Educational expansion, intergenerational mobility and over-education**

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Abstract

There is a vast literature on intergenerational mobility in sociology and economics. Similar interest has emerged for the phenomenon of over-education in both disciplines. There are no studies, however, linking these two research lines. We study the relationship between social mobility and over-education in a context of educational expansion. Our framework allows for the evaluation of several policies, including those affecting social segregation, early intervention programs and the power of unions. Results show the evolution of social mobility, over-education, income inequality and equality of opportunity under each scenario.

Keywords: good jobs, intergenerational cultural transmission, aggregate productivity, matching model, over-education

J.E.L. Classification Code: J21, J24, J62, I24

Resum:

Hi ha una àmplia literatura sobre la mobilitat intergeneracional a les àrees de sociologia i economia. El fenomen de la sobreeducació ha suscitat interès similar en ambdues disciplines. No hi ha estudis, però, que uneixin aquestes dues línies d’investigació. En aquest article s’estudia la relació entre la mobilitat social i la sobre-educació en un context d’expansió de l’educació. El nostre marc permet l’avaluació de diverses polítiques, com les que afecten la segregació social, els programes d’intervenció primerenca i el poder dels sindicats. Els resultats mostren l’evolució de la mobilitat social, la sobre-educació, la desigualtat i la igualtat d’oportunitats en cada escenari.

Paraules clau: transmissió cultural, mobilitat intergeneracional, funció de matching, sobre-educació.

Classificació JEL: J21, J24, J62, I24
1 Introduction

There is a vast literature on inter-generational mobility (see reviews of the empirical literature in Solon, 1999; Björklund and Jäntti, 2009 and Black and Devereux, 2011). Social mobility is often studied in relationship to the process of development, wealth distribution, inequality and economic growth. These variables are usually linked via individuals’ investment decisions in education and its return. Education is positively related to development and growth and explains at least partially wealth distribution and inequality. The cost of education is commonly defined in monetary terms and the introduction of some type of capital market imperfections explain why kids from poor families have less access to education than kids from richer families (Becker and Tomes, 1986; Galor and Tsiddon, 1997; Loury, 1981; Maoz and Moav, 1999).\footnote{An exception is Galor and Tsiddon (1997), who study the effect of technological progress on intergenerational mobility under the assumption of perfect capital markets. Technological progress increases incentives to invest in education overtime, leading to higher mobility. We differ from them in that we do not include technological progress, but cultural transmission of education and frictions in the labor market.}

Lefgren, Lindquist and Sims (2012), however, estimate that no more than 37 percent of the income persistence between fathers and sons is attributable to the causal effect of financial resources. Therefore, there is room for other mechanisms to explain the persistence in income across generations.

The so called ’mechanistic persistence’ refers to the transmission of human capital independently of the level of financial investment. It includes the transmission of genetic traits and other non-financial investments. Understanding the transmission mechanisms in place is crucial to develop effective policies to improve equality of opportunities and social mobility. Income redistribution might be useful if financial constraints are binding (Benabou, 2002), but they might be futile otherwise. Mayer (2008), for instance, assumes the presence of heterogeneous abilities, which are transmitted across generations, generating a positive correlation between fathers’ and sons’ incomes via self-selection into education. Early childhood intervention programs to improve ability of children from low-income families would then lead to better results than subsidizing college education.

Our paper proposes a cultural transmission mechanism similar to the one proposed in Bisin and Verdier (2001). Both direct parental effort in terms of time devoted to children and socialization with neighbors affect the probability of the kid to obtain high education. Several papers provide empirical evidence on the importance of early parental attention (Heckman, Pinto and Savelyev, 2012; Restuccia and Urrutia, 2004;) and neighborhood effects (Case and Katz, 1991; O’Regan and Quigley, 1996; Cutler and Glaeser, 1997) on kids’ future educational outcomes.

We combine the cultural transmission of education with the presence of frictions in the labor market. This allows us to introduce another phenomenon in the analysis of social mobility: over-education. Over-education occurs when individuals’ job require a lower level of
education than the one acquired. It is a consequence of the fast educational expansion that occurred in most developed countries during last decades and it has been studied since the late 70s’ (Brunello, Garibaldi, and Wasmer, 2007; Freeman, 1976; Sicherman and Galor, 1990; Kalleberg and Sorensen, 1979). There are no studies, however, linking over-education and inter-generational mobility. We study the relationship between social mobility and over-education in a context of educational expansion. Our framework allows for the evaluation of several policies, including those affecting social segregation, early intervention programs and the power of unions.

The paper is organized as follows. In the next section we develop the theoretical model, which comprises of three main parts: the education transmission mechanism, the job allocation process and the individual’s problem. In section 3 we first define the variables of interest to then perform several policy evaluations using simulation techniques. Finally, section 4 concludes giving directions for further research.

2 The Model

We set up two initially independent mechanisms in our model. One refers to the educational attainment of the population and the other explains how individuals get allocated to jobs. Then we study how the interaction of these two mechanisms determines several measures of interest of a country. These measures include the level of social mobility, over-education, income inequality and aggregate productivity level.

The first mechanism consists on the transmission of education from parents to children. This is thought to occur as the inter-generational cultural transmission in Bisin and Verdier (2001) adapted to the transmission of education. We follow Patacchini and Zenou (2011). All parents prefer high education for their kids. With some probability they succeed in transmitting their preferences, otherwise children get the education level of a random individual of the population. We depart from any capital market imperfections story or wealth transmission since these effects have been already widely analyzed in the literature (Becker and Tomes, 1986; Galor and Tsiddon, 1997; Loury, 1981; Maoz and Moav, 1999).

The second mechanism in our model explains how individuals get allocated to jobs using a simple matching model. The number of good jobs is endogenously determined as well as the productivity level. Moreover, individuals with parents in a good job are more likely to find a good job than those individuals whose parents have a bad job. This is related to the fast expanding literature on networks, that emphasizes the importance of having the right connections to obtain the right information (Corak and Piraino, 2011; Ioannides and Loury, 2004).

We study the steady state equilibrium as well as transition dynamics, which are very relevant since transition might take several generations.
2.1 Education Transmission

Let us start describing one generation of individuals. Each generation will be denoted by the time when they are born. Each generation consists of a continuum of individuals of finite measure \( \Lambda \). Individuals are distinguishable according to their education level and their network type. We assume two education levels and two types of networks. Let the education level be Low or High (\( i \in \{ L, H \} \)) and the network type Bad or Good (\( j \in \{ B, G \} \)). When the individual is still not working his/her network type is determined by the job type of his/her parent. Once the individual starts working his/her network type is determined by his/her job type. Let \( \mu_{t}^{ij} \) be the population measure of educational type \( i \) working in type \( j \) job for generation \( t \). In other words, \( \mu_{t}^{ij} \) represents the population measure of educational type \( i \) and network type \( j \) of the generation \( t \). All individuals with low education are employed in bad jobs and therefore have bad networks (\( \mu_{t}^{LG} = 0 \)), while individuals with high education may be employed in good or bad jobs. Hence \( \Lambda = \mu_{t}^{HG} + \mu_{t}^{HB} + \mu_{t}^{LB} \).

Each individual has one offspring. Parents transmit their preferences for education to their children in the following fashion. In general all parents prefer the high education level for their kids. They exert however different effort in transmitting this preference. Parents do not always succeed in transmitting their preference. We apply a similar model to the one developed in Bisin and Verdier (2001). Parents make a direct effort to transmit their educational preference. If they fail, then the kid gets the education type of a random individual within the population (oblique socialization). The main difference is that all parents in our model have the same preferences for their kids, independently of their type, while in Bisin and Verdier individuals want to transmit their type. We introduce an index of segregation of the country (\( I_{S} \)) that will indicate whether the oblique socialization occurs only within networks or it can also occur across networks.

Let \( d_{t}^{ij}(\alpha_{t}^{j}) \) represent the direct effort exerted by parents of generation \( t \) with education type \( i \) and network \( j \) to transmit their educational preference to their children, where

\[
\alpha_{t}^{j} = \alpha_{global,t}(1 - I_{S}) + \alpha_{local,t}^{j} I_{S}, \tag{1}
\]

\[
\alpha_{global,t} = \frac{\mu_{t}^{HB} + \mu_{t}^{HG}}{\Lambda}, \tag{2}
\]

\[
\alpha_{local,t}^{B} = \frac{\mu_{t}^{HB}}{\mu_{t}^{HB} + \mu_{t}^{LB}}, \tag{3}
\]

\[
\alpha_{local,t}^{G} = 1. \tag{4}
\]

The direct effort depends on the proportion of people with high education in the network and the level of segregation of the society. In the case of perfect segregation across networks, \( I_{S} = 1 \) and then only the proportion of high educated individuals in the network matters for socialization. In contrast, for a non-segregated society, the index of segregation of the country \( I_{S} \) is zero and
\( \alpha^j_t \) corresponds to the proportion of highly educated individuals in the whole population. We also allow for intermediate levels of segregation, \( I_S \in [0, 1] \).

Whether \( d^i_{jt}(\alpha^j_t) \) is increasing or decreasing in \( \alpha^j_t \) will tell us whether direct and oblique socialization are complementaries or substitutes. Pattacchini and Zenou (2011) estimate this relationship for the UK and find that neighborhood and parental involvement in kids education are complementary.

The direct effort exerted by parents translates into a probability of success in direct socialization, defined by the function \( f \). Let this probability of success depend on the direct effort exerted by parents, \( d \), and the proportion of high educated individuals, \( \alpha^j_t \), that is, \( f^i(d^i_{jt}(\alpha^j_t), \alpha^j_t) \).

We assume that \( f^i(0, \alpha) = 0 \), that is, without effort there is no direct socialization. The function \( f \) is increasing in effort \( d \) and \( \alpha \). This means that higher effort by parents increases the probability of success in transmitting their preferences and given a level of effort, having better neighborhood translates into higher probability of success in direct socialization. We assume therefore some kind of spillovers in the direct socialization process. Spillovers are needed in order to allow for complementarity between direct and oblique socialization.

Based on the empirical evidence in Guryan, Hurst and Kearney (2008) and Pattacchini and Zenou (2011) we assume that parents with low education have lower quality direct effort than high education individuals.

\[
 f^L(d^L_{jt}(\alpha^j_t), \alpha^j_t) = \delta f^H(d^H_{jt}(\alpha^j_t), \alpha^j_t) \tag{5}
\]

where \( \delta < 1 \).

We assume that the probability of oblique socialization is an increasing and convex function on the proportion of highly educated individuals in the society.\(^2\) Let’s denote \( g(\alpha^j_t) \) the probability of oblique socialization. \( g(.) \) is such that \( g' > 0 \) and \( g'' > 0 \). This implies that there is a reinforcing effect of having more educated individuals in the society. Increasing the proportion of human capital has a stronger effect on the probability of socialization when there are already many highly educated individuals. Moreover, if everybody has high education, then oblique socialization is successful with certainty, \( g(1) = 1 \).

Then the probabilities of education transition are the following:

\[
\Pi^H_{jt+1} = f^i(d^i_{jt}(\alpha^j_t), \alpha^j_t) + (1 - f^i(d^i_{jt}(\alpha^j_t), \alpha^j_t))g(\alpha^j_t), \tag{6}
\]

\[
\Pi^L_{jt+1} = (1 - f^i(d^i_{jt}(\alpha^j_t), \alpha^j_t))(1 - g(\alpha^j_t)). \tag{7}
\]

\( \Pi^H_{jt+1} \) is the probability of a parent with education \( i \) and network \( j \) to transmit his/her preference for high education to his/her kid. It has two components, the probability of success in

\(^2\)This assumption is needed in order to obtain an interior equilibrium (or equivalently, a non-degenerate population distribution).
direct transmission plus the probability of success in oblique socialization if direct transmission fails. $\Pi_{j_{t+1}}^{i_L}$ represents the probability of a parent with education $i$ and network $j$ to fail to transmit the high education to the kid. This happens in the event of failure in direct and oblique transmission.

Given these transition probabilities, we can find the measure of young individuals for each type of education and network. Let $\gamma_{i_{t+1}}^{j}$ be the measure of young individuals with education type $i$ and network type $j$ just before entering the labor market. Recall that the network type of this individuals is determined by the job type of their parents.

$\gamma_{L_{t+1}}^{i_L} = \Pi_{t+1}^{LL}+\Pi_{t+1}^{HL}+\Pi_{t+1}^{HG}$, \hspace{1cm} (8)

$\gamma_{H_{B_{t+1}}}^{i_B} = \Pi_{t+1}^{LB}+\Pi_{t+1}^{HB}$, \hspace{1cm} (9)

$\gamma_{H_{G_{t+1}}}^{i_G} = \Pi_{t+1}^{HG}$. \hspace{1cm} (10)

or in matrix notation:

$$
\begin{bmatrix}
\gamma_{L_{t+1}}^{i_L} \\
\gamma_{H_{B_{t+1}}}^{i_B} \\
\gamma_{H_{G_{t+1}}}^{i_G}
\end{bmatrix} =
\begin{bmatrix}
\Pi_{t+1}^{LL} & \Pi_{t+1}^{HL} & \Pi_{t+1}^{HG} \\
0 & \Pi_{t+1}^{LB} & 0 \\
0 & 0 & \Pi_{t+1}^{HG}
\end{bmatrix}
\begin{bmatrix}
\mu_{t+1}^{LB} \\
\mu_{t+1}^{HB} \\
\mu_{t+1}^{HG}
\end{bmatrix}.
$$

Once the education transmission is completed we observe three types of children. Next we study how this young individuals, given their education and network type, are allocated to jobs.

### 2.2 Job allocation

Let us now describe the labor demand. As already mentioned above, there are two types of job: good and bad. A job exists when a suitable worker fills a vacancy created by a firm. Each firm can open only one vacancy. The vacancy cost is $\kappa > 0$ for a good job and 0 for a bad job. Firms are expected profit maximizers and there is free entry. Whereas there are search frictions in the market for good jobs, the market for bad jobs is assumed perfectly competitive. Therefore, everybody can instantly find a bad job and there are as many bad jobs as required. On the other hand, the matching of highly educated individuals to good vacancies is not without effort. Individuals must search in order to find a good vacancy. The total number of good jobs created is determined by a matching function $m(u_t, v_t)$, where $v_t$ denotes the number of good vacancies opened and $u_t$ denotes the total efficiency units of search devoted to the good job market (as in Bentolila, Michelacci and Suarez, 2008).

Similar to Bentolila et al (2008) we normalize to one the maximum efficiency units of search for a good job that an individual can use including formal channels (employment agencies, newspaper adds, internet search, etc.) and networks. Each individual has access to a fraction of this maximum efficiency units of search. We assume that young individuals whose parents are in
good jobs have better networks than young individuals with parents in bad jobs. Therefore, the former are endowed with a larger fraction of efficiency units of search than the latter. In other words, the network type is associated with different endowments of efficiency units of search, $S$ for type $G$, $s$ for type $B$ with $1 \geq S > s > 0$. Therefore, the total efficiency units of search in the good jobs market is $u_t = S_t^{HG} + s_t^{HB}$.

Assume that the matching function for the good job, $m(u, v)$, is increasing in both arguments, continuous function and upper bounded by $\min\{u_t, v_t\}$.

Denote the production obtained in a bad job by $y_{B,t}$ and the production in a good job by $y_{G,t}$. The production of the bad job is given entirely to the worker as wage. In contrast, in the good job, workers and firms negotiate under a Nash bargaining game. The solution is that each player gets a proportion of the total surplus created by a good match, which is $y_{G,t} - y_{B,t}$, plus the outside option. Let workers power be represented by $\beta$. Then the wage of a worker employed in a good job is $w_t^G = \beta (y_{G,t} - y_{B,t}) + y_{B,t}$. The following condition ensures that all individuals with high education prefer a good job rather than a bad job:

$$spw_t^G > y_{B,t},$$

where $p = \frac{m(u_t, v_t)}{u_t} \in [0, 1]$ is the probability per efficiency unit of search to find a good job. Notice that the number of realized matches corresponds to the number of individual with high education in a good job, $\mu_t^{HG} = m(u_t, v_t)$.

The production function in each sector is assumed to have the following specification:

$$y_{G,t} = A_t \alpha_{global,t}^\varepsilon = A_t \left( \frac{\mu_t^{HB} + \mu_t^{HG}}{\Lambda} \right)^\varepsilon,$$

$$y_{B,t} = A_t \left( \alpha_{local,t}^\varepsilon = A_t \left( \frac{\mu_t^{HB}}{\Lambda - \mu_t^{HG}} \right)^\varepsilon,$$

where $\varepsilon < 1$ and $A_t$ represents the productivity level. Notice that with this specification of the production function we are assuming that production of a good job depends on the proportion of highly educated individual in the whole population, meanwhile production of a bad job depends on the proportion of highly educated people in the bad job holders. It follows that $y_{G,t} > y_{B,t}$.

We assume free entry of firms, and hence, firms will open new vacancies in the good jobs market until the expected profit of opening a vacancy equals its cost:

$$q(1 - \beta) (y_{G,t} - y_{B,t}) = \kappa.$$

where $q = \frac{m(u_t, v_t)}{m} \in [0, 1]$ is the probability for a firm of getting a vacancy filled.

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3 Ribo and Vilalta-Buï (2012) analyze which properties are necessary for particular matching functions to be bounded from above by the number of search units and vacancies.

4 To be precise, by the expected profit of opening a vacancy we mean the proportion of the expected surplus generated by the creation of a new job that goes to the firm.
Firms decide to open vacancies after all educational decisions have been taken, so they know how many students of each type there are \((\gamma^L, \gamma^H_G, \gamma^H_B)\), which determines the supply side of the labor market (the amount of efficiency units of search). They also know which will be the production level of the good job, since it is a function of the percentage of highly educated individuals in the whole economy \((\frac{\gamma^H_G + \gamma^H_B}{A})\). They cannot know, however, how many students will end up over-educated. We assume that they expect the production in the bad sector to be equal to that of the previous period \((E(y_{B,t}) = y_{B,t-1})\). Therefore, the following free entry condition determines the number of vacancies of good jobs.

\[
m(u_t, v_t) = \frac{\kappa}{(1 - \beta) (y_G - y_{B,t-1})}.
\]

Given this amount of vacancies, the probabilities of obtaining a job type \(z\) conditional on having high education and being in the network type \(j\) \((P_{t}^{jz})\) for individuals of generation \(t\) are the following:

\[
P_{t}^{BB} = 1 - sp,
\]

\[
P_{t}^{GB} = 1 - Sp,
\]

\[
P_{t}^{BG} = sp,
\]

\[
P_{t}^{GG} = Sp.
\]

Recall that in our model, bad jobs are not rationed. We therefore have no unemployment in our model.

The dynamics of population measures of each type are defined by:

\[
\begin{align*}
\mu_{t+1}^{LB} &= \Pi_{t+1 \mid t}^{LL} L_{t}^{LB} + \Pi_{B,t+1 \mid t}^{HL} H_{t}^{LB} + \Pi_{G,t+1 \mid t}^{HL} G_{t}^{LB} \\
\mu_{t+1}^{HB} &= \Pi_{t+1 \mid t}^{LL} L_{t}^{HB} + \Pi_{B,t+1 \mid t}^{HH} H_{t}^{HB} + \Pi_{G,t+1 \mid t}^{HH} G_{t}^{HB} \\
\mu_{t+1}^{HG} &= \Pi_{t+1 \mid t}^{LL} L_{t}^{HG} + \Pi_{B,t+1 \mid t}^{HH} H_{t}^{HG} + \Pi_{G,t+1 \mid t}^{HH} G_{t}^{HG}
\end{align*}
\]

or in matrix form:

\[
\begin{bmatrix}
\mu_{t+1}^{LB} \\
\mu_{t+1}^{HB} \\
\mu_{t+1}^{HG}
\end{bmatrix} = 
\begin{bmatrix}
\Pi^{LL} & \Pi^{HL} & \Pi^{HL} \\
\Pi^{LB} & \Pi^{HH} & \Pi^{GB} \\
\Pi^{LB} & \Pi^{HB} & \Pi^{HG}
\end{bmatrix}
\begin{bmatrix}
\mu_{t}^{LB} \\
\mu_{t}^{HB} \\
\mu_{t}^{HG}
\end{bmatrix}.
\]

2.3 The individual’s problem

Individuals, when choosing their effort in transmitting their education preferences, maximize a composed utility function which is made up of two parts: their own utility and the expected utility that their kids will get. They derive utility from consumption and leisure.\(^5\) Each individual

\(\footnotesize{^5}\)Individuals consume all their earnings in our model because we want to abstract from any monetary transfer to the kids, which has been already widely studied in the literature.
is endowed with 1 unit of leisure and has to decide how much of it will be devoted to him/herself and how much will be devoted to the kid. The time devoted to the kid corresponds to the direct effort of socialization. There is empirical evidence that supports the idea that parental care of the kid is a determinant of the future education level and other socioeconomic variables of the kid (Heckman, Pinto and Savelyev, 2012; Restuccia and Urrutia, 2004).

When parents decide how much time to invest in educating their kids, they do not know the salaries their kids will get. We assume their best guess is to consider they will face the same situation as in their generation. Moreover, we assume that they do not take into account the effect of their decision in the aggregate outcome of the economy.

Then, the problem that parents of education $i$; having a job $j$ have to solve is

$$\text{Max}_{\{l_t^{ij}, d_t^{ij}\}} \quad w_t + u(l_t^{ij}, d_t^{ij}) + \Pi_{j+1}^{ijH}E_t(V_{j+1}^{iH}) + \Pi_{j+1}^{ijL}E_t(V_{j+1}^{iL})$$

subject to

$$w_t = y_{B,t} + \|_{i=G}\beta(y_{G,t} - y_{B,t}),$$

$$l_t^{ij} = 1 - d_t^{ij},$$

where $d_t^{ij}$ and $l_t^{ij}$ is how much of leisure is devoted to children and to oneself respectively and

$$E_t(V_{j+1}^{iL}) = E_t(w_{t+1}^{LB}) = y_{B,t},$$

$$E_t(V_{j+1}^{iH}) = E_t(w_{t+1}^{H}) = y_{B,t} + \Pi_t^{ijG\beta}(y_{G,t} - y_{B,t}),$$

$$\Pi_{j+1}^{ijH} = f_i(d_t^{ij}, \alpha_t^{ij}) + (1 - f_i(d_t^{ij}, \alpha_t^{ij}))g(\alpha_t^{ij}),$$

$$\Pi_{j+1}^{ijL} = (1 - f_i(d_t^{ij}, \alpha_t^{ij}))(1 - g(\alpha_t^{ij})).$$

Here, we are assuming that parents do not take into account the future cost of socializing children that their kids will have to bear. Parents control variable is the time investment in education, $d_t$.

The first order condition to the general problem is the following:

$$-u_1 + u_2 + \frac{\partial f_i}{\partial d}(1 - g(\alpha_t^{ij}))E_t(V_{t+1}^{iH} - V_{t+1}^{iL}) = 0.$$  \hspace{1cm} (22)

Notice here that since $u_1 > 0$ and $\frac{\partial f_i}{\partial d} > 0$, we have that $u_1 > u_2$ in an interior equilibrium. The economic interpretation of the first order condition is that the marginal cost of spending time with the kid (which is the lost utility of leisure) must be equal to the benefit of spending time with the kid (the utility/disutility you get from spending time with the kid plus the increase in expected utility of your kid).

The second order condition to the general problem is:

$$u_{11} - 2u_{12} + u_{22} + \frac{\partial^2 f_i}{\partial d^2}(1 - g(\alpha_t^{ij}))E_t(V_{t+1}^{iH} - V_{t+1}^{iL}) < 0.$$  \hspace{1cm} (23)
We assume a negative SOC in order to ensure that there is a maximum and the problem is well defined. We assume satiation in leisure \((u_{11} < 0)\).

Using the implicit function theorem we can check whether there is complementarity or substitutability between direct effort and quality of the society.

\[
\frac{\partial d_t}{\partial \alpha_t^j} &= -\left[ \frac{\partial^2 f^i}{\partial d \partial \alpha_t^j} (1 - g(\alpha_t^j)) - \frac{\partial f^i}{\partial \alpha_t^j} g'(\alpha_t^j) \right] E_t(V_{iH}^{t+1} - V_{iL}^{t+1}) \\
&\quad + \frac{\partial^2 f^i}{\partial \alpha_t^j \partial \alpha_t^j} (1 - g(\alpha_t^j)) E_t(V_{iH}^{t+1} - V_{iL}^{t+1}). \tag{24}
\]

**Proposition 1.** If \(\frac{\partial^2 f^i}{\partial d \partial \alpha_t^j} (1 - g(\alpha_t^j)) > \frac{\partial f^i}{\partial \alpha_t^j} g'(\alpha_t^j)\), then there is complementarity between direct effort and oblique socialization. That is, \(\frac{\partial d_t}{\partial \alpha_t^j} > 0\). If \(\frac{\partial^2 f^i}{\partial d \partial \alpha_t^j} (1 - g(\alpha_t^j)) < \frac{\partial f^i}{\partial \alpha_t^j} g'(\alpha_t^j)\), then there is substitutability between direct effort and oblique socialization. That is, \(\frac{\partial d_t}{\partial \alpha_t^j} < 0\).

Notice that for the case \(f^i(d_t^j, \alpha_t^j) = d_t^j\), we have only substitutability between direct effort and \(\alpha_t^j\).

Notice that these conditions involve \(d\) and \(\alpha\). Therefore, depending on the values \(d\) and \(\alpha\) (endogenous to the model) we will observe substitutability or complementarity between direct and oblique socialization. Moreover, an economy may change from one state to the other overtime.

To allow for the possibility of complementarity, we need that the marginal effect of effort on \(f\) is higher on societies with higher \(\alpha\). That is, given two societies with different proportion of highly educated people whose citizens exert exactly the same level of effort in educating kids, the society with higher proportion of educated parents will have a higher marginal effect of effort. In words of Bisin and Verdier (p.308): "direct socialization is in this case more efficient, other things equal, when the trait to be transmitted is held by a majority of the population, and hence when oblique transmission is more efficient". For getting complementarity of direct and oblique socialization we need a minimum level of these spillovers.

### 2.4 Timing

Let us summarize the model by stating clearly the timing within each period. The economy starts with an initial distribution of the population: \(\mu_t^L, \mu_t^H, \mu_t^{HG}\). This initial condition determines the production level of good and bad jobs in this period. Each individual decides how much direct effort to exert given initial conditions and their expectations, which we assume that are determined based on the initial conditions. These decisions on direct effort plus the process of oblique socialization give as an outcome the distribution of young individuals for next generation: \(\gamma_{t+1}^L, \gamma_{t+1}^{HB}, \gamma_{t+1}^{HG}\).
Once the distribution of young individuals is revealed, the number of units of search for the good job is determined \((u_{t+1})\). Moreover, the amount of young individuals with high education is also known \((\gamma_{t+1}^{HG} + \gamma_{t+1}^{HB})\), which must coincide with the number of old individuals in this generation with high education \((\gamma_{t+1}^{HG} + \gamma_{t+1}^{HB} = \mu_{t+1}^{HG} + \mu_{t+1}^{HB})\). Therefore, the number of individuals with low education in generation \(t+1\) can be identified \((\mu_{t+1}^{LB} = \gamma_{t+1}^{L})\).

At this point, firms decide how many vacancies of the good job to open \((v_{t+1})\). Notice, however, that firms do not know how many over-educated individuals there will be in this generation. This means that they cannot forecast the production level of bad jobs (although they know the production level of good jobs, as they know the number of people with high education). We assume that, similarly to individuals’ expectations, the production level of bad jobs next period will be the same as in this period. Given this, they decide how many vacancies to open.

Once vacancies are decided, the matching function tells us how many matches in the good job there will be: \(\mu_{t+1}^{HG} = m(u_{t+1}, v_{t+1})\). Therefore, now we also know how many overeducated individuals there will be in this generation \((\mu_{t+1}^{HB} = A - \mu_{t+1}^{LR} - \mu_{t+1}^{HG})\). Notice then that we already know the distribution of individuals of generation \(t+1\) in the labor market and therefore, we can compute aggregate production and any other measure of interest.

### 3 Simulation exercises

In this section we do comparative static analysis of the model by means of numerical exercises. In all the following simulations we analyze an economy that starts with 70% of the population with low education, 20% of the population with high education and employed in a good job and the other 10% is over-educated (high education and bad job). We also assume that the probability to find a good job for this generation was 0.8 for those in a good network.

The values of the baseline economy are the following: \(\kappa = 0.5; \delta = 0.6; A = 10; \beta = 0.5; \varepsilon = 0.8; I_S = 0.5; S = 1\) and \(s = 0.8\). Moreover, the functional forms used are\(^6\)

\[
\begin{align*}
    f^H(h_H^{ij}, \alpha^j_t) &= (d_H^{ij})^{0.6} \left(\alpha^j_t\right)^{0.4}, \\
    u(l_t^{ij}, d_t^{ij}) &= (l_t^{ij})^{0.8} \left(\alpha^j_t\right)^{0.2}, \\
    m(u_t, v_t) &= (u_t^{\rho} + v_t^{\rho})^{1/\rho}, \text{ where } \rho = -1.27, \\
    g(\alpha^j) &= \left(\alpha^j\right)^2.
\end{align*}
\]

\(^6\)The value of \(\rho\) in the matching function is taken from the calibration in Den Haan, Ramey and Watson (2000).
3.1 Definition of aggregate variables

We are interested in measuring several aggregate variables. For measuring social mobility and equality of opportunity we base on Erikson and Goldthorpe (1993, pp. 55-59). The rest of variables are self-explanatory.

**Measures of social mobility:** Absolute mobility rates refer to the proportions of individuals in some base category who are mobile.

Upward mobility: Probability to change from a Bad Network to a Good Network.

\[ UM = \frac{\Pi_{t+1}^{LB}P_{t+1}^{BG} + \Pi_{B,t+1}^{BG}P_{t+1}^{HG} + \Pi_{t+1}^{HH}P_{t+1}^{LB}}{\mu_{LB} + \mu_{HB}} \]

We can compute the same measure for each type of family:

Upward mobility-low education: Probability to change from a family with low educated parent to a good job.

\[ UM_L = \Pi_{t+1}^{LB}P_{t+1}^{BG} \]

Upward mobility-high education: Probability to change from a family with highly educated parent to a good job.

\[ UM_H = \Pi_{B,t+1}^{HH}P_{t+1}^{BG} \]

Downward mobility: Probability to change from a Good Network to a Bad Network.

\[ DM = \Pi_{G,t+1}^{HL} + \Pi_{G,t+1}^{HH}P_{t+1}^{GB} \]

**Measures of equality of opportunity:** They evaluate the existence of discrimination across groups.

Relative mobility= \( \frac{\text{Probability of change from Bad to Good Network}}{\text{Probability of transition from Good to Good Network}} = RM = \frac{UM}{1-DM} \)

Relative mobility-low education= \( \frac{\text{Probability of change from low educated family to Good Network}}{\text{Probability of transition from Good to Good Network}} = RM_L = \frac{UM_L}{1-DM} \)

Relative mobility-high education= \( \frac{\text{Probability of change from educated family with bad network to Good Network}}{\text{Probability of transition from Good to Good Network}} = RM_H = \frac{UM_H}{1-DM} \)

Perfect equality of opportunity occurs when all measures of relative mobility are equal to one. Then family origins do not matter for outcome.

**Other measures of interest:**

Aggregate production= \( Y = \mu_t^{HG}y_{Gt} + (\Lambda - \mu_t^{HG})y_{Bt} \).

Measure of over-education= % of over-educated workers= \( \mu_t^{HB}/\Lambda \).
Measure of aggregate human capital = % of people with high education = \((\mu_t^{HB} + \mu_t^{HG})/\Lambda\).

Measure of income inequality = Gini Index of wages:

\[
Gini = 1 - \frac{y_B\Lambda}{\Lambda y_B + \mu^{HG}\beta (y_G - y_B)} = \frac{\mu^{HG}\beta (y_G - y_B)}{\Lambda y_B + \mu^{HG}\beta (y_G - y_B)}.
\]

### 3.2 Segregation index (new housing policy, public housing, school policy...)

In this exercise we compare an economy with perfect segregation with a fully integrated economy. As Figure 1 reveals, full segregation slows down the transition towards the long run equilibrium. While the economy with zero segregation converges after 15 generations, the fully segregated economy needs double time to get to the long run equilibrium. This affects strongly the initial years of transition. Educational expansion is faster without segregation, although in the long run equilibrium, the segregated society achieves a larger proportion of individuals with a good job and less individuals with low education. This dynamic path is the result of parental decisions on direct effort, which is much larger in the first generations when there is no segregation (see Figure 2). In contrast, in the fully segregated society, the pick of parental effort for individuals in the bad network happens after 22 generations. Notice also that in the fully segregated society the parents from the good network (HG) spend only time with their kids because it directly enters their utility function. The transmission of high education is guaranteed for their children with oblique socialization alone since they will meet someone with high education with probability 1.
Figure 1. Evolution of the population distribution for fully segregated and fully integrated societies.
Figures 3 and 4 show the measures of social mobility, equality of opportunity and inequality for these two economies. Regarding social mobility (Figure 3), the long run equilibrium level of social mobility is similar for both economies (around 70% of individuals born in a bad network will end up in the good network). However, the transition paths are very different. The society with zero segregation presents a steep increase in social mobility in the first ten generations, while for the fully segregated economy, social mobility changes marginally over the same period of time. Equality of opportunity is also much higher in the fully integrated society, with faster convergence to the long run equilibrium (Figure 4). Differences in equality of opportunity are, however, small in the long run.

The Gini index comes up very large for both economies and while it is larger for the segregated economy during the first 23 generations, the situation reverses afterwards (Figure 4).
Figure 3. Probability of upward mobility for a fully integrated and a fully segregated society.
3.3 Quality of time of low educated parents ($\delta$) (courses on how to take care of your kid, early education programs,...)

In this section we compare an economy where the time quality of parents with low education is half the time quality of parents with high education with an economy where the time quality of parents with low education is seventy percent that of parent with high education. As it can be seen in Figure 5, policies devoted to improve parental time quality of disadvantaged families reduces the transition time by half in our example. It does not affect, however, the level of over-education that is stable and equal across time. This is mostly due to an advancement of the level of effort to the first generations and an increase in the level of effort of low educated families when their time quality is higher (Figure 6).
Figure 5. Evolution of population distribution for societies with different quality of time of low educated parents.
Figure 6. Parental time devoted to children for two societies with different time quality of low educated parents.

Obviously, the levels of social mobility and equality of opportunity are larger in the society where time parental quality is larger (Figures 7 and 8). This keeps true even in the long run for those individuals with low educated parents. Surprisingly, though, the level of inequality is significantly larger for the first 10 generations in the society with larger time parental quality. This is due to the rapid increase of individuals in good jobs in this society during this period.
Figure 7. Probability of upward mobility for two societies with different time quality of low educated parents.
Figure 8. Measures of equality of opportunity and income inequality in two societies with different time quality of low educated parents.

3.4 $s/S$: efficiency units of search for each network.

Here we analyze an economy where probability of finding a good job if you come from a bad network is 70% that of someone from a good network and compare it to an economy where this probability is 90% that of someone from a good network. It turns out that convergence to the long run equilibrium is faster in the latter economy (Figure 9). Moreover, the level of over-education is lower when the probability to find a job is less dependent on the type of network you are in. This is due to the fact that a higher $s$ translates into more efficiency units of search, and therefore, more vacancies are open. Consequently, the number of matches in the good job sector is larger the larger is $s$. 
Figure 9. Evolution of the population distribution for two societies with different networking influence in the labor market.

In Figure 10 we can see that the long run direct effort of individuals in a bad job does not depend on the network effects in the labor market. Notwithstanding, during transition, direct effort of these individuals is increased when \( s = 0.9 \), since the expected value of their kid having high education is larger because the probability of getting a good job increased. Individuals in the good network seem to decrease their effort from the seventh generation on, suggesting a substitution effect of their effort by the oblique socialization that occurs more often in a more educated society.
Figure 10. Parental time devoted to children in two societies that differ in their networking importance in the labor market.

Upward mobility increases strongly when the efficiency units of search associated to individuals from a bad network increase since it rises aggregate efficiency units of search, enhancing the creation of new vacancies and creating a larger amount of good jobs. As a consequence, it is more likely for anyone to change from a bad network to the good network (Figure 11). Moreover, since changing from $s = 0.7$ to $s = 0.9$ we are reducing the differences across types of family, the equality of opportunity increases significantly (Figure 12).
Figure 11. Probability of upward mobility for two societies with different networking influence in the labor market.
3.5 Cost of creating a vacancy: $\kappa$ (reduction of bureaucracy).

Figure 13 shows the evolution of the population distribution in two economies that differ in the cost of creating a vacancy for a good job. An increase in vacancy costs clearly rises the amount of over-educated individuals in the economy, via a reduction in the amount of vacancies open. While the transition period length is similar in both cases, changes in the population distribution are more intense in the first generations for the economy with lower vacancy costs.
Figure 13. Evolution of population distribution for two societies with different vacancy costs.
While social mobility is higher in the case of low vacancy costs during transition and in the long run (Figure 15), equality of opportunity is only significantly larger in the first 15 generations and inequality (Gini index) is also larger (Figure 16).
Figure 15. Probability of upward mobility for two societies with different vacancy costs.
Figure 16. Measures of equality of opportunity and income inequality in two societies with different vacancy costs.

3.6 Worker bargaining power $\beta$ (presence of unions for instance).

An increase in workers’ bargaining power in the economy results in a similar transition period as with the case of a reduction in vacancy costs, except that over-education now increases (Figure 17). Moreover, in the long run equilibrium, the number of good jobs is lower, since firms have less incentives to open vacancies. Similarly as in the previous case, the parental effort increases for the first generations, and specially for the over-educated individuals (Figure 18). This leads to higher social mobility and a drastic increase in equality of opportunity, with small changes in inequality mostly for the first 10 generations (Figure 20).
Figure 17. Evolution of the population distribution for two societies with different workers' bargaining power.
Figure 18. Parental time devoted to children in two societies with different workers’ bargaining power.
Figure 19. Probability of upward mobility for two societies with different workers’ bargaining power.
4 Conclusions

We study the interaction of the cultural transmission of education from parents to kids and the effect of networks in a labor market with frictions to analyze the relationship between social mobility, over-education and inequality. We develop a general equilibrium framework that allows for the realization of counter-factual analyses. We perform several simulation exercises to understand the effect of several policies in a simulated fiction economy. We learned for instance that the transition to the equilibrium might take even 30 generations and in general, takes longer in the presence of social segregation than without it.

Policies devoted to decrease the degree of social segregation (housing or school policies) enhance the education expansion, social mobility and the equality of opportunity at the cost of
a smaller good sector and larger inequality in the long run.

Policies devoted to improve the job matching process (public employment offices, unemployed training, reduction of administrative costs to open vacancies) decrease over-education levels and have a positive impact on social mobility while the effect on equality of opportunity and inequality is not clear.

Policies that affect the bargaining power of workers have a positive effect on social mobility and equality of opportunity at the cost of increasing over-education, while leaving inequality unchanged.

Further work will consist on calibrating the model to real economies in order to verify the explanatory power of the model. We aim at then using it to assess the effect of alternative policies on real economies.

References


