

Covid-like epidemic spreading on networks

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Abstract: The COVID-19 pandemic has had significant impacts on health, economies, and societies. Mathematical models in networks play a crucial role in understanding and mitigating the spread of infectious diseases. This study develops a customized SIR model to simulate the spread of COVID-19, incorporating non-pharmaceutical interventions (NPIs) to limit transmission. Self-protection and mobility restrictions are complementary, allowing hygiene measures to serve as an alternative to strict mobility limitations. Gillespie algorithm is used to simulate infection and recovery events. To combat the late response in the first wave of infection in Spain, the study explores different lockdown strategies, including Intense and Multi-phase approaches. Intense lockdowns effectively reduce cases, but may be challenging to sustain due to population fatigue with prolonged restrictions. On the other hand, Multi-phase lockdowns have limited impact on final case numbers but aid in the recovery of the health system between waves, minimizing the impact on the economy, fatalities and people's mental health.

I. INTRODUCTION

The outbreak of the COVID-19 pandemic has had far-reaching consequences, impacting global health, economies and societies on a massive scale. To effectively respond to such pandemics, it is essential to comprehend their dynamics and develop strategies to mitigate their spread and minimize their impact. Mathematical models are invaluable tools for analyzing and predicting the behavior of infectious diseases, providing crucial insights for public health interventions. This study aims to develop and implement a mathematical model specifically tailored to simulate the spread of a pandemic, with a focus on the COVID-19 outbreak. The chosen model is the SIR (Susceptible-Infectious-Recovered) model, a widely-used framework renowned for its ability to capture the dynamics of infectious diseases. In this model, *non-pharmaceutical interventions* (NPIs) are incorporated as essential measures to limit the spread of the virus.

The spread of a pandemic in complex networks is mathematically defined by *nodes* (N), representing individuals, and *edges* or *links* (E), which symbolize the physical connections between them. The *degree* (k) of a node corresponds to the number of connections it possesses, reflecting the level of social interactions an individual engages in. This study focuses on undirected networks, where interactions between individuals have no inherent directionality. Social networks exhibit heterogeneity, meaning that there are nodes with a large degree while others have only a few links. This heterogeneity in connectivity patterns reflects the real-world scenario, where some individuals have more extensive social networks and interactions than others. It has been considered an email network kind network [7] which consist of 33,696 nodes, 180,811

edges and a degree probability distribution $P(k) \sim k^{-\delta}$ where $\delta \approx 1.77$.

II. MODEL

A. Raw model

This project is based on the SIR model. In this model, the population is divided into three compartments: *susceptible* (S), *infected* (I), and *recovered* (R). The dynamics of the pandemic are governed by two key parameters: the *infection rate* (β), which represents the probability per unit of time of a susceptible individual getting infected when coming into contact with an infected individual, and the *recovery rate* (μ), which represents the probability per unit of time of an infected individual recovering and becoming immune or death. Both immunity and fatalities have the same effect in terms of transitioning individuals out of the infected state. Despite the presence of two parameters, the transition dynamics only depend on the ratio between them[1]. To simplify the analysis, the recovery rate is standardized to 1, and the *transmissivity rate* is defined as ($\lambda \equiv \beta/\mu$).

The model is composed of three distinct regions shown in Fig. (1). Region (I), referred to as the *exponential growth phase*, is characterized by the unhindered spread of the virus in the absence of recovered nodes as "barriers". Region (II) corresponds to the *inertia phase*, during which the diminishing number of susceptible nodes reaches a threshold that disrupts the sustained propagation, causing the daily case count to decline, reaching the peak of the pandemic. Lastly, Region (III) denotes the *recovery phase*, wherein the count of recovered nodes increases, subsequently resulting in a reduction in the number of infected individuals. Eventually, all the infected nodes recover, marking the conclusion of the simulation.

The progression of the pandemic follows a stochastic

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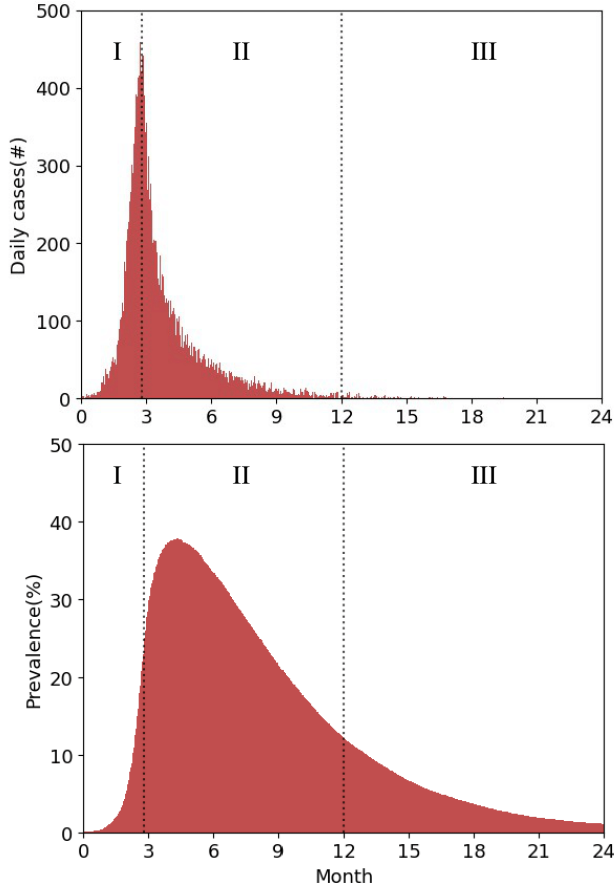


FIG. 1: Simulation of the SIR model's evolution on the network considered in the study, where vertical dashed lines separates the three distinct regions. The prevalence and daily cases are shown on the y-axis, while months are displayed on the x-axis. ($\lambda = 0.8$, $\tau = 140$)

process on a *Gillespie algorithm*, where each event is determined by the probability of infection (Eq. (1)) and recovery (Eq. (2)). The number 1 in each equation represents the normalization of the *recovery rate* (μ).

$$P_I(t) = \frac{E_A(t) \cdot \lambda}{E_A(t) \cdot \lambda + N_I(t) \cdot 1} \quad (1)$$

$$P_R(t) = \frac{N_I(t) \cdot 1}{E_A(t) \cdot \lambda + N_I(t) \cdot 1} \quad (2)$$

Probabilities are recalculated at each time step using two variables: the number of *active links*, E_A , which represents edges connecting infected nodes to susceptible ones, and the number of *infected nodes*, N_I . The event that occurs next is determined by a uniform random number between 0 and 1, selecting one of the two probabilities. To provide an estimation of real-time progression, the time intervals between events are modeled using a Poisson distribution, Eq. (3). To convert the simulation time

to days, it is necessary to multiply it by the definition of a new parameter named *day length* (τ), Eq. (4).

$$t_i = t_{i-1} + \delta t, \quad \delta t \in \{T \sim \text{Poisson}(\gamma)\} \quad (3)$$

$$\gamma = E_A \cdot \lambda + N_I \cdot 1$$

$$t_{days} = \tau \cdot t_{sim} \quad (4)$$

B. Adding restrictions

Focusing on NPIs, measures such as mandatory masks, hygiene practices, discipline, and mobility restrictions have a significant impact on reducing virus transmission[2][3]. In the model, *mobility restrictions* and *personal precautions* work together. Mobility restrictions are represented by the parameter (ϕ_m), reducing active links, while self-protection measures are represented by (ϕ_v), regulating transmissivity[6]. Both effects are combined into a single parameter, *restriction coefficient* ($\phi \equiv \phi_m \cdot \phi_v$), reflecting the overall level of restriction. Due to this proportional relationship, proper hygiene and discipline can achieve the same level of restriction without affecting mobility significantly[3].

$$\phi \begin{cases} < 1 & \text{restrictions} \\ = 1 & \text{no restrictions} \\ > 1 & \text{increased transmissivity} \end{cases}$$

The equations to determine the following event are now modified to introduce restrictions (Eqs. (5) and (6)).

$$P_I(t) = \frac{E_A(t) \cdot \phi \cdot \lambda}{E_A(t) \cdot \phi \cdot \lambda + N_I(t) \cdot 1} \quad (5)$$

$$P_R(t) = \frac{N_I(t) \cdot 1}{E_A(t) \cdot \phi \cdot \lambda + N_I(t) \cdot 1} \quad (6)$$

Probabilities in each step are calculated over the actual evolution of the simulation. However, in reality, it is challenging to detect all infections. Therefore, measures are taken by estimating the number of *infected nodes*, $N_I^{est}(t)$. This calculation is represented by Eq. (7). The number of *simulation daily cases*, $n_{cases}^s(t)$, is reduced by a correction factor referred to as *testing capacity* (ϵ), accounting for the system's capacity to detect new cases, and resulting in *reported daily cases*, $n_{cases}^r(t)$. The number of infected nodes at a given time is underestimated due to factors such as the capacity to detect positives through PCR tests and the difficulty of tracking all infected nodes over time [2]. To estimate the number of infected nodes, it is necessary to sum the reported daily cases from previous days. The probability of cases from earlier days remaining active decreases going further back in time, following a Poisson distribution.

$$N_I^{est}(t) = \sum_{i=1}^t n_{cases}^r(t_i) \cdot e^{(t-i) \cdot \frac{1}{\tau}} \quad (7)$$

$$n_{cases}^r(t) = \epsilon \cdot n_{cases}^s(t)$$

TABLE I: Description of all the parameters that characterize the model. Both (ϕ_v) and (ϕ) can occasionally be greater than 1, according to the model, but in practice it has not been tested.

Name	Symbol	Range	Unit
Transmissivity rate	λ	\mathbb{R}^+	-
Day length	τ	$\mathbb{R}^+ > 1$	T
Reported cases capacity	ϵ	$[0,1]$	-
Mobility restriction	ϕ_m	$[0,1]$	-
Self-protection effectivity	ϕ_v	$[0,1]$ (\mathbb{R}^+)	-
Restriction coefficient	ϕ	$[0,1]$	-

III. SIMULATIONS

A. Reproducing the peak of the first wave

The first reported case in Spain occurred on *January 31st* [2]. The lockdown measures were implemented on *March 15th* [2], although some communities and individuals had already taken precautionary measures starting from the beginning of March, such as staying at home to prevent infection. When the lockdown started, there was at least a 50% reduction in mobility [2], which corresponds to a value of the mobility restriction coefficient of $(\phi_m = 0.5)$. By practicing self-protection measures, such as wearing masks with an effectiveness of 50%, it is possible to significantly reduce mortality and, therefore, transmission [6]. Although some individuals may not have worn masks correctly during the early stages of the pandemic, the effectiveness of masks is still considered to be the same value $(\phi_v = 0.5)$, due to hygiene and individual responsibility. Therefore, the overall restriction coefficient is $(\phi = 0.25)$. At that time, the medical system's capacity to test infected cases was around 10% [2], thus the testing capacity is set to $(\epsilon = 0.1)$. This value is an average estimation corresponding to the fluctuation of testing capacity at the beginning of the pandemic due to periodic shortages in PCR supplies [2]. Finally, the values for the transmissivity rate and day length are determined to adjust the evolution of the real data to the simulation. In the first wave of the pandemic in Spain, the peak was measured around *March 27th* with roughly, 8500 cases. Therefore, considering $(\lambda = 1.35)$, $(\tau = 135)$ and starting with 3 randomly chosen simulation infected nodes with 256 active links, it reproduces the peak of the first wave.

The total number of population at *January 1st*, N_{real} , is estimated to be 47,450,795 [4], while the number of nodes in the considered network, N_{sim} . To transform the number of reported daily cases, n_{cases}^r , to *escalated reported daily cases*, $n_{cases}^{r'}$, it is necessary to keep the

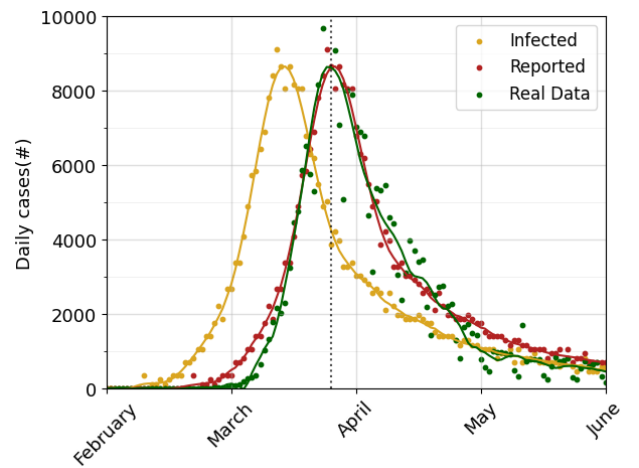


FIG. 2: Daily cases over time. The *infected* curve represents the simulation conducted on the considered parameters when nodes are infected. The *reported* curve takes into account the infected curve but is shifted forward by 14 days [2] to reflect the time it takes for infected cases to be detected. Both curves are escalated according to Eq. (8). The *real data* curve represents the data reported by the Spanish government [5]. Dots represent the actual data points, while the continuous line corresponds to an interpolation, and the dotted vertical line indicates a peak around *March 27th*.

same proportion of population/network nodes.

$$n_{cases}^{r'}(t) = \frac{N_{real}}{N_{sim}} \cdot \epsilon \cdot n_{cases}^s(t) \quad (8)$$

The model does not consider the duration of exposure required for an individual to become infectious. To address this limitation, Fig. (2) illustrates the process in which the model monitored the actual progression of the pandemic, with reported cases occurring approximately two weeks later. Although this is a simplified approximation, it aligns with the real scenario [2] and provides insights into the progression of the pandemic.

When the lockdown started on *March 15th*, the trajectory of daily cases was already showing a decrease (Region (II)). This suggests that even though measures were implemented when the number of tests being conducted was increasing, the actual rate of infections was already declining. As a result, the effectiveness of the measures at that point may have been limited. According to the real data available, the total number of reported infections diagnosed prior to *July 1st* was 257,696 [5], which represents 0.6% of the susceptible population considered. The reported curve shown in Fig. (2) predicts 295,378(0.68%) infections, hence the simulation provides a closer percentage match. The following two simulations are done in the infected curve, advanced 14 days afterwards and escalating according to Eq. (8), becoming into the reported curve.

B. Intense Lockdown Strategy

Lockdown suggests a way to reduce mobility among the population. If only one lockdown is applied and restrictions are maintained until the end of the pandemic, it is named an *Intense Lockdown strategy*. In Fig. (3), a simulation is shown where this type of strategy is applied. With this strategy, the number of infections drops to 197,974(0.46%) before *July 1st*. When measures are implemented, it takes approximately two weeks for their effects to manifest. This allows to potentially shift the peak of the pandemic and reduce its height significantly. Although reported daily cases do not drop to zero, they are effectively controlled. It is evident that restrictions

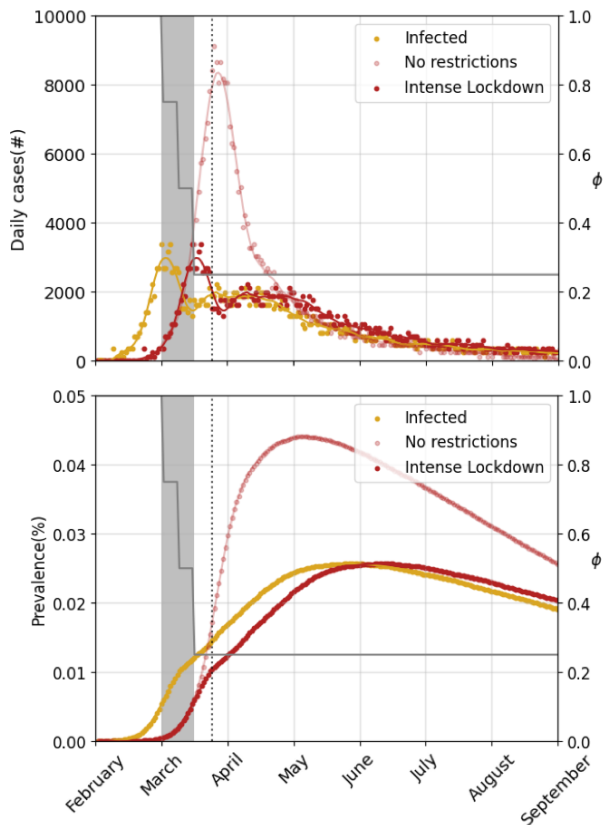


FIG. 3: Simulation of Intense Lockdown vs No restrictions vs Infected: Daily Cases and Prevalence. Dots represent simulation data, while the continuous line correspond to an interpolation. Gray area indicates measure implementation (*March 1st - March 14th*). Escalation consists of 3 gradual relaxation steps (from $\phi = 1$ to $\phi = 0.25$). Dotted vertical line represents full lockdown effects (*March 23rd*). The gray continuous line represents the value of (ϕ) at each time.

have a significant impact on the number of infections, but maintaining restrictions for an extended period would have a profound impact on the economy and people's mental health.

C. Multi-phase Lockdown Strategy

In contrast to the Intense Lockdown strategy, the *Multi-phase Lockdown strategy* splits the pandemic process into multiple peaks, alternating between periods of infection and periods of restrictions. Although the overall duration of the pandemic is longer, this approach allows the healthcare system to better prepare for each subsequent wave during the restriction periods. Fig. (4) provides an example of this process. Although the number of infections without restrictions is only slightly higher than 273,519 (0.63%) infections with that strategy prior to *July 1st*, the healthcare system remains stable, enabling better treatment for patients and ultimately reducing the number of fatalities. At the end of the pandemic, there would be more infections but fewer fatalities.

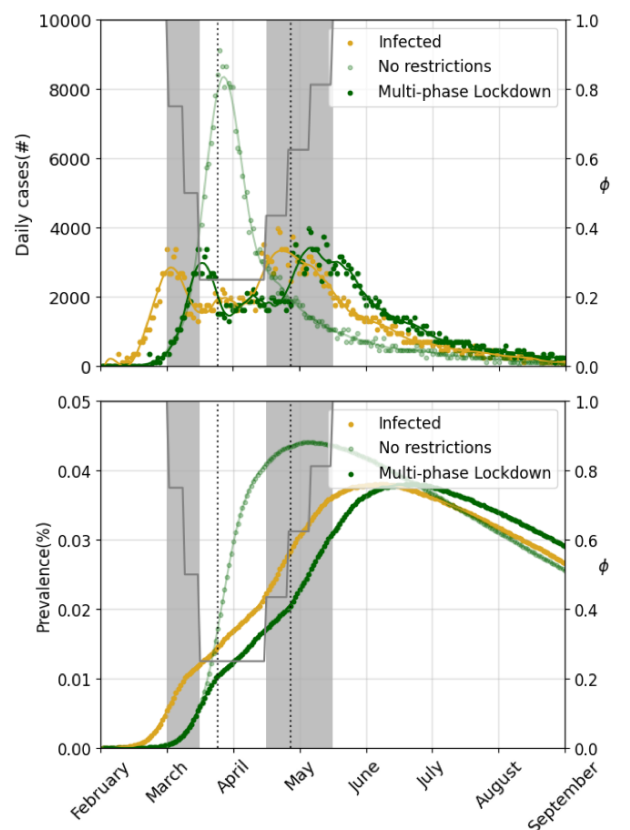


FIG. 4: Simulation of Multi-phase Lockdown vs No restrictions vs Infected: Daily Cases and Prevalence. Dots represent simulation data, while the continuous line correspond to an interpolation. Gray areas indicate measure implementation (*March 1st - March 14th*) and de-escalation (*April 15th - May 14th*). Escalation consists of 3 gradual relaxation steps (from $\phi = 1$ to $\phi = 0.25$), while de-escalation follows 4 phases (from $\phi = 0.25$ to $\phi = 1$) according to government guidelines. Dotted vertical lines represent full lockdown effects (*March 23rd*) and first de-escalation effects (*April 27th*). The gray continuous line represents the value of (ϕ) at each time.

During the first wave of the pandemic, there was a shortage of PCR tests [2], resulting in a loss of information regarding the true evolution of the virus. However, with the Multi-phase Lockdown strategy, there is enough time for the government to procure and supply an adequate number of PCR tests. This ensures that the testing capacity aligns with the needs of the population, allowing for a more accurate assessment of the pandemic's progression.

IV. CONCLUSIONS

- Self-protection measures, such as practicing good hygiene, being responsible, and properly using masks, reduce the transmissivity of the virus. Similarly, mobility restrictions decrease the number of active links. These effects complement each other, meaning that maintaining good hygiene practices and using masks effectively can compensate for the need to restrict mobility.
- During the first wave of COVID-19 in Spain, restrictions were implemented after the peak of daily cases had already occurred. As a result, the restrictions were introduced relatively late, making it challenging to effectively control and reduce the number of daily cases.
- Two strategies for pandemic restrictions are considered: a Intense lockdown and Multi-Phase lockdowns. An Intense lockdown not only advances the

peak of the pandemic and lowers its height, but infected cases at the end of the pandemic are significantly reduced. However, it can have significant economic and mental health impacts. The other strategy is Multi-phase lockdowns, splitting the peak into multiple waves, it provides intervals for healthcare system preparation and offering some relief for individuals. Infections may be higher, but the system has time to respond. The choice depends on healthcare capacity, pandemic severity, economic impact, and population well-being. It's a balance between minimizing healthcare burden and mitigating economic and mental health effects.

- Finally, we wanted to incorporate the reproduction number, R_t , into the study to determine the timing of implementing measures based on this variable. However, it was not possible as the expected results differed significantly from the calculations, at least by an order of magnitude. I would have also liked to introduce an exposition compartment to the model to make a better approximation on the delay.

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