Correlation functions of 2D Black Holes in JT gravity

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Abstract: This work shows the possibility that the information loss paradox might be solved when the higher order of the steepest descent expansion in the gravity path integral is taken into account. The decaying behaviour of the correlation functions, which are the main signal of this information paradox, is replaced by a linear growing at large Lorentzian times, which is a signal of an information recovery. This has been shown in the 2D Jackiw-Teitelboim gravity.

I. INTRODUCTION

In 1974 Stephen Hawking found that Black Holes behave as a dissipative system so they evaporate in thermal black body radiation.
This radiation only depends on the BH's mass (considering uncharged and not rotating BH), and it is independent of its initial configuration. If one considers the complete evaporation of the BH through Hawking radiation, it leads to the Information Paradox. This final state, when the entire BH is evaporated, would only have information about the total initial mass. Several states have the same mass, so there are several compatible configurations which could evolve into the final scenario we are considering, which means that the complete information about the BH before the evaporation would be irretrievable. This clearly violates the quantum principle asserting that information persists from the initial state to the final. One way to see this is to compute correlation functions of test fields on a BH spacetime. The main characteristic of the Information Paradox is an exponential decay with time of those correlation functions.
We want to show a possible procedure which solves the Information Paradox in 2D gravity, suggesting it could be solved similarly in 4D gravity.
To achieve this, we will use the path integral formalism to compute the correlation function at next to leading order in the saddle point approximation and analyse the behaviour obtained.
What we expect to find is an exponential decay at leading order and at next to leading order a manifestation of the emergence of the information.
All this work has been done in the Jackiw-Teitelboim gravity, a two dimensional dilaton gravity.
JT gravity is interesting because it captures some essential aspects of gravitational dynamics, within a simplified setting. The analysis of gravitational properties in 2D systems can be helpful in studying 4D features.

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II. PATH INTEGRAL FORMALISM

The path integral formalism is a mathematical formulation based on functional methods which can be a powerful tool to describe quantum systems and their properties.
Instead of describing the systems' evolution by wave functions, the path integral provides a different approach, integrating all possible paths that a system can take.
The object we will like to consider is the correlation function of the fields ϕ,

$$\langle \phi(x')\phi(x)\rangle = N \int \mathcal{D}\phi \ \phi(x') \ \phi(x) \ e^{\frac{i}{\hbar}S[\phi]}. \tag{1}$$

The normalization constant, N, is computed as

$$N = \left(\int \mathcal{D}\phi \ e^{iS[\phi]} \right)^{-1}.$$

This constant's purpose is to cancel the vacuum contribution (which we know is an ill-defined object) to the correlator.
To evaluate this integral we will first represent the field values by discrete Fourier modes [9]

$$\phi(x) = \sum_n e^{-ik_n \cdot x} \phi(k_n).$$

And then, we will divide the continuous time interval into short time slices and decompose the paths into classical paths (which satisfies the boundary conditions, $\phi_{cl}(x) = \phi_i$ and $\phi_{cl}(x') = \phi_f$) and a fluctuating part (which vanishes at the boundaries), $\phi(x) = \phi_{cl}(x) + \xi(x)$.
Doing this decomposition, the action can be expressed as a sum of a classical and a fluctuating action up to second order in the field.

$$S = S_{cl} - \frac{1}{2}\int_x^{x'} dx \ \xi \left(m\frac{d^2}{dx^2} + V''(\phi_{cl}) \right) \xi.$$

The first contribution is easy to evaluate. To compute the contributions of this fluctuations we will expand them in Fourier series, $\xi(x) = \sum_{n=1}^{\infty} a_n \xi_n(x)$.
Where $\xi_n(x)$, the eigenfunctions, must obey

$$\left(\frac{d^2}{ds^2} + V''(\phi_{cl}) \right) \xi_n(x) = \lambda_n \xi_n(x), \tag{2}$$

and λ_n are the eigenvalues.
With this decomposition, the correlation function takes the form [10]

$$\langle\phi(x')\phi(x)\rangle = N\ e^{iS_{cl}} \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-x')} \int \left(\prod_n^\infty da_n\right) e^{-\frac{-im}{2}\sum_n^\infty \lambda_n a_n^2}.$$

When we substitute the corresponding action and determine the eigenvalues of ξ_n this integral will be Gaussian and easy to compute.
In curved spacetimes, the expression for the correlator is slightly different. Aside from the contribution of the field itself, we also have to consider the contribution from the metric field,

$$\langle\phi(x')\phi(x)\rangle = N \int \mathcal{D}\phi\ \mathcal{D}g_{\mu\nu}\ \phi(x')\ \phi(x)\ e^{iS[\phi,g]}, \quad (3)$$

here S denotes the scalar-gravity system's action and N is the normalization constant which is determined by

$$N = \left(\int \mathcal{D}\phi\ \mathcal{D}g_{\mu\nu}\ e^{iS[\phi,g]}\right)^{-1}.$$

For our study we will consider real scalar fields, specifically massless fields.

III. SADDLE POINT APPROXIMATION

Starting from the two point function expression (3), the saddle point approximation can be used in the gravitational part of the action.
Suppose we want to evaluate an integral like

$$I(\alpha) = \int_{-\infty}^{\infty} dx\ g(x)\ e^{i\alpha f(x)},$$

in the limit of large α. Assume that x_0 is the dominating stationary point of f(x), expanding this function around x_0 and evaluating the test function g(x) at x_0, we obtain

$$I(\alpha) \approx g(x_0)\ e^{i\alpha f(x_0)} \int_{-\infty}^{\infty} dx\ e^{\frac{i}{2}\alpha f''(x_0)(x-x_0)^2},$$

we have neglected higher orders of the Taylor series. This integral is Gaussian and

$$I(\alpha) \approx \sqrt{\frac{2\pi}{|f''(x_0)|}} g(x_0)\ e^{i\alpha f(x_0) + i\frac{\pi}{4}\ \mathrm{sgn}(f''(x_0))}.$$

Returning to the two point function, the saddle point of the gravitational action, when we decoupled ϕ (we consider ϕ as a test field), is the one that makes $\delta S_{grav} = 0$. This defines the semiclassical approximation [2]. So the correlation function can be expressed as

$$\langle\phi(x')\phi(x)\rangle \simeq \bar{N} \int \mathcal{D}\phi\ \mathcal{D}g_{\mu\nu}\ \phi(x')\ \phi(x)\ e^{i\bar{S}[\phi,\bar{g}]}, \quad (4)$$

where $\bar{N}$ is the new normalization constant (computed like the other one), $\bar{S}$ is the action evaluated in the saddle point metric, $\bar{g}_{\mu\nu}$.
The correlation function in the surroundings of a BH with spherical symmetry can be computed considering the metric $ds^2 = -f(r)\ dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2$, the point r_0, which satisfies $f(r_0) = 0$, indicates the horizon of the BH.
Expanding this metric around the horizon, the Rindler metric is obtained $ds^2 = -\left(\frac{2\pi}{\beta}\right)^2 x^2 dt^2 + dx^2 + dl_2^2$ with $x^2 = \frac{4}{f'(r_0)}(r - r_0)$, $\beta \equiv \frac{4\pi}{f'(r_0)}$ and $dl_2^2 = r_0^2 d\Omega_2^2$.
Before continuing, let's do some definitions:
The Rindler space is a Minkowski space adapted to an accelerated observer.
The constant β is known as the inverse of the Bekenstein-Hawking temperature which is the temperature associated to the Hawking radiation.
Following [3], the correlation function is computed by rescaling the field $\hat{\phi} = x^2\phi$, expanding in Fourier-Bessel modes as $\hat{\phi}(x,\tau) = \sum_{n,w}\psi_w(x)e^{-i2\pi n\tau}$ with

$$\psi_w(x) = \frac{\sqrt{2w\sinh(\pi w)}}{(2\pi)^{3/2}\pi} e^{p_i l^i} x\ K_{iw}(px),$$

where K is a Bessel function of second kind. Around the horizon we can do a Wick rotation such that $iS \to -S_E$ [11] with $S_E = \sum_{n,w} = S_E(n,w)$, and

$$S_E = 4\pi^2(n^2 + w^2)\ c_{n,w}^2.$$

Once we reached this point, the correlation function is simply a product of Gaussian integrals. Evaluating at coincident spatial points and large Lorentzian times ($t' - t \gg \beta$):

$$\langle\phi(t',x)\phi(t,x)\rangle \xrightarrow{t'-t\to\infty} e^{-\frac{2\pi}{\beta}(t'-t)}. \quad (5)$$

This exponential decay is the essence of the information paradox.

IV. A 2D APPROACH: JT GRAVITY

When considering the leading order in the saddle point approximation, the BH correlator decays exponentially. Now, we want to suggest that the information encoded in the correlation function starts to manifest when the dominant term in the expansion of the gravitational path integral, obtained through the steepest descent method, becomes comparable to the next term in the expansion.

In order to support this, we will consider the 2D Jackiw-Teitelboim gravity. The reason to work with a 2D system is that this allows us to understand many properties and behaviour of gravity, but with a simplified point of view, which with a 4D theory can result in a challenging and complex work.
The Jackiw-Teitelboim gravity consists of a dilaton field coupled to gravity in one spatial and one time dimension. The JT gravity has the following action:

$$S_{JT} = \frac{1}{16\pi G}\left[\int d^2x\ \Phi\sqrt{-g}\ (R+2) + 2\int_{bdy} \Phi_b\ K\right], \tag{6}$$

where Φ is the dilaton field, and the second integral is the Gibbons–Hawking term multiplied by Φ_b (the value of the dilaton at the boundary). The dilaton's role is that of a Lagrange multiplier which fixes the metric to be locally AdS_2.
The BH metric we are considering, which is also a solution of $\delta S = 0$, is $ds^2 = -4(r^2 - \frac{\pi^2}{\beta^2})\ d\bar{t}^2 + \frac{dr^2}{r^2-\frac{\pi^2}{\beta^2}}$ and the connection with the dilaton is made through $\Phi^2 = 1 + r$. The points satisfying $r = \pm\frac{\pi^2}{\beta^2}$ describe the horizons of two copies of BH. By writing this metric in a conformal form (with which we are going to work) and rotating to Euclidean time, $t_E = i\frac{2\pi}{\beta}t$, it is obtained

$$ds_E^2 = \frac{4\pi^2}{\beta^2}\frac{dt_E^2 + dz^2}{\sinh^2\frac{2\pi}{\beta}z}. \tag{7}$$

We define, in Poincaré coordinates, the boundary as a closed curve, $(f(\tau), \zeta(\tau))$ with τ as the boundary time. The proper length of the boundary curve is determined by $\frac{1}{\varepsilon^2} = \frac{f'(\tau)^2+\zeta'(\tau)^2}{\zeta(\tau)^2}$. [5]
We can approximate the JT action into a Schwarzian one at leading order. Using the fact that we are in AdS_2 space (imposed by the Φ equations of motion), the first term in (6) vanishes and the action is reduced to the boundary term. [5]

$$S_{JT} \sim \frac{1}{8\pi G}\int \frac{dt}{\epsilon}\frac{\Phi_r(t)}{\epsilon}K,$$

here the boundary value for the dilaton is $\Phi_b = \frac{\Phi_r}{\epsilon}$ and Φ_r the renormalized value of the dilaton, finite when $\epsilon \to 0$. The extrinsic curvature, K, is related to the Schwarzian derivative as $K = 1 + \epsilon^2 \mathrm{Sch}(f,\tau)$.
Doing the Wick rotation once more and substituting the value of the extrinsic curvature, we get

$$S_{JT} \to \frac{i}{2g^2}\int d\tau\ \mathrm{Sch}(f,\tau), \tag{8}$$

we have defined a new constant $g^2 = \frac{4\pi G}{\phi_r} \ll 1$ and

$$\mathrm{Sch}(f,\tau) = -\frac{1}{2}\frac{f''^2}{f'^2} + \left(\frac{f''}{f'}\right)'.$$

Varying the action, $\frac{\delta S_{JT}}{\delta f}$, one finds a saddle point (a periodic solution), $f = \tan(\frac{\tau}{2})$. With this solution the bulk metric [6] can be rewritten as

$$ds^2 = \frac{f'(u)f'(v)}{(f(u)-f(v))^2}dudv, \tag{9}$$

where u, v are the bulk Euclidean coordinates defined as $u = i\frac{2\pi}{\beta}(t+z)$ and $v = i\frac{2\pi}{\beta}(t-z)$. Using the periodic solution of f, we recover (7), the BH conformal metric.
The two point function we'd like to solve is

$$\langle\phi(t',z)\phi(t,z)\rangle = N\int \mathcal{D}\phi\,\mathcal{D}f\ \phi(t',z)\ \phi(t,z)\ e^{iS_{JT}}e^{iS_\phi[\phi,f]},$$

where the $S_\phi[\phi, f]$ refers to the massless scalar field action.
The procedure will be to start by doing the path integral of Φ, which will fix the metric $g_{\mu\nu}$ to be AdS_2, and note that AdS_2 has different time reparametrizations that are inequivalent due to the existence of a boundary. Then, the integrals in the fields ϕ correspond to the AdS_2 correlators [6]. Finally, we will have to integrate over all reparametrizations defined by f.
The result of all of this is

$$\langle\phi(t',z)\phi(t,z)\rangle = -N\int_v^u d\tau_1\int_{v'}^{u'} d\tau_2\int \mathcal{D}f\frac{f'(\tau_1)f'(\tau_2)}{(f(\tau_1))-f(\tau_2))^2}\ e^{-\frac{1}{2g^2}\int du\ \mathrm{Sch}(f,u)}. \tag{10}$$

To evaluate the Schwarzian integral we have to define two new variables, $\psi(u)$ a bosonic variable and $\eta(u)$ a (Majorana) fermionic one, the latter is defined $\eta = \frac{d\psi}{\psi'}$, following the discussion of [4] about the measure of the disk, which we won't discuss here, we find the final expression we will use to compute the correlation function

$$\langle\phi(t',z)\phi(t,z)\rangle = -N\int_v^u d\tau_1\int_{v'}^{u'} d\tau_2 \int \frac{\mathcal{D}\psi\mathcal{D}\eta}{\mathrm{SL}(2,\mathcal{R})}\frac{f'(\tau_1)f'(\tau_2)}{(f(\tau_1))-f(\tau_2))^2}\; e^{-\frac{1}{2g^2}\int du\left(\frac{\psi''^2}{\psi'^2}-\psi'^2+g^2\frac{\eta''\eta'}{\psi'^2}-g^2\eta'\eta\right)}. \tag{11}$$

To calculate the bulk correlator, we will use the steepest descent expansion up to g^2 order.

V. CORRELATION FUNCTION WITH THE STEEPEST DESCENT APPROXIMATION

As we already mentioned, we will compute the correlation function, proposing the saddle $\psi(\tau)=\tau$ and then we will define the expansion $\psi(\tau)=\tau+g\gamma(\tau)$.
What we will find is an exponentially decay at leading order in g and then at next to leading order, a linear growth.[4]
At zeroth order in the expansion of (11), we take $\psi(\tau)=\tau$, which is, in fact, the saddle point.

At this order, the correlator is

$$\langle\phi(t',z)\phi(t,z)\rangle = -\int_v^u d\tau_1\int_{v'}^{u'} d\tau_2\; \frac{e^{-\frac{\pi}{\beta g^2}}}{2-2\cos(\tau_1-\tau_2)}, \tag{12}$$

integrating and then applying the large Lorentzian times limit, we obtain the exponential decay typical of the information paradox

$$\langle\phi(t',z)\phi(t,z)\rangle \xrightarrow{\frac{2\pi}{\beta}\Delta t\to\infty} \left(\cosh\frac{4\pi z}{\beta}-1\right)e^{-\frac{2\pi}{\beta}\Delta t}, \tag{13}$$

where we have defined $\Delta t\equiv(t'-t)>0$.
Doing the same procedure and expanding up to g^2 and defining $\mathrm{G}(\Delta t,z)\equiv\mathrm{Re}(\langle\phi(t',z)\phi(t,z)\rangle)$ we find

$$\begin{aligned}\mathrm{G}(\Delta t,z) = \langle\phi(t',z)\phi(t,z)\rangle|_{g=0} - \frac{g^2}{8}\int_v^u d\tau_1\int_{v'}^{u'} d\tau_2 \csc\left(\frac{\tau_1-\tau_2}{2}\right)^4\cdot\Bigg(\left(\langle\gamma(\tau_1)^2\rangle+\langle\gamma(\tau_2)^2\rangle\right)\left(1-\frac{1}{2}\cos(\tau_1-\tau_2)\right)-\\ -\sin(\tau_1-\tau_2)\left(\langle\gamma(\tau_1)\gamma'(\tau_1)\rangle+\langle\gamma(\tau_2)\gamma'(\tau_2)\rangle+\langle\gamma(\tau_1)\gamma'(\tau_2)\rangle+\langle\gamma(\tau_2)\gamma'(\tau_1)\rangle\right)\\ +\langle\gamma'(\tau_1)\gamma'(\tau_2)\rangle\left(1-\cos(\tau_1-\tau_2)\right)-\langle\gamma(\tau_1)\gamma(\tau_2)\rangle\left(\cos(\tau_1-\tau_2)+2\right)+\mathcal{O}(\gamma^4)\Bigg).\end{aligned} \tag{14}$$

The first correlation function $\langle\gamma(\tau_1)\gamma(\tau_2)\rangle$ is computed by expanding the Schwarzian action up to second order as mentioned before $\psi(\tau)=\tau+g\gamma(\tau)$.
Specifically, if

$$I\equiv-\frac{1}{2g^2}\int du\;\mathrm{Sch}(f,u),$$

then

$$I=-\frac{\pi}{\beta g^2}-\frac{1}{2g^2}\int_0^{\frac{2\pi}{\beta}} du\; g^2(\gamma''^2-\gamma'^2+\eta''\eta'-\eta'\eta). \tag{15}$$

As already mentioned in previous sections, to calculate the correlators one has to expand in Fourier modes of the field. In particular, $\gamma=\sum_n e^{-in\tau}k_n$ ([5],[7]) and then the integral is Gaussian

$$\langle\gamma(\tau_1)\gamma(\tau_2)\rangle=\frac{1}{2\pi}\sum_{n\neq0,\pm1}\frac{e^{-\frac{2\pi}{\beta}in\tau}}{n^4-n^2}=\frac{1}{2\pi}\Bigg[-\frac{(\Delta\tau-\pi)^2}{2} +(\Delta\tau-\pi)\sin(\Delta\tau)+1+\frac{\pi^2}{6}+\frac{5}{2}\cos(\Delta\tau)\Bigg]. \tag{16}$$

The last three terms of the expression are proportional to the SL(2) zero modes fixed as

$$\int_0^{\frac{2\pi}{\beta}} du\;\gamma(u)=\int_0^{\frac{2\pi}{\beta}} du\; e^{\pm iu}\gamma(u)=0. \tag{17}$$

The other two correlators can be computed similarly.
The exact expression of $\mathrm{G}(\Delta t,z)$ is not included because of its length, but in the large Lorentzian times limit the exponential decay behaviour is the one computed in (13) and the linear growing one is

$$|\mathrm{G}(\Delta t,z)|\sim 32\pi^2g^2\frac{z^2\;\Delta t}{\beta^3}.$$

In Fig.1, we have plotted the behaviour of $\mathrm{G}(\Delta t,z)$ for the same value of $\frac{z}{\beta}$, but different values of g, g=0.010 and g=0.005.
As we can see, there are two behaviours, the exponential decay and the linear growth as mentioned before, that implies the emergence of the information of the BH.

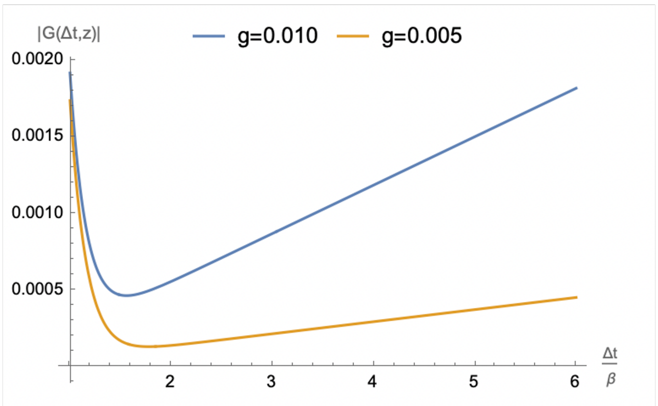

FIG. 1: Behaviour of the correlation function at large Lorentzian times. The blue line is for g=0.010 and the orange line is for g=0.005, both for $\frac{z}{\beta}$=0.1. We can see the two behaviours mentioned, the exponential decay and then the linear growth.

VI. CONCLUSIONS

First of all, we have developed the mathematical framework to calculate the analytical expressions of general correlation functions using the Path Integral formalism.

In this report, we have considered a massless scalar field and uncharged and not rotating Black Holes. We checked that the correlation function in the vicinity of the BH horizon (considering a 4D gravity), using the saddle point approximation, leads to an exponential decay showing the loss of information.

In a 2D JT gravity, we proposed that when the dominant term in the expansion of the gravitational path integral, obtained via the steepest descent method, becomes comparable to the subsequent term in the expansion, the correlator starts to exhibit a linear growing behaviour, which means, that the information emerges.

We, indeed, find that the leading order exhibits an exponential decay. However, the next-to-leading order displays a linear growth that eventually surpasses the decay, despite being initially suppressed by a small coupling g.

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[9] Here we use the notation $x = (t, \mathbf{x})$.
[10] From now on, we will use $\hbar = 1$.
[11] $it \to \tau$