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How Do Aggregate Fluctuations Depend on the Network Structure of the Economy?

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Abstract

In this paper we analyze the aggregate volatility of a stylized economy where agents are networked. If strategic relations connect agents' actions, idiosyncratic shocks can generate nontrivial aggregate fluctuations. We show that the aggregate volatility depends on the network structure of the economy in two ways. On the one hand, the more connected the economy, the lower the aggregate volatility. On the other hand, the more concentrated the network, the higher the aggregate volatility. We provide an application of our theoretical predictions using US data on intersectoral linkages and firms' diversification patterns.

Keywords: Aggregate fluctuations, Networks, Firms, Intersectoral linkages.

JEL Classification: E32, C67, D57.

Abstract

En este artículo, analizamos la volatilidad agregada de una economía estilizada donde los agentes están conectados en redes. Si hay relaciones estratégicas entre las acciones de los agentes, choques idiosincráticos pueden generar fluctuaciones agregadas. Demonstramos que la volatilidad agregada depiende de la estructura de redes de la economía de dos maneras. Por un lado, si hay más conexiones en la economía en su conjunto, la volatilidad agregada es más baja. Por otro lado, si las conexiones están más concentradas, la volatilidad agregada es más alta. Presentamos una aplicación de nuestras predicciones teóricas que utiliza datos de EEUU de conexiones intrasectoriales y de diversificación de las empresas.

1 Introduction

This paper sheds light on how aggregate fluctuations can originate from idiosyncratic shocks. The economic crisis started in 2007 posed the need of discretionary interventions by the public authorities aimed at stabilizing the financial markets. As the different destinies of Lehman Brothers and AIG illustrate, the rationale behind these interventions was not based on the size of the institutions that needed financial help but rather on their systemic relevance. The public bail-out of AIG was justified on the basis that AIG was "too interconnected to fail." Apart from the natural consequences for normative analysis and policy design, the main theoretical question behind these interventions is whether aggregate fluctuations are simply the results of inherently aggregate shocks that hit all the agents of an economy at the same time, or they are rather the consequence of idiosyncratic shocks occurring at the micro level and then propagating through the economy. In this paper we present a theoretical toolbox to analyze the transmission of idiosyncratic shocks to the aggregate level when the economy presents a network structure.

Aggregate fluctuations can be the result of intrinsic aggregate shocks like in Kydland and Prescott [1982] or extrinsic aggregate shocks (sunspots) like in Cass and Shell [1983]. In contrast with these two approaches, our shocks are independent across agents and none of them is the necessary culprit of aggregate fluctuations. Our paper is close in spirit to the seminal work of Jovanovic [1987], who showed that theoretically any amount of aggregate risk can originate from idiosyncratic shocks. In an economy composed by a large number of agents, the standard diversification argument à la Lucas [1977] maintains that idiosyncratic shocks cannot result in aggregate volatility because the Law of Large Numbers (LLN) applies and independently distributed shocks cancel each other out. However, although the shocks to the agents may be independent per se, the strategic complementarities across agents' actions make the actions to be not independent in equilibrium. Thus, aggregate shocks can be the result of the aggregation of idiosyncratic shocks.

From an empirical point of view, if the assumptions of size homogeneity or independence behind the LLN do not hold, then nontrivial aggregate fluctuations can be the result of idiosyncratic shocks. For example, Gabaix [2011] shows that the shocks to the biggest 100 firms can account for up to one third of aggregate fluctuations for the US economy. The main reason is the existence of a power-law size distribution for US firms, so that the aggregate consequences of an idiosyncratic shock to a big firm are considerably different from the consequences of a shock to a small firm. Moreover, Carvalho [2007] and Acemoglu et al. [2011] point at the importance of the input-output structure of the US economy to understand the transmission of sector-specific shocks to the aggregate level. In particular, the presence of sectors that work as hubs to the economy, providing intermediate goods to almost all other sectors, permits the generation of aggregate fluctuations that could potentially account for up to two thirds of aggregate volatility. In line with these approaches, we do not claim that idiosyncratic shocks are the only driving force of aggregate fluctuations. We simply maintain that a sizable share of the aggregate volatility that cannot be explained by inherently aggregate shocks derives from the aggregation of independent micro shocks, and that such a share can be analyzed and quantified once we know the network structure of the economy.

We present a stylized economy composed of N agents. We model it on the basis of Ballester et al. [2006]. The effort of each agent can be either a strategic complement or a strategic substitute to the effort of other agents. These strategic relations across agents describe the network structure of the economy. For example, firms involved in joint R&D activities may see the research effort of their partners as a strategic complement, while firms that compete in the same market with other firms may see the production of their competitors as a strategic substitute.[1] When information is perfect, the effort of each agent at equilibrium depends on the network structure of the economy, that is, on the whole set of bilateral strategic relations across agents. Hence, the dispersion of effort levels at equilibrium depends mainly on the position of each agent in the network structure. An idiosyncratic shock to a single agent of the economy can transmit to the aggregate level depending on whether that agent is highly interconnected with other agents but also on whether there exists the possibility of another idiosyncratic shock originated somewhere else in the economy to countervail the first one. The aggregate volatility of the economy is the aggregation of the equilibrium consequences of any idiosyncratic shock that hit any agent in the economy.

We are able to decompose the aggregate volatility of the economy into three components. The first component reflects the usual LLN. According to this component, aggregate volatility should decrease at pace $\sqrt{N}$ as the number N of agents increases. The second component measures the overall connectedness of the economy. We find that the more connected the economy, the lower the volatility. This is due to the fact that the links between agents imply the emergence of preliminary LLNs that partially diversify the idiosyncratic risk before the market interaction and the realization of the equilibrium. The third component accounts for the concentration of the network. We provide a mapping between a commonly used centrality measure

[1] On R&D and collaboration networks see for example Goyal and Moraga-Gonzalez [2001] and Goyal and Joshi [2003].

developed in Bonacich [1987] and the eigendecomposition of the adjacency matrix, that is, the eigenvectors and eigenvalues of the matrix that represents the network structure of the economy. We find that the more concentrated the network, the higher is aggregate volatility. The intuition is that the more central some agents are with respect to other agents within the network, the more likely is an idiosyncratic shock to the former agents to propagate through the economy and to generate an aggregate fluctuation. The third component reflects therefore that the aggregate volatility depends on how the linkages across agents are structured rather than on their number or their strength.

Our paper is related to the analyses of systemic risk, especially if we consider the case of networks of financial liabilities.[2] For example, Nier et al. [2007] finds that more concentrated banking systems are conducive of larger systemic risk. This conclusion is similar to our finding about the positive relation between the concentration of the network and the aggregate volatility. However, our work has a difference theoretical focus with respect to the literature on financial contagion and systemic risk. We analyze the aggregate volatility as it results from idiosyncratic shocks that hit all the agents of the economy, and not from the propagation of a particular shock to a particular agent. Moreover, most of the analyses of systemic risk devote particular attention to the banking sector or to the wider financial sector, and usually within a partial equilibrium framework. Our results instead may apply to the economy as a whole and are designed for general equilibrium applications.

The paper is organized as follows. In Section 2, we present the general set-up of our economy and its equilibrium. In Section 3, we analyze the connection between the network structure and aggregate volatility. In Section 4, we provide an application of the theoretical predictions of the general model to the case of establishment-level shocks. In Section 5, we draw the conclusions and present possible lines of research for the future. Proofs, tables, figures, and numerical exercises can be found in the Appendix.

2 The Model

Consider the economy composed of N agents. Each agent's payoff is linear quadratic, concave in the agent's own effort and such that other agents' efforts are strategically complementary or substitutable to her own. Under mild conditions, this framework yields a unique interior Nash equilibrium that expresses each player's effort as a linear function of her weighted Bonacich centrality, as in Ballester et al. [2006]. The Bonacich centrality is a measure

[2] A review of network models of financial contagion is provided by Wims et al. [2011].

of how the single agent is at the center of a nexus of paths within the network of the economy. Let us first review the main insights of Ballester et al. [2006] and then extend this framework to the analysis of aggregate volatility.

2.1 Set-up and equilibrium

Each Agent i in $\mathscr{N} \equiv \{1, ..., N\}$ chooses the effort $q_i \geq 0$ that maximizes the payoff,

$$\pi_i \equiv \varepsilon_i q_i + \frac{\gamma_{ii}}{2} q_i^2 + \sum_{j \in \mathscr{N} \setminus \{i\}} \gamma_{ij} q_i q_j, \tag{1}$$

where $\varepsilon_i > 0$ and $\gamma_{ii} < 0$ for every i in $\mathscr{N}$, and $\gamma_{ij} \in \mathbb{R}$ for every (i, j) in $\mathscr{N}^2$. We call ε_i the idiosyncratic endowment of Agent i. Note that each agent's payoff is concave in own effort and that there are bilateral strategic relations among players which are pair-specific and can be both positive and negative. The effort of Agent j is a strategic complement to the effort of Agent i when $\gamma_{ij} > 0$, for example if i and j are firms involved in a common R&D activity and q_i and q_j are the research efforts of Firm i and Firm j. The effort of Agent j is a strategic substitute to the effort of i when $\gamma_{ij} < 0$, for example if i and j are firms that compete in quantities where the effort is the quantity produced by each firm. The strategic relation γ_{ij} does not need to coincide with γ_{ji} unless $i = j$, for every (i, j) in $\mathscr{N}^2$. Nevertheless, for the clarity of exposition we simplify our framework. We discuss in Remark 1 the general case.

Assumption 1. The concavity in own effort γ_{ii} is the same for all agents, that is, $\gamma_{ii} = \gamma$ for every i in $\mathscr{N}$. Moreover, the strategic relations among players are symmetric, that is, $\gamma_{ij} = \gamma_{ji}$ for every (i, j) in $\mathscr{N}^2$.

We can represent the First Order Conditions (FOCs) of all players in matrix form, that is,

$$-\mathbf{\Gamma} \bar{q} = \bar{\varepsilon}, \tag{2}$$

where $\bar{q}$ and $\bar{\varepsilon}$ are vectors of length N and $\mathbf{\Gamma}$ is a $N \times N$ square matrix. The i-th elements of $\bar{q}$ and $\bar{\varepsilon}$ are q_i and ε_i, respectively. The (i, j)-th element of $\mathbf{\Gamma}$ is γ_{ij}, and in particular $\gamma_{ii} = \gamma$ when $i = j$. We can decompose $\mathbf{\Gamma}$ into a concavity component, a global substitutability component, and a local complementarity component. In order to do this, we make the following simplifying assumptions. Let us define $\gamma_{min} \equiv \min\{\gamma_{ij} | i \neq j\}$ and $\gamma_{max} \equiv \max\{\gamma_{ij} | i \neq j\}$.

Assumption 2. The concavity of payoff is less than the minimal bilateral strategic relation, that is, $\gamma < \gamma_{min}$. Moreover, there exist at least a pair of agents whose efforts are strategic substitutes, that is, $\gamma_{min} < 0$, and at least a pair of agents whose efforts are strategic complements, that is, $\gamma_{max} > 0$.

This implies that $\gamma_{max} - \gamma_{min} > 0$ and $\gamma_{min} - \gamma > 0$. We define

$$g_{ij} \equiv \frac{\gamma_{ij} - \gamma_{min}}{\gamma_{max} - \gamma_{min}},$$

for every (i, j) in $\mathscr{N}^2$ such that $i \neq j$ and $g_{ii} \equiv 0$ for every i in $\mathscr{N}$. By construction, $0 \leq g_{ij} \leq 1$. Thus, the FOCs in (2) become

$$[(\gamma_{min} - \gamma)\mathbf{I} - \gamma_{min}\mathbf{U} - (\gamma_{max} - \gamma_{min})\mathbf{G}]\, \bar{q} = \bar{\varepsilon}, \tag{3}$$

where $\mathbf{I}$ is the $N \times N$ identity matrix, $\mathbf{U}$ is the $N \times N$ matrix of ones, and $\mathbf{G} \equiv [g_{ij}]$ is an $N \times N$ nonnegative square matrix with zeros on the diagonal. The component $(\gamma_{min} - \gamma)\mathbf{I}$ reflects the concavity of payoffs in own effort, the component $-\gamma_{min}\mathbf{U}$ reflects the uniform substitutability of effort across players, and the component $-(\gamma_{max} - \gamma_{min})\mathbf{G}$ reflects the complementarity of effort across players relative to the benchmark of global substitutability. We can rewrite (3) as

$$\left[\mathbf{I} - \frac{\gamma_{max} - \gamma_{min}}{\gamma_{min} - \gamma}\mathbf{G}\right] \bar{q} = \frac{1}{\gamma_{min} - \gamma}\, [\bar{\varepsilon} + \gamma_{min}\mathbf{U}]\,.$$

In order to characterize the equilibrium, we have to find conditions that permit us to express $\bar{q}$ as a function of the rest. Let us therefore define

$$a \equiv \frac{\gamma_{max} - \gamma_{min}}{\gamma_{min} - \gamma} > 0, \tag{4}$$

which by construction is strictly positive. Moreover, let us call λ_{max} the maximal eigenvalue of $\mathbf{G}$.

Assumption 3. The maximal eigenvalue λ_{max} of $\mathbf{G}$ is strictly smaller than the inverse of a, that is,

$$a\lambda_{max} < 1,$$

where a is defined in (4).

The intuition behind this assumption is that the maximal relative complementarity across the efforts $(\gamma_{max} - \gamma_{min})\lambda_{max}$ must be less than the relative concavity of payoff in own effort $\gamma_{min} - \gamma$. Otherwise, the feasible set of efforts might be nonconvex for some agents. Note that by the Perron-Frobenius theorem all eigenvalues of $\mathbf{G}$ are real numbers due to symmetry implied by Assumption 1. Moreover, Assumption 2 guarantees that $\mathbf{G}$ has at least one nonnil entry, so that $\lambda_{max} > 0$. Given that $a > 0$ and all values of $\mathbf{G}$ are nonnegative, Theorem III* in Debreu and Herstein [1953] states that the matrix $[\mathbf{I} - a\mathbf{G}]^{-1}$ has only nonnegative values if and only if Assumption 3 holds. Moreover, under Assumption 3 we can rewrite the inverse of $[\mathbf{I} - a\mathbf{G}]^{-1}$ in its Neumann series form. Hence, we can define

$$\mathbf{M} \equiv [\mathbf{I} - a\mathbf{G}]^{-1} = \sum_{k=0}^{+\infty} a^k \mathbf{G}^k, \tag{5}$$

where $\mathbf{G}^k$ is the k-th power of $\mathbf{G}$. In particular, $\mathbf{G}^0 = \mathbf{I}$. Let $\bar{1}^T$ denote the horizontal vector of ones of length N. Thus, we can characterize the vector $\bar{q}^*$ of equilibrium efforts.

Proposition 1. *Suppose that Assumption 1, Assumption 2, and Assumption 3 hold. Then, there exists a unique (Nash) equilibrium. Moreover, the vector $\bar{q}^*$ of equilibrium efforts is*

$$\bar{q}^* = \frac{1}{\gamma_{min} - \gamma} \left[\mathbf{M}\bar{\varepsilon} + \frac{\gamma_{min} \bar{1}^T \mathbf{M} \bar{\varepsilon}}{\gamma_{min} - \gamma - \gamma_{min} \bar{1}^T \mathbf{M} \bar{1}} \mathbf{M}\bar{1} \right], \tag{6}$$

where $q_i^* > 0$ for every i in $\mathscr{N}$ and M is defined in (5).

The equilibrium effort of each agent is a function of all idiosyncratic endowments $\bar{\varepsilon}$ and the structure of direct and indirect strategic relations across agents $\mathbf{M}$.

2.2 Equilibrium and network centrality

We can interpret $\mathbf{G}$ as the adjacency matrix of a network $\mathbf{g}$ of relative payoff complementarities across pairs. If the complementarity induced by the presence of a linkage from Agent i to Agent j within the network $\mathbf{g}$ is strong enough to countervail the uniform substitutability in efforts, then effort of Agent j is a strategic complement to Agent i. Otherwise, the effort of Agent j is a strategic substitute to Agent i. The network $\mathbf{g}$ can be represented as a graph with neither loops nor multiple links. Given that we suppose that the original $\mathbf{\Gamma}$ is symmetric, then $\mathbf{G}$ is symmetric and therefore the underlying network $\mathbf{g}$ is an undirected network. We define a network centrality measure introduced by Bonacich [1987] and applied to our framework in Ballester et al. [2006].

Definition 1 (Weighted Bonacich centrality)**.** The vector of weighted Bonacich centralities of the economy is

$$\bar{b}(\bar{x}) = \sum_{k=0}^{+\infty} a^k \mathbf{G}^k \bar{x},$$

where $\bar{x}$ is a length-N vector of nonnegative weights.

According to (5), this implies that

$$\bar{b}(\bar{x}) = \mathbf{M}\bar{x}. \tag{7}$$

Let us call $g_{ij}^{[k]}$ the (i,j)-th element of $\mathbf{G}^k$. Thus, the i-th element of $\bar{b}(\bar{x})$ is

$$b_i(\bar{x}) = \sum_{j \in \mathcal{N}} \left(\sum_{k=0}^{+\infty} a^k g_{ij}^{[k]} \right) x_j = \sum_{j \in \mathcal{N}} m_{ij} x_j,$$

where x_j is the j-th element of $\bar{x}$. Since $\mathbf{G}$ is the adjacency matrix of the network $\mathbf{g}$, the nonnil elements of $\mathbf{G}$ account for the direct connections between agents, that is, the paths of length one within the network. Consequently, the nonnil elements of the k-th power of $\mathbf{G}$ account for the indirect connections, that is, the paths of length k within the network. In other words, the element $g_{ij}^{[k]}$ is equal to the number of paths of length k that exist between i and j. If $g_{ij}^{[k]} = 0$, then there does not exist any path of length k between i and j. If instead $g_{ij}^{[k]} > 0$, then there exists at least a path of length k between i and j, and the intensity of this indirect connection is exactly $g_{ij}^{[k]}$.

Example 1. Suppose that adjacency matrix of $\mathbf{g}$ is

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

You can see a graphical representation of this network in Figure 1. The network is such that there are a link between 1 and 2 and a link between 1 and 3, but no link between 2 and 3. The second power of $\mathbf{G}$ is

$$\mathbf{G}^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

There are two paths of length two that go from 1 to herself, either from 1 to 2 and back or from 1 to 3 and back. Moreover, there is one path of length two that goes from 2 to 3, one from 2 to herself, and one from 3 to herself.

The weighted sum $\sum_{k=0}^{+\infty} a^k g_{ij}^{[k]}$ accounts for all paths of any length between i and j. The weighted Bonacich centrality of Agent i sums up all the paths of any length across all agents of the economy, weighing each Agent j who is connected to i by her weight x_j. Thus, the Bonacich centrality measures how much Agent i is at the center of a nexus of paths from any other agent within the network of the economy. Figure 11 reports an example of Bonacich centralities for a network structure we describe later on.

Proposition 2. *Suppose that Assumption 1, Assumption 2, and Assumption 3 hold. Then, we can rewrite the equilibrium solution as*

$$\bar{q}^* = \frac{1}{\gamma_{min} - \gamma} \left[\bar{b}(\bar{\varepsilon}) + B\gamma_{min}\bar{1}^T\bar{b}(\bar{\varepsilon})\bar{b}(\bar{1}) \right],$$

where

$$B \equiv \frac{1}{\gamma_{min} - \gamma - \gamma_{min}\bar{1}^T\bar{b}(\bar{1})} > 0.$$

In other words, the equilibrium effort of each agent is a linear function of her Bonacich centrality with weights $\bar{\varepsilon}$ and $\bar{1}$.

The equilibrium effort of each agent depends on how that agent is at the center of a nexus of paths coming from all other agents, who are themselves weighted either by their idiosyncratic endowments $\bar{\varepsilon}$ or by the uniform $\bar{1}$.

3 Network structure and aggregate volatility

3.1 Eigendecomposition of network centrality

Let us consider the adjacency matrix $\mathbf{G}$. Since it is symmetric due to Assumption 1, it has N distinct eigenvalues and is therefore diagonalizable. Hence, there exists an invertible matrix $\mathbf{V}$ such that

$$\mathbf{V}^{-1}\mathbf{G}\mathbf{V} = \mathbf{\Lambda},$$

where $\mathbf{\Lambda}$ is a diagonal matrix with the eigenvalues of $\mathbf{G}$ on its diagonal, and $\mathbf{V}$ is an invertible matrix whose colums are the eigenvectors of $\mathbf{G}$. Thus,

$$\mathbf{G} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}. \tag{8}$$

Moreover, the eigenvectors of $\mathbf{G}$ can be chosen to form an orthonormal basis of $\mathbb{R}^N$. Thus, we set $\mathbf{V}$ to be an orthogonal matrix, which implies $\mathbf{V}^{-1} = \mathbf{V}^T$, where $\mathbf{V}^T$ is the transpose of $\mathbf{V}$. This implies that $\mathbf{V}\mathbf{V}^T = \mathbf{V}^T\mathbf{V} = \mathbf{V}^{-1}\mathbf{V} = \mathbf{I}$.

Remark 1. Suppose that Assumption 1 does not hold. If the bilateral strategic relations across agents are not symmetric, that is, γ_{ij} is not necessarily equal to γ_{ji} for every i and j in $\mathscr{N}$, then the resulting adjacency matrix $\mathbf{G}$ is not symmetric either. Hence, the matrix does not necessarily have N distinct eigenvalues, in which case we could not decompose it into the diagonal matrix $\mathbf{\Lambda}$ of eigenvalues and its eigenvector matrices, $\mathbf{V}$ and $\mathbf{V}^{-1}$. However, there exists an invertible matrix $\mathbf{P}$ of generalized eigenvectors of $\mathbf{G}$ such that

$$\mathbf{P}^{-1}\mathbf{G}\mathbf{P} = \mathbf{J},$$

where $\mathbf{J}$ is called the Jordan normal (or canonical) form of $\mathbf{G}$. The matrix $\mathbf{J}$ is a $N \times N$ block diagonal matrix, that is,

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_1 & & & \\ & \mathbf{J}_2 & & \\ & & \ddots & \\ & & & \mathbf{J}_L \end{bmatrix},$$

where $L \leq N$ is the number of distinct eigenvalues of $\mathbf{G}$ and $\mathbf{J}_i$ is a so-called Jordan block, for every i in $\mathscr{L} \equiv \{1, \cdots, L\}$. Each Jordan block $\mathbf{J}_i$ is defined by two elements, the eigenvalue λ_i of $\mathbf{G}$ and its multiplicity μ_i, for every i in $\mathscr{L}$. The Jordan block $\mathbf{J}_i$ is a $\mu_i \times \mu_i$ square matrix. The diagonal of each Jordan block $\mathbf{J}_i$ is filled with the fixed element λ_i and the superdiagonal is filled with ones. All other entries of $\mathbf{J}_i$ are zeros, so that

$$\mathbf{J}_i = \begin{bmatrix} \lambda_i & 1 & 0 & \cdots & 0 \\ 0 & \lambda_i & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \lambda_i & 1 \\ 0 & 0 & 0 & 0 & \lambda_i \end{bmatrix},$$

for every i in $\mathscr{L}$. In fact, the diagonal form $\mathbf{\Lambda}$ of $\mathbf{G}$ in (8) is the special case of Jordan normal form of $\mathbf{G}$ when $\mathbf{G}$ is symmetric. If $\mathbf{G}$ is symmetric, the $L = N$ and each Jordan Block $\mathbf{J}_i$ consists of a 1×1 matrix with a single entry, λ_i. We let Assumption 1 hold and we restrict $\mathbf{G}$ to be symmetric only for simplicity. All propositions and intuitions can be reformulated with minor changes using the Jordan normal form of $\mathbf{G}$.

Proposition 3. *Suppose that Assumption 1, Assumption 2, and Assumption 3 hold. Then, the matrix* $\mathbf{M}$ *defined in (5) is such that*

$$\mathbf{M} = \mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1},$$

where $\mathbf{V}$ *is the matrix of eigenvectors of* $\mathbf{G}$ *and* $\tilde{\mathbf{\Lambda}}$ *is a diagonal matrix whose* l*-th diagonal element is a convex function of the* l*-th eigenvalue in* $\mathbf{\Lambda}$*, that is,*

$$\tilde{\lambda}_l \equiv \frac{1}{1 - a\lambda_l} > 0,$$

where a *is defined in (4), for every* l *in* $\mathscr{N}$*.*

The connections of any length between the N agents of the economy describe a rotation of the N-dimensional space of direct connections described by the original adjacency matrix $\mathbf{G}$. Each dimension l in this new space is associated to an eigenvalue λ_l of the original $\mathbf{G}$, which measures how much the dimension l accounts for the structure of indirect connections in $\mathbf{G}$. We call such a dimension a *latent group*. The (i, j)-th element of $\mathbf{M}$ is then

$$m_{ij} = \sum_{k=0}^{+\infty} a^k g_{ij}^{[k]} = \sum_{l \in \mathscr{N}} v_{il} v_{lj}^{-1} \tilde{\lambda}_l,$$

where v_{il} is the (i, l)-th element of $\mathbf{V}$ and v_{lj}^{-1} is the (l, j)-th element of $\mathbf{V^{-1}}$. Since we choose $\mathbf{V}$ such that $V^{-1} = V^T$, then $v_{lj}^{-1} = v_{jl}$. Hence,

$$m_{ij} = \sum_{l \in \mathscr{N}} v_{il} v_{jl} \tilde{\lambda}_l.$$

The element v_{il} of $\mathbf{V}$ accounts for how much Agent i contributes to the relevance of latent group l in capturing the structure of indirect connections contained implicitly in $\mathbf{G}$, while the element v_{jl} does the same for Agent j. Hence, the element m_{ij}, which measures how many paths of any length pass between i and j, can be obtained by summing all the contributions of Agent i and Agent j to any latent group l, weighing each pair of contributions (v_{il}, v_{jl}) by the (duly transformed) relevance $\tilde{\lambda}_l$ of the latent group l in capturing the structure of indirect connections.

We can express the unweighted Bonacich centrality of Agent i,

$$b_i(\bar{1}) = \sum_{j \in \mathscr{N}} \left(\sum_{k=0}^{+\infty} a^k g_{ij}^{[k]} \right) = \sum_{j \in \mathscr{N}} m_{ij},$$

in terms of the contributions of Agent i to all the latent groups of $\mathbf{G}$, that is,

$$b_i(\bar{1}) = \sum_{j \in \mathscr{N}} \sum_{l \in \mathscr{N}} v_{il} v_{jl} \tilde{\lambda}_l.$$

For example, suppose we add a link of some weight between Agent i and Agent j. On the one hand, the Bonacich centrality of i increases because any path that could arrive up to j now arrives until i. On the other hand, the contribution of i to some latent group l to which also j contributes increases, as well as the relevance $\tilde{\lambda}_l$ of that latent group in explaining the structure of indirect connections across agents, that is, $\mathbf{M}$. Using Definition 1 and Proposition 3, we can express the vector of weighted Bonacich centralities with weights $\bar{x}$ as

$$\bar{b}(\bar{x}) = \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \bar{x}. \tag{9}$$

In other words, the network centrality of the agents in a networked economy can be decomposed into the eigenvectors and the (transformed) eigenvalues of the adjacency matrix. The intuition behind the eigenvalues and the eigenvectors is that they describe a series of latent groups implied by the adjacency matrix. The single (transformed) eigenvalue $\tilde{\lambda}_l$ associated to the latent group l in $\mathscr{N}$ measures how much of the indirect connections implied by the adjacency matrix can be accounted for by that latent group, while the corresponding eigenvectors measure the contribution of each agent to that latent group. Once we know the adjacency matrix, we can pin down all indirect connections and therefore also their representation through the eigendecomposition. Central agents in the network are agents that contribute relevantly to sizeable latent groups, that is, latent groups that account for large portions of the indirect connections. Peripheral agents instead are agents that contribute little to the latent groups, and isolated agents are agents that contribute perhaps relevantly but to relatively small latent groups. Hence, there is a mapping between the network centrality of each agent and her contributions to latent groups of different sizes.

3.2 Aggregate volatility and network centrality

We want to study the aggregate volatility of efforts in our stylized economy. In order to do this, suppose that the idiosyncratic endowment ε_i of an agent's payoff is the realization of a random variable, $\tilde{\varepsilon}_i$.

Assumption 4. The random variable $\tilde{\varepsilon}_i$ is independently and identically distributed for every i in $\mathscr{N}$. It follows a distribution with support set $E \subseteq (0, +\infty)$, finite mean $\mu \equiv E[\tilde{\varepsilon}_i]$, and finite variance $\sigma^2 \equiv E[(\tilde{\varepsilon}_i - \mu)^2]$.

The idiosyncratic endowment ε_i in (1) is then the realization of $\tilde{\varepsilon}_i$. Hence, the vector $\bar{\varepsilon}$ is the vector of realizations of a random vector whose i-th element is $\tilde{\varepsilon}_i$, for every i in $\mathscr{N}$. The dispersion of equilibrium efforts $\bar{q}^*$ simply reflects the dispersion of realizations of $\tilde{\varepsilon}_i$ across the agents filtered by the equilibrium interaction.[3] Since our economy is inherently static and the time dimension corresponds simply to the repetition of the vector of idiosyncratic shocks,

[3]Suppose that there are realizations of the random vector through time. According to (6), different realizations $\bar{\varepsilon}$ through time yield different vectors $\bar{q}^*$ of equilibrium efforts. If we define the aggregate volatility as the dispersion of the first moment of $\bar{q}^*$ through the time dimension, there is no necessary relation between the dispersion of q_i^* across agents at a certain point in time and the dispersion of, say, the average of elements in $\bar{q}^*$ through time. Nevertheless, if the random vector whose realization is $\bar{\varepsilon}$ is iid through time, then the cross-sectional dispersion in equilibrium levels is tighly connected to the aggregate volatility.

from now on we use interchangeably the concepts of cross-sectional variance and volatility.[4] First, we define and analyze the variance-covariance matrix of equilibrium efforts. Second, we provide a measure of aggregate volatility derived from the variance-covariance matrix.

Definition 2. The variance-covariance matrix $\mathbf{\Sigma}(\mathbf{G})$ is the $N \times N$ real-valued matrix given by

$$\mathbf{\Sigma}(\mathbf{G}) \equiv E\left[\left(\bar{q}^* - E\left[\bar{q}^*\right]\right)\left(\bar{q}^* - E\left[\bar{q}^*\right]\right)^T\right],$$

where the expectation operator $E[\cdot]$ is defined over the probability distributions of $\tilde{\varepsilon}_i$, for every i in $\mathscr{N}$.

In our economy under Assumption 2 agents are necessarily networked and the equilibrium effort is described by (6). Hence, the variance-covariance matrix of equilibrium efforts depends on the idiosyncratic volatility but also on how the agents are linked to each other. The pattern of direct and indirect connections across agents determines the structure of the variance-covariance matrix, and the potential correlation in equilibrium efforts is a product of some direct or indirect connection between agents.

Let us analyze the variance-covariance matrix. The diagonal entries of the variance-covariance matrix account for the volatility of each agent, while the off-diagonal entries account for the comovement between different agents. Since there exist links between agents, there exists covariance in the equilibrium effort across agents although the idiosyncratic shocks ε_i for every i in $\mathscr{N}$ are independently distributed. Moreover, the volatility of the individual equilbrium efforts itself is affected by the presence of the linkages, because the single agents are subject not only to their own idiosyncratic shocks but also to the shocks that hit directly or indirectly connected agents. Hence, the variance-covariance matrix depends on the network structure of the economy.

Proposition 4. *Suppose Assumption 1, Assumption 2, Assumption 3, and Assumption 4 hold. Then, the variance-covariance matrix $\mathbf{\Sigma}(\mathbf{G})$ of the equilibrium efforts is the sum of an idiosyncratic component $\mathbf{\Sigma_I}$, a uniform component $\mathbf{\Sigma_U}$, and a network component $\mathbf{\Sigma_N}$, that is,*

$$\mathbf{\Sigma}(\mathbf{G}) = \left(\frac{\sigma}{\gamma_{min} - \gamma}\right)^2 \left[\mathbf{\Sigma_I} + \mathbf{\Sigma_U} + \mathbf{\Sigma_N}\right]. \tag{10}$$

[4] A dynamic model would yield different harmonics depending on the network structure of the economy and would permit the spectral decomposition of volatility. We leave the dynamic formulation to future research.

In particular,

$$\boldsymbol{\Sigma}_{\mathbf{I}} \equiv \mathbf{MIM} = \mathbf{M}^2,$$

where $\mathbf{M}$ *is defined in (5),*

$$\boldsymbol{\Sigma}_{\mathbf{U}} \equiv (B\gamma_{min})^2 \left[\bar{1}^T \mathbf{M}^2 \bar{1}\right] \mathbf{MUM},$$

and

$$\boldsymbol{\Sigma}_{\mathbf{N}} \equiv B\gamma_{min} \mathbf{M} \left[\mathbf{UM} + \mathbf{MU}\right] \mathbf{M},$$

where B *is the scalar defined in Proposition 2.*

Since the presence of a link between two agents implies a correlation in their equilibrium efforts, the variance-covariance matrix of the equilibrium efforts of all agents depends on the adjacency matrix. In particular, it depends on the structure of paths of connections of any length between agents. If any two agents are connected by a path of some degree, their equilibrium efforts will be correlated. The degree of correlation will depend not only on how long the path of connection is but also on the general structure of paths in the whole economy, given that the equilibrium efforts are a result of agents' interactions in equilibrium. The first component $\boldsymbol{\Sigma}_{\mathbf{I}}$ of the variance-covariance matrix measures the volatility of agents that comes directly from the idiosyncratic volatility σ^2. If there were no links, this component would simply be $\mathbf{I}$ and the other components would not exist, as we discuss in Example 2. The only volatility would be the idiosyncratic volatility and aggregate volatility would decrease with the number of agents according to the LLN. Given that there are links, the idiosyncratic volatility is distributed across all agents depending on the existence of paths of connection, that is, according to $\mathbf{M}$. The second component $\boldsymbol{\Sigma}_{\mathbf{U}}$ of the variance-covariance matrix simply scales up or down the correlation across agents. The higher the number and the intensity of connections, the higher the scalar $\left[\bar{1}^T \mathbf{M}^2 \bar{1}\right]$ that multiplies the $N \times N$ matrix of ones, $\mathbf{U}$. Any uniform intensity of the network is filtered by the matrix $\mathbf{M}$ of paths of connections. The third component $\boldsymbol{\Sigma}_{\mathbf{N}}$ of the variance-covariance matrix takes into account the structure of the network. The matrix $[\mathbf{UM} + \mathbf{MU}]$ differs across pairs of agents, since its (i,j)-th element is

$$\sum_{k \in \mathcal{N}} m_{ik} + \sum_{k' \in \mathcal{N}} m_{k'j} = \sum_{k \in \mathcal{N}} m_{ik} + \sum_{k' \in \mathcal{N}} m_{jk'} = \sum_{k \in \mathcal{N}} (m_{ik} + m_{jk}).$$

In other words, the correlation between two agents i and j depends on the centralities of both Agent i and Agent j. If both are central to the overall network structure, the correlation of their efforts is likely to be high given that the likelihood of a path of connection between the two agents is high. If the two agents are more peripheral, the correlation is lower. Again, any pattern of correlation measured by $\mathbf{\Sigma_N}$ is filtered by the actual structure of the connections $\mathbf{M}$.

The analysis of the variance-covariance matrix sheds light on the composition of the aggregate volatility.

Definition 3. The aggregate volatility $\sigma_Y^2(\mathbf{G})$ is the scalar given by

$$\sigma_Y^2(\mathbf{G}) \equiv E\left[\left(\frac{1}{N}\sum_{i=1}^{N}(q_i - E[q_i])\right)^2\right],$$

where the expectation operator $E[\cdot]$ is defined over the probability distribution of $\tilde{\varepsilon}_i$ for every i in $\mathscr{N}$.

The aggregate volatility measures the expected square of the average deviation of the equilibrium efforts from their expected level. This definition of aggregate volatility corresponds to a norm of the diagonal elements of the variance-covariance matrix under the assumption of underlying iid shocks. Consider the variance of the random process $\bar{\varepsilon}_i$. Given that $\tilde{\varepsilon}_i$ is identically distributed for every i in $\mathscr{N}$, the idiosyncratic volatility (variance) is the same through time (across all agents) and equal to $\sigma^2 \geq 0$. The aggregare volatility $\sigma_Y^2(\mathbf{G})$ of equilibrium efforts will depend on the idiosyncratic volatility σ^2 but also on the number N of agents and on whether these agents are linked through some strategic interaction, that is, on $\mathbf{G}$.

Example 2. Suppose that Assumption 1 holds and that there are no strategic interactions between agents, that is, $\gamma_{ij} = 0$ for every $i \neq j$. Note that this condition is incompatible with Assumption 2 and that therefore we cannot decompose Γ into its three components. Nevertheless, (2) boils down to

$$-\gamma\bar{q} = \bar{\varepsilon}.$$

Then, $q_i^* = -\varepsilon_i/\gamma > 0$ for every i in $\mathscr{N}$ and the variance-covariance matrix is simply

$$\mathbf{\Sigma}(\mathbf{G}) = (\sigma/\gamma)^2\mathbf{I},$$

that is, an $N \times N$ diagonal matrix with $(\sigma/\gamma)^2$ on all diagonal entries. The aggregate volatility instead is

$$\sigma_Y^2(\mathbf{G}) = \frac{(\sigma/\gamma)^2}{N}.$$

Besides the usual negative effect of the payoff concavity γ on aggregate volatility, when the shocks are iid the aggregate volatility decreases as the number N of agents increases, as predicted by the Law of Large Numbers (LLN). This is the standard diversification argument, that is, idiosyncratic shocks do not transmit to the aggregate level because they cancel each other out as the number of agents grows. Hence, as long as the number of agents is high enough, the average effort is not affected by the idiosyncratic volatility and the resulting aggregate volatility is close to zero.

Since the measure of the aggregate volatility corresponds to a norm of the diagonal elements of the variance-covariance matrix, also the aggregate volatility can be decomposed into three components, each of them reflecting a different aspect of the (static) propagation of shocks.

Proposition 5. *Suppose Assumption 1, Assumption 2, Assumption 3, and Assumption 4 hold. Then, the aggregate volatility $\sigma_Y(\mathbf{G})$ of equilibrium efforts is the product of an idiosyncratic component $\sigma_{\mathbf{I}}$, a uniform component $\sigma_{\mathbf{U}}$, and a network component $\sigma_{\mathbf{N}}$, that is,*

$$\sigma_Y(\mathbf{G}) = \sigma_{\mathbf{I}}\sigma_{\mathbf{U}}\sigma_{\mathbf{G}}. \tag{11}$$

In particular,

$$\sigma_{\mathbf{I}} \equiv \frac{\sigma^2}{N},$$

$$\sigma_{\mathbf{U}} \equiv B^2 = \left(\frac{1}{\gamma_{min} - \gamma - \gamma_{min} b(\bar{1})}\right)^2,$$

and

$$\sigma_{\mathbf{N}} \equiv \frac{1}{N}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sum_{l\in\mathcal{N}} v_{il}v_{jl}\tilde{\lambda}_l^2.$$

The idiosyncratic component $\sigma_{\mathbf{I}}$ of aggregate volatility represents the standard diversification argument. The aggregate volatility is a function of the idiosyncratic volatility σ^2 and decreases at rate $\sqrt{N}$ as the number N of agents increases. The second component $\sigma_{\mathbf{U}}$ captures the idea that the aggregate volatility decreases as the number of links increases, other things equal. By adding any link between a pair of agents, there is at least one additional path of connection between all agents of the economy, that is, the network centrality of some agent necessarily increases. Hence, the sum of all unweighted network centralities $b(\bar{1})$ increases and therefore the uniform component of aggregate volatility decreases. The third component $\sigma_{\mathbf{N}}$ of aggregate volatility refers more directly to how the network is structured.

Recall that the eigendecomposition of $\mathbf{M}$ identifies a rotation of the space spanned by the original adjacency matrix $\mathbf{G}$. We call each dimension of the new space a latent group, and each latent group l accounts for a certain portion of the indirect connections across agents. This portion is measured by the transformed eigenvalue $\tilde{\lambda}_l$ defined in Proposition 3, for every l in $\mathscr{N}$. The contribution of Agent i to latent group l is measured by the entry v_{il} in the corresponding eigenvector. On the one hand, $\tilde{\lambda}_l$ is strictly positive and it is a convex function of the original eigenvalue λ_l, for every l in $\mathscr{N}$. On the other hand, the diagonal elements of $\mathbf{G}$ are nil, so the trace of $\mathbf{G}$ is zero. This implies that the sum of all eigenvalues of $\mathbf{G}$ is nil as well, that is,

$$\sum_{l \in \mathscr{N}} \lambda_l = tr(\mathbf{\Lambda}) = 0,$$

where $tr(\mathbf{\Lambda})$ is the trace of $\mathbf{\Lambda}$.[5] Hence, any increase in the standard deviation of the eigenvalues of $\mathbf{G}$ corresponds to a mean-preserving spread of the eigenvalues.

Proposition 6. *Suppose Assumption 1, Assumption 2, Assumption 3, and Assumption 4 hold. Then, the network component $\sigma_{\mathbf{N}}$ of aggregate volatility increases if the standard deviation of the eigenvalues of* $\mathbf{G}$ *increases, other things equal.*

The aggregate volatility of a networked economy has three components, each of them capturing a different aspect of the network structure. The idiosyncratic component $\sigma_{\mathbf{I}}$ reflects the standard diversification mechanism of the Law of Large Numbers, the uniform component $\sigma_{\mathbf{U}}$ decreases as the number of links increases and more in general as the number of paths of connection of any length increases, and the network component $\sigma_{\mathbf{N}}$ increases as the concentration of network increases. The standard deviation of the eigenvalues of $\mathbf{G}$, which according to Proposition 6 drives the level of the network component of aggregate volatility, represents the dispersion of the magnitudes of the latent groups of the adjacency matrix. The more condensed the network, the more of the overall dispersion in equilibrium efforts can be accounted by just a few latent groups, with each agent contributing differently to these latent groups. The more evenly distributed into different loosely connected groups (or even into completely isolated components) the agents, the less dispersed the eigenvalues and the lower the network component of aggregate volatility. On the one hand, more links imply more smoothing out of the idiosyncratic volatility before the equilibrium interaction, as measured

[5]This is due to the fact that the trace is similarity-invariant, that is, $tr(\mathbf{G}) = tr(\mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1}) = tr(\mathbf{\Lambda}\mathbf{V}^{-1}\mathbf{V}) = tr(\mathbf{\Lambda})$.

by the uniform component $\sigma_{\mathbf{U}}$ of aggregate volatility. On the other hand, if the links are organized so that agents are grouped together into just a few latent groups, the possibility of an aggregate fluctuation arising as a consequence of an idiosyncratic shock is higher, since the contagion to a relevant portion of agents from the same idiosyncratic shock is more likely. Hence, there are two countervailing effects of the presence of linkages across agents reflected by the uniform component and the network component. On the one hand, the more connected the network, the lower the aggregate volatility. On the other hand, the more concentrated the network, the higher the aggregate volatility. In Appendix B we provide an intuition for this decomposition.

4 An application: firms and sectors

4.1 The origins of fluctuations: grains and networks

A recent stream of literature tries to provide a microfoundation for the existence of aggregate shocks as the result of idiosyncratic volatility. We refer to this stream as the granular hypothesis (GH) literature, as in Gabaix [2011]. In this literature there are at least two types of "grains" from which aggregate fluctuations may originate. First, idiosyncratic shocks to firms have the potential to trigger aggregate fluctuations. For example, Gabaix [2011] notices that the empirical size distribution of firms is fat-tailed, and more specifically follows a power law. Hence, the homogeneity in size necessary for the standard LLN to apply does not hold in the case of firm-specific shocks. As a consequence, the variability in sales of the 100 top US firms can explain as much as 1/3 of aggregate volatility. Other studies that look at firms to at least partially explain aggregate fluctuations are, among others, Jovanovic [1987], Durlauf [1993], Bak et al. [1993], and Nirei [2006]. Second, idiosyncratic shocks to sectors can contribute to the origin of aggregate fluctuations as well. The seminal paper of Long and Plosser [1983] started a series of contributions that use multisectoral RBC models to generate aggregate volatility. Some examples are Horvath [1998], Horvath [2000], Conley and Dupor [2003], Dupor [1999], Shea [2002], Scheinkman and Woodford Scheinkman and Woodford [1994], Carvalho and Gabaix [2010], and Acemoglu et al. [2011]. The main idea is that if we look at the input-output tables we can notice some sectors that provide intermediate inputs to almost all other sectors of the economy. The presence of these hub sectors make idiosyncratic shocks that would normally be irrelevant propagate to the aggregate level. More recent works like Carvalho [2007], Acemoglu et al. [2010], and Acemoglu et al. [2011] exploit the same intuition but with the explicit use of network theory tools applied to the analysis of the input-output tables, linking aggregate volatility to the network structure of intersectoral trade.

These explanations of the micro-origins of aggregate fluctuations may be overlapping. On the one hand, the existence of big firms allows for the transmission of micro-level shocks to macro-level variables. On the other hand, aggregate fluctuations can originate from idiosyncratic shocks to sectors that spread through the economy via the input-output structure of production. Aggregate fluctuations are therefore either alternatively or jointly facilitated by idiosyncratic shocks to sectors and firms. In this section we consider sector- and firm-specific volatilities as two aggregations of the same fundamental uncertainty, the volatility coming from idiosyncratic shocks to establishments. The baseline intuition of our model is that big firms are not sector-specific. In other words, firms are intersectoral networks of sector-specific establishments. Each establishment produces a sector-specific commodity, and it can be part of a firm. We make shocks originate at the establishment level, so that firm- or sector-wide fluctuations result as aggregations of multiple establishment-specific shocks. Moreover, sectors per se are linked through input-output relations. Hence, the network of intersectoral linkages and the network of proprietary relations overlap and constitute the network structure of the economy.

4.2 A stylized framework

We present a static economy with a final sector and multiple intermediate sectors. The final sector aggregates multiple intermediate goods in order to produce a unique final consumption good. On the preference side, a continuum of households consumes the unique consumption good. On the production side, the final sector is populated by a continuum of perfectly competitive and identical final good producers. They all produce the same final good using different intermediate good varieties. The varieties can be more or less complementary in the production of the final good. These production complementarities across varieties of intermediate goods reflect the input-output structure of production and constitute the network structure of intersectoral linkages. Within each intermediate sector, a finite number of sector-specific establishments compete in quantities for the sector-specific demand expressed in the final good sector. We suppose that each establishment can have proprietary realtions with other establishments. If a link exists, then the two linked establishments are part of the same firm.[6] In

[6]In the real world firms have several establishments that operate in the same sector. We can think of the establishments here as an agglomeration of all establishments belonging

the terminology of network theory, the establishments represent the vertices of a graph where we can note different components, that is, distinct path-connected subnetworks of establishments. Each component is a firm. The network structure of proprietary relations overlaps independently with the network structure of intersectoral linkages.

Each household owns a symmetric share of the profits realized on the production side. Each final good producer aggregates all intermediate good varieties into a unique final good. There are $S \in \mathbb{N}$ intermediate commodities produced in S intermediate sectors, with $s \in \mathscr{S} = \{1, \cdots, S\}$.[7] Each sector s is populated by n_s establishments, and each establishment i in sector s produces an undifferentiated quantity q_i of good s competing à la Cournot with the other establishments within the same sector. The total production Q_s of intermediate good s in $\mathscr{S}$ is simply the sum of the production of all establishments in sector s, that is,

$$Q_s = \sum_{i \in \mathscr{N}_s} q_i, \tag{12}$$

where $\mathscr{N}_s$ is the set of establishments that operate in sector s. The set $\mathscr{N}$ of all establishments of the economy is $\mathscr{N} \equiv \bigcup_{s \in \mathscr{S}} \mathscr{N}_s$, and the total number N of establishments in the economy is $N \equiv \sum_{s \in \mathscr{S}} n_s$. The production function of the final good producer is linear-quadratic, that is,

$$Q \equiv \sum_{s \in \mathscr{S}} \alpha_s Q_s - \frac{1}{2} \sum_{s \in \mathscr{S}} \beta Q_s^2 + \sum_{s \in \mathscr{S}} \sum_{s' \neq s} \beta_{ss'} Q_s Q_{s'},$$

where $\alpha_s > 0$, $\beta > 0$, and $\beta_{ss'} \in [0, 1/(S-1)]$ for all s and s' in $\mathscr{S}$.[8] The value of α_s measures the size of sector s, that is, α_s is high enough so that the final good production function is strictly increasing and strictly concave separately with respect to each intermediate good quantity Q_s. The parameter β measures the concavity of the final good production function separately with respect to each good, and $\beta_{ss'}$ parametrizes the degree of technological complementarity between different intermediate goods. A possible interpretation for these technological complementarities is the direct requirement values we can extract from an input-output table. If $\beta_{ss'} = 0$, the goods are independent, that is, the intermediates from sector s are not necessary for the production in sector s'. If $0 < \beta_{ss'} \leq 1/(S-1)$, the goods are complements at different degrees, that is, for each intermediate unit of sector s we need

to the same firm that operate in the same sector.

[7] From now on, we use the term commodity and sector interchangeably because we assume that only sector s produces commodity s.

[8] We omit the index for the generic final good producer.

a certain amount of intermediate goods produced in sector s'. The upper bound on the possible values of $\beta_{ss'}$ avoids aggregate increasing returns in the model. For simplicity, we assume that $\beta_{ss'} = \beta_{s's}$, that is, the relation of complementarity between two good types is symmetric. This assumption relates to Assumption 1.[9]

There is a unique final good, so the problem of the final good producer is

$$\max_{\{Q_s\}_{s\in\mathscr{S}}} \sum_{s\in\mathscr{S}} \alpha_s Q_s - \frac{1}{2}\sum_{s\in\mathscr{S}} \beta Q_s^2 + \sum_{s\in\mathscr{S}}\sum_{s'\neq s} \beta_{ss'} Q_s Q_{s'} - \sum_{s\in\mathscr{S}} p_s Q_s, \tag{13}$$

taking the price p_s of intermediate good s as given. The FOC yields a linear inverse demand function for each commodity, that is,

$$p_s = \alpha_s - \beta Q_s + \sum_{s'\neq s} \beta_{ss'} Q_{s'}. \tag{14}$$

For a discussion of the parameter values for which we have positive prices and quantities, see Bloch [1995]. Within each intermediate sector s there are n_s establishments. They compete à la Cournot and share the sector-specific demand expressed by the final good sector. The maximization problem of establishment i in intermediate sector s is

$$\max_{q_i} \quad \pi_i^s \equiv p_s q_i - m_i q_i, \tag{15}$$

subject to (14), where m_i is the marginal cost of producing one unit of intermediate s. The marginal cost m_i of establishment i in sector s is

$$m_i = \frac{\delta}{2} q_i - \sum_{j\in\mathscr{N}\setminus\mathscr{N}_s} \delta_{ij} q_j - \xi_i, \tag{16}$$

where $\delta > 0$ parameterizes the concavity of each establishment's profits in own production. The idiosyncratic component ξ_i is the realization of an iid random variable with mean μ and finite variance σ^2, as in Assumption 4. The element $-\sum_{j\in\mathscr{N}\setminus\mathscr{N}_s} \delta_{ij} q_j$ represents how the marginal cost of an establishment decreases linearly with the production of the establishments that are linked to it. If i does not have any proprietary relation with j, then $\delta_{ij} = 0$. If instead i owns a share of j or viceversa, then $\delta_{ij} > 0$. We can think of δ_{ij} as proportional to how much of the total shares of j i owns, or

[9] In reality, the input-output table are asymmetric at any aggregation level, and Acemoglu et al. [2011] show that this asymmetry has important implications for aggregate volatility. We discuss the case of asymmetric strategic relations in Remark 1.

viceversa. By construction, $\delta_{ij} = \delta_{ji}$ and $0 \leq \delta_{ij} \leq 1$ for every i and j in $\mathscr{N}$. In particular, $\delta_{ii} = 0$.[10]

There exists a proprietary relations (a link) between i and j if establishment i owns some shares of establishment j, or viceversa. Hence, if i and j are part of the same firm, then there exists a series of bilateral proprietary relations that connects i to j. Figure 2 reports an example of firm composed of different establishments linked to each other through proprietary relations. Note that according to our definition of proprietary relation, two establishments do not need to share a direct link in order to belong to the same conglomerate firm. Moreover, we suppose that a firm cannot have more than one establishment in each sector. This is not the case in the real world where firms tend to have several establishments within the same sector, especially if the aggregation level is high enough. We can think of this assumption as implying that the establishments in our model are alerady the aggregation of all establishments belonging to the same firm that operate in a given sector. This assumption implies that the maximum degree of any establishment i is $S - 1$, that is, the number of sectors other than its own. Since a firm can have at most one establishment in each sector, two establishments that belong to the same firm cannot compete with one another within the same sector. Hence, if i and j operate in sector s, then $\delta_{ij} = 0$. Moreover, $\delta_{ij} = 0$ also if there exists a path of proprietary relations between i and j such that an establishment in the middle of the path belongs to either the sector of firm i or the sector of firm j.

Our set-up and specifically (15) and (16) imply a positive correlation in production levels among establishments linked by proprietary relations. One way to think about this is that establishments are in general credit constrained. Hence, whenever an establishment within the same firm is hit by a positive efficiency shock, the cash flow generated by this establishment propagates through the corporate structure, relaxing the constraints of all neighboring establishments. This generates a strategic complementarity in the actions of different agents of the same type that we analyzed in the general framework.

According to (12), (14), (15), and (16), the payoff function of establishment i in sector s is

$$\pi_i^s = \left(\alpha_s - \beta \sum_{j \in \mathscr{N}_s} q_j + \sum_{s' \neq s} \beta_{ss'} \sum_{j \in \mathscr{N}_{s'}} q_j \right) q_i - \left(\frac{\delta}{2} q_i - \sum_{j \in \mathscr{F} \setminus \mathscr{F}_s} \delta_{ij} q_j - \xi_i \right) q_i,$$

[10] The case of cross-ownerships is very limited by specific accounting regulations in most developed countries. In general, the permitted amounts of cross-ownerships are negligible. Moreover, self-links are ruled out by the fact that in most real-world regulatory settings a juridical entity cannot own itself, so $\delta_{ii} = 0$ for every i in $\mathscr{N}$.

which implies the FOC

$$(\delta+\beta)q_i+\beta\sum_{j\in N_s}q_j-\sum_{s'\neq s}\beta_{ss'}\left(\sum_{j\in N_{s'}}q_j\right)-\sum_{\substack{j\in\mathscr{N}\\ j\notin\mathscr{N}_s}}\delta_{ij}q_j=\alpha_s+\xi_i.$$

Hence, the equilibrium allocation satisfies the system of equations in matrix form

$$\left[(\delta+\beta)\mathbf{I}-\left(\hat{\mathbf{B}}+\mathbf{\Delta}\right)\right]\bar{q}^*=\bar{\alpha}+\bar{\xi}, \tag{17}$$

where $\bar{q}^*$ is a vector of length N whose i-th element is the equilibrium production q_i^* of establishment i. We order the establishments by sector, so the first $n_s=n_1$ elements of $\bar{q}^*$ refer to the equilibrium production of the establishments in sector $s=1$, the following $n_s=n_2$ elements of $\bar{q}^*$ refer to establishments in sector $s=2$, and so on, for all s in $\mathscr{S}$. Consequently, $\hat{\mathbf{B}}\equiv[B_{ss'}]$ is an $N\times N$ block matrix whose (s,s')-th block $B_{ss'}$ is a $n_s\times n_{s'}$ matrix. Every diagonal block B_{ss} has all entries equal to $-\beta$, while every (s,s')-th block $B_{ss'}$ for $s\neq s'$ has all entries equal to $\beta_{ss'}$. The matrix $\mathbf{\Delta}\equiv[\delta_{ij}]$ is the $N\times N$ matrix of proprietary relations, so the (i,j)-th element of $\mathbf{\Delta}$ is δ_{ij} for every i and j in $\mathscr{N}$. The vector $\bar{\alpha}\equiv[\alpha_s]$ is the block vector of length N with S blocks, where each block s is of length n_s and has all entries equal to α_s, for every s in $\mathscr{S}$. The vector $\bar{\xi}\equiv[\xi_i]$ is a vector of length N whose i-th element is ξ_i. The FOCs in (17) reflect the same equilibrium interaction as the FOCs in (2). In fact, if we substitute $\left[(\delta+\beta)\mathbf{I}-\left(\hat{\mathbf{B}}+\mathbf{\Delta}\right)\right]$ for $-\mathbf{\Gamma}$ and $\bar{\alpha}+\bar{\xi}$ for $\bar{\varepsilon}$, we obtain (2) from (17). We can decompose $\left[(\delta+\beta)\mathbf{I}-\left(\hat{\mathbf{B}}+\mathbf{\Delta}\right)\right]$ as in (3) such that

$$\left[(\delta+\beta)\mathbf{I}-(\overline{\theta}+\beta)\mathbf{G}\right]\bar{q}^*=\bar{\alpha}+\bar{\xi}-\beta\left(\sum_{i\in\mathscr{N}}q_i^*\right)\bar{1},$$

where $\overline{\theta}$ is the maximal entry of $\hat{\mathbf{B}}+\mathbf{\Delta}$ and the generic element g_{ij} of $\mathbf{G}$ is

$$g_{ij}\equiv\frac{\theta_{ij}+\beta}{\overline{\theta}+\beta}\in[0,1],$$

the element θ_{ij} being the (i,j)-th element of $\hat{\mathbf{B}}+\mathbf{\Delta}$. Thus, we can characterize the equilibrium allocation as in Proposition 1 and given that Assumption 1, Assumption 2, and Assumption 4 hold by construction we can apply all the propositions of the previous sections, provided that Assumption 3 holds.[11]

[11] For Assumption 3 to hold we simply need β or δ large enough.

4.3 The numerical results

We use our stylized framework to conduct a numerical exercise with US data. First, we use BEA data for sectoral sizes $\bar{\alpha}$ and intersectoral linkages $\hat{\mathbf{B}}$, and US Census Bureau data for the number n_s of establishments for each intermediate sector s. Second, we use a random graph algorithm to create a network structure $\mathbf{\Lambda}$ of proprietary relations that is consistent with our stylized framework and that reproduces some aggregate features of the diversification of US firms across sectors. Third, we check that the joint adoption of actual and simulated data yields aggregate statistics that are consistent with some key aggregate features of the US economy like the size distribution of establishments. Fourth, we conduct a counterfactual exercise to give a numerical intuition of the relation between the network structure of an economy and its aggregate volatility.

First, we derive data on the US economy. We take into account the year 2002. We choose to use the highest aggregation level for the BEA data and the US Census Bureau, which accounts for 14 sectors once we exclude government and the residual category. Thus, $S = 14$ and $\mathscr{S}$ is the ordered set of IO codes from 1 to 14. Table 1 reports the sectors to which each code corresponds. For each sector s, we use its gross output as a proxy for α_s and the number of employer establishments as a proxy for n_s. Moreover, we derive the intersectoral linkage $\beta_{ss'}$ for each pair of sectors (s, s') from a transformation of BEA's direct requirements table. In general, the direct requirements table is asymmetric and this feature has important consequences for the aggregate volatility.[12] The numerical results below hold also in case we simply use the (s, s')-th entry in the input-output table as a proxy for $\beta_{ss'}$, and the theoretical predictions would not change relevantly either as discussed in Remark 1. Nevertheless, in order to be consistent with the simplified framework implied by Assumption 1, we apply the following transformation on the input-output table. First, we call $\tilde{\beta}_{ss'}$ the (s, s')-th entry in the input-output table. Then, we derive our symmetric proxy for $\beta_{ss'}$ as

$$\beta_{ss'} = \beta_{s's} \equiv \frac{1}{S-1} \frac{\tilde{\beta}_{ss'} + \tilde{\beta}_{s's}}{\max_{(s,s')\in\mathscr{S}^2}\{\tilde{\beta}_{ss'} + \tilde{\beta}_{s's}\}},$$

for every $s \neq s'$ in $\mathscr{S}$. In other words, we add the two corresponding entries in the input-output table and we normalize the sum in order to obtain values of $\beta_{ss'}$ in the interval $[0, 1/(S-1)]$. The intuition for this transformation is that we capture both the upstream and the downstream diffusion of shocks. Shocks to the production of a certain intermediate can affect the

[12] See for example Acemoglu et al. [2011].

production of other intermediates either downstream, that is, from a sector that supplies the intermediate to other sectors that demand the intermediate, or upstream, that is, from a sector that demands the intermediate to other sectors that supply the intermediate. By summing both entries of the input-output table, we have a partial proxy for this transmission mechanism. We must though bear in mind that this transformation dismisses part the information contained in the input-output table and it is simple meant to simplify the exposition. Moreover, we ignore the diagonal elements of the input-output table. Table 2 reports the values of our $\beta_{ss'}$'s. If we considered this table as the adjancency matrix of the network of intersectoral linkages, the network structure would look like in Figure 3 and Figure 4.

Second, we create the proprietary relations, that is, the matrix $\mathbf{\Delta}$. There exist databases that have detailed information on the proprietary relations and the sectoral specialization of each establishment. One example is the WorldBase database compiled by Dun & Bradstreet for 2005 and used by Alfaro and Chen [2009], among others. A thorough empirical exercise would require the use of such a database. Given the illustrative purpose of our numerical exercise, our alternative is to construct a random graph algorithm that generates a network of proprietary relations consistent with the assumptions of the theoretical framework and mimicking key aggregate characteristics of the intersectoral diversification of US firms. The key assumption we adopt to construct the network of proprietary relations is that firms cannot have more than one establishment in each sector. This assumption implies not only that there cannot exist a path of proprietary relations between two establishments that operate in the same sector, but also that any path between two establishments i and j cannot pass through an intermediate establishment k that operates in the sector of either i or j. As a consequence, there exists an upper bound on the degree, that is, on the number of proprietary relations, that an establishment can have. This upper bound is the number of sectors where the establishment does not operate, that is, $S - 1 = 13$. Moreover, an establishment cannot have links with two different establishments that operate in the same sector, because that would mean that they belong to the same firm. These limitations lead to the network structure reported in Figure 5, where each node of the graph is an establishment and each component, that is, each path-connected group of nodes, is a firm. The components that arise from this algorithm have different sizes. Components with several nodes are firms that have establishments in several sectors, that is, firms that are diversified across sectors. Components with only one or few nodes are instead firms specialized in one or few businesses. Hence, the size distribution of components, that is, the frequency at which components of different sizes occur, mirrors the distribution of firms' intersectoral diversification. Figure 6 reports this distribution as it arises from the algorithm we use.[13] The reason for the emergence of such a distribution is the interaction between the upper bound on firms' diversification and the fixed number of establishments per sector. The shape of the distribution that we obtain through our algorithm resembles the actual diversification distribution for the Fortune 500 firms for the years 1980 and 1990 reported in Figure 7 taken from Davis et al. [1994].

Third, we merge the actual data on intersectoral linkages with the generated data on proprietary relations in order to obtain the (normalized) network structure of the economy. We provide a graphical representation of the resulting adjacency matrix **G** in Figure 8. Note that the intersectoral linkages are diffuse across establishments but not particularly intense, while the proprietary relations are relatively sparse with respect to the number of establishments in the economy but quite intense individually. Since we have the actual data on the α_s's, the n_s's, and the $\beta_{ss'}$'s, and the generated data on the δ_{ij}'s, we can derive values for δ and β that respect Assumption 2 and Assumption 3. This permits us to compute the Bonacich centrality of each establishment as in Definition 1, which we represent for the case of equal weights in Figure 9. Note that the most path-central establishments in the US economy appear to be the establishments that operate in the "manufacturing" sector. This is due to two reasons. On the one hand, the manufacturing sector provides considerable amounts of intermediate goods to all the other sectors of the economy, so that it is central in the network of intersectoral linkages. On the other hand, there are only a few big establishments in the manufacturing sector, which means low competition and high likelihood to be part of all firms that are diversified through many sectors. This intepretation is confirmed by the fact that two sectors similar in size and intersectoral linkages to the "manufacturing" sector, like the "professional and business services" sector and the "finance, insurance, real estate, rental, and leasing" sector, are not populated by establishments with centrality measures comparable with the manufacturing sector. This is due to the fact that these two sectors are populated by high numbers of establishments. In order to compute the vector $\bar{q}^*$ of production at equilibrium that solves (17), we only need a realization of $\bar{\xi}$ which we draw from an iid uniform distribution between 0 and 1 which respects Assumption 4. Thus, we can compute the equilibrium production q_i^* of each establishment i given the network structure, which is associable with the amount of sales that each establishment realizes. Sup-

[13] A future exercise that targeted the average firm diversification may use the theoretical results of Newman et al. [2001] on the relation between the degree distribution and the component size distribution.

pose we consider the sales as a proxy for the size of an establishment. Then, if we analyze the size distribution of these establishments, that is, the frequency at which establishments of different sizes occur, we obtain Figure 10. Our exercise generates a size distribution of establishments, and of firms if we were to aggregate production at the firms level, that is similar in shape to the power law distribution that we observe in the data, as described by Luttmer [2007] among many others. This is not the target of the algorithm used to generate the network structure of proprietary relations, and should be looked at as an encouraging collateral result.

Fourth, we conduct a counterfactual exercise in order to understand the relation between the network structure of the economy and its aggregate volatility. The results of this exercise are reported in Table 3. In particular, we compute the level of aggregate volatility $\sigma_Y(\mathbf{G})$ as defined in Definition 3 for different cases. In Case 1, we consider a benchmark economy with neither intersectoral linkages nor proprietary relations. This economy is similar to the one described in Example 2. We compute what would be the aggregate volatility in this case and normalize its value to 1. In Case 2, we introduce the intersectoral linkages based on the US data that we derived above, and we compute the corresponding aggregate volatility. As predicted in Proposition 5, the presence of linkages decreases the second component $\sigma_{\mathbf{U}}$ due to the smoothing of idiosyncratic shocks across establishments within the same paths of connection that occurs before the equilibrium interaction. The drop in volatility implied by the presence of intersectoral linkages is around 10%. This is due to the decrease in the second component $\sigma_{\mathbf{U}}$ of aggregate volatility. Nevertheless, the third component $\sigma_{\mathbf{G}}$ increases because the concentration of the network increases with the introduction of the intersectoral linkages. We pass from a situation with no linkages to a situation where we can distinguish between more central and less central sectors, as Figure 3 and Figure 4 illustrate. In Case 3, we introduce the proprietary relations generated through the algorithm described above, that is, we compute the aggregate volatility of our fully networked economy. The volatility decreases again but the drop is not as relevant as in Case 2. The reason for such a difference in magnitude is mainly due to the sparseness of the network of proprietary relations with respect to the overall number of potential linkages across establishments. Moreover, the presence of firms that are diversified across sectors adds paths of connections across establishments in different sectors, partially increasing the concentration of the network around the three central sectors in the network of intersectoral linkages. This is captured by the third component $\sigma_{\mathbf{N}}$ which increases as predicted by Proposition 6.

5 Conclusion

We propose a stylized framework for the analysis of the aggregate volatility in a networked economy. Aggregate fluctuations can be the result of idiosyncratic shocks that transmit to the aggregate level through the strategic complementarities across agents and the market interaction. We show that aggregate volatility depends on the network structure of the economy. On the one hand, the more connected the network the lower the aggregate volatility, because the existence of a linkage partially diversifies the idiosyncratic risk. On the other hand, the more concentrated the network the higher the volatility, because the high centrality of some agents within the network structure makes the economy susceptible to the propagation of micro shocks through the economy.

This paper helps explaining how the observed correlations across agents in microeconomic data can be partially accounted for by knowing the network structure of the economy. Part of the autocorrelation across agents may be simply due to strategic complementarities across agents that translate perfectly independent shocks into correlated equilibrium outcomes. This paper suggests analytical tools to identify how much of the total correlation between the actions of two agents is due to their relative positions within the network structure of the economy.

Our contribution focuses on the analysis of aggregate volatility taking as given the network structure of the economy. There are at least two ways in which we can proceed further in this line of research. First, we can consider non-interior solutions. Our stylized set-up only admits interior solutions. Nevertheless, an arguably relevant part of the aggregate fluctuations consists of corner solutions that propagate through the economy. For example, a negative shock to an establishment might induce its parent company to shut down the establishment once and for all rather than conveying resources from other establishments in order to buffer the impact of the negative shock. Second, we can consider equilibrium networks. In our set-up the existence of a link is part of the technology of the model. If our bilateral relations were the result of an equilibrium interaction, we could understand how the network structure of the economy responds to different shocks and consequently have a better understanding of propagation mechanisms and aggregate volatility. In this way, we could understand the reverse causality between aggregate volatility and network structure. For example, we could investigate whether a drop in aggregate volatility like the Great Moderation triggered a shift in the network structure like the change of US top firms' diversification pattern presented in Figure 7.

The formation of the network structure is a promising line of future research. We could set up a two-stage game where the model presented so far would represent the second stage, being the first stage devoted to network formation. There would be a trade-off between the cost of forming a link and the benefits of contributing to a latent group. Agents would act strategically and decide with whom to share a link depending on the potential equilibrium outcomes in the second stage. Another possibility is a dynamic network formation model as in König et al. [2009], where the timing of the two stages is inverted. First, the agents realize their equilibrium production given previous period's network. Second, given the equilibrium result they choose which other agents to share a link with. With a payoff structure similar to ours, different network structures arise and it is possible to identify stationary network structures that follow the properties of nested split graphs.[14] These networks are also called interlink stars in Goyal and Joshi [2003] and Goyal et al. [2006]. Their main property is the core-periphery structure, which partially reminds us of the ownership structure of firms, with a central parent establishment that specializes the firm into a core business and peripheral subsidiaries that diversify the production to smooth out sector-specific fluctuations.

The model has also several policy implications. For instance, future work may use our framework to instruct discretionary policy interventions. If idiosyncratic shocks can transmit to the aggregate level and can generate aggregate fluctuations, then discretionary policy interventions may play a role in stabilizing output. If the public authority bails out a troubled establishment, it stabilizes the performance of all the establishments directly or indirectly connected to it by paths of bilateral ties. Each intervention involves a public cost, as the bailout of AIG exemplified. Hence, a key question is which economic agent we should stabilize first in order to obtain the most substantial drop in aggregate volatility. Our model suggests that the establishment to be stabilized is the most central establishment of the economy. Moreover, the mapping between centrality and volatility suggests a practical way of identifying the key establishments to stabilize. The highest eigenvalue of the network matrix identifies the largest latent group. The highest value in the eigenvector that correspond to the highest eigenvalue tells us which establishment contributes the most to aggregate volatility.[15] This method has important analogies with the static Principal Component Analysis (PCA). Future research could explore further the link between centrality measures and PCA in networks.

[14] On nested split graphs see, for example, [Mahadev and Peled, 1995, Chapter 5].

[15] Related works on the importance of the eigendecomposition of the adjacency matrix in identifying key players in the network is, e.g., Bramoullé et al. [2010], Golub and Jackson [2010], and Banerjee et al. [2012]

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Appendix A: Proofs

Proof of Proposition 1. The proof is a variation of Theorem 1 and Remark 1 in Ballester et al. [2006]. □

Proof of Proposition 2. The proof is straightforward once we consider Proposition 1, Definition 1, and (7). □

Proof of Proposition 3. The eigendecomposition in (8) implies that the matrix $\mathbf{M}$ can be expressed as

$$\mathbf{M} = \sum_{k=0}^{+\infty} a^k \mathbf{G}^k = \sum_{k=0}^{+\infty} a^k \left(\mathbf{V} \mathbf{\Lambda}^k \mathbf{V}^{-1} \right) = \mathbf{V} \left(\sum_{k=0}^{+\infty} a^k \mathbf{\Lambda}^k \right) \mathbf{V}^{-1}.$$

Moreover, since $\mathbf{\Lambda}$ is a diagonal matrix, the l-th diagonal element of $\sum_{k=0}^{+\infty} a^k \mathbf{\Lambda}^k$ is

$$\sum_{k=0}^{+\infty} (a\lambda_l)^k ,$$

where a is defined in (4), for every l in $\mathscr{N}$. If Assumption 3 holds, then

$$a\lambda_l \leq a\lambda_{max} < 1,$$

so the l-th element of $\sum_{k=0}^{+\infty} a^k \mathbf{\Lambda}^k$ converges to

$$\sum_{k=0}^{+\infty} (a\lambda_l)^k = \frac{1}{1 - a\lambda_l} > 0,$$

for every l in $\mathscr{N}$. □

Proof of Proposition 4. Let us call

$$b(\bar{x}) \equiv \bar{1}^T \bar{b}(\bar{x}),$$

for every $\bar{x}$ in $\mathbb{R}^N$. Given (6) and Proposition 3, we can express the equilibrium efforts as

$$\bar{q}^* = \frac{1}{\gamma_{min} - \gamma} \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \left[\bar{\varepsilon} + B \gamma_{min} b(\bar{\varepsilon}) \bar{1} \right],$$

where

$$B \equiv \frac{1}{\gamma_{min} - \gamma - \gamma_{min} \bar{1}^T \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \bar{1}}$$

and

$$b(\bar{\varepsilon}) = \sum_{i \in \mathcal{N}} b_i(\bar{\varepsilon}) = \bar{1}^T \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \bar{\varepsilon}$$

are scalars. Then, the expectation of $\bar{q}^*$ is

$$E\left[\bar{q}^*\right] = \frac{1}{\gamma_{min} - \gamma} \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \left[\mu\bar{1} + B\gamma_{min} b(\mu\bar{1})\bar{1}\right],$$

where

$$b(\mu\bar{1}) = \bar{1}^T \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \mu\bar{1}$$

The vector of deviations from the expected values is

$$\bar{q}^* - E\left[\bar{q}^*\right] = \frac{1}{\gamma_{min} - \gamma} \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \left[(\bar{\varepsilon} - \mu\bar{1}) + B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\bar{1}\right], \tag{18}$$

where

$$b(\bar{\varepsilon} - \mu\bar{1}) = \bar{1}^T \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \left(\bar{\varepsilon} - \mu\bar{1}\right).$$

Hence, the variance-covariance matrix is

$$\begin{aligned}
&\mathbf{\Sigma}(\mathbf{G}) = E\left[(\bar{q}^* - E\left[\bar{q}^*\right])(\bar{q}^* - E\left[\bar{q}^*\right])^T\right] \\
&= \left(\frac{1}{\gamma_{min} - \gamma}\right)^2 \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} E\Bigg[\left[(\bar{\varepsilon} - \mu\bar{1}) + B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\bar{1}\right] \\
&\qquad\qquad \left[(\bar{\varepsilon} - \mu\bar{1}) + B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\bar{1}\right]^T\Bigg] \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \\
&= \left(\frac{1}{\gamma_{min} - \gamma}\right)^2 \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} E\Bigg[\left[(\bar{\varepsilon} - \mu\bar{1}) + B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\bar{1}\right] \\
&\qquad\qquad \left[(\bar{\varepsilon} - \mu\bar{1})^T + \left[B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\bar{1}\right]^T\right]\Bigg] \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \\
&= \left(\frac{1}{\gamma_{min} - \gamma}\right)^2 \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} E\Bigg[\left(\bar{\varepsilon} - \mu\bar{1}\right)\left(\bar{\varepsilon} - \mu\bar{1}\right)^T + \\
&\quad + \left(B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\right)^2 \bar{1}\bar{1}^T + \\
&\quad + B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\bar{1}\left(\bar{\varepsilon} - \mu\bar{1}\right)^T + B\gamma_{min} b(\bar{\varepsilon} - \mu\bar{1})\left(\bar{\varepsilon} - \mu\bar{1}\right)\bar{1}^T\Bigg] \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \\
&= \left(\frac{1}{\gamma_{min} - \gamma}\right)^2 \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1} \Bigg[E\left[\left(\bar{\varepsilon} - \mu\bar{1}\right)\left(\bar{\varepsilon} - \mu\bar{1}\right)^T\right] + \\
&\quad + (B\gamma_{min})^2 E\left[\bar{1}^T \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1}\left(\bar{\varepsilon} - \mu\bar{1}\right)\left(\bar{\varepsilon} - \mu\bar{1}\right)^T \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1}\bar{1}\right] \mathbf{U} + \\
&\quad + B\gamma_{min} E\left[\bar{1}^T \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1}\left(\bar{\varepsilon} - \mu\bar{1}\right)\left(\bar{1}\left(\bar{\varepsilon} - \mu\bar{1}\right)^T + \left(\bar{\varepsilon} - \mu\bar{1}\right)\bar{1}^T\right)\right]\Bigg] \mathbf{V} \tilde{\mathbf{\Lambda}} \mathbf{V}^{-1}
\end{aligned}$$

$$= \left(\frac{1}{\gamma_{min}-\gamma}\right)^2 \mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\Bigg[E\left[(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\right]+$$

$$+(B\gamma_{min})^2 E\left[\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\bar{1}\right]\mathbf{U}+$$

$$+B\gamma_{min}E\Big[\bar{1}\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T+$$

$$+(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\bar{1}\bar{1}^T\Big]\Bigg]\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}.$$

$$= \left(\frac{1}{\gamma_{min}-\gamma}\right)^2 \mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\Bigg[E\left[(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\right]+$$

$$+(B\gamma_{min})^2\left[\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}E\left[(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\right]\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\bar{1}\right]\mathbf{U}+$$

$$+B\gamma_{min}\Big[\mathbf{U}\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}E\left[(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\right]+$$

$$+E\left[(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\right]\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\mathbf{U}\Big]\Bigg]\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}.$$

By Assumption 4, $E\left[(\bar{\varepsilon}-\mu\bar{1})(\bar{\varepsilon}-\mu\bar{1})^T\right]=\sigma^2\mathbf{I}$, so

$$\mathbf{\Sigma}(\mathbf{G}) = E\left[(\bar{q}^*-E\left[\bar{q}^*\right])(\bar{q}^*-E\left[\bar{q}^*\right])^T\right]$$

$$= \left(\frac{1}{\gamma_{min}-\gamma}\right)^2 \mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\Bigg[\sigma^2\mathbf{I}+(B\gamma_{min})^2\left[\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\sigma^2\mathbf{I}\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\bar{1}\right]\mathbf{U}+$$

$$+B\gamma_{min}\left[\mathbf{U}\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\sigma^2\mathbf{I}+\sigma^2\mathbf{I}\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\mathbf{U}\right]\Bigg]\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}$$

$$= \left(\frac{1}{\gamma_{min}-\gamma}\right)^2 \sigma^2\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\Bigg[\mathbf{I}+(B\gamma_{min})^2\left[\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}^2\mathbf{V}^{-1}\bar{1}\right]\mathbf{U}+$$

$$+B\gamma_{min}\left[\mathbf{U}\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}+\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\mathbf{U}\right]\Bigg]\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}$$

$$= \left(\frac{1}{\gamma_{min}-\gamma}\right)^2 \sigma^2\mathbf{M}\Bigg[\mathbf{I}+(B\gamma_{min})^2\left[\bar{1}^T\mathbf{M}^2\bar{1}\right]\mathbf{U}+B\gamma_{min}\left[\mathbf{U}\mathbf{M}+\mathbf{M}\mathbf{U}\right]\Bigg]\mathbf{M}$$

□

Proof of Proposition 5. According to (9) and (18), Agent i's deviation from her expected equilibrium effort is

$$\bar{q}_i^*-E\left[\bar{q}_i^*\right]=\frac{1}{\gamma_{min}-\gamma}\left[b_i\left(\bar{\varepsilon}-\mu\bar{1}\right)+B\gamma_{min}b(\bar{\varepsilon}-\mu\bar{1})b_i\left(\bar{1}\right)\right].$$

Hence,

$$\frac{1}{N}\sum_{i\in\mathcal{N}}\left(\bar{q}_i^* - E\left[\bar{q}_i^*\right]\right) = \frac{1}{N(\gamma_{min}-\gamma)}\left[\sum_{i\in\mathcal{N}} b_i\left(\bar{\varepsilon}-\mu\bar{1}\right) + B\gamma_{min}b(\bar{\varepsilon}-\mu\bar{1})\sum_{i\in\mathcal{N}} b_i\left(\bar{1}\right)\right],$$

that is,

$$\frac{1}{N}\sum_{i\in\mathcal{N}}\left(\bar{q}_i^* - E\left[\bar{q}_i^*\right]\right) = \frac{1}{N(\gamma_{min}-\gamma)}\left[b\left(\bar{\varepsilon}-\mu\bar{1}\right) + B\gamma_{min}b(\bar{\varepsilon}-\mu\bar{1})b\left(\bar{1}\right)\right],$$

where $b(\bar{x}) \equiv \sum_{i\in\mathcal{N}} b_i(\bar{x})$, for every vector $\bar{x}$ in $\mathbb{R}^N$. Thus,

$$\frac{1}{N}\sum_{i\in\mathcal{N}}\left(\bar{q}_i^* - E\left[\bar{q}_i^*\right]\right) = \frac{1+B\gamma_{min}b(\bar{1})}{N(\gamma_{min}-\gamma)}b\left(\bar{\varepsilon}-\mu\bar{1}\right),$$

which by the definition of B is

$$\frac{1}{N}\sum_{i\in\mathcal{N}}\left(\bar{q}_i^* - E\left[\bar{q}_i^*\right]\right) = \frac{1}{N}Bb\left(\bar{\varepsilon}-\mu\bar{1}\right).$$

Hence, the aggregate volatility of equilibrium efforts is

$$\begin{aligned}
\sigma_Y^2(\mathbf{G}) &\equiv E\left[\left(\frac{1}{N}\sum_{i=1}^{N}\left(q_i - E\left[q_i\right]\right)\right)^2\right] \\
&= E\left[\left(\frac{1}{N}Bb\left(\bar{\varepsilon}-\mu\bar{1}\right)\right)^2\right] \\
&= E\left[\frac{1}{N^2}\left(B\right)^2 b\left(\bar{\varepsilon}-\mu\bar{1}\right)^2\right] \\
&= \frac{1}{N^2}\left(B\right)^2 E\left[\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\left(\bar{\varepsilon}-\mu\bar{1}\right)\left(\bar{\varepsilon}-\mu\bar{1}\right)^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\bar{1}\right] \\
&= \frac{1}{N^2}\left(B\right)^2\left[\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\sigma^2\mathbf{I}\mathbf{V}\tilde{\mathbf{\Lambda}}\mathbf{V}^{-1}\bar{1}\right] \\
&= \frac{\sigma^2}{N}\left(B\right)^2\frac{1}{N}\left[\bar{1}^T\mathbf{V}\tilde{\mathbf{\Lambda}}^2\mathbf{V}^{-1}\bar{1}\right] \\
&= \frac{\sigma^2}{N}\left(B\right)^2\frac{1}{N}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sum_{l\in\mathcal{N}} v_{il}\tilde{\lambda}_l^2 v_{jl} \\
&= \frac{\sigma^2}{N}\left(\frac{1}{\gamma_{min}-\gamma-\gamma_{min}b(\bar{1})}\right)^2\frac{1}{N}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}}\sum_{l\in\mathcal{N}} v_{il}\tilde{\lambda}_l^2 v_{jl}.
\end{aligned}$$

$\square$

Proof of Proposition 6. Given the definition of $\sigma_{\mathbf{N}}$,

$$\sigma_{\mathbf{N}} = \frac{1}{N} \sum_{i \in \mathscr{N}} \sum_{j \in \mathscr{N}} \sum_{l \in \mathscr{N}} v_{il} \tilde{\lambda}_l^2 v_{jl},$$

and the definition of $\tilde{\lambda}_l$,

$$\tilde{\lambda}_l \equiv \frac{1}{1 - a\lambda_l},$$

for every l in $\mathscr{N}$, then the network component of aggregate volatility is a sum of the values of a convex function of the eigenvalues of $\mathbf{G}$, that is,

$$\sigma_{\mathbf{N}} = \frac{1}{N} \sum_{i \in \mathscr{N}} \sum_{j \in \mathscr{N}} \sum_{l \in \mathscr{N}} v_{il} v_{jl} \left(\frac{1}{1 - a\lambda_l} \right),$$

where a is defined in (4). Consider an alternative adjacency matrix $\hat{\mathbf{G}}$ that satisfies Assumption 1, Assumption 2, and Assumption 3. Suppose, on the one hand, that the eigenvector matrices of $\hat{\mathbf{G}}$ and $\mathbf{G}$ are the same, that is, $\hat{\mathbf{V}} = \mathbf{V}$. On the other hand, suppose that the matrix $\hat{\mathbf{\Lambda}}$ of eigenvalues is such that

$$\frac{1}{N} \sum_{l \in \mathscr{N}} \hat{\lambda}_l^2 > \frac{1}{N} \sum_{l \in \mathscr{N}} \lambda_l^2,$$

where $\hat{\lambda}_l$ is the l-th diagonal element of $\hat{\mathbf{\Lambda}}$. In other words, suppose that the standard deviation of the eigenvalues of $\hat{\mathbf{G}}$ is higher than the standard deviation of the eigenvalues of $\mathbf{G}$. Given that

$$\frac{1}{N} \sum_{l \in \mathscr{N}} \lambda_l = \frac{1}{N} \sum_{l \in \mathscr{N}} \hat{\lambda}_l = 0,$$

this is equivalent of saying that the alternative adjacency matrix $\hat{\mathbf{G}}$ has eigenvalues that are a mean-preserving spread of the eigenvalues of $\mathbf{G}$. Given that $\tilde{\lambda}_l$ is an increasing and convex function of λ_l for every l in $\mathscr{N}$, then

$$\frac{1}{N} \sum_{i \in \mathscr{N}} \sum_{j \in \mathscr{N}} \sum_{l \in \mathscr{N}} v_{il} v_{jl} \left(\frac{1}{1 - a\hat{\lambda}_l} \right)^2 > \frac{1}{N} \sum_{i \in \mathscr{N}} \sum_{j \in \mathscr{N}} \sum_{l \in \mathscr{N}} v_{il} v_{jl} \left(\frac{1}{1 - a\lambda_l} \right)^2,$$

that is,

$$\sigma_{\hat{\mathbf{G}}} > \sigma_{\mathbf{N}},$$

where $\sigma_{\hat{\mathbf{G}}}$ is the network component of the aggregate volatility when the adjacency matrix is $\hat{\mathbf{G}}$. □

Appendix B: Decomposing aggregate volatility

This exercise illustrates how the aggregate volatility does not depend on the number of connections in the economy nor on their strength but rather on the disposition of the links, that is, on the topology of the network.

Suppose that the economy is composed by 10 agents with a benchmark (complex) network structure. Suppose that γ_{ij} can be either $\gamma_{max} > 0$ or $\gamma_{min} < 0$. Then, $g_{ij} = 1$ if $\gamma_{ij} = \gamma_{max}$ or $g_{ij} = 0$ if $\gamma_{ij} = \gamma_{min}$. The graph representation of the benchmark $\mathbf{G}$ is portrayed on the left side of Figure 11. Moreover, we set $\gamma = k(\gamma_{min} - (\gamma_{max} - \gamma_{min})\lambda_{max}) < (\gamma_{min} - (\gamma_{max} - \gamma_{min})\lambda_{max})$ with $k > 1$, so as to satisfy Assumption 1, Assumption 2, and Assumption 3.

The right side of Figure 11 reports the implied unweighted Bonacich centralities of each agent, that is, how much each agent is central to the network structure of the economy. We set $\gamma_{max} = 2$, $\gamma_{min} = -1$, and $k = 1.5$ simply in order to have small magnitudes. We can distinguish some peripheral agents such as Agents $\{1, 2, 7, 8, 9, 10\}$, and some central agents such as Agents $\{3, 4, 5, 6\}$. This distinction is based on the level of their Bonacich centralities, that is, on their relative position within the network structure of the economy. Moreover, we assume that the idiosyncratic volatility σ is equal to $N = 10$, so that the first component $\sigma_{\mathbf{I}}$ of aggregate volatility as defined in Proposition 5 is equal to 1. This network structure implies an aggregate volatility equal to $\sigma_Y = 0.00474$, which is the product of the second component $\sigma_{\mathbf{U}} = 0.00076$ and the third component $\sigma_{\mathbf{N}} = 6.24975$. We take these values as the benchmark and we normalize them to 1. First, we add a link between Agents 1 and 2. Second, we add a link between Agents 4 and 6. Third, we remove the link between Agents 3 and 5 and add a link between Agents 1 and 2. Fourth, we remove the link between Agents 2 and 3 and add a link between Agents 4 and 6. We illustrate these exercises in Figure 12 and Table 4.

First, we add a link between two peripheral agents, Agent 1 and Agent 2, that is, $g_{12} = g_{21} = 1$. This means that the efforts between the two agents become strategic complements. On the one hand, this increases the general connectiveness of the economy, thus decreasing the second component of aggregate volatility which drops to 88.73% of its initial value. On the other hand, given that Agent 1 and Agent 2 pass from being relatively peripheral to being quite central, the concentration of the network increases since now only Agents $\{7, 8, 9, 10\}$ are peripheral. Hence, the third component jumps to 118.64% of the initial value. The result is that the aggregate volatility increases to 105.27% of its initial value.

Second, we add a link between two central agents, Agent 4 and Agent 6, that is, $g_{46} = g_{64} = 1$. The connectiveness increases and brings down the second component to 72.57% of the initial value. Nevertheless, given that Agents 4 and 6 are central, a link between them increases the concentration of the network substantially, that is, the gap in centrality between the most central and the least central agents widens. This results in a considerable increase (21.26% of its initial value) in the aggregate volatility.

Third, we remove a link between central agents, Agent 3 and Agent 5, and add a link between peripheral agents, Agent 1 and Agent 2. As expected from Proposition 6, this redistribution of centrality decreases the third component by almost 25% and leads the aggregate volatility to 87.30% of its initial value.

Fourth, we remove a link between peripheral agents, Agent 2 and Agent 3, and add a link between peripheral agents, Agent 4 and Agent 6. The effect is to decrease the (latent) concentration of the network and therefore to increase aggregate volatility to 117.86% of its initial value.

Appendix C: Figures and Tables

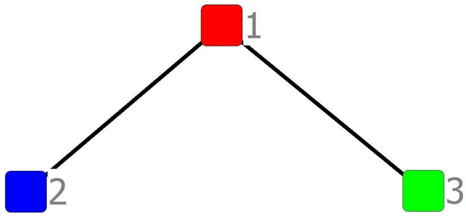

Figure 1: An example of a network structure with three agents and two links. Given that the adjacency matrix is symmetric, the links are undirected. Moreover, the diagonal elements of the adjacency metrix are nil, so there are no self-links. The software used for this figure is Borgatti et al. [2002].

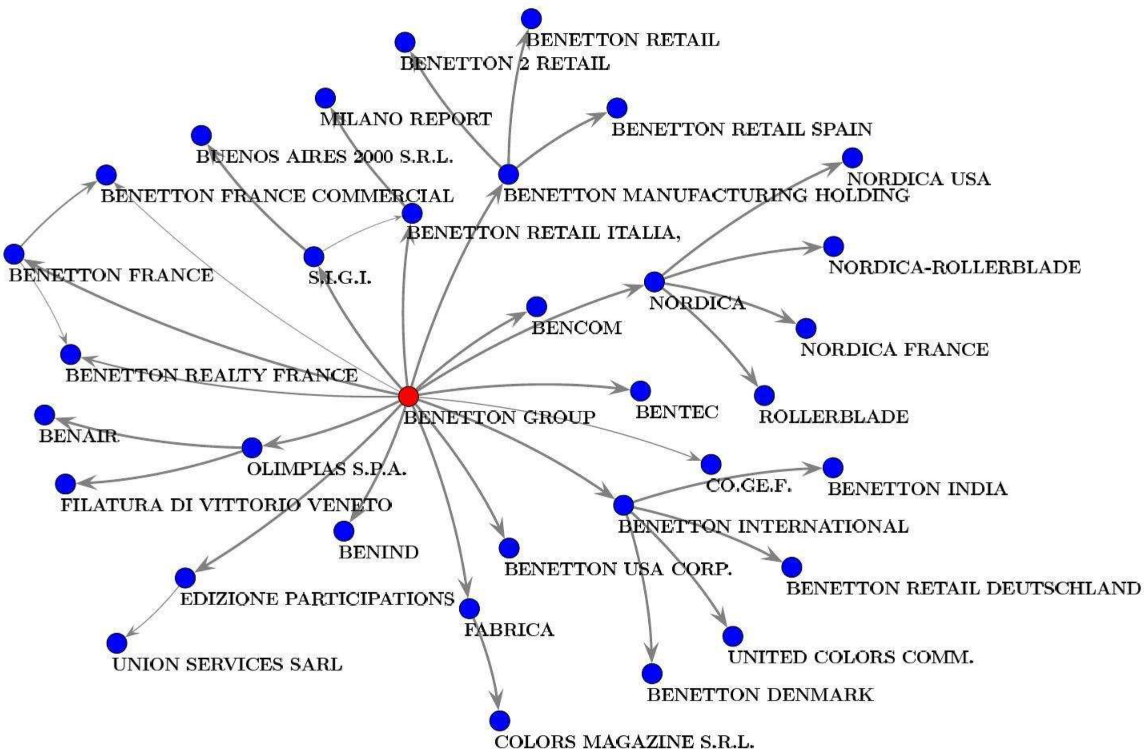

Figure 2: The network structure of the Benetton group in Vitali et al. [2011].

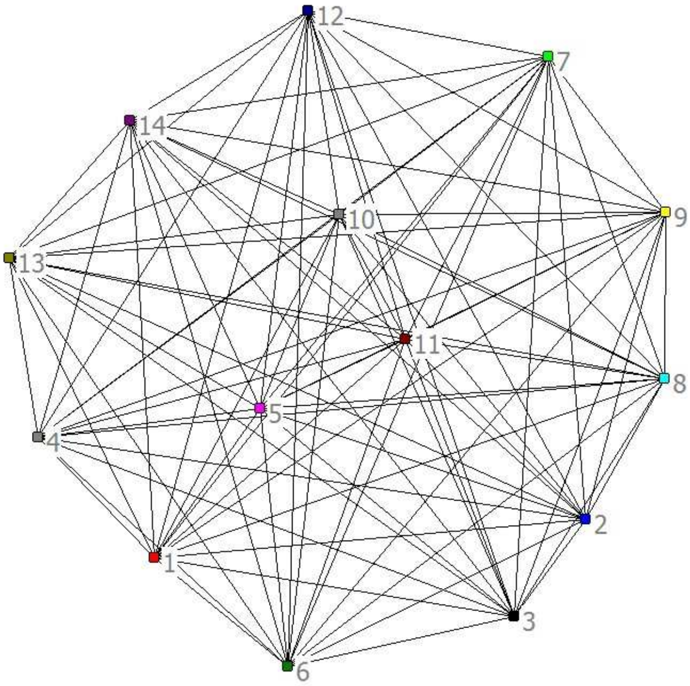

Figure 3: The network structure of intersectoral linkages. The most central sectors are Manufacturing (5), Finance (10), and Professional and business services (11). The software used for this figure is UCINET by Borgatti et al. [2002].

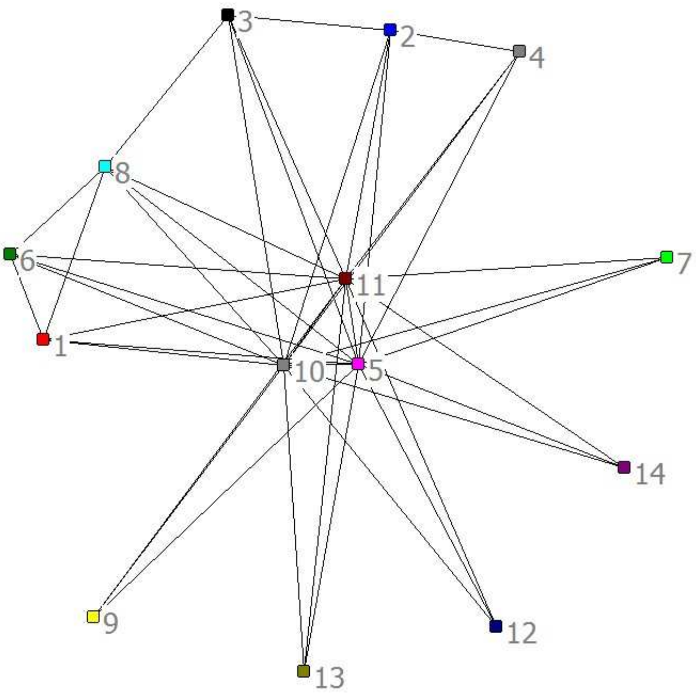

Figure 4: The dichotomized network structure of intersectoral linkages. The numbers correspond to the IO codes listed in Table 1. The software used for this figure is UCINET by Borgatti et al. [2002].

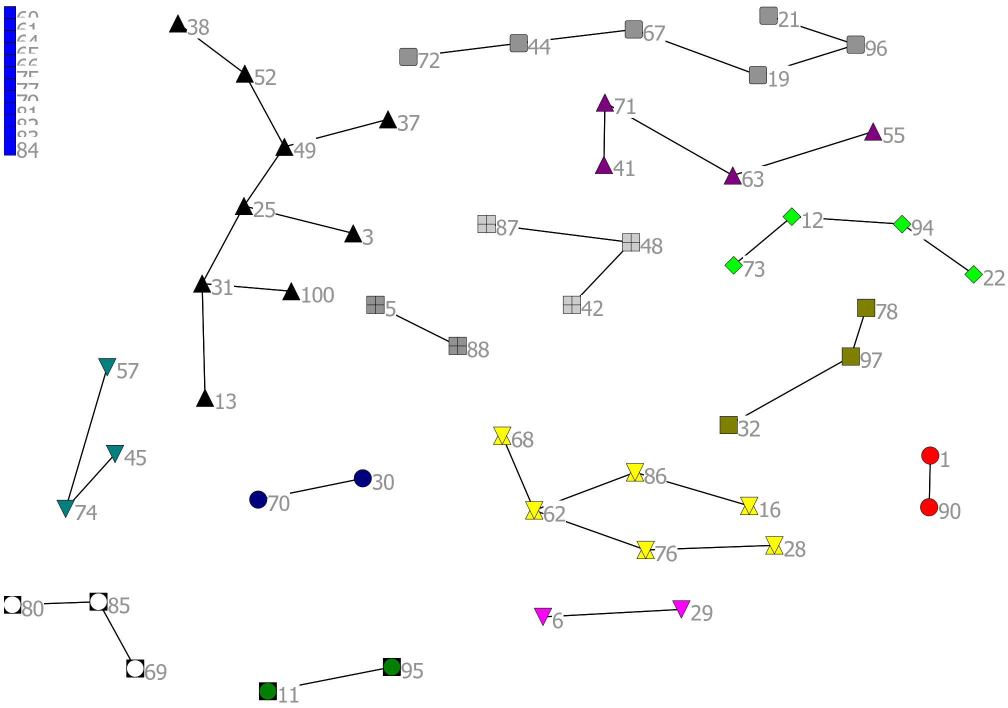

Figure 5: An example of the network structure of proprietary relations. Each group of connected establishments is a firm. Firms can be more or less sectorally diversified depending on whether they are composed of several or a few establishments. There are firms that operate in 9 different sectors and others that operate in just one. The software used for this figure is UCINET by Borgatti et al. [2002].

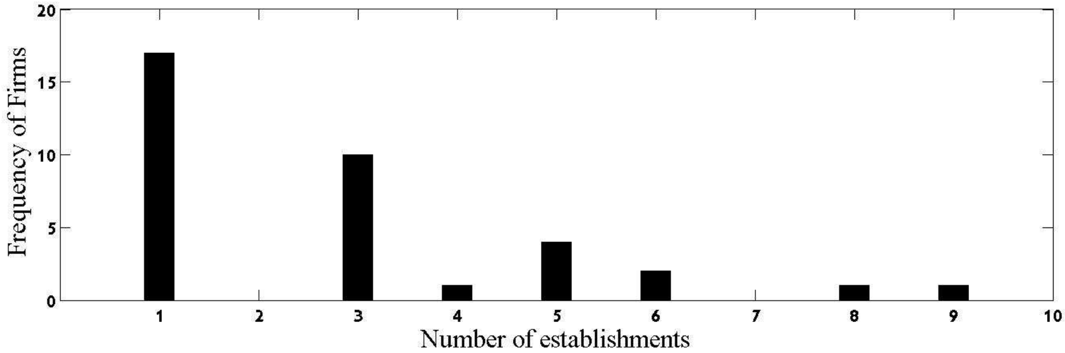

Figure 6: An example of diversification distribution of firms generated with our algorithm.

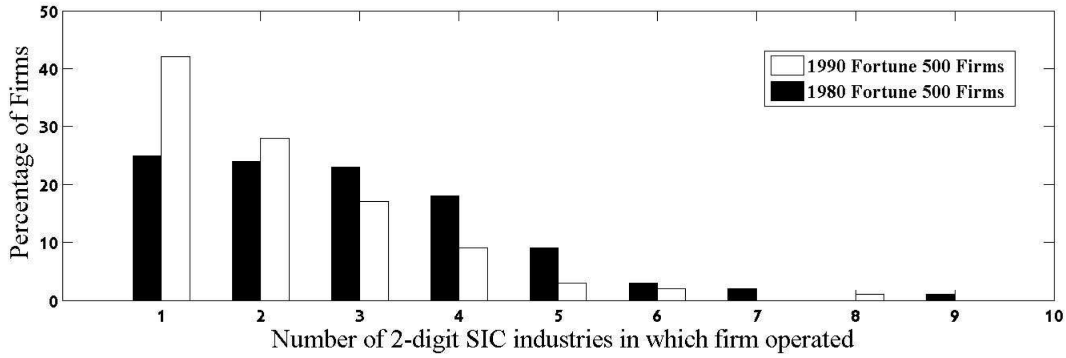

Figure 7: Frequency distribution of diversification: *Fortune* 500. Source: Davis, Diekman, and Tinsley [Davis et al., 1994, Figure 2].

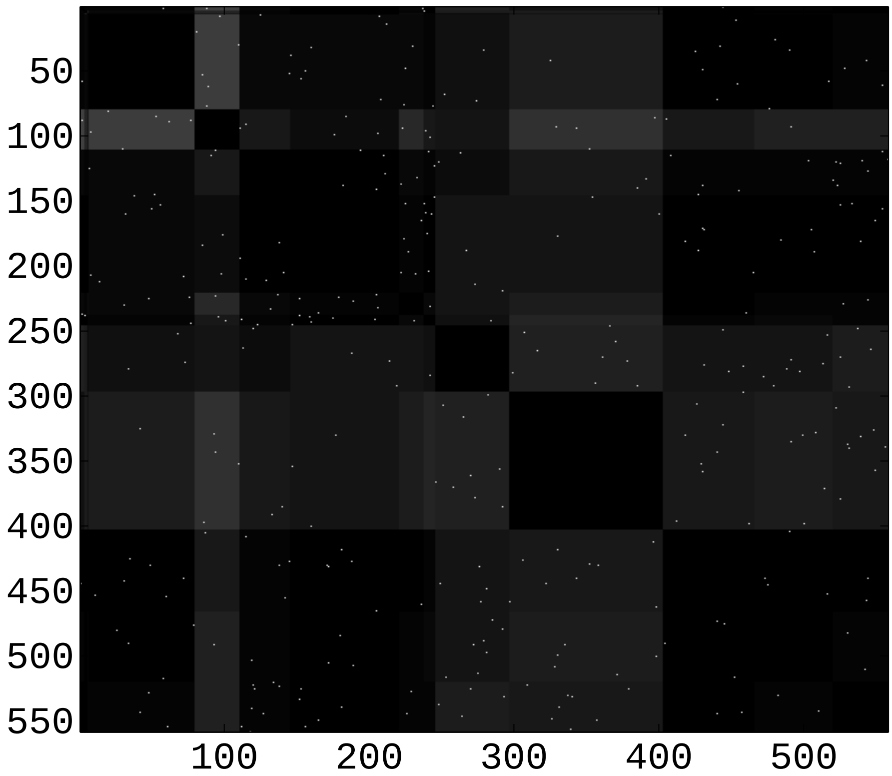

Figure 8: A representation of the network matrix $\mathbf{G}$ of the economy. Data sources: BEA's direct requirements tables for intersectoral linkages, number of establishments per sector from the US Census Bureau's County Business Patterns. Year: 2002. On the horizontal and vertical axis there are 558 establishments ordered by sector of activity. These represent the 5524784 establishments distributed across the 14 sectors of the US economy, expressed in tens of thousands and rounded up within each sector. Elements in the matrix represent whether there is a connection or not. The different shading represent the different degree of complementarity, from low complementarity (darker) to high complementarity (lighter). The blocks represent the 14 different sectors. A dark block means that between block s's sector and block s''s sector there is low complementarity, a light block that there is high complementarity. The white dots correspond to the existence of a proprietary relation between column i's establishment and row j's establishment. These dots are almost white because the intensity of the bilateral relation between two establishments within the same firm is much higher with respect to any other pair of establishments that do not share a link. The most important feature of this representation is that it highlights the sparseness of the matrix of proprietary relations Γ with respect to the matrix of intersectoral linkages $\hat{\mathbf{B}}$.

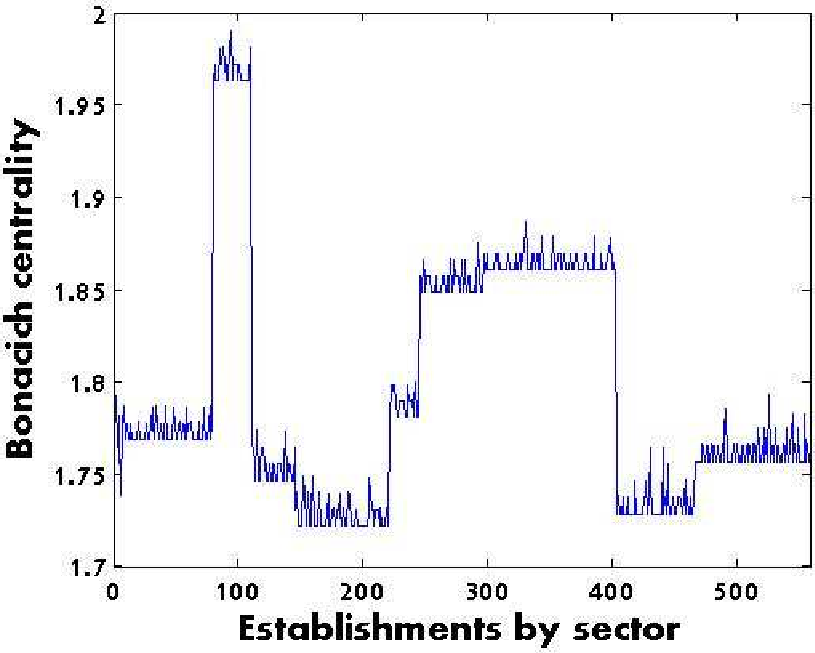

Figure 9: The unweighted Bonacich centrality measures of US establishments implied by the network structure described in Figure 8. On the horizontal axis there are the 558 representative establishments for the US economy grouped by sector of activity. The different floors represent the average centrality measure of establishments that operate in the same sector, while the spikes within the same sector are due to which firm each establishment belongs. According to this graph, the most central establishments of the US economy are the ones that operate in Manufacturing. This is due on the one hand to the centrality of the Manufacturing sector per se and on the other hand on the small number of establishments operating in the sector.

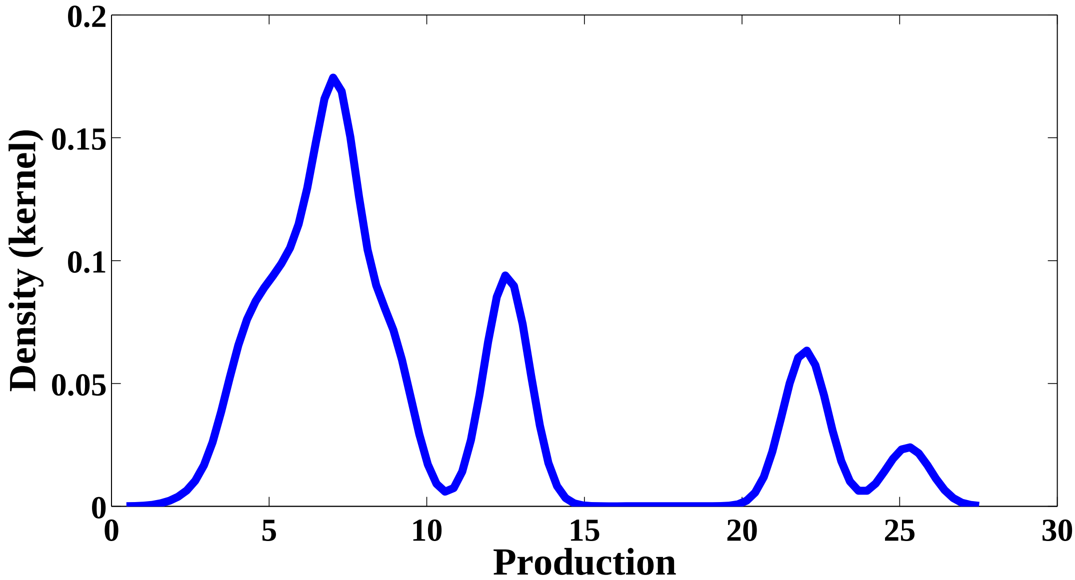

Figure 10: The size distribution of establishments implied by the network structure represented in Figure 8. The horizontal axis is the production q_i of each establishment i, while the vertical axis measures at which frequency (smoothed at its kernel) each production level occurs.

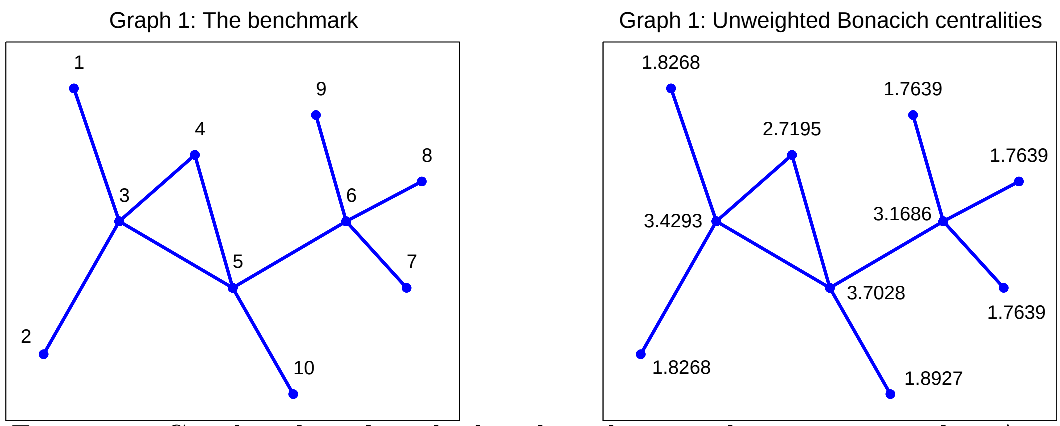

Figure 11: Graph 1 describes the benchmark network structure used in Appendix B. On the left side we can distinguish the position of each agent within the network structure of the economy. On the right side we report the implied (unweighted) Bonacich centrality measures of each agent.

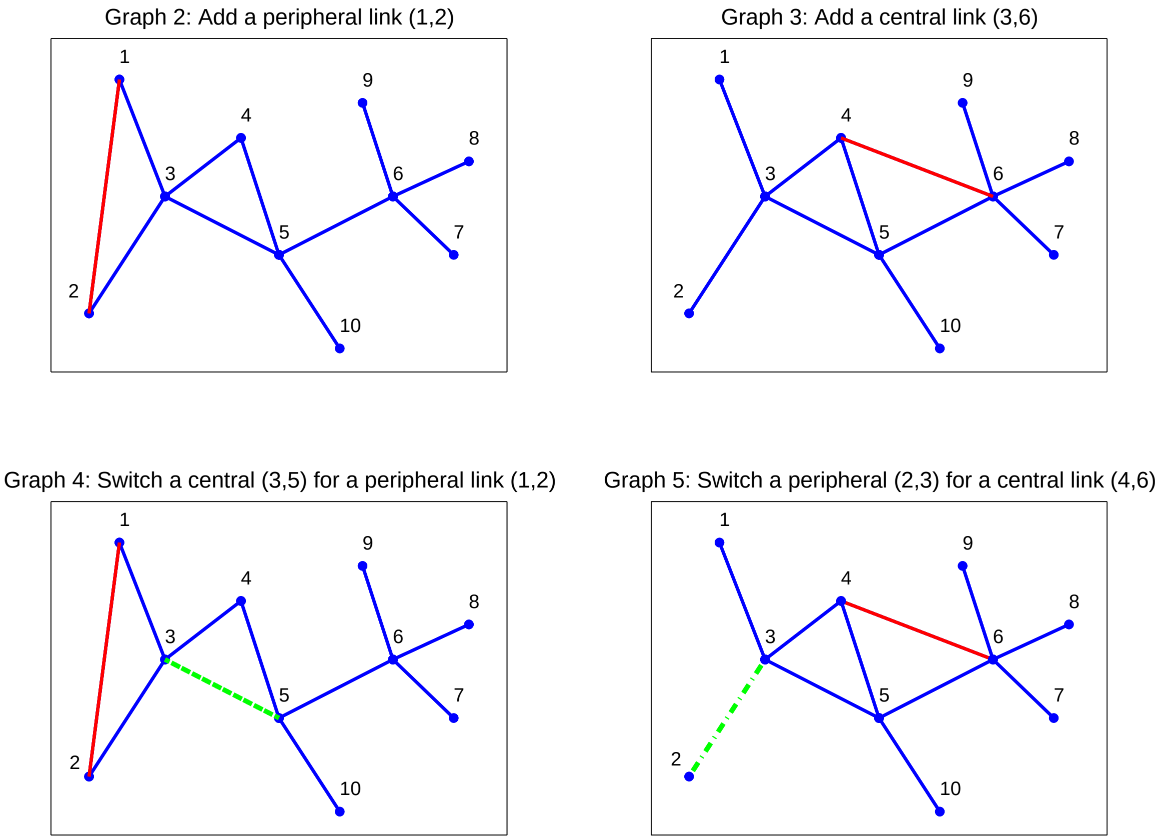

Figure 12: Graph 2 represents the network structure we obtain if we add the link between Agent 1 and Agent 2 to the benchmark network presented in Graph 1. Given that both Agent 1 and Agent 2 are peripheral with respect to the network structure of the economy, we can consider the link (1,2) a peripheral link. Graph 3 represents the addition of the central link (4,6) to the benchmark, that is, a link between two already central agents. Graph 4 reports the network structure that we obtain if we severe the central link (3,5) and we add the peripheral link (1,2) to the benchmark. Graph 5 reports the network structure we obtain if we severe instead the peripheral link (2,3) and we add the central link (4,6).

IO code	**Sector**	**Gross Output**	**Employer establishments**	**Companies**
1	Agriculture, forestry, fishing, and hunting	240.8	29250	249290
2	Mining	188.7	19324	102029
3	Utilities	320.4	6223	18896
4	Construction	970.6	729842	2780323
5	Manufacturing	3848.3	310821	601181
6	Wholesale trade	894.0	347319	711083
7	Retail trade	1030.9	745872	2584689
8	Transportation and warehousing	579.2	167865	976826
9	Information	959.6	76443	309117
10	Finance, insurance, real estate, rental, and leasing	3438.4	507281	3047522
11	Professional and business services	1780.6	1061706	4877023
12	Educational services, health care, and social assistance	1295.7	629550	2430839
13	Arts, entertainment, recreation, accommodation, and food services	704.9	538265	1645857
14	Other services, except government	464.0	392656	2677613
	Total	16716.1	5524784	22974655

Table 1: Gross output (in billions of dollars), number of employer establishments, and number of nonemployer companies by industry. Year: 2002. Sources: BEA (accounts), US Census Bureau (County Business Patterns and Survey of Business Owners). We report the number of nonemployer companies only for illustrative purposes. In fact, these companies constitute three quarters of all establishments in the economy but account for only around 3% of total sales and receipts data. Hence, we consider only employer establishments in the analysis. We do not consider the residual category "Industries not classified" in the list of industries because it is not present in the list of industries used by the BEA.

IO code	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	560	722	567	6250	1156	137	1008	336	3279	1716	223	444	360	219	612	096
2	560	0	3032	1033	3891	519	180	759	379	2953	3018	166	358	302	263	546	147
3	722	3032	0	706	2427	430	317	1399	454	1486	2114	328	721	402	375	403	116
4	567	1033	706	0	6187	867	840	788	518	1734	2760	143	313	476	396	276	082
5	6250	3891	2427	6187	0	2529	1243	4127	2447	2230	4787	2492	3481	3239	2832	5245	173
6	1156	519	430	867	2529	0	309	1084	575	1403	2557	409	685	617	559	779	170
7	137	180	317	840	1243	309	0	662	352	2179	2078	081	228	317	163	059	060
8	1008	759	1399	788	4127	1084	662	0	790	2297	2995	335	658	725	850	642	485
9	336	379	454	518	2447	575	352	790	0	1843	3658	451	851	706	736	552	248
10	3279	2953	1486	1734	2230	1403	2179	2297	1843	0	3257	2288	2122	3076	1170	1378	126
11	1716	3018	2114	2760	4787	2557	2078	2995	3658	3257	0	2413	3118	2530	2804	3028	116
12	223	166	328	143	2492	409	081	335	451	2288	2413	0	276	274	237	077	071
13	444	358	721	313	3481	685	228	658	851	2122	3118	276	0	536	449	281	074
14	360	302	402	476	3239	617	317	725	706	3076	2530	274	536	0	374	305	070
15	219	263	375	396	2832	559	163	850	736	1170	2804	237	449	374	0	094	173
16	612	546	403	276	5245	779	059	642	552	1378	3028	077	281	305	094	0	166
17	096	147	116	082	173	170	060	485	248	126	116	071	074	070	173	166	0
Mean	1040	1065	908	1040	3152	862	542	1153	876	1931	2526	604	859	842	688	850	140
Std	1557	1273	878	1486	1815	724	677	1046	931	961	1155	866	1033	1032	854	1341	106

Table 2: Complementarity matrix. All the entries are in 10^{-5}. Year: 2002. Source: BEA commodity-by-commodity direct requirements tables. We do not consider in the simulation exercises the following categories: "Government" (15), "Scrap, used and secondhand goods" (16), and "Other inputs" (17). The reason is that they are not reported in the list of industries of US Census Bureau's County Business Patterns.

	σ_Y	$\sigma_{\mathbf{U}}$	$\sigma_{\mathbf{G}}$
Case 1: no networks	1	1	1
Case 2: only intersectoral linkages	0.7765	0.7743	1.0029
Case 3: intersectoral and proprietary relations	0.7701	0.7678	1.0031

Table 3: Numerical application with US data. Sources: BEA and US Census Bureau. Year: 2002. Case 1 reports the components of aggregate volatility for the US economy in case there were no intersectoral linkages and no proprietary relations. Case 2 reports the same components with only the intersectoral linkages. Case 3 reports the components when both intersectoral linkages and proprietary relations are into place.

	σ_Y	$\sigma_{\mathbf{U}}$	$\sigma_{\mathbf{G}}$
1) Benchmark	1	1	1
2) Add peripheral link (1,2)	1.0527	0.8873	1.1864
3) Add central link (4,6)	1.2126	0.7257	1.6709
4) Remove central link (3,5), add link (1,2)	0.8730	1.1593	0.7530
5) Remove peripheral link (2,3), add link (4,6)	1.1786	0.8251	1.4285

Table 4: Numerical exercise of Appendix B. The Benchmark refers to the network structure illustrated in Graph 1 of Figure 11. 2) refers to Graph 2 in Figure 12, 3) to Graph 3 in Figure 12, 4) to Graph 4 in Figure 12, and 5) to Graph 5 in Figure 12.