Equation of state of the running vacuum in quantum field theory

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Abstract: Within the framework of Quantum Field Theory in Curved Spacetime we obtain a Running Vacuum Model (RVM) by considering a scalar field nonminnimally coupled to gravity. With the aid of this result, we derive the equation of state of the quantum vacuum in the early universe, where it is close to $w_{\rm vac} = -1$, and in the FLRW regime, where remarkably it evolves adopting radiation, dust and quintessence behaviours. Finally, we discuss the RVM inflation mechanism.

I. INTRODUCTION

The cosmological constant term introduced by Einstein in [1] to allow a static universe has been a key ingredient of cosmology ever since. The concordance ΛCDM cosmology considers it to be a constant that accounts for the energy of vacuum. Nonetheless, in the recent Running Vacuum Model, RVM (see for instance [10]) it is taken as a dynamic quantity. This generalization allows to alleviate some cosmological tensions [12]. However, it was not until very recent [7] that it was justified on theoretical Quantum Field Theory in Curved Spacetime grounds. In that theory one quantizes the matter fields but treats the gravitational field as a classical field, yielding a semiclassical theory of gravity that was proved successful to explain phenomena like particle creation in cosmological and black hole spacetimes (see for instance [2]).

In this work we aim to explain the origin of the RVM in the context of QFT in curved spacetime as well as some implications of the RVM to the equation of state (EoS) of the vacuum. In section II we introduce the QFT in curved spacetime framework for a scalar field nonminimally coupled to gravity, defining the Ultraviolet (UV) divergent Vacuum Energy Density (VED). In section III we mention how to renormalize this quantity following the approach in [8]. Finally, in section IV, using the previous results and following [11] we derive the EoS of the vacuum in the FLRW regime and in an early inflationary epoch, discussing also the RVM inflation.

II. QFT CALCULATION OF THE VED

A. Classical equations for a scalar field nonminimally coupled to gravity

We consider a single matter field ϕ nonminimally coupled to gravity. Hence, the action is:

$$S = S_{EH} + S_m = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}(R-2\Lambda) + \hat{\mathcal{L}}_m\right] \tag{II.1}$$

Where S_{EH} is the Einstein-Hilbert action with a Λ term and $S_m = \int d^4x\sqrt{-g}\hat{\mathcal{L}}_m$ is the action of the matter field. R is the Ricci Scalar, g is the determinant of the metric tensor $g_{\mu\nu}$ and $\hat{\mathcal{L}}_m$ is the scalar lagrangian density of ϕ:

$$\hat{\mathcal{L}}_m = -\frac{1}{2}\left(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + (m^2+\xi R)\phi^2\right) \tag{II.2}$$

Which is nothing but the lagrangian of the neutral scalar field in Minokowski spacetime (with metric $\eta_{\mu\nu}$) generalized by the usual prescriptions ($\eta^{\mu\nu} \to g^{\mu\nu}$ and $\partial_\mu \to \nabla_\mu$) with an additional coupling term ($\xi R\phi^2$).

Imposing that variations respect to the inverse metric $g^{\mu\nu}$ of the action in (II.1) vanish, one gets Einstein's equations (see [6]):

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(-\rho_\Lambda g_{\mu\nu} + T^\phi_{\mu\nu}) \tag{II.3}$$

Where $R_{\mu\nu}$ is the Ricci tensor and $T^\phi_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$ is the energy-momentum tensor (EMT) of ϕ, given by:

$$T^\phi_{\mu\nu} = (1-2\xi)\partial_\mu\phi\partial_\nu\phi + \left(2\xi - \frac{1}{2}\right)g_{\mu\nu}\partial^\alpha\phi\partial_\alpha\phi - 2\xi\nabla_\mu\nabla_\nu\phi + 2\xi g_{\mu\nu}\phi\Box\phi + \xi G_{\mu\nu}\phi^2 - \frac{1}{2}m^2 g_{\mu\nu}\phi^2 \tag{II.4}$$

Here $\Box \equiv g^{\mu\nu}\nabla_\mu\nabla_\nu$. In (II.3) we have also identified the "typical" VED $\rho_\Lambda \equiv \frac{\Lambda}{8\pi G}$. It is important to highlight (see sections II C and III B) that in the current work ρ_Λ is a bare parameter and not the physical VED.

Using Euler-Lagrange equations one gets the classical equation of motion for ϕ (see [4]):

$$(\Box - m^2 - \xi R)\phi = 0 \tag{II.5}$$

The metric under consideration is the spatially flat FLRW, i.e: $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$, where $a(t)$ is the scale factor. Using conformal coordinates (see [8]) the metric can be written as $ds^2 = a^2(\tau)\eta_{\mu\nu}dx^\mu dx^\nu$, where $\eta_{\mu\nu} = \text{diag}\,(-1,1,1,1)$ is the Minkowski metric. In these coordinates (II.5) reads as:

$$\phi'' + 2\mathcal{H}\phi' - \nabla^2\phi + a^2(m^2+\xi R)\phi = 0 \tag{II.6}$$

Where the prime means derivative respect to the conformal time τ and $\mathcal{H} \equiv \frac{a'}{a}$. Seeking for mode solutions of the form $\phi_{\vec{k}}(\tau,\vec{x}) = e^{i\vec{k}\cdot\vec{x}}\phi_k(\tau)$ and introducing a new variable $\psi_k \equiv \phi_k a$ one arrives to:

$$\psi_k'' + \left[w_k^2(m) + a^2\left(\xi - \frac{1}{6}\right)R\right]\psi_k = 0 \tag{II.7}$$

By definition $w_k^2(m) \equiv k^2 + m^2a^2$. Note that the mode solutions are no longer plane waves (as in a Minkowskian Klein-Gordon). The general solution of (II.5) is a linear combination of modes (which we now write as h_k):

$$\psi(\tau, \vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[A_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} h_k(\tau) + A^*_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}} h^*_k(\tau) \right] \tag{II.8}$$

Where $A_{\vec{k}}$ are (at this stage) linear coefficients.

B. WKB expansion of mode functions

In general, the modes solution to (II.7) must be found approximately through a recursive process. The method we introduce can be found in [4]. Here we follow the notation in [8], starting by the ansatz:

$$h_k(\tau) = \frac{1}{\sqrt{2W_k(\tau)}} \exp\left(-i\int^\tau W_k(\tilde{\tau})d\tilde{\tau}\right) \tag{II.9}$$

Introducing (II.9) in (II.7) and defining $\Omega_k^2(\tau) \equiv \omega_k^2(m) + a^2(\xi - 1/6)R$, we obtain a non-linear differential equation for W_k:

$$W_k^2(\tau) = \Omega_k^2(\tau) - \frac{1}{2}\frac{W_k''}{W_k} + \frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2 \tag{II.10}$$

It can be solved using a WKB expansion, writing $W_k = \sum_j \omega_k^{(j)}$, where $\omega_k^{(j)}$ indicates a contribution of adiabatic order j. Throughout this manuscript an adiabatic contribution of order n will be denoted by $O(n)$. As explained in [7], k^2 and a are $O(0)$; a' and $\mathcal{H} \equiv \frac{a'}{a}$ $O(1)$; a'', a'^2 and R of $O(2)$ and so on. In general a time derivative increases the adiabatic order by one.

To illustrate the iteration we calculate explicitly the terms up to $\omega_k^{(2)}$. As in [8] we employ an off-shell prescription for w_k, that is $w_k \equiv w_k(M) = \sqrt{k^2 + a^2M^2}$. As in [9] $\Delta^2 \equiv m^2 - M^2$ is $O(2)$. To $O(0)$ using (II.10) $(\omega_k^{(0)})^2 = k^2 + a^2m^2 = k^2 + a^2M^2 + O(2) = w_k^2$. $\omega_k^{(1)}$ vanishes because the RHS of (II.10) has no $O(1)$ terms (R, W_k'' and $(W_k')^2$ are $O(2)$).
To $O(2)$ expanding the LHS we get to $w_k^2 + 2w_k w_k^{(2)}$ and the RHS $k^2 + a^2m^2 + a^2\left(\xi - \frac{1}{6}\right)R - \frac{1}{2}\frac{w_k''}{w_k} + \frac{3}{4}\frac{(w_k')^2}{w_k^2}$. Solving for $w_k^{(2)}$ we obtain $w_k^{(2)} = \frac{a^2\Delta^2}{2w_k} - \frac{1}{4}\frac{w_k''}{w_k^2} + \frac{3}{8}\frac{(w_k')^2}{w_k^3} + \frac{a^2}{2w_k}\left(\xi - \frac{1}{6}\right)R$.

The full expressions for $\omega_k^{(j)}$ up to $O(6)$ can be found in [8]. We shall remark that only even adiabatic terms are non-zero due to the general covariance of the action that only allows terms with an even number of derivatives.

C. Quantization and definition of the VED

The theory is quantized by upgrading $A_{\vec{k}}$ and $A^*_{\vec{k}}$ to annihilation and creation operators respectively, that obey the canonical commutation relations $\left[A_{\vec{k}}, A_{\vec{k}'}\right] = 0$ and $\left[A_{\vec{k}}, A^\dagger_{\vec{k}'}\right] = \delta(\vec{k} - \vec{k}')$.

The EMT of vacuum $T_{\mu\nu}^{\text{vac}}$ by definition has contributions from the Vacuum Expectation Value (VEV) of the EMT of ϕ, given by $\left\langle T_{\mu\nu}^\phi \right\rangle \equiv \left\langle 0|T_{\mu\nu}^\phi|0\right\rangle$ and from the cosmological term in (II.3):

$$T_{\mu\nu}^{vac} \equiv -\rho_\Lambda g_{\mu\nu} + \left\langle T_{\mu\nu}^\phi \right\rangle \tag{II.11}$$

Where $|0\rangle$ is the vacuum state, which in curved spacetime is not trivial to define. In this work we employ the adiabatic vacuum defined by $A_{\vec{k}}|0\rangle = 0 \ \forall \vec{k}$ (see [4] and [5]). We consider the vacuum as a perfect fluid, i.e: $T_{\mu\nu}^{\text{vac}} = P_{\text{vac}} g_{\mu\nu} + (\rho_{\text{vac}} + P_{\text{vac}})U_\mu U_\nu$, where ρ_{vac} and P_{vac} are the Vacuum Energy Density (VED) and Vacuum Energy Pressure, respectively. If the vacuum is itself a comoving observer, we have $U^\mu = (1/a, \vec{0})$, so we obtain:

$$\rho_{\text{vac}} = \frac{T_{00}^{\text{vac}}}{a^2} = \rho_\Lambda + \frac{\left\langle T_{00}^\phi \right\rangle}{a^2} \tag{II.12}$$

Below we see that these quantities are ill-defined, so renormalization is required.

D. Calculation of ZPE

To calculate the Zero Point Energy (ZPE) of the field, given by $\left\langle T_{00}^\phi \right\rangle$, we insert the mode expansion (II.8) in (II.4) and compute the VEV. The result is (see [7]):

$$\begin{aligned}\left\langle T_{00}^\phi \right\rangle =& \frac{1}{4\pi^2a^2}\int dk k^2 \left[|h_k'|^2 + \left(w_k^2 + a^2\Delta^2\right)|h_k|^2 \right. \\ &\left. + \left(-6\mathcal{H}^2|h_k|^2 + 6\mathcal{H}(h_k' h_k^* + h_k^{*'} h_k)\right)\right]\end{aligned} \tag{II.13}$$

Obviously we cannot perform the exact calculation and the adiabatic expansion in (II.9) is required again. The full integrand in (II.13) can be recast in terms of the $w_k^{(j)}$, e.g: $|h_k|^2 = \frac{1}{2W_k} = \frac{1}{2\left(w_k + w_k^{(2)} + w_k^{(4)}\right)} + O(6) = \frac{1}{2w_k}\left(1 - \frac{w_k^{(2)}}{w_k} - \frac{w_k^{(4)}}{w_k} + \frac{\left(w_k^{(2)}\right)^2}{w_k^2}\right) + O(6)$. The remaining terms in (II.13) can be modified in a similar fashion (see (4.7) and (4.11) in [8]).

III. RENORMALIZATION SCHEME: ADIABATIC SUBSTRACTION

A. Renormalized ZPE

In the UV limit ($k \gg aM$), clearly $w_k \sim k$, so $k^2 w_k^{-n} \sim k^{2-n}$. Therefore, terms containing powers of $1/w_k$ up to order 3 are UV-divergent. It turns out that all UV-divergent terms are on the pieces of order $O(2)$ and $O(4)$

in the adiabatic expansion. This allows us to follow the Adiabatic Regularization (AR) that was introduced in [3]. Here we employ an off-shell version used in [8]. The key of this procedure is to subtract all the $O(2)$ and $O(4)$ contributions term by term inside the integral, computed off-shell to an on-shell calculation. Namely:

$$\left\langle T^{\phi}_{\mu\nu}\right\rangle^{(6)}_{\rm ren}(M) \equiv \left\langle T^{\phi}_{\mu\nu}\right\rangle^{(6)}(m) - \left\langle T^{\phi}_{\mu\nu}\right\rangle^{(4)}(M) \quad \text{(III.1)}$$

Where the superscript (j) refers to the adiabatic order the computation is performed. By this prescription we readily see the necessity of all the $O(6)$ calculations in [8], limiting to $O(4)$ would have produced a vanishing result when using the mass of the field as the renormalization point. The off-shell result of (III.1) is presented in equation (5.13) in [8].

B. Renormalized VED

The physical VED is obtained from (II.12) using the ZPE and ρ_Λ renormalized at the scale M. Then $\rho_{\rm vac}(M)$ is the sought physical VED. It should be understood that $\rho_{\rm vac}(M)$ is a dynamic quantity as it depends not only on M but also on $\mathcal{H}, \mathcal{H}'$ and so. However, (II.12) is not sufficient to compute the VED, since we do not know the value of $\rho_\Lambda(M)$, one shall compute the difference of the VED between two scales, i.e: $\rho_{\rm vac}(M) - \rho_{\rm vac}(M_0)$. A more general equation is the following (see [8]) which compares the VED at two scales and two cosmic times to $O(2)$:

$$\rho_{\rm vac}(M,H) - \rho_{\rm vac}(M_0,H_0) = \frac{3\left(\xi - \frac{1}{6}\right)}{16\pi^2} \left[H^2\left(M^2 - m^2 + m^2 \ln\frac{m^2}{M^2}\right) - H_0^2\left(M_0^2 - m^2 + m^2\ln\frac{m^2}{M_0^2}\right)\right] \quad \text{(III.2)}$$

$H \equiv \frac{\dot a}{a}$ is the Hubble Parameter (the point means derivative respect the cosmic time).

IV. EOS OF THE QUANTUM VACUUM

Considering again the vacuum as a perfect fluid, we have $P_{\rm vac} = T^{\rm vac}_{11}/a^2$. Using the 11-th component of (II.11) we find $P_{\rm vac}(M) = -\rho_\Lambda + \left\langle T^{\phi}_{11}\right\rangle/a^2$. To compute $\left\langle T^{\phi}_{11}\right\rangle$, we need the trace ($T^\phi$) of the EMT. Using spatial isotropy we arrive to:

$$\frac{\left\langle T^{\phi}_{11}\right\rangle_{\rm ren}(M)}{a^2} = \frac{1}{3}\left(\left\langle T^{\phi}\right\rangle_{\rm ren}(M) + \frac{\left\langle T^{\phi}_{00}\right\rangle_{\rm ren}(M)}{a^2}\right) \quad \text{(IV.1)}$$

Using (II.12) we get rid of $\rho_\Lambda(M)$:

$$P_{\rm vac}(M) = -\rho_{\rm vac}(M) + \frac{1}{3}\left(\left\langle T^{\phi}\right\rangle_{\rm ren} + 4\frac{\left\langle T^{\phi}_{00}\right\rangle_{\rm ren}}{a^2}\right) \quad \text{(IV.2)}$$

As usual we define $w_{\rm vac} \equiv \frac{P_{\rm vac}}{\rho_{\rm vac}}$. Clearly, it is no longer -1 as it has corrections to even adiabatic orders:

$$P_{\rm vac}(M) = -\rho_{\rm vac}(M) + f_2(M,\dot H) + f_4(M,H,\dot H,\ddot H,\dddot H) + O(6) \quad \text{(IV.3)}$$

The expressions for f_j can be found in [8].

A. FLRW regime

The most physical choice for the renormalization point is taking $M = H$ at each cosmic time. Substituting in (III.2), we get to (ignoring terms of order $O(4)$ and above):

$$\rho_{\rm vac}(H) \approx \rho^0_{\rm vac} + \frac{3\nu_{\rm eff}(H)}{8\pi}(H^2 - H_0^2)m^2_{\rm Pl} + O(4) \quad \text{(IV.4)}$$

This constitutes the canonical form for a low energy RVM. We have identified $\rho^0_{\rm vac} \equiv \rho_{\rm vac}(H_0)$ (the current measured value for the VED), $m_{Pl} \equiv G_N^{-1/2}$ (Planck mass) and the effective running parameter $\nu_{\rm eff}(H)$:

$$\nu_{\rm eff}(H) \equiv \epsilon\left(-1 + \ln\frac{m^2}{H^2} - \frac{H_0^2}{H^2 - H_0^2}\ln\frac{H^2}{H_0^2}\right) \quad \text{(IV.5)}$$

By definition $\epsilon \equiv \frac{\left(\xi - \frac{1}{6}\right)}{2\pi}\frac{m^2}{m^2_{\rm Pl}}$. In the FLRW regime we should only consider terms of $O(2)$ in (IV.3), hence:

$$\begin{aligned} w^{\rm FLRW}_{\rm vac}(H) &\approx -1 + \frac{f_2(H,\dot H)}{\rho_{\rm vac}(H)} \\ &\approx -1 + \frac{1}{\rho_{\rm vac}(H)}\frac{\left(\xi - \frac{1}{6}\right)}{8\pi^2}\dot H m^2\left(1 - \ln\frac{m^2}{H^2}\right) \end{aligned} \quad \text{(IV.6)}$$

Where we have neglected a term H^2/m^2 in the expression for f_2 as far as $H \ll m$. In the following we derive a compact expression for $w^{\rm FLRW}_{\rm vac}(z)$ (z is the cosmological redshift).

So far we have learned that the VED runs with the cosmic time. If one wants to preserve the Bianchi identity then the gravitational coupling itself runs with H, $G(H) = \frac{G_N}{1 - \epsilon\ln H^2/H_0^2}$ (see [11]), where G_N is its current value. These runnings imply that in our model the Hubble rate is no longer given by the ΛCDM:

$$H^2_{\Lambda CDM}(z) = H_0^2[\Omega^0_m(1+z)^3 + \Omega^0_r(1+z)^4 + \Omega^0_{\rm vac}] \quad \text{(IV.7)}$$

First we derive the $O(\epsilon)$ corrections to H^2 and $\dot H$ respect to the ΛCDM value. Starting from the generalized (where we introduce runnings in G and $\rho_{\rm vac}$) 1st Friedmann equation:

$$\begin{aligned} H^2(z) \approx& \frac{8\pi G(H)}{3}(\rho_r + \rho_m + \rho_{\rm vac}(H)) \\ \approx& \frac{8\pi G_N}{3}\left(1 + \epsilon\ln\frac{H^2}{H_0^2}\right)\left(\rho_r(z) + \rho_m(z) + \rho^0_{\rm vac} + \frac{3\nu_{\rm eff}(H)}{8\pi}m^2_{\rm Pl}(H^2 - H_0^2)\right) \\ \approx& H^2_{\Lambda CDM}\left(1 + \epsilon\ln\frac{H^2}{H_0^2}\right) + \epsilon\left(-1 + \ln\frac{m^2}{H^2}\right)(H^2 - H_0^2) - \epsilon H_0^2\ln\frac{H^2}{H_0^2} \\ \approx& H^2_{\Lambda CDM} + \epsilon(H^2_{\Lambda CDM} - H_0^2)\left(-1 + \ln\frac{m^2}{H_0^2}\right) \end{aligned} \quad \text{(IV.8)}$$

Where we have ignored $O(\epsilon^2)$ terms, used (IV.5) between the second and third line, and that $H^2 = H^2_{\Lambda CDM} + O(\epsilon)$ so whenever H^2 is already multiplied by ϵ we can replace it by $H^2_{\Lambda CDM}$ as far as the calculation remains at $O(\epsilon)$.

Proceeding analogously and using the following Friedmann equation $\dot{H} = -4\pi G(H)\sum_i (1+w_i)\rho_i$ (see [6]) where ρ_i is the energy density of matter, radiation or vacuum, one easily arrives to:

$$\dot{H} = \dot{H}_{\Lambda CDM} + \epsilon \dot{H}_{\Lambda CDM}\left(-1 + \ln\frac{m^2}{H_0^2}\right) + O(\epsilon^2) \quad \text{(IV.9)}$$

Taking the time derivative of (IV.7):

$$\dot{H}_{\Lambda CDM} = \frac{-HH_0^2}{H_{\Lambda CDM}}\frac{3}{2}\left[\Omega_m^0(1+z)^3 + \frac{4}{3}\Omega_r^0(1+z)^4\right] \quad \text{(IV.10)}$$

We have used $\frac{d}{dt}(1+z) = \frac{d}{dt}\frac{1}{a} = -\frac{\dot{a}}{a^2} = -H(1+z)$. Putting together expressions (IV.4)-(IV.10) we obtain:

$$w_{vac}^{\text{FLRW}}(z) = -1 + \frac{\epsilon\frac{-H}{H_{\Lambda CDM}}\left[\Omega_m^0(1+z)^3 + \frac{4}{3}\Omega_r^0(1+z)^4\right]\left(1 - \ln\frac{m^2}{H^2}\right)\left(1 + \epsilon\left(-1+\ln\frac{m^2}{H_0^2}\right)\right)}{\Omega^0_{\text{vac}} + \nu_{\text{eff}}\left(-1 + \frac{H^2_{\Lambda\text{CDM}}}{H_0^2}\left(1+\epsilon\left(-1+\ln\frac{m^2}{H_0^2}\right)\right)\right)} \quad \text{(IV.11)}$$

Which does not include any additional approximation. To arrive to our final expression we consider:

- Our calculation goes to $O(\epsilon)$ so therefore we can ignore the terms $\epsilon(-1+\ln\frac{m^2}{H_0^2})$ in both the numerator and denominator and also neglect the $O(\epsilon)$ term in $H/H_{\Lambda\text{CDM}}$.
- We collect $-\epsilon\left(1 - \ln\frac{m^2}{H^2}\right)$ into ν_{eff} using an approximate form of (IV.5).

Hence, the EoS for the quantum vacuum in the FLRW regime is given by:

$$w_{vac}^{\text{FLRW}}(z) = -1 + \frac{\nu_{\text{eff}}\left(\Omega_m^0(1+z)^3 + \frac{4}{3}\Omega_r^0(1+z)^4\right)}{\Omega^0_{\text{vac}} + \nu_{\text{eff}}\left(-1 + \frac{H^2_{\Lambda\text{CDM}}}{H_0^2}\right)} \quad \text{(IV.12)}$$

In Figure 1 we plot equation (IV.12) for the entire FLRW regime. We see 3 asymptotic behaviours that can be derived analytically from (IV.12) (z_{eq} in the redshift at the matter-radiation equality, which is obtained from equating the mass and radiation energy densities, i.e: $\Omega_r^0(1+z_{\text{eq}})^4 = \Omega_m^0(1+z_{\text{eq}})^3 \rightarrow z_{\text{eq}} = \Omega_m^0/\Omega_r^0 - 1$):

1. $z \gg z_{\text{eq}}$. In this case the radiation density is much larger than the mass density, hence $\Omega_r^0(1+z) \gg \Omega_m^0$, so we only keep the radiation density term in both the numerator and denominator, obtaining $w_{vac}^{\text{FLRW}}(z \gg z_{\text{eq}}) = \frac{1}{3}$. This corresponds to the EoS of radiation.
2. $z_{\text{eq}} \gg z \gg 1$. In this case we only keep the matter density term, obtaining $w_{vac}^{\text{FLRW}}(z_{\text{eq}} \gg z \gg 1) = 0$. This is the defining behaviour of dust.

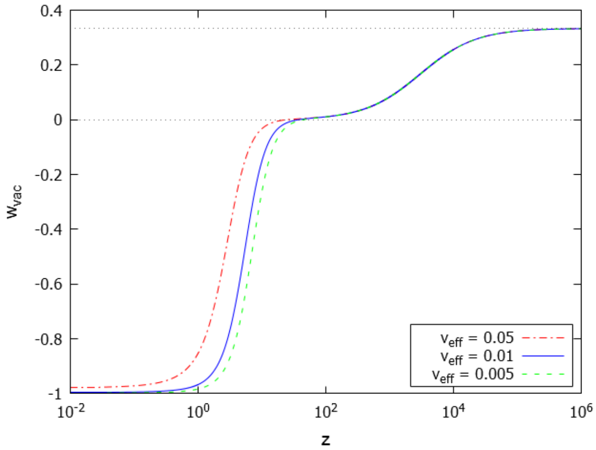

FIG. 1: EoS of the quantum vacuum as a function of the cosmological redshift in the FLRW regime using three different values of ν_{eff} obtained by fitting observational data (see [12]).

3. $-1 < z < O(1)$. $w_{vac}^{\text{FLRW}}(|z| \sim O(1)) = -1 + \nu_{\text{eff}}\frac{\Omega_m^0}{\Omega_v^0}(1+z)^3$, thus behaving as quintessence.

So the QFT in curved spacetime treatment directly leads to an evolving vacuum with no need of quintessence fields or whatsoever.

B. RVM Inflation

As it has been shown before (see [10]), a running vacuum can trigger inflation in the early universe, by a short period where $H = \text{const}$. In this scenario, we have $w_{\text{vac}} \approx -1$ as a traditional vacuum, because all terms in expressions for f_n in (IV.3) are proportional to some time derivative of H which in the RVM inflation are null.

In [10] a RVM inflation was analysed for a phenomenological RVM of the type:

$$\rho_{\text{vac}} = \frac{3}{8\pi G}\left(A + \nu H^2 + \alpha\frac{H^{n+2}}{H_I^n}\right) \quad \text{(IV.13)}$$

Where A, ν, α and H_I are constants and $n \geq 2$ is an even (again for general covariance requirements) number. As one expects a large value for H during inflation, it suffices to keep only the H^{n+2} contribution. Now we can derive such a behaviour from first principles. By our prescription, it turns out that the renormalized VED at the scale of the mass of the field which has now leading $O(6)$ terms appears as:

$$\rho_{\text{vac}}^{\text{inf}} \approx \frac{\left\langle T_{00}^{\phi}\right\rangle_{\text{ren}}^{(6)}(m)}{a^2} \approx \frac{\tilde{\xi}}{80\pi^2 m^2}H^6 \quad \text{(IV.14)}$$

Where we have ignored, as stated, time derivatives and $\tilde{\xi} \equiv \left(\xi - \frac{1}{6}\right) - \frac{2}{63} - 360\left(\xi - \frac{1}{6}\right)^3$. Introducing (IV.14) in Friedmann equations one gets to the following expressions for the VED and the radiation energy density as

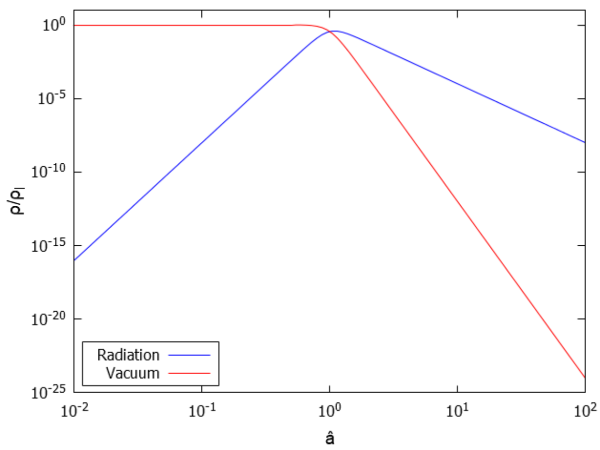

FIG. 2: Radiation and vacuum energy densities in an early inflation as function of the normalized scale factor.

a function of $\hat{a} \equiv \frac{a}{a^*}$ (a^* is at the transition time from vacuum to radiation domination):

$$\rho_{\rm vac}(\hat{a}) = \rho_I(1+\hat{a}^8)^{-\frac{3}{2}} \quad \text{(IV.15)}$$

$$\rho_r(\hat{a}) = \rho_I\hat{a}^8(1+\hat{a}^8)^{-\frac{3}{2}} \quad \text{(IV.16)}$$

In Figure 2 we plot these results. At the beginning there is no radiation and the VED is maximal. Initially the VED remains barely constant, until the transition time where the radiation takes over. Hereafter, the VED decays very fast, whereas from (IV.16) we trivially retrieve the concordance result $\rho_{\rm rad} \sim a^{-4}$ for $\hat{a} \gg 1$.

V. CONCLUSIONS

- We have learned how to perform a calculation of the Zero Point Energy (ZPE) of a neutral scalar field coupled to gravity in the framework of QFT in curved spacetime, treating the gravitational field as a classical field and solving difficulties as the non-exactness of the mode solutions by using a WKB expansion.

- The Vacuum Energy Density (VED), that has contributions from the ZPE and the cosmological constant, is an UV divergent quantity, that can be renormalized using the Adiabatic Regularization.

- After renormalization, the VED gets a dynamic behaviour. In particular, in the low energy regime one retrieves the canonical RVM energy density.

- From the canonical low energy RVM, the EoS of the vacuum in the FLRW regime has been obtained. Remarkably enough, we see it is not -1 as usually stated and it evolves with the cosmic time, behaving as radiation, dust and quintessence. All of this comes naturally from the QFT procedure, with no additional ad hoc fields.

- On the other hand, during the inflationary time, the EoS of vacuum is indeed close to -1. Inflation is another consequence of the RVM, with no need of an inflaton field. RVM inflation is characterized by an initial dominant vacuum energy that decays on radiation. Not only that, but the RVM also provides the graceful exit of inflation to the radiation dominated epoch.

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