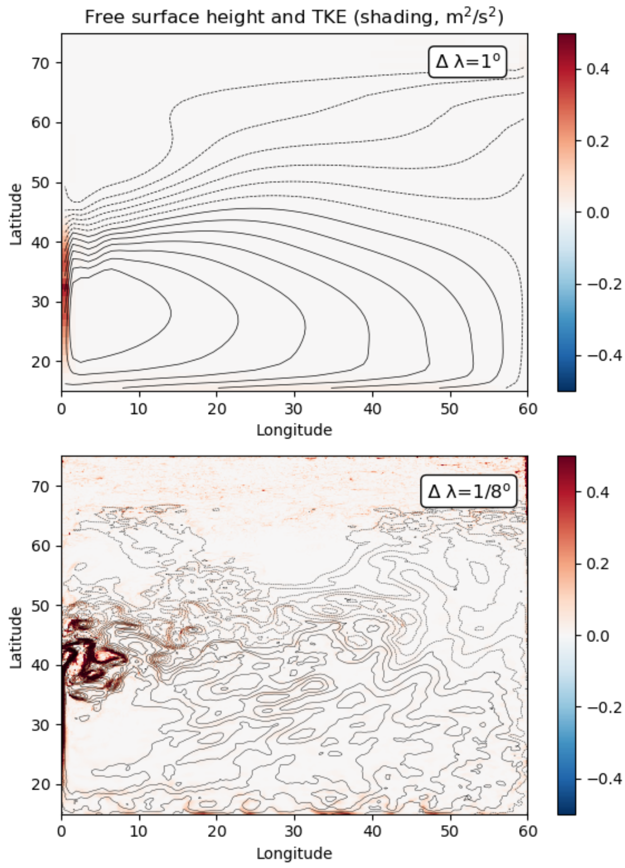

Figure 4: Shading of the surface total kinetic energy, TKE, and contouring of the free surface height for the coarse resolution, $\Delta\lambda = 1°$, and the eddy permitting resolution, $\Delta\lambda = 1/8°$.

With the objective of comparing how the different results of our experiments show the heat transportation of the Gulf Stream, we have plotted in fig. 5 the sea surface temperature (SST) for the coarse resolution ($\Delta\lambda = 1°$) and for the eddy-resolving one ($\Delta\lambda = 1/8°$).

The non-eddying ocean ($\Delta\lambda = 1°$), shows a stratified image of the sea surface temperature where there is little horizontal mixing. This emphasises the fact that coarse-resolution solutions look like stationary solutions without interactions between scales, which is inconsistent with observations.

On the other hand, the eddying ocean ($\Delta\lambda = 1/8°$) seems much more energetic and interacting between scales as medium-size eddies interact with the basic large-scale circulation. Although this last result starts to resemble the observations, it is far from showing an accurate description of the ocean dynamics, what suggests there must be smaller scale features that affect the large scale dynamics of our ocean.

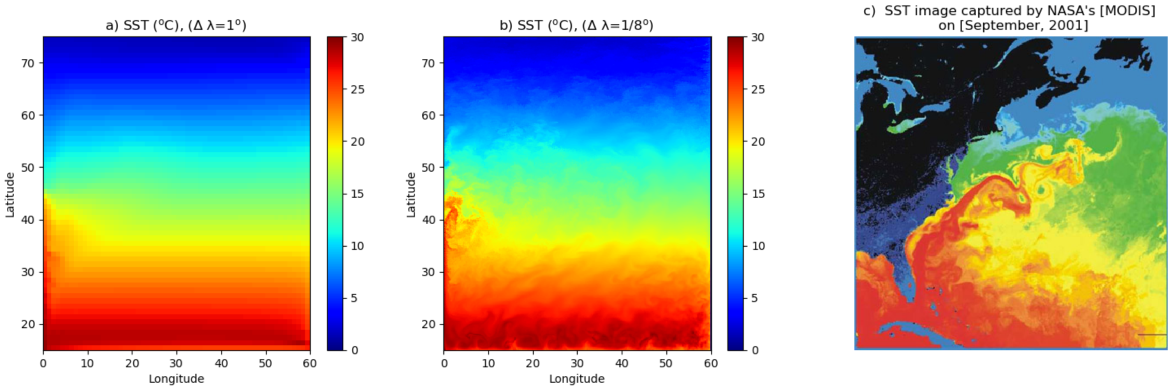

Figure 5: Sea surface temperature for the coarse resolution experiment (on the left), the eddy-permitting resolution experiment (in the center) and for an image captured by NASA's satellite (on the right).

IV. CONCLUSIONS

In summary, we have simulated a baroclinic, wind and buoyancy-forced, double-gyre North Atlantic Ocean circulation. We have seen how the outcomes change drastically as the horizontal resolution goes from coarse to eddy-resolving. This suggests that further increasing the resolution, till submesoscale physics appear, would lead to more realistic results. Unfortunately, increasing the resolution comes at a cost, both in terms of electricity and computational time. For our experiment conducted on a personal computer, simulating just three months of an eddy-resolving ocean required approximately 72 hours of running time. As a solution to this technological issue, I believe we should start studying smaller regions to understand what processes we may be missing out and determine if increasing the resolution indeed leads to more realistic solutions. Focusing on smaller regions leads to more detailed analysis of localized dynamics, which may provide valuable insights into the underlying processes that influence the large scale ocean circulation.

Having a comprehensive understanding of the ocean is crucial for comprehending the impacts of climate changes on coastal communities, marine resources, and ecosystems. The question researches must ask is if the numerical set-up is able to solve the key dynamical processes at work. Without a model that simulates correctly the ocean dynamics, we cannot make a good prediction of the future evolution of climate.

Overall, we have observed that by increasing the resolution, we can reduce the magnitude of eddy viscosity and begin to consider the nonlinear terms of (HPEs). By incorporating mesoscale effects, we start to simulate an ocean that is more energetic and starts to resemble the real one. This highlights the importance of accounting for smaller scale processes and considering the nonlinearity of ocean dynamics to improve the accuracy of our models and gain a better understanding of the complex interactions within the ocean.

Acknowledgments

I want to thank my advisor, Dr. Joaquim Ballabrera, for introducing me to the world of ocean modeling and patiently teaching me how fascinating it can be and to Prof. Pilar Queralt for the expertise and guidance. I also want to thank my friends for their support during the most critical and stressful moments. Lastly, I am grateful to my family, even though they may not fully understand what my Bachelor's Thesis is about, for always being there to support me.

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