

Title:

Estimating slope and level change in N=1 designs

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ABSTRACT

The current study proposes a new procedure for separately estimating slope change and level change between two adjacent phases in single-case designs. The procedure eliminates baseline trend from the whole data series prior to assessing treatment effectiveness. The steps necessary to obtain the estimates are presented in detail, explained, and illustrated. A simulation study is carried out to explore the bias and precision of the estimators and compare them to an analytical procedure matching the data simulation model. The experimental conditions include two data generation models, several degrees of serial dependence, trend, level and/or slope change. The results suggest that the level and slope change estimates provided by the procedure are unbiased for all levels of serial dependence tested and trend is effectively controlled for. The efficiency of the slope change estimator is acceptable, whereas the variance of the level change estimator may be problematic for highly negatively autocorrelated data series.

Key words: single-case designs, level change, slope change, autocorrelation, trend

In recent years it has become widely accepted that psychological studies based on single-case designs need to provide scientific evidence on the interventions applied, which would allow subsequent $N = 1$ studies to have a solid foundation (Jenson, Clark, Kircher, & Kristjansson, 2007; Olive & Smith, 2005; Parker & Brossart, 2006; Parker & Hagan-Burke, 2007; Schlosser & Sigafoos, 2008; Shadish, Rindskopf, & Hedges, 2008). The evidence on the effectiveness of intervention techniques can be accumulated integrating individual studies in a quantitative way, for instance, considering different levels of analysis and incorporating study characteristics as effect moderators (van den Noortgate & Onghena, 2003; 2008).

The availability of useful summary measures is essential for quantifying the results of an individual study and also for conducting meta-analyses (Busk & Serlin, 1992; Cohen, 1990; 1994; Kirk, 1996; Kromrey & Foster-Johnson, 1996; Rosnow & Rosenthal, 1989; Wilkinson & Task Force on Statistical Inference, 1999). A “useful” effect size index needs to meet the following criteria: 1) to represent correctly the true data characteristics; 2) to offer valuable and easily interpretable information; and 3) to be easily applicable by researchers with scarce statistical expertise. Regression-based techniques (e.g., Allison & Gorman, 1993; Center, Skiba, & Casey, 1985-1986; Gorsuch, 1983; White, Rusch, Kazdin, & Hartmann, 1989) have been found to perform unsatisfactorily with respect to criterion 1 (Beretvas & Chung, 2008b; Brossart, Parker, Olson, & Mahadevan, 2006; Manolov & Solanas, 2008; Parker & Brossart, 2003), while also being deficient regarding criterion 2, due

to the large effect size estimates yielded (Campbell, 2004; Scruggs & Mastropieri, 1998). Interpretability is also problematic for ITSE (Gottman, 1981) and ITSACORR (Crosbie, 1993), since the meaningfulness of their results is compromised by conceptual and computational issues (Huitema, 2004; Huitema, McKean, & Laraway, 2007). As regards criterion 3, the procedures based on visual analysis (i.e., Ma, 2006; Parker, Hagan-Burke, & Vannest, 2007; Scruggs, Mastropieri, & Casto, 1987) can be easily applied even by hand calculation. Although the percentages yielded by these techniques are also readily interpretable (criterion 2), trend and autocorrelation have been shown to jeopardize the completion of criterion 1 (Manolov, Solanas, & Leiva, in press).

In the following section a new procedure for assessing treatment effectiveness is proposed, taking into consideration the need for separate quantification of slope change and level change in order to represent more fully the information contained in single-case studies (Beretvas & Chung, 2008a). The procedure is designed to yield meaningful results (criterion 2) expressed in terms of the behavioral measurement units used in each individual study. Subsequently, the interpretation of the estimates produced by the procedure is discussed and illustrated with an example. In order to meet criterion 3 different software solutions are proposed to researchers and practitioners. Monte Carlo simulation is used to evaluate how well the estimates represent known data parameters, that is, to obtain information about the degree of achievement of criterion 1.

A procedure for estimating slope and level change (SLC)

Rationale

The objective of the proposed procedure is to estimate slope and level change (being present either of them or both) eliminating baseline data linear trend whenever it is present. After a potential phase A trend is removed, it can be logically assumed that the level of behavior in that phase presents zero slope. It is conjectured that the procedure may also deal with positive serial dependence, since the presence of large positive autocorrelation in data can be represented by an upward or a downward trend. In contrast, when measurements are negatively autocorrelated, data present greater variability rather than trend. The slope and level change estimates obtained for the detrended data express the shifts in terms of the measurement units. For instance, if the frequency of behavior is measured, a slope change of 3 would mean that at each measurement time during phase B the experimental unit produces an average of three behaviors more than in the previous observation point. A level change of 3 would imply that with the introduction of the intervention (i.e., with the change in phase) the experimental unit produces an average of 3 behaviors more in the treatment phase than in the baseline phase. It has to be remarked that this average change in level is computed after estimating and controlling slope change.

Steps required for computing SLC

In the present paragraph the computation of the SLC estimates is explained verbally, whereas the mathematical expressions and an example can be found in Appendix I. Since SLC is designed to control general trend prior to assessing intervention effectiveness, an initial data correction step involves eliminating phase A trend from data. Trend is estimated only for the baseline phase, since it allows avoiding confusion between trend and potential intervention effects taking place in phase B (Allison & Gorman, 1993). Trend estimation is not carried out by means of ordinary least squares, since unstable estimates are expected on theoretical basis when few data points are available (Weisberg, 1980). Additionally, there is evidence on the inappropriate performance of Gorsuch's (1983) trend analysis which uses this kind of estimation (Manolov & Solanas, 2008; Parker & Brossart, 2003). Instead, the phase A trend is estimated as the mean of the differenced phase A measurements. Afterwards trend is removed from both phase A and phase B, using a method that has been shown to be useful for dealing not only with trend but also with autocorrelation (Manolov & Solanas, 2009).

The second step involves estimating slope change as the trend present in the phase B data, from which baseline trend has already been removed. The average value is assumed to represent an estimation of slope change, considering that the phase A trend has been previously removed and, thus, the baseline is supposed to have a zero slope.

The third step consists in the estimation of level change. Firstly, the already estimated change in slope is removed from the treatment phase data, without removing the intercept. That is, the phase B slope is eliminated from the detrended phase B data, while maintaining potential shifts taking place at the first measurement time of the treatment phase. Level change is estimated subtracting the detrended baseline data mean from the detrended and slope-change-controlled treatment data mean.

The procedure described is not restricted to AB designs and can be applied to any combination of a baseline and treatment phase which is included in more complex design structures (e.g., multiple-baselines designs, ABAB designs).

Software availability

The example in Appendix I shows that SLC can be applied using hand calculations. However, for longer data series this can become tedious. Therefore, the procedure has been implemented in an MS-DOS executable file and in two well-known and widely-used statistical packages: R (version 2.9.2) and SAS (version 9.1).

The *slc.exe* file (available from the authors) asks for series length, baseline phase length, and the name of the *.dat* file containing the data points separated by spaces. The estimates are printed in an output file, whose name needs to be specified by the user. However, this program does not permit graphing data and, thus, the R and SAS/IML codes were developed.

R is a freeware language which has already been used to make automatic the application of other techniques for analyzing single-case data (Bulté & Onghena, 2008; Manolov & Solanas, 2009). For SLC, a package and a plug-in (available upon request and from the <http://cran.r-project.org> website) containing the R function presented in Appendix II were developed. Both the function and the packages compute the slope and level change estimates and represent graphically the original and detrended data (e.g., Figure 1). One of the versions of the package requires using three commands to obtain the results, whereas the other one is based on menus. Appendix II explains the use of the two packages

INSERT FIGURE 1 ABOUT HERE

The SAS/IML code presented in Appendix III also permits obtaining the slope change and level change estimates, by simply inputting the measurements and baseline phase length. The graph of the detrended data which is drawn using the code allows complementing numerical analysis with visual inspection.

The diversity of possibilities mentioned above makes the application of SLC straightforward. In fact, the use any piece of software requires fewer steps than computing, for instance, the Percentage of all nonoverlapping data effect size estimate, as described in Schneider, Goldstein, and Parker (2008).

Method

AB series lengths

Short data series were included in the present study, since those are more feasible in applied settings: a) $N = 10$ with $n_A = n_B = 5$; b) $N = 15$ with $n_A = 5$; $n_B = 10$; c) $N = 15$ with $n_A = 7$; $n_B = 8$; d) $N = 20$ with $n_A = n_B = 10$; e) $N = 30$ with $n_A = n_B = 15$; and f) $N = 40$ with $n_A = n_B = 20$, where N denotes whole series length and n_A and n_B represent the number of measurements in phase A and B, respectively.

Data generation models

For each combination of n_A and n_B data were generated according to the model proposed by Huitema and McKean (2000): $y_t = \beta_0 + \beta_1 \cdot T_t + \beta_2 \cdot LC_t + \beta_3 \cdot SC_t + \varepsilon_t$, where y_t is the value of the dependent variable at moment t , β_0 is the intercept (set to zero), β_1 , β_2 , and β_3 are the coefficients associated with trend, level change, and, slope change respectively, T_t is the value of the time variable at moment t (taking values from 1 to N), LC_t is a dummy variable for level change (equal to 0 for phase A and to 1 for phase B), SC_t is a dummy variable for slope change being equal to 0 for phase A, and taking values from 0 to $(n_B - 1)$ for phase B, and ε_t is the error term. This data generation and variables specification method has been previously used in studies related to

simulating single-case behavioral data (Beretvas & Chung, 2008b; Huitema & McKean, 2007; McKnight, McKean, & Huitema, 2000).

The beta parameters related to trend, level and slope change were set to 1 and 10 to represent a small and a large effect, respectively. The aim of these parameter values was to explore the impact of effect size on the bias and variance of the estimators. It has to be adverted that a permanent level change of one behavior implies a smaller effect than a progressive slope change of one behavior per measurement time.

The error term (ε_t) was generated following two different models which are assumed to represent adequately the greater part of behavioral data (Harrop & Velicer, 1985): a) the first-order autoregressive model $\varepsilon_t = \varphi_1 \cdot \varepsilon_{t-1} + u_t$, with φ_1 ranging from $-.9$ to $.9$ in steps of $.1$; and b) the first-order moving average model $\varepsilon_t = u_t - \theta_1 \cdot u_{t-1}$ (as presented in McCleary and Hay, 1980) with 19 values of θ_1 : $-.9(.1).9$, leading to autocorrelation ranging from $.4972$ to $-.4972$.

For both models the random variable u_t was generated following three different distributions (exponential, normal, and uniform) with mean equal to zero and standard deviation equal to 1, 2, and 3, constituting a total of nine different u_t distributions. These conditions permit studying the effect of skewness, kurtosis, and data variability on the performance of the procedure proposed.

Data analysis

For each experimental condition defined by the combination of ε_t model, u_t distribution, and β_1 , β_2 , and β_3 values the mean and variance of the two estimators was computed on the basis of 100,000 samples. The SLC level and slope change estimates were compared to simultaneous multiple regression (SMR; Huitema & McKean, 2000) coefficient estimates in terms of bias and variance. The bias of the estimators was obtained as the difference between the simulation parameters and the estimates for slope and level change. The variance of the estimators was computed as an indicator of the efficiency and a comparison was performed between SLC and SMR. The SMR procedure can be considered a gold standard, since it matches perfectly the data generation model used.

Simulation

The specific steps that were implemented in the FORTRAN programs (one for each of the six series' length) were the following ones:

- 1) Systematic selection of each of the 19 values of φ_1 or θ_1 autoregressive or moving average models, respectively.
- 2) Systematic selection of the (β_1 , β_2 , and β_3) parameters for data generation, leading to 8 different data patterns – absence of effect or trend; presence of trend; level change; slope change; trend and level change; trend and slope change; combined level and slope change; trend and combined level and slope change.
- 3) 100,000 iterations of steps 4 through 11.

- 4) Generate the u_t term according to an exponential, a normal, or a uniform distribution with different values of the standard deviation parameter.
- 5) Establish $\varepsilon_1 = u_1$.
- 6) Obtain the error term ε_t out of the random variable u_t using the AR(1) model $\varepsilon_t = \varphi_1 \cdot \varepsilon_{t-1} + u_t$ or the MA(1) model $\varepsilon_t = u_t - \theta_1 \cdot u_{t-1}$.
- 7) Obtain the time array $T_t = 1, 2, \dots, N$.
- 8) Obtain the dummy treatment variable array LC_t , where $LC_t = 0$ for phase A and $LC_t = 1$ for phase B.
- 9) Obtain the slope change array according to: $SC_t = [T_t - (n_A + 1)] \cdot LC_t$.
- 10) Obtain the y_t array containing measurements (i.e., dependent variable): $y_t = \beta_0 + \beta_1 \cdot T_t + \beta_2 \cdot LC_t + \beta_3 \cdot SC_t + \varepsilon_t$.
- 11) Apply SLC and SMR.
- 12) Obtain the mean and variance of the slope change and level change estimates the 100,000 replications of each experimental condition.
- 13) Compute the bias of the estimates.

For data generation NAG libraries *nag_rand_neg_exp*, *nag_rand_normal*, and *nag_rand_uniform* were used. In order to guarantee suitable simulated data, the 50 values previous to each simulated data series were eliminated in order to reduce artificial effects (Greenwood & Matyas, 1990) and to avoid dependence between successive data series (Huitema, McKean, & McKnight, 1999).

Results

Detection of treatment effects

Both SLC and SMR proved to be unbiased, that is, null estimates were obtained when the simulation parameter beta was zero. Complementarily, when $\beta_2 = 1$ (or 10) or $\beta_3 = 1$ (or to 10), the estimates were equal to one or ten, respectively. In fact, the estimators' bias and variance for small and large effect differed only at the third decimal level and in the results presented in following sections are applicable to both cases. Thus, SLC detects both level and slope changes, whenever either of them or both are present. Moreover, when the intervention is ineffective, the estimates are equal to zero, indicating that the change in phase is not associated with an alteration in the behavior of interest. The size (0, 1, or 10) of either type of treatment effect does not alter the variance of the estimates of SLC and SMR. This implies that the importance of the variability decreases for greater effects as the relative efficiency of the procedure increases for stronger interventions. For instance, for first-order autoregressive series with normal unitary error, 15 measurements in each phase, level change of 1, and $\phi_I = .3$, the coefficient of variation of the SLC level change estimate is $1.241/1 = 124.1\%$. For the same case and a level change of 10, the coefficient of variation is $1.245/10 = 12.45\%$, according to the results obtained.

Distortion due to trend

According to the results obtained, it can be stated that SLC controls effectively for general trend, since it does not distort the estimates obtained. Whenever the intervention is not effective, the presence of trend in the desired direction does not lead to erroneously inferring treatment effectiveness. Additionally, when treatment is effective, its effect is not overestimated due to the presence of trend. The level and slope change estimates of SMR are also not affected by trend, whose magnitude is estimated precisely, without bias. The presence or absence of trend does not alter the variance of the estimates of SLC and SMR.

Distortion due to autocorrelation

The presence of autocorrelation in data is not associated with biased estimates. In contrast, serial dependence is relevant for the variance of the estimates obtained. The variance of the estimators is greater for higher degrees of negative and to a lesser extent for positive autocorrelation (see Tables 1 and 2 for SLC and Tables 3 and 4 for SMR). Another finding common to SLC and SMR and both data generation processes is the greater variability of the level change estimator in comparison to the slope change estimator.

INSERT TABLES 1, 2, 3, AND 4 ABOUT HERE

Regarding variability, SLC and SMR differ to a greater degree in the case of the level change estimate where the sign of the serial dependence becomes especially relevant. Negative autocorrelation distorts more the SLC estimator

and positive one has greater impact on the SMR estimator for both autoregressive (Figure 2) and moving average (Figure 3) processes. Apart from the greater distortion in relative terms (comparing sequentially related to independent data), SMR also shows greater absolute estimator variability for $\varphi_I \geq .3$ in AR(1) processes and for $\varphi_I \geq 0.275$ for $n_A = n_B = 5$ data generated using a MA(1) process.

INSERT FIGURES 2 AND 3 ABOUT HERE

Influence of series length

Although both estimators are unbiased for all series lengths, it is important to have as much measurements as possible. That is so, due to the fact that the efficiency of the estimators of both SMR and SLC improves for longer series, as Tables 1 to 4 show. The increase of N is associated with especially important reduction of variance for the slope change estimator.

Influence of error model and random variable distribution

The difference between AR(1) and MA(1) data is mainly in the variance of the estimators, which is somewhat greater in the case of the latter. Generating the random variable u_t following an exponential, a normal, or a uniform distribution does not seem to affect the performance of SLC or SMR in terms of bias or variance.

Discussion

The present investigation proposes a new procedure for estimating slope change and level change, in that order, after controlling for linear baseline phase trend. The estimates obtained are expressed in terms of the measurement units used to quantify the dependent variable, rather than in terms of standardized mean difference (d) or association (R^2). The performance of the procedure proposed is assessed in the context of two-phase data series representing a wide set of experimental conditions including autocorrelation, trend, and two different types of treatment effect. Considering the potential usefulness of SLC it was implemented in both free and commercial software requiring few inputs to produce the magnitude of change estimates.

The results suggest that the procedure proposed is practically unbiased both for first-order autoregressive and moving average processes and regardless of the distributional shape of the random variable; its initial data correction step controls effectively for linear trend. In the case of unbiased estimators, variance is an important indicator of their efficiency and the SLC estimators are generally more efficient for positively autocorrelated data and less efficient for $\rho_1 \leq .0$. When comparing both procedures it has to be kept in mind that SMR implies a perfectly specified regression model corresponding exactly to the data parameters. It has to be highlighted that the least favorable conditions for SLC are defined by a) high negative serial dependence; and b)

immediate and permanent changes in the response rate with the introduction of the intervention. Regarding point “a”, evidence suggests that high negative autocorrelation is not frequent in behavioral data (Matyas & Greenwood, 1997; Parker, 2006). On a substantive basis, it is also more logical to expect an individual or a group to show consistent behavior over time (i.e., positive autocorrelation). As far as point “b” is concerned, in psychological studies an abrupt and sustained (level) change in the behavior is less likely to occur than a progressive change representing a gradual improvement of the individual or group treated. Another point to be remarked is that the relevance of the estimators’ variability is subjected to the magnitude of the intervention effect. For large effects, which seem to be typical for $N = 1$ studies (Campbell, 2004; Matyas & Greenwood, 1990; Parker et al., 2005), a variability of two behavioral units, such as the one observed for the level change estimator for most levels of serial dependence, may not be crucial for determining treatment effectiveness. Taking into account these considerations, it can be stated that SLC will perform well for the majority of applied studies, where positive autocorrelation, slope changes, and/or sizeable level changes are likely to be present.

The results of estimators’ greater variance for ϕ_1 values diverging from 0 can be explained considering the fact that the error term (and consequently the data series) variability increases when the degree of (negative and positive) autocorrelation is higher. These relationship can be explained through the following steps for the AR(1) processes. The data generation model for the

error term is $\varepsilon_t = \varphi_1 \varepsilon_{t-1} + u_t$, which implies that $\text{Var}(\varepsilon_t) = \varphi_1^2 \cdot \text{Var}(\varepsilon_{t-1}) + \text{Var}(u_t)$. Since the variability of the series at each measurement time is constant $\text{Var}(\varepsilon_t) = \varphi_1^2 \cdot \text{Var}(\varepsilon_t) + \text{Var}(u_t)$ and thus $\text{Var}(\varepsilon_t) - \varphi_1^2 \cdot \text{Var}(\varepsilon_t) = \text{Var}(u_t)$. Therefore, $\text{Var}(\varepsilon_t) \cdot (1 - \varphi_1^2) = \text{Var}(u_t)$ and $\text{Var}(\varepsilon_t) = \text{Var}(u_t) / (1 - \varphi_1^2)$. Thus, the greater the absolute value of the autocorrelation parameter, the greater the variance of the error term. An additional implication of this expression is that a greater variability of the u_t term would have resulted in more variable series. In both cases this increased data variability entails lower reliability in the estimation of the behavioral change.

Apart from the greater variance observed for $\varphi_1 \neq 0$, the results suggest that the level change estimates vary to a considerably greater degree than the slope change estimates in both SLC and SMR. The fact that this finding is common to both procedures implies that it is not a consequence of the stepwise nature of SLC in which the level change is estimated after controlling for a potential slope change.

Although testing the performance of SMR was not the main aim of the present study, the results obtained suggested that it can be useful for estimating behavioral change. However, our preliminary results on the statistical significance of the regression coefficients suggest that conventional 5% alpha does not permit using p values as a reliable criterion for assessing the existence of level changes as small as one behavior. Additionally, autocorrelation was also shown to be problematic, as negative one makes more difficult the rejection of the null hypothesis and positive one makes SMR too

liberal, doubling the probability of committing a Type I error. Until more evidence is available on statistical decision making with SMR, the assessment of treatment effectiveness can be done using solely the regression coefficients.

In summary, SLC can readily be applied to single-case data presenting trend and/or positive autocorrelation, since it is not affected by these data features providing unbiased estimates. The procedure can be complemented by the visual inspection of the graphed original and detrended data in order to enhance the assessment of treatment effectiveness. In fact, the technique only quantifies the amount of slope and level change in measurement units and, thus, allows the decision on the relevance of the changes to be made according to practitioner's substantive criteria, considering the behavior of interest, the particular client and setting, etc.

A specific drawback of the procedure proposed is that it is conceived for correcting linear trend in data and, therefore, its performance might not be optimal when behavioral data present curvilinear trends. Further research is needed in order to explore whether the data correction present in SLC can attenuate the effect of nonlinear trends. Future efforts may also focus on estimating the sampling distribution of the slope and level change estimators, due to its utility for obtaining statistical significance and confidence intervals. The transformation of the SLC estimates into common effect size metrics is another issue to be tackled in the subsequent investigations.

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Appendix I

Formulae for computing SLC

Equation (1): Differencing

$$\Delta A_t = A_{t+1} - A_t$$

where A_t represents the original phase A measurement at time t and ΔA_t represents the differenced phase A data.

Equation (2): Obtaining the phase A trend estimate β_A as the mean of the differenced phase A measurements

$$\beta_A = \sum_{t=1}^{n_A-1} \Delta A_t / (n_A - 1)$$

where n_A is the number of observations in the baseline phase.

Equation (3): Obtaining the detrended phase A data (A_i)

$$A_t = A_t - \beta_A \cdot t$$

Equation (4): Obtaining the detrended phase B data (B_i)

$$B_t = B_t - \beta_A \cdot t$$

Equation (5): Differencing the detrended phase B data

$$\Delta B_t = B_{t+1} - B_t$$

Equation (6): obtaining the phase B slope estimate SC as the mean of the differenced phase B measurements

$$SC = \sum_{t=n_A+1}^{n_A+n_B-1} \Delta B_t / (n_B - 1)$$

where n_B is the number of observations in the treatment phase.

Equation (7): Eliminating phase B slope from the detrended phase B data, maintaining potential level changes at time $t = n_A + 1$.

$$B_t = (B_t - SC \cdot (t - 1))$$

Equation (8): Estimating level change subtracting the detrended baseline data mean from the detrended and slope-change-controlled treatment data mean

$$LC = \bar{B} - \bar{A} = \sum_{t=n_A+1}^{n_A+n_B} B_t / n_B - \sum_{t=1}^{n_A} A_t / n_A$$

An illustrative example

In order to illustrate the application of SLC, consider the fictitious data set consisting of the following measurements: 1, 2, 3, 4, and 5 for phase A and 7, 9, 11, 13, and 15 for phase B.

Data correction. First, the phase A data are differenced: $\Delta A_1 = A_2 - A_1 = 2 - 1 = 1$, $\Delta A_2 = A_3 - A_2 = 3 - 2 = 1$, $\Delta A_3 = A_4 - A_3 = 4 - 3 = 1$, $\Delta A_4 = A_5 - A_4 = 5 - 4 = 1$. Then, the average of the differenced phase A data is used to represent phase A trend $\beta_A = (1 + 1 + 1 + 1) / (5 - 1) = 1$. The phase A trend is removed from the whole data series, obtaining the detrended baseline data as $A_1 = A_1 - 1 \cdot 1 = 1 - 1 = 0$, $A_2 = A_2 - 1 \cdot 2 = 2 - 2 = 0$, $A_3 = A_3 - 1 \cdot 3 = 3 - 3 = 0$, $A_4 = A_4 - 1 \cdot 4 = 4 - 4 = 0$,

and $A_5 = A_5 - 1 \cdot 5 = 5 - 5 = 0$. The detrended treatment data are computed through

$$B_1 = B_1 - 1 \cdot 6 = 7 - 6 = 1, \quad B_2 = B_2 - 1 \cdot 7 = 9 - 7 = 2, \quad B_3 = B_3 - 1 \cdot 8 = 11 - 8 = 3, \\ B_4 = B_4 - 1 \cdot 9 = 13 - 9 = 4, \text{ and } B_5 = B_5 - 1 \cdot 10 = 15 - 10 = 5.$$

Slope change estimation. First, the detrended phase B data are differenced:

$$\Delta B_1 = B_2 - B_1 = 2 - 1 = 1, \quad \Delta B_2 = B_3 - B_2 = 3 - 2 = 1, \quad \Delta B_3 = B_4 - B_3 = 4 - 3 = 1, \text{ and } \Delta B_4 = B_5 - B_4 = 5 - 4 = 1. \text{ Then, the average of the differenced and previously detrended phase B data is computed and used to represent the slope change estimate: } SC = (1 + 1 + 1 + 1) / (5 - 1) = 1. \text{ Hence, in the treatment phase the experimental unit produces an average of one behavior more in each successive measurement time, considering that baseline trend has been removed.}$$

Level change estimation. Phase B slope is removed from the detrended phase B data, without removing the intercept. The detrended slope-change-controlled treatment data are obtained through: $B_1 = B_1 - \beta_B \cdot (1 - 1) = 1 - 1 \cdot 0 = 1$, $B_2 = B_2 - \beta_B \cdot (2 - 1) = 2 - 1 \cdot 1 = 1$, $B_3 = B_3 - \beta_B \cdot (3 - 1) = 3 - 1 \cdot 2 = 1$, $B_4 = B_4 - \beta_B \cdot (4 - 1) = 4 - 1 \cdot 3 = 1$, and $B_5 = B_5 - \beta_B \cdot (5 - 1) = 5 - 1 \cdot 4 = 1$. The level change estimate is computed subtracting the A-detrended phase A mean from the detrended slope-change-controlled phase B data mean: $LC = \bar{B} - \bar{A} = (1 + 1 + 1 + 1 + 1) / 5 - (0 + 0 + 0 + 0 + 0) / 5 = 1 - 0 = 1$. Therefore, after controlling for potential baseline trend and slope change between the two phases, the

experimental unit produces an average of one behavior more during phase B than during phase A (level change).

The data were constructed to present no random fluctuations, only general trend in data – on each measurement time the experimental unit increases its response rate by one behavior. When the treatment phase starts, there is an immediate and permanent change in level of one behavior: the data point at time 6 is 7 instead of 6, as the mere continuation of the phase A trend would imply. Additionally, during phase B the response rate increases by 2 behaviors at a time and, therefore, there is a slope change of one behavior. In summary, the level change and slope change parameters are known and are both equal to 1 behavior. As it can be seen, both slope change and level change were precisely estimated.

Appendix II

The R code for SLC requires copy-pasting the function it in the R console. Afterwards, data can be input reading a file “filename.det” containing the measurements separated by spaces in `info <- array(scan("filename.dat"))` or writing the values directly in `info <- array(c(value1,value2,value3))`. Then, the baseline phase length is specified in `n_a <- length`. Finally, the expression `results <- slcestimates(info,n_a)` needs to be written in order to obtain the output. The same specifications need to be done after installing and loading the `SLC_0.1.tar.gz` package based on commands.

```
-----  
# R function for estimating slope and level change  
slcestimates <- function(info,n_a) {  
  slength <- length(info)  
  n_b <- slength-n_a  
  phaseA <- info[1:n_a]  
  phaseB <- info[(n_a+1):slength]  
  # Estimate phase A trend  
  phaseAdiff <- c(1:(n_a-1))  
  for (iter in 1:(n_a-1))  
    phaseAdiff[iter] <- phaseA[iter+1] - phaseA[iter]  
  trendA <- mean(phaseAdiff)  
  # Remove phase A trend from the whole data series  
  phaseAdet <- c(1:n_a)  
  for (timeA in 1:n_a)  
    phaseAdet[timeA] <- phaseA[timeA] - trendA * timeA  
  phaseBdet <- c(1:n_b)  
  for (timeB in 1:n_b)  
    phaseBdet[timeB] <- phaseB[timeB] - trendA * (timeB+n_a)  
  # Compute the slope change estimate  
  phaseBdiff <- c(1:(n_b-1))  
  for (iter in 1:(n_b-1))  
    phaseBdiff[iter] <- phaseBdet[iter+1] - phaseBdet[iter]  
  trendB <- mean(phaseBdiff)  
  print ("Slope change estimate = "); print(trendB)  
  # Compute the level change estimate  
  phaseBddet <- c(1:n_b)  
  for (timeB in 1:n_b)  
    phaseBddet[timeB] <- phaseBdet[timeB] - trendB * (timeB-1)  
  level <- mean(phaseBddet) - mean(phaseAdet)  
  print ("Level change estimate = "); print(level)
```

```

# Represent graphically
time <- c(1:length)
par(mfrow=c(2,1))
plot(time,info, xlim=c(1,length), ylim=c((min(info)-1),(max(info)+1)), xlab="Measurement
time", ylab="Variable of interest", font.lab=2)
abline(v=(n_a+0.5))
lines(time[1:n_a],info[1:n_a])
lines(time[(n_a+1):length],info[(n_a+1):length])
axis(side=1, at=seq(0,length,1),labels=TRUE, font=2)
axis(side=2, at=seq((min(info)-1),(max(info)+1),2),labels=TRUE, font=2)
points(time, info, pch=24, bg="black")
title (main="Original data")
transf <- c(phaseAdet,phaseBdet)
plot(time,transf, xlim=c(1,length), ylim=c((min(transf)-1),(max(transf)+1)),
xlab="Measurement time", ylab="Variable of interest", font.lab=2)
abline(v=(n_a+0.5))
lines(time[1:n_a],transf[1:n_a])
lines(time[(n_a+1):length],transf[(n_a+1):length])
axis(side=1, at=seq(0,length,1),labels=TRUE, font=2)
axis(side=2, at=seq((min(transf)-1),(max(transf)+1),2),labels=TRUE, font=2)
points(time, transf, pch=24, bg="black")
title (main="Detrended data")
list(trendB,level) }

# Input data
info <- array(scan("info.dat"))
n_a <- 3

# Obtain estimates
results <- slcestimates(info,n_a)

```

The plug-in needs both R and R Commander to be installed. The use of the plug-in also requires installing and loading *SLC_0.1.zip* package. The *RcmdrPlugin.SLC_0.1.tar.gz* plug-in needs also to be installed and loaded. Afterwards, in the R console the expression *library(Rcmdr)* needs to be written in order to open R Commander. The plug-in is loaded in R Commander by choosing *Tools ► Load R Cmdr plug-in(s)* and the SLC tab appears in the main menu. Using the menus, the input data file is selected and the length of the baseline phase (obligatorily longer than 1 measurement) is specified prior to executing.

Appendix III

The SAS/IML code for SLC requires copy-pasting the module in the SAS console. Afterwards, data should be input in the statement *measurements = {value1 value2 value3}*; between the curly brackets, separating the values by spaces. Then the baseline phase length is specified in *n_a = length*;. The slope change and level change estimates are obtained pasting the last three lines and executing the whole code. Furthermore, the detrended data is graphed.

```
-----  
proc iml;  
* Module SLC;  
start slc(series, n_a);  
* Obtain phase B length;  
n_b = ncol(series)-n_a;  
* Difference phase A;  
adiff = j(n_a-1,1,1);  
do i = 1 to (n_a-1);  
adiff[i]=series[i+1]-series[i];  
end;  
* Estimate trend;  
aslope=sum(adiff)/(n_a-1);  
* Remove trend from phase A;  
adet = j(n_a,1,1);  
do i = 1 to n_a;  
adet[i]=series[i]-aslope#i;  
end;  
* Remove trend from phase B;  
bdet = j(n_b,1,1);  
do i = 1 to n_b;  
bdet[i]=series[i+n_a]-aslope*(i+n_a);  
end;  
* Difference phase B;  
bdiff = j(n_b-1,1,1);  
do i = 1 to (n_b-1);  
bdiff[i]=bdet[i+1]-bdet[i];  
end;  
* Graph the detrended data;  
time_a=j(n_a,1,1);  
do i = 1 to n_a;  
time_a[i] = i;  
end;
```

```

time_b=j(n_b,1,1);
do i = 1 to n_b;
time_b[i] = i+n_a;
end;
dims=j(4,1,1);
dims[1] = 0;
dims[2] = min(min(adet),min(bdet))-1;
dims[3] = n_a + n_b + 1;
dims[4] = max(max(adet),max(bdet))+1;
start_pt = j(2,1,1);
start_pt[1] = 0;
start_pt[2] = min(min(adet),min(bdet))-1;
num_y = max(max(adet),max(bdet)) - min(min(adet),min(bdet)) + 2;
num_x = nrow(time_a) + nrow(time_b) + 1;
call gstart;
call gopen;
call gwindow(dims);
call gport({ 15 15, 85 85});
call gyaxis(start_pt,num_y,num_y, ,2.,1.5);
call gxaxis(start_pt,num_x,num_x, ,2.,1.5);
call gpoint(time_a,adet,"circle","red");
call gdraw(time_a,adet,1,"red");
call gpoint(time_b,bdet,"square","green");
call gdraw(time_b,bdet,1,"green");
call gshow;
* Estimate slope change;
bslope=sum(bdiff)/(n_b-1);
* Control slope change;
bclear = j(n_b,1,1);
do i = 1 to n_b;
bclear[i]=bdet[i]-bslope*(i-1);
end;
* Estimate level change;
alevel=sum(adet)/n_a;
blevel = sum(bclear)/n_b;
levelchange=blevel-alevel;
* Save estimates;
estimates = j(2,1,1);
estimates[1] = bslope;
estimates[2] = levelchange;
return(estimates);
finish slc;

* Obtain estimates;
measurements = { 1 2 5 6 9};
n_a = 3;
results = slc(measurements,n_a);
print results;
quit;

```

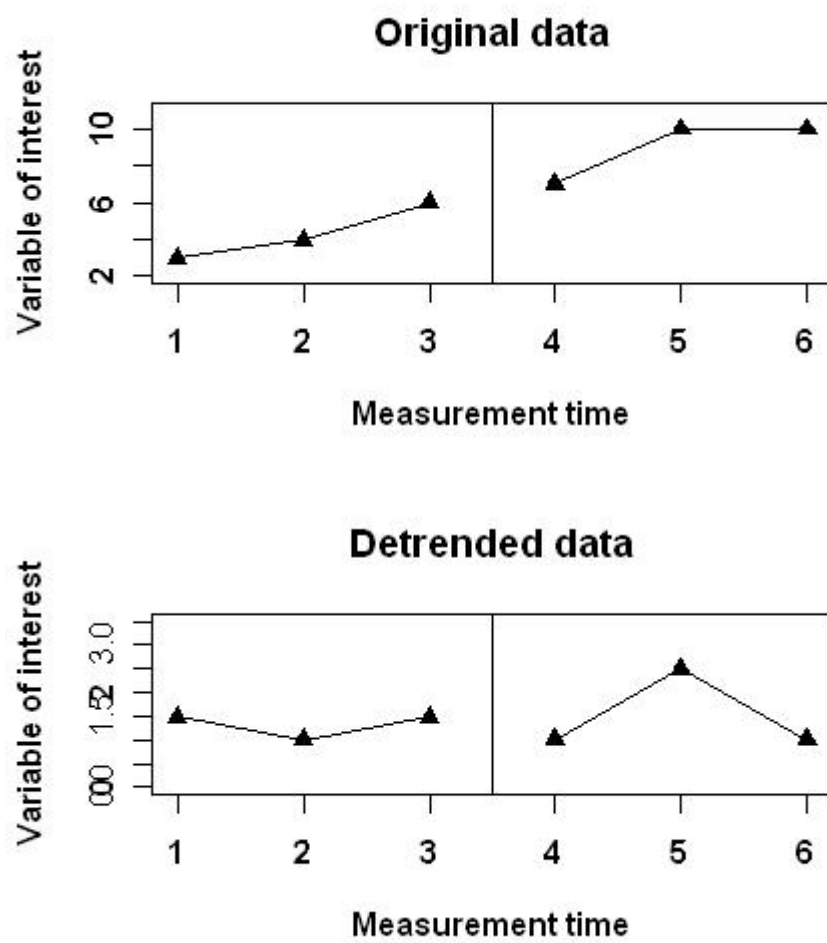


Figure 1. Example of the graphical output of the R function and package “SLC”.

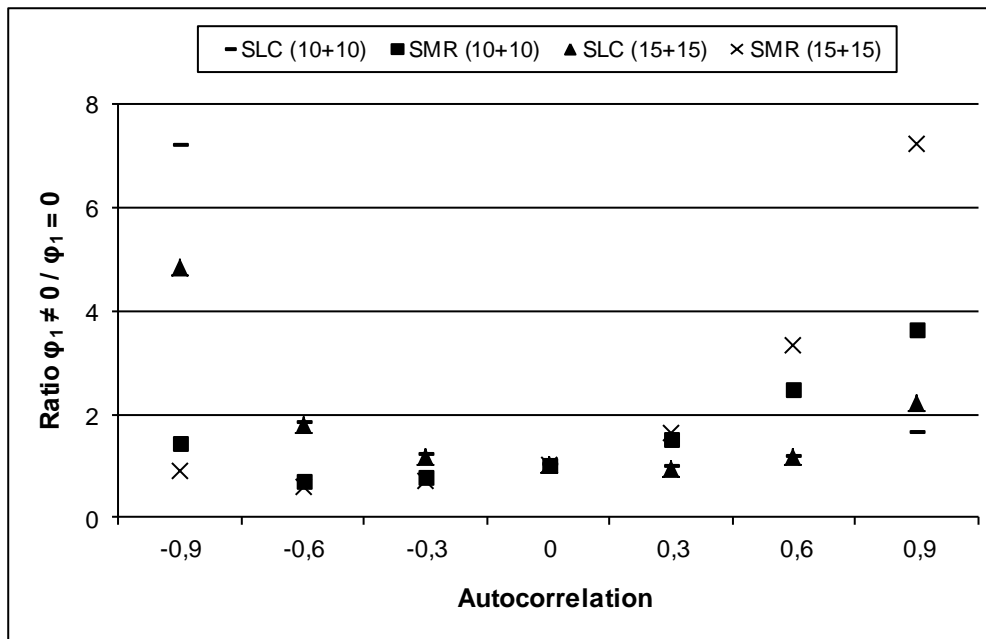


Figure 2. $\varphi_1 \neq 0 / \varphi_1 = 0$ ratios of the variability of the level change estimators of SMR and SLC for two series lengths and data generated using an AR(1) process, exponential error, and null beta parameters.

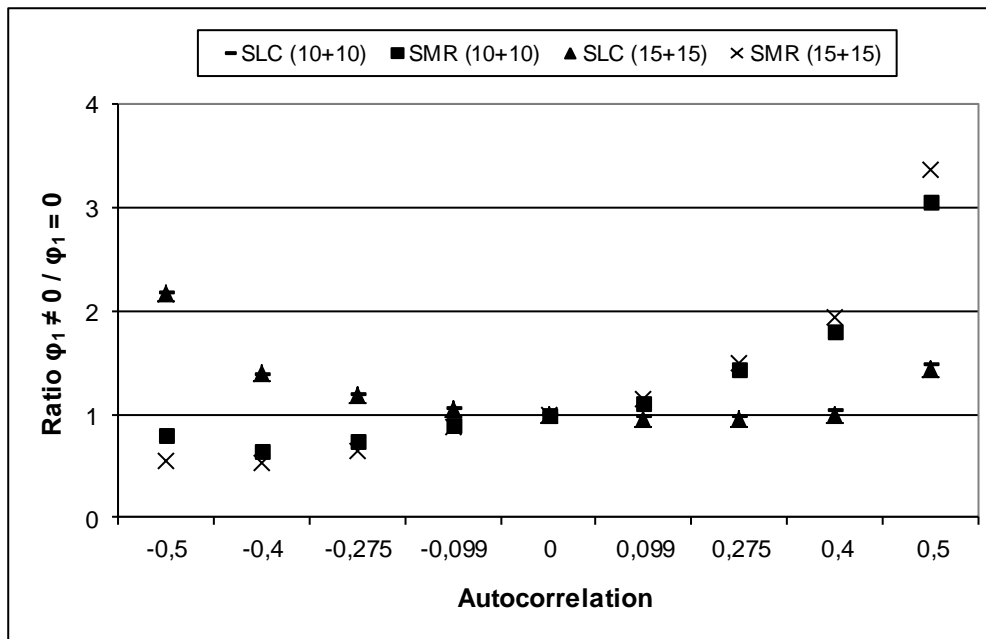


Figure 3. $\phi_1 \neq 0 / \phi_1 = 0$ ratios of the variability of the level change estimators of SMR and SLC for two series lengths and data generated using an MA(1) process, normal error, and null beta parameters.

Table 1. Variance of the SLC estimators for data series generated using an AR(1) model with exponential error and with null beta parameters. LC and SC denote the level change and slope change estimators, respectively.

	$n_A = n_B = 5$		$n_A = n_B = 10$		$n_A = n_B = 15$	
Autocorrelation	LC	SC	LC	SC	LC	SC
−.9	4.71	.30	10.44	.09	6.30	.04
−.6	3.15	.22	2.63	.05	2.32	.02
−.3	2.38	.23	1.73	.05	1.54	.02
.0	2.04	.25	1.45	.05	1.30	.02
.3	1.92	.31	1.41	.06	1.24	.03
.6	1.95	.41	1.69	.09	1.51	.04
.9	1.79	.52	2.36	.20	2.89	.10

Table 2. Variance of the SLC estimators for data series generated using an MA(1) model with normal error and with null beta parameters. LC and SC denote the level change and slope change estimators, respectively.

	$n_A = n_B = 5$		$n_A = n_B = 10$		$n_A = n_B = 15$	
Autocorrelation	LC	SC	LC	SC	LC	SC
-.500	4.40	.31	3.14	.06	2.80	.03
-.400	2.80	.23	2.01	.05	1.79	.02
-.275	2.39	.23	1.72	.05	1.52	.02
-.099	2.10	.24	1.51	.05	1.35	.02
.000	2.03	.25	1.45	.05	1.29	.02
.099	1.95	.26	1.40	.05	1.24	.02
.275	1.98	.30	1.41	.06	1.24	.03
.400	2.11	.36	1.50	.07	1.29	.03
.500	3.03	.56	2.14	.11	1.86	.05

Table 3. Variance of the SMR estimators for data series generated using an AR(1) model with exponential error and with null beta parameters. LC and SC denote the level change and slope change estimators, respectively.

	$n_A = n_B = 5$		$n_A = n_B = 10$		$n_A = n_B = 15$	
Autocorrelation	LC	SC	LC	SC	LC	SC
−.9	2.38	.13	1.15	.01	.49	.00
−.6	1.65	.12	.57	.01	.32	.00
−.3	1.56	.15	.62	.02	.38	.01
.0	1.70	.20	.81	.02	.54	.01
.3	1.93	.29	1.23	.04	.89	.01
.6	2.07	.41	2.02	.09	1.80	.03
.9	1.92	.53	2.96	.22	3.92	.12

Table 4. Variance of the SMR estimators for data series generated using an MA(1) model with normal error and with null beta parameters. LC and SC denote the level change and slope change estimators, respectively.

	$n_A = n_B = 5$		$n_A = n_B = 10$		$n_A = n_B = 15$	
Autocorrelation	LC	SC	LC	SC	LC	SC
-.500	2.52	.12	.65	.01	.29	.00
-.400	1.72	.12	.53	.01	.28	.00
-.275	1.63	.14	.60	.01	.35	.00
-.099	1.64	.18	.73	.02	.47	.01
.000	1.70	.20	.82	.02	.54	.01
.099	1.79	.23	.91	.03	.62	.01
.275	2.05	.29	1.17	.04	.81	.01
.400	2.45	.36	1.48	.05	1.05	.02
.500	3.98	.60	2.51	.09	1.82	.03