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Labor Mobility and Productivity Growth

Xavier Raurich

Fernando Sánchez-Losada

Montserrat Vilalta-Bufí

Departament de Teoria Econòmica and CREB
Universitat de Barcelona

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Correspondence address: Montserrat Vilalta-Bufi. Universitat de Barcelona. Departament de Teoria Econòmica. Facultat d.Economia i Empresa. Avinguda Diagonal 690. 08034 Barcelona. Spain. Phone: (+34) 934020107. E-mail: montsevilalta@ub.edu

Abstract

Els models de creixement amb aprenentatge suposen que el coneixement après en producció es transmet de forma lliure i instantània a tota l'economia. En conseqüència, l'economia presenta economies d'escala creixents i el creixement de la productivitat (TFP) és endògena. No obstant, el supòsit de difusió instantània del coneixement és poc realista. La difusió del coneixement necessita temps i algun canal de transmissió. En aquest article suposem que el coneixement es transmet amb la contractació de treballadors nous (learning-by-hiring). En el nostre model la difusió instantània i lliure de coneixement pot ocórrer només dins d'un sector. La difusió de coneixement entre sectors pot ocórrer només a través de la mobilitat de treballadors, i per tant, el mercat de treball determina el nivell i la taxa de creixement de productivitat (TFP). Estudiem com els costos de mobilitat laboral modifiquen l'equilibri sota dos escenaris: creixement endogen i exogen. A més, demostrem que d'altres ineficiències del mercat laboral, com són les taxes o els costos de cerca, poden reduir la mobilitat laboral, i per tant, modificar la TFP.

Abstract

Growth models of learning-by-doing assume that knowledge learned in production gets freely and instantly spread to the whole economy. As a result, the economy exhibits aggregate increasing returns and the total factor productivity (TFP) growth is endogenous. However, the assumption of instant diffusion of knowledge seems unrealistic. Diffusion of knowledge takes time and requires some channel of transmission. In this paper we assume this transmission channel is learning-by-hiring, since knowledge is embodied in workers. We present a model where the free and instant diffusion of knowledge may exist only within sectors, but not across sectors. Diffusion of knowledge across sectors can only occur through the mobility of labor and, therefore, the labor market determines both the level and growth of TFP. We investigate how labor mobility costs modify the equilibrium outcome of such an economy considering two scenarios: endogenous and exogenous growth. Moreover, we show that other labor market inefficiencies, such as labor income taxes or labor search costs, may reduce labor mobility and therefore modify TFP.

JEL classification codes: O41.

Keywords: Learning-by-doing, learning-by-hiring, labor mobility, economic growth.

1. Introduction

It has been established that differences in income per capita among developed countries cannot be accounted for by differences in capital per worker, but by differences in Total Factor Productivity (Prescott, 1998). Thus, it must be a factor or factors other than accumulation of capital that account for differences in worker productivities across countries. Examples are different work practices, different within firm work organization, different resistance to the adoption of new technologies, or simply a different policy arrangement each society employs. In this paper we propose and analyze labor mobility as one of these factors. We use worker mobility as a mechanism of knowledge diffusion across sectors. Different degrees of knowledge diffusion imply different exploitation of available knowledge and, therefore, different productivity growth across countries.

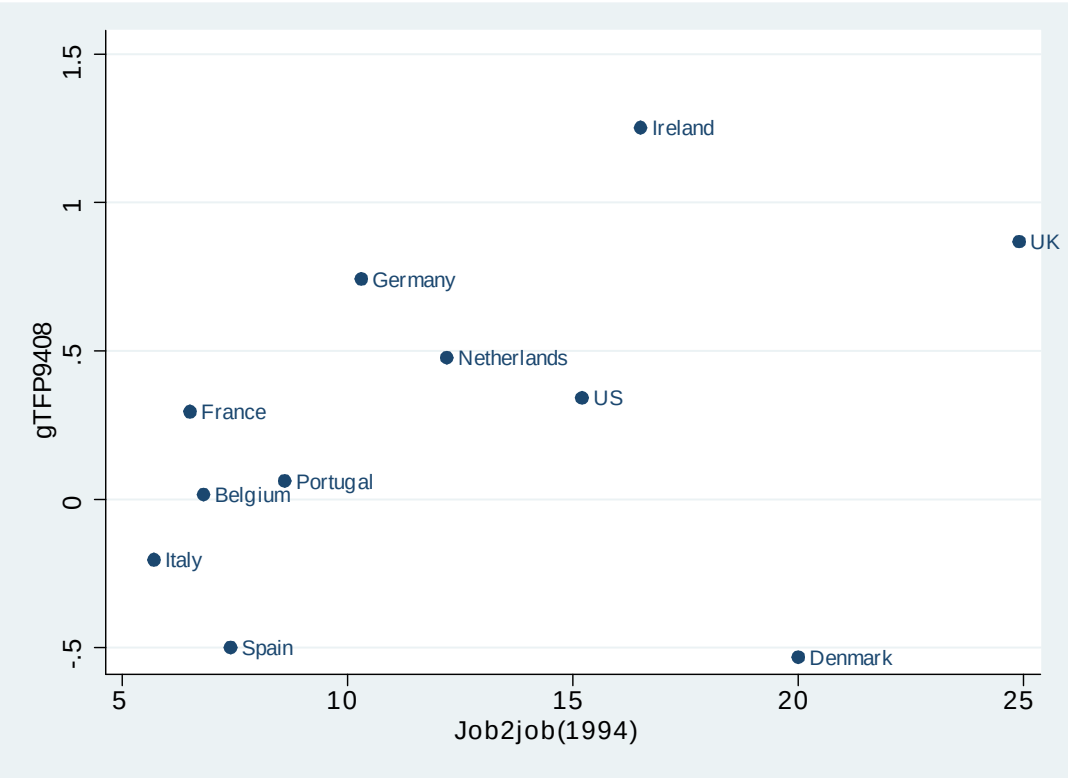

Figure 1. Labor mobility (% of job-to-job mobility in a 3 year spell) and TFP growth. Source: data on labor mobility from Jolivet, Postel-Vinay et al (2006) and TFP growth is own computations based on The Conference Board Total Economy Database, January 2010, http://www.conference-board.org/economics/database.cfm.

Figure 1 plots a measure of labor mobility for 10 European countries and the US in 1994 against the TFP growth rate in the period 1994-2008. Data on labor mobility is taken from Jolivet, Postel-Vinay et al (2006), who compute percentage of individuals who changed job in a 3-year-spell using ECHP and PSID databases. This measure allows us to distinguish three groups of countries according to their labor mobility level. The UK and Denmark are highly mobile, with at least one out of five employees

changing jobs in 3 years. In the other end, Portugal, Spain, Belgium, France and Italy have a very immobile labor force. In the middle range there is Ireland, the US, the Netherlands and Germany. Clearly, there is a cultural component to labor mobility: countries with Germanic language have more mobile labor than countries with romance language. Figure 1 shows a positive relationship between worker mobility and TFP growth. Only Denmark, which has a very particular labor market, appears as an outlier, with high mobility and low TFP growth.[1] The correlation between labor mobility and TFP growth is as high as 0.72 and significant at the 2% level when Denmark is excluded, consistent with our thesis that labor mobility plays a major role in explaining TFP growth.

We build on the seminal paper of Arrow (1962), where learning-by-doing is the main determinant of firms productivity growth. The main idea in Arrow's paper is that by producing a product workers gain experience and become more efficient in production. Arrow assumes that knowledge learned in production gets freely and instantly spread to the whole economy. As a result, he obtains aggregate increasing returns and therefore endogenous TFP and economic growth. Hence, countries with the same available knowledge will have the same TFP and growth rate. Certainly, however, the assumption of instant diffusion of knowledge seems unrealistic. We assume instead that diffusion of knowledge across sectors takes time and requires some channel of transmission. In particular we assume that knowledge is embodied in workers and firms have to hire external workers to learn (learning-by-hiring). Hence, hiring decisions and thus worker mobility across sectors affect knowledge exploitation and thus the TFP.

Labour mobility in our model is constrained by a labour market inefficiency, namely mobility costs. Mobility costs may represent training costs when hiring new workers, loss of productivity with moving if some knowledge was firm/sector specific, hiring costs, search cost, reduction of severance pay (firing costs) when changing job, harassment and non-cooperation of insiders (see Lindbeck and Snower, 2001), relocation expenses, psychological costs, preference for some location, or time-consuming labor mobility (opportunity cost to look for a new house, etc., which is usually higher if the individual has a higher wage). We show that mobility costs affect directly firms' hiring decisions and, therefore, diffusion of knowledge and TFP growth.

We study an economy with a final goods sector and a continuum of intermediate sectors. The only inputs used in production of the final good are the intermediate goods. We describe the production function of the intermediate goods sectors as a Cobb-Douglas function with two inputs: labor and physical capital. Moreover, the labor measure is a CES function of all types of workers hired in that sector (retained and poached from other sectors) weighted by the amount of knowledge they have (as in Vilalta-Bufi, 2010). In Arrow's analysis, externalities are only due to learning-by-doing. Since we consider the possibility of learning-by-hiring, workers are heterogenous and then complementarities are likely to arise. We have included the possibility of these

[1] Denmark is characterized by having a very particular labor market, with highly flexible individual employment relationship combined with a generous unemployment benefits system. This labor market has been named 'flexicurity' and it is found to enhance labor mobility. A full description of the Danish labor market is provided in Madsen (2003). Labor mobility occurs mostly in the unskilled blue-collar workers, which could mean that most labor mobility in Denmark does not imply knowledge diffusion and therefore, does not necessarily foster economic growth.

complementarities through the CES function. As in Arrow (1962), the learning of one sector is a function of the investment made the last period in that sector. As a result, the level of knowledge in a sector is going to be the accumulated stock of physical capital in that sector. Nevertheless, firms do not take into account this externality in their decision making.

The properties of the symmetric equilibrium of this economy depend on the assumption on the aggregate production function. We distinguish between two particular cases. First, we assume that the parameters of the economy are such that the reduced form of the aggregate production is an AK function. As a consequence, the equilibrium path does not exhibit transition and coincides with a Balanced Growth Path (BGP) along which the Gross Domestic Product (GDP) and consumption grow at the same constant rate. Interestingly, TFP is endogenous and depends on labor mobility, which at the same time depends on the substitution among types of labor.

Second, we assume that the aggregate production function exhibits decreasing returns to scale (DRTS). In this case, the economy converges to a steady state where GDP and consumption remain constant. The equilibrium exhibits transition and we compare this transition with the one obtained in a model with no labor mobility.

Finally, we analyze how other labor market inefficiencies may reinforce or even be the cause of mobility costs. We show that mobility costs are increasing in labor income taxes. This implies that labor income taxes may directly affect TFP. As a consequence, labor income taxes are not only the cause of a lower individual labor supply, as showed in Prescott (2004), but also of a lower productivity and then a lower wage. We also show, by introducing labor search in our economy, that not only search costs amplify labor mobility costs, but also take the role of these mobility costs. Thus, search costs may directly affect TFP. We find that countries with higher labor market tightness will be more affected by mobility costs than countries with low market tightness and, similarly, mobility costs will matter more in the periods of high labor market tightness (economic booms) and less in recessions.

The paper is organized as follows. In Section 2 we describe the full model. In Section 3 we study the equilibrium of the economy. In Section 4 we introduce two labor market inefficiencies into the model other than mobility costs and analyze how they interact with these mobility costs. In particular we consider labour income taxes and search costs. Conclusions are summarized in Section 5.

2. The model

Consider an economy with a perfect competitive final goods sector and a continuum of intermediate sectors with constant measure S. As in Ethier (1982) or Kim (2004), the technology in the final goods sector is defined by the following constant elasticity of substitution production function:

$$Y = \left(S^{\mu(1+v)-1} \int_0^S Y_i^{\mu} di \right)^{\frac{1}{\mu}}, \tag{2.1}$$

where Y denotes final goods production and Y_i is the amount of intermediate good of sector i used in the production of the final good, the elasticity of substitution between

two intermediate products is measured by $1/(1-\mu)$, with $\mu \leq 1$, and v measures scale effects. As Romer (1990) and many others have shown, these scale effects modify the growth rate. In order to focus on the growth effects of labor mobility, we eliminate these scale effects by assuming that $v = 0$.

Any final goods firm solves the following profit maximization problem:

$$\max_{\{Y_i\}} Y - \int_0^S p_i Y_i$$

subject to (2.1), where p_i is the price of the intermediate good of sector i in units of the final good. From the first order conditions of this maximization problem, we obtain the inverse demand of any intermediate good as

$$p_i = \left(\frac{Y}{Y_i}\right)^{1-\mu} S^{\mu-1}. \tag{2.2}$$

Intermediate goods sectors use labor and physical capital. Workers are infinitely lived and in each period learn the knowledge of the sector where they are employed without any cost (learning-by-doing). Firms are interested in the knowledge a worker has learned in the last period. Following Arrow (1962), knowledge of sector i is related to the stock of capital in that sector. In particular, we assume that the knowledge of sector i coincides with the average per worker stock of physical capital in that sector, $\overline{k}_i$.[2] Since learning is appropriated by workers, it is not internalized by companies when taking their own investment decisions. However, the amount of knowledge accumulated is a determinant of the hiring decisions of the firm. In fact, by hiring workers from other sectors, each period firms can learn from the investment decisions made in other sectors. Denote by λ_i^j the amount of workers from sector j that are hired in sector i. As already stated above, they have embodied knowledge of sector j, $\overline{k}_j$. We call them poached workers. Similarly, let η_i be the amount of workers of sector i hired by the same sector i, which have knowledge of sector i. We call them retained workers.

In order to include the possibility of learning-by-doing and of learning-by-hiring from poached workers, we assume that the production function of sector i is

$$Y_i = \left[\left(\eta_i \overline{k}_i^{\xi}\right)^{\sigma} + q\int_0^S \left(\lambda_i^j \overline{k}_j^{\xi\phi}\right)^{\sigma} dj\right]^{\frac{\alpha}{\sigma}} K_i^{1-\alpha},$$

where K_i is the stock of physical capital in sector i, $\xi \in [0,1]$ measures the return from learning, $\phi \in (0, 1/\xi)$ measures the differences in the returns from learning between poaching and hiring, $1/(1-\sigma) \geq 0$ is the elasticity of substitution between different types of workers, with $\sigma \leq 1$ and $\alpha \in (0,1)$ measures the labor income share. The

[2] Arrow (1962) assumes that learning in sector i is a function of the investment made in the last period in that sector. As workers do not move across sectors in the seminal paper by Arrow, this assumption implies that the accumulated knowledge in a given sector coincides with the stock of capital. In order to make comparisons with the original paper by Arrow, we must assume that: i) firms are interested in the knowledge a worker has learned in the last period and ii) learning is accumulated from the stock of capital. These two assumptions imply that, even with mobility across sectors, the knowledge of workers coincides with the average stock of capital in the last sector where they have been employed. Thus, these two assumptions allow a direct comparison with the Arrow (1962)'s original paper.

parameter $q \geq 0$ measures the ability of learning-by-hiring. This function can be rewritten as

$$Y_i = \underbrace{\left[\left(\frac{\eta_i}{N_i} \bar{k}_i^{\xi} \right)^{\sigma} + q \int_0^S \left(\frac{\lambda_i^j}{N_i} \bar{k}_j^{\xi\phi} \right)^{\sigma} dj \right]^{\frac{\alpha}{\sigma}}}_{\psi_i} N_i^{\alpha} K_i^{1-\alpha}. \tag{2.3}$$

where N_i is total employment in sector i. The variable ψ_i measures TFP. When $q = 0$, $\eta_i = N_i$ and $\psi_i = \bar{k}_i^{\alpha\xi}$. In this case, ψ_i is an exogenous variable that increases with the capital stock as in the seminal papers of the endogenous growth literature (see Arrow, 1962; Barro, 1990; Rebelo, 1991; and Romer, 1986, among many others). In contrast, when $q > 0$, the TFP is endogenous and depends on the hiring decisions and, specifically, on the ability to both retain workers and to hire workers from other sectors. This occurs because if $q > 0$, sectors can always learn from other sectors through labor mobility. In this case, the labor market affects TFP by means of modifying labor mobility.

Firms in each sector maximize profits in a perfect competitive market, i.e.

$$\max_{\{\eta_i, \lambda_i^j, K_i\}} p_i Y_i - (r + \delta) K_i - \int_0^S w_i^j \lambda_i^j dj - w_i^i \eta_i,$$

subject to (2.3), where r is the rental cost of capital, $\delta \in (0, 1)$ is the depreciation rate, w_i^j is the salary paid in sector i to those workers hired from sector j and w_i^i is the salary paid in sector i to those workers retained in the same sector. The first order conditions with respect to η_i, λ_i^j, and K_i are, respectively,

$$\alpha p_i \left[1 + q \int_0^S \left(\frac{\lambda_i^j \bar{k}_j^{\xi\phi}}{\eta_i \bar{k}_i^{\xi}} \right)^{\sigma} dj \right]^{\frac{\alpha}{\sigma} - 1} \eta_i^{\alpha-1} \bar{k}_i^{\xi\alpha} K_i^{1-\alpha} \leq w_i^i, \tag{2.4}$$

$$\alpha p_i \left[1 + q \int_0^S \left(\frac{\lambda_i^j \bar{k}_j^{\xi\phi}}{\eta_i \bar{k}_i^{\xi}} \right)^{\sigma} dj \right]^{\frac{\alpha}{\sigma} - 1} q \left(\lambda_i^j \right)^{\sigma-1} \bar{k}_j^{\xi\phi\sigma} \bar{k}_i^{(\alpha-\sigma)\xi} K_i^{1-\alpha} \eta_i^{\alpha-\sigma} \leq w_i^j, \tag{2.5}$$

$$(1 - \alpha) p_i \left[1 + q \int_0^S \left(\frac{\lambda_i^j \bar{k}_j^{\xi\phi}}{\eta_i \bar{k}_i^{\xi}} \right)^{\sigma} dj \right]^{\frac{\alpha}{\sigma}} K_i^{-\alpha} \eta_i^{\alpha} \bar{k}_i^{\alpha\xi} = r + \delta, \tag{2.6}$$

where equations (2.4) and (2.5) hold with equality whenever $\eta_i > 0$ and $\lambda_i^j > 0$, respectively.

We assume that there are mobility costs and that these mobility costs are proportional to the wage. In order to hire an external worker, the firm has to pay her at least the same wage as in her initial firm plus mobility costs, i.e.

$$w_i^j \geq m w_j^j, \tag{2.7}$$

where $m - 1 > 0$ measures mobility costs as a percentage of the wage. Perfect competition in the labor market implies that the previous relations holds in exact equality, i.e. $w_i^j = m w_j^j$ for all j. Note that the labor mobility cost introduces an inefficiency in the labor market as the cost of a worker depends on the sector from where she has been

hired. In fact, the labor income net of mobility costs obtained by a poached worker is w_j^j. This implies that the net labor income w_j^j does not depend on the particular sector a worker is employed, but on the sector she has been employed, since this determines the specific knowledge of the worker.

The economy is populated by a large family with N members. Each member supplies inelastically one unit of labor. The family net labor income is $\int_0^S w_i^i \eta_i di + \int_0^S \int_0^S w_j^j \lambda_i^j didj$, where note that w_j^j is the wage net of labor mobility costs. This labor income can either be consumed or invested. Then, the budget constraint of the family is

$$Nc + \dot{A} = rA + \int_0^S w_i^i \eta_i di + \int_0^S \int_0^S w_j^j \lambda_i^j didj, \tag{2.8}$$

where c is individual consumption and A are financial assets. The members' utility function is

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \tag{2.9}$$

where $1/\theta > 0$ measures the intertemporal elasticity of substitution. The family maximizes $\int_0^\infty Ne^{-\rho t} u(c) dt$ subject to equations (2.8) and (2.9), where ρ is the discount factor. From this maximization problem, we obtain that the consumption growth rate satisfies

$$\frac{\dot{c}}{c} = \frac{r-\rho}{\theta}. \tag{2.10}$$

The capital market clearing condition is $A = K$, where $K = \int_0^S K_i di$ is the aggregate capital stock. The final goods market clearing condition implies that

$$Y = C + \dot{K} + \delta K + M, \tag{2.11}$$

where Y is GDP, $C = Nc$ is aggregate consumption and M measures the aggregate mobility costs in units of final goods production. These mobility costs are defined as

$$M = \int_0^S \int_0^S (m-1) w_j^j \lambda_i^j didj. \tag{2.12}$$

The model is closed with the labor market clearing conditions. Let N_i be the number of workers that has been employed in sector i. Then, the market clearing condition for these workers is

$$\eta_i + \int_0^S \lambda_i^j dj = N_i, \text{ for all } i. \tag{2.13}$$

The market clearing condition in the labor market also implies that

$$\int_0^S N_i di = N. \tag{2.14}$$

Finally, combining equations (2.13) and (2.14) we obtain

$$\int_0^S \eta_i di + \int_0^S \int_0^S \lambda_i^j didj = N. \tag{2.15}$$

3. Equilibrium

In this section we characterize the symmetric equilibrium path, where the intermediate sectors are of identical size and, thus, $K_j = K_i$ for all i and j. This assumption and equations (2.4), (2.5) and (2.6) imply that $w_i^i = w_j^j = w$, $\eta_i = \eta$ and $\lambda_i^j = \lambda$. Equation (2.1) implies that GDP is $Y = SY_i$ and the price level of the intermediate good is $p = 1$. Moreover, equation (2.14) implies that $N_i = N/S$ workers are employed in each sector. The per capita stock of capital then satisfies $k = K/N = SK_i/SN_i = k_i = \bar{k}_i$ for all i. To characterize this equilibrium, we define the variable $x = \lambda/\eta$ as a measure of labor mobility. Then, in a symmetric equilibrium, the first order conditions (2.4), (2.5) and (2.6) simplify as follows:

$$\alpha \left[1 + qSx^{\sigma}k^{\xi(\phi-1)\sigma}\right]^{\frac{\alpha}{\sigma}-1} \left(\frac{S}{N}\right)^{\alpha-1} \eta^{\alpha-1}k^{1-\alpha(1-\xi)} \leq w, \tag{3.1}$$

$$\alpha \left[1 + qSx^{\sigma}k^{\xi(\phi-1)\sigma}\right]^{\frac{\alpha}{\sigma}-1} \left(\frac{S}{N}\right)^{\alpha-1} qx^{\sigma-1}\eta^{\alpha-1}k^{1-\alpha(1-\xi)-\xi\sigma(1-\phi)} \leq mw, \tag{3.2}$$

$$(1-\alpha) \left[1 + qSx^{\sigma}k^{\xi(\phi-1)\sigma}\right]^{\frac{\alpha}{\sigma}} \left(\frac{S}{N}\right)^{\alpha} \eta^{\alpha}k^{\alpha(\xi-1)} = r + \delta, \tag{3.3}$$

where equations (3.1) and (3.2) hold with equality whenever $\eta > 0$ and $\lambda > 0$, respectively. Moreover, the labor market clearing condition (2.15) becomes

$$\eta = \left(\frac{N}{S}\right)\left(\frac{1}{1+Sx}\right), \tag{3.4}$$

and, from equation (2.3), GDP per capita is equal to

$$y = \left[\frac{\left(1 + qSx^{\sigma}k^{\xi(\phi-1)\sigma}\right)^{\frac{1}{\sigma}}}{1+Sx}\right]^{\alpha} k^{1-\alpha(1-\xi)}. \tag{3.5}$$

Finally, combining equations (2.11) and (2.12), and noting that $\dot{k} = \dot{K}/N$, we obtain

$$\dot{k} = y - c - \delta k - \left(\frac{S}{N}\right)(m-1)wSx\eta, \tag{3.6}$$

Let us use equation (3.5) to summarize the main assumptions of the model and emphasize the main differences with respect to Arrow's analysis. In fact, this production function coincides with the AK production function in the seminal paper by Arrow (1962) when $q = 0$. We depart from Arrow's analysis by considering the possibility of learning-by-hiring (positive q). Obviously, we need to assume heterogeneity of workers to allow for learning-by-hiring. Moreover, as there are different types of workers, complementaries are likely to arise. We have included the possibility of these complementarities in the model through the parameter σ.

Definition 3.1. *A dynamic equilibrium of this economy is a path $\{x, \eta, c, k, w, r, y\}$ that, given the initial stock of capital k_0, solves the system of differential equations (2.10) and (3.6), and satisfies equations (3.1), (3.2), (3.3), (3.5), (3.4) and the transversality condition $\lim_{t\to\infty} e^{-\rho t}c^{-\theta}k = 0$.*

We claim that the equilibrium path is interior when the amount of retained and poached workers are both positive. In this case, equations (3.1) and (3.2) hold with equality and, from them, we obtain a non-trivial expression of labor mobility:

$$x = \left[\left(\frac{q}{m}\right) k^{\xi\sigma(\phi-1)}\right]^{\frac{1}{1-\sigma}}. \tag{3.7}$$

However, when the equilibrium path is non-interior, then either $x = 0$ or $x \to \infty$. The following proposition characterizes the conditions making the path of the dynamic equilibrium interior.

Proposition 3.2. *When $\sigma < 1$ the path of the dynamic equilibrium is interior and labor mobility is measured by equation (3.7), and when $\sigma = 1$ the path of the dynamic equilibrium is non-interior and there exists a value of per capita aggregate capital, $\widetilde{k} = (m/q)^{1/\xi\sigma(\phi-1)}$, such that,*

a) If $k > \widetilde{k}$ then $\eta = 0$, $\lambda = N/S^2$ and $x \to \infty$.

b) If $k < \widetilde{k}$ then $\eta = N/S$, $\lambda = 0$ and $x = 0$.

c) If $k = \widetilde{k}$ then $\eta > 0$, $\lambda > 0$ and x is indeterminate.

Proof. When $\sigma < 1$ there is a value of $x > 0$ such that equations (3.1) and (3.2) hold simultaneously in equality. This value does not exist when $\sigma = 1$. In this case, if $k > \widetilde{k}$ then only equation (3.2) holds with equality, which implies that $\eta = 0$. If $k < \widetilde{k}$ then only equation (3.1) holds with equality, which implies that $\lambda = 0$. Finally, when $k = \widetilde{k}$ then equations (3.1) and (3.2) hold with equality for any value of x.■

This proposition shows that the value of the elasticity of substitution determines labor mobility. Obviously, the relevant case is that of an interior equilibrium. Therefore, from now on we assume that $\sigma < 1$ and then labor mobility is determined by equation (3.7). As follows from this equation capital accumulation modifies labor mobility when $\sigma \neq 0$. Capital accumulation increases labor mobility when it increases (decreases) the ratio between the marginal productivity of own workers and the marginal productivity of hired workers. The effect of capital on this ratio depend on the value of σ and of ϕ. If $\phi > (<)\, 1$ capital accumulation increases the learning from workers hired from other sectors (the same sector). This causes an increase (decrease) in this ratio when workers are highly substitutes ($\sigma > 0$) and a reduction when workers are highly complementaries ($\sigma < 0$). Finally, if $\sigma = 0$ then capital accumulation does not change the ratio of marginal productivities and then it does not modify labor mobility.

Using equations (3.3) and (3.4), and afterwards equation (3.7), the interest rate can be written as

$$r + \delta = (1-\alpha)\underbrace{\left(\frac{[1+Smx]^{\frac{1}{\sigma}}}{1+Sx}\right)^{\alpha}}_{h(x)} k^{\alpha(\xi-1)} \tag{3.8}$$

Note that in this dynamic interior equilibrium path, the interest rate is not constant and the equilibrium exhibits transition. However, this transition is driven by two different forces: the diminishing returns to capital and learning-by-hiring. The first force arises when $\xi < 1$ and the second one arises when labor mobility is not constant, which requires $q > 0$, $\phi \neq 1$ and $\sigma > 0$.

We combine equations (3.5), (3.4) and (3.7) to obtain

$$y = h(x)\, k^{1+\alpha(\xi-1)}, \tag{3.9}$$

and we combine equations (3.1), (3.5) and (3.7) to obtain

$$w = \alpha \left(\frac{1+Sx}{1+Smx} \right) y. \tag{3.10}$$

We use equations (2.10) and (3.8) to obtain a differential equation governing the path of consumption,

$$\frac{\dot{c}}{c} = \frac{(1-\alpha)h(x)\, k^{\alpha(\xi-1)} - \delta - \rho}{\theta}, \tag{3.11}$$

and we substitute equations (3.9) and (3.10) into equation (3.6) to obtain

$$\frac{\dot{k}}{k} = \left[1 - \alpha \left(\frac{(m-1)Sx}{1+Sxm} \right) \right] h(x)\, k^{\alpha(\xi-1)} - \frac{c}{k} - \delta, \tag{3.12}$$

where x is defined as a function of capital in equation (3.7).

Definition 3.3. *Assume that $\sigma < 1$. Then, a dynamic interior equilibrium is a path $\{c, k, x, y\}$ that solves the system of differential equations (3.11) and (3.12), and satisfies equations (3.7) and (3.9) and the transversality condition $\lim_{t\to\infty} e^{-\rho t} c^{-\theta} k = 0$.*

The properties of the dynamic equilibrium will strongly depend on the operativeness of the sources of transition. In the following section we assume that $\xi = \phi = 1$ and the equilibrium will not exhibit transition and the long run equilibrium will exhibit sustained endogenous growth. In Section 5, we assume that $\xi < 1$ and $\phi \neq 1$ and the equilibrium exhibits transition but, because of the decreasing returns to capital, growth is exogenous.

4. Endogenous Growth

In this section we characterize the dynamic equilibrium when $\xi = 1$ and $\phi = 1$. As follows from Proposition 3.2 and equation (3.7), when $\phi = 1$ labor mobility is constant even along a dynamic interior equilibrium path. This implies that the interest rate is constant. Obviously, this implies that the consumption growth rate, defined in equation (3.12), is also constant. Moreover, from equation (3.9), we obtain that GDP per capita simplifies as

$$y = h(x)\, k, \tag{4.1}$$

which is an AK production function. TFP is measured by $h(x)$ and it is endogenous as it depends on labor mobility. Finally, as it is standard in AK models, the final goods market clearing condition implies that along the dynamic equilibrium the capital stock grows at the same constant growth rate than consumption, which implies that the equilibrium does not exhibit transition. Then, we have a BGP along which GDP and consumption grow at the same constant growth rate.

Proposition 4.1. *Assume that $\xi = 1$ $\phi = 1$ and $\sigma < 1$. Then, the equilibrium is interior and the economic growth rate γ is*

$$\gamma = \frac{(1-\alpha)h(x) - \delta - \rho}{\theta}.$$

Using the growth rate in Proposition 4.1, it can be shown that a larger mobility increases growth when $\sigma > 0$ and reduces growth when $\sigma < 0$. The intuition is simple. If $\sigma > 0$, workers are substitutes and then an increase in mobility enhances the marginal product of capital which rises the growth rate. The opposite occurs when $\sigma < 0$ as in this case workers are complementaries. This implies that a large mobility that reduces labor mobility will increase growth if $\sigma < 0$ and it will reduces growth when $\sigma > 0$. Similarly, an increase in the number of sectors, S, will increase the growth rate if and only if $\sigma > 0$. Thus, increasing the number of sectors accelerates growth only when different types of workers are sufficiently substitutes. The intuition is that increasing S reduces workers from any sector but increases the total number of workers in the rest of sectors. This implies that the chance of substitution from retained workers to poached workers from other sectors increases. This substitution enhances growth if these workers are sufficiently substitutes and reduces growth otherwise. Finally, note that if $q = 0$ and thus there is no labor mobility then $h(0) = 1$ and $\gamma = (1-\alpha)-\delta-\rho/\theta$. This is the expression of the long run growth rate in the Arrow's economy.

5. Exogenous Growth

When $\xi < 1$, the production function exhibits decreasing returns to scale (DRTS) and the economy converges to a steady state where capital and consumption remain constant.

Proposition 5.1. *Assume that $\xi < 1$, $\phi \leq 1$ and $0 < \sigma < 1$. Then, there is a unique and saddle path stable steady state.*

6. Growth effects of other labor market inefficiencies

In this section we show that differences in either taxes or the labor search tightness among countries can explain differences in both the growth rate of the economy and labor mobility. Moreover, we show that even in the case of absence of mobility costs, search costs take the role of these mobility costs.

6.1. Labor income taxes

Let $\tau \in (0,1)$ be the labor income tax rate. In this case, the non-arbitrage condition in the labor market implies that workers employed in sector j must obtain the same wage net of taxes and of labor mobility costs in any sector where they are hired, so that

$$(1-\tau)\, w_i^j = (1-\tau)\, w_j^j + n,$$

where n amounts for the labor mobility cost. As in the previous sections, we assume that this mobility cost is proportional to the gross wage, $n = dw_j^j$, and then

$$(1-\tau)\, w_i^j = (1-\tau)\, w_j^j + dw_j^j.$$

Note that this equation implies that $w_i^j = \widehat{m} w_j^j$, where $\widehat{m} = 1 + \frac{d}{1-\tau}$. Thus, labor mobility cost is increasing in the labor income tax rate, which implies that the tax rate increases the distortion due to labor mobility cost. Therefore, taxes may indirectly affect TFP through labor mobility.

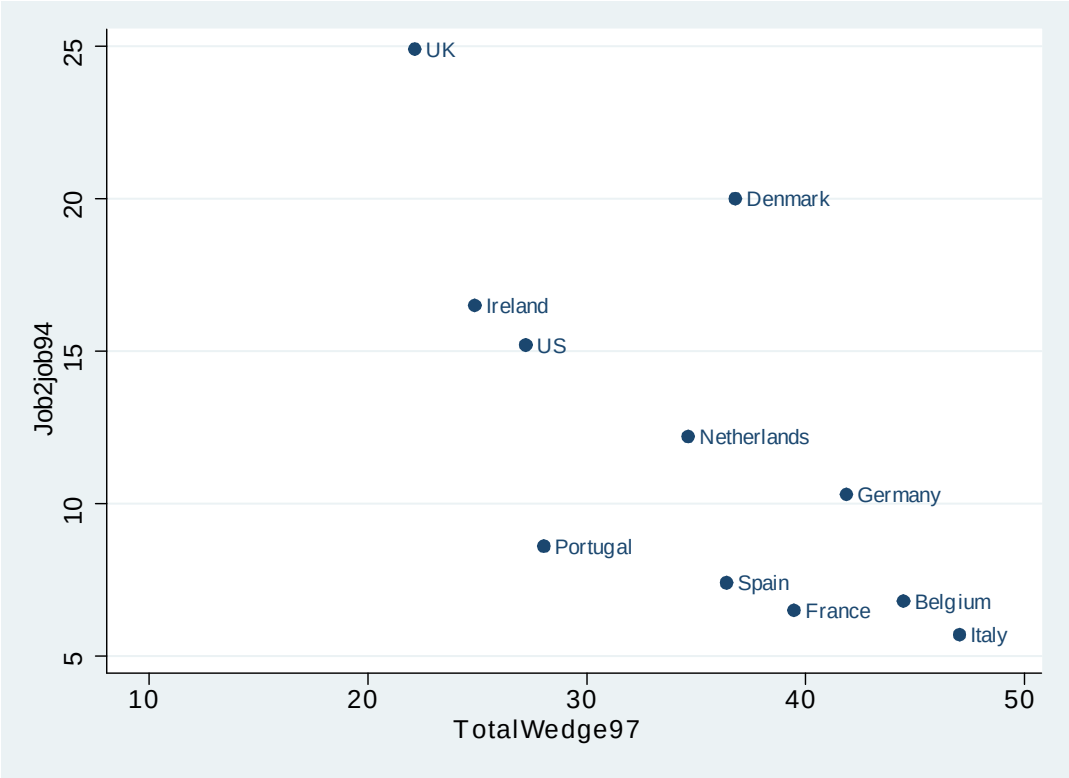

Figure 2. Labor taxes and labor mobility.
Source: data on labor mobility from Jolivet, Postel-Vinay et al (2006) and Total tax wedge from OECD year 1997 (Total tax wedge including employer payroll taxes (average rate in % based on two-earner married couple, one at 100% of average earnings and the other at 33 %, 2 children))

Assume also that government revenues are returned to consumers as a lump-sum transfer so that there is no wealth effect. Then, the only distortion due to taxes is the substitution effect that, as mentioned, increases the labor mobility cost. This conclusion and the analysis in this paper imply that those countries with a larger tax rate are also those countries with a larger mobility cost and a smaller growth rate. This theoretical conclusion is supported by empirical evidence. Although not significant, the correlation between total tax wedge and TFP growth in the period 1994-2008 is negative (-0.318). Moreover, the relationship between total tax wedge and labor mobility is clearly negative and significant, with a correlation of -0.7058. Figure 2 shows that countries with low labor taxes such as US, UK or Ireland have high labor mobility, while countries with high labor taxes such as Italy, Belgium and France have low mobility.

6.2. Labor search costs

Let us introduce labor search in our economy. Contrary to the typical search models, we have neither unemployment nor separation rate (firms that close down). Instead, we have on-the-job-search: individuals who work in a firm may be actively searching for another job. In this environment, a firm in sector i opens a vacant V_i^j when it wants to poach a worker from sector j. This vacancy has an associated cost, π_i^j. Search is costless for workers. The intertemporal profits maximization problem of a representative firm of sector i is now

$$\max_{\{I_i, K_i, V_i^j, \lambda_i^j, \eta_i\}} \quad \int_0^\infty e^{-rt} \left\{ p_i Y_i - w_i^i \eta_i - \int_0^S w_i^j \lambda_i^j dj - \int_0^S \pi_i^j V_i^j dj - I_i \right\}$$

$$s.t. \quad \dot{K}_i = I_i - \delta K_i,$$

$$\dot{\eta}_i = -h_i \eta_i + \int_0^S \left(1 - h_i^j\right) \lambda_i^j dj, \tag{6.1}$$

$$\dot{\lambda}_i^j = -\lambda_i^j + g^j V_i^j, \tag{6.2}$$

and equation (2.3), where I_i denotes capital investment. The novelty with respect to our baseline economy is that since opening a vacancy has a cost, firms always want to retain poached workers. This means that poached workers will become retained workers in the following period. Therefore, the choice of retained and poached workers is now a cumulative process. Equations (6.1) and (6.2) describe the labor flows in the firm, where h_i is the proportion of retained workers that leave the firm, h_i^j is the proportion of poached workers of type j that leave firm i and g^j is the probability that a vacancy type j is filled in.[3]

From the first order conditions with respect to I_i and K_i we have that the value of the marginal productivity has to be equal to the capital cost and depreciation, i.e. $p_i F_{K_i} = r + \delta$, where F_a is the derivative of Y_i with respect to a. Combining the first order conditions with respect to V_i^j, λ_i^j and η_i evaluated in the (symmetric) BGP, where $p_i = 1$, $\dot{v}_i^j = \dot{h}_i^j = \dot{h}_i = 0$, and assuming that the vacancy costs are proportional to the hiring wage, $\pi_i^j = z_i^j w_i^j$, we obtain

$$\frac{\left(F_{\eta_i} - w_i^i\right)}{\left(h_i + r - \gamma\right)} = -\frac{\left(F_{\lambda_i^j} - w_i^j \left[1 + \frac{z_i^j (1+r-\gamma)}{g^j}\right]\right)}{\left(1 - h_i^j\right)}.$$

This equation says that the expected actual value of the surplus arising from poaching a marginal worker has to be zero, i.e. the expected actual value of the marginal productivity of a poached worker has to cover the expected actual value of wages and firm's search cost.

Assuming that the mobility cost is a proportion d of the wage, the non-arbitrage condition in the labor market implies that $w_i^j = (1+d)\, w_j^j$. Applying this condition

[3] Note that the probability of filling in a vacancy of type j depends on the total number of type j vacancies opened in the economy and the number of individuals of type j looking for a job.

to the previous equation implies that the hiring cost for a poached worker is $\widehat{m} = (1+d)\left[1+\frac{z_i^j(1+r-\gamma)}{g^j}\right]$ times the wage of a retained worker in firm j.

Define the matching rate function for workers of type j as $f^j = f\left(u_j, v^j\right)$ where u_j is the worker's rate that is on-the-job-searching in sector j and v^j is the number of vacancies for workers of sector j over the total amount of workers of type j, and let f be homogeneous of degree one, increasing in both arguments and concave. Noting that $g^j = \frac{f^j N_j}{v^j N_j}$ and defining the labor market tightness by $\theta^j = \frac{v^j}{u_j}$, we have that $g^j = f\left(\frac{u_j}{v^j}, 1\right) = g\left(\theta^j\right)$, so that $g'\left(\theta^j\right) < 0$, that is, there is a congestion externality: the more vacancies per worker available, the lower the probability of filling them in. Then, $\widehat{m}$ can be rewritten as $\widehat{m} = (1+d)\left[1+\frac{z_i^j(1+r-\gamma)}{g\left(\theta^j\right)}\right]$. The higher the labor market tightness, the higher the hiring costs. In words of Pissarides (1994), if there is a drop in search activity then the total wage increases. Thus, vacancy costs not only amplify labor mobility costs, but also take the role of these mobility costs. Therefore, vacancy costs may directly affect TFP growth. The implications of this result is that countries with higher labor market tightness will be more affected by the mobility costs than countries with low market tightness and, similarly, mobility costs will matter more in the periods of high labor market tightness (economic booms) and less in recessions.

7. Concluding remarks

We have considered labor mobility as one of the factors that accounts for differences in TFP growth across countries. In doing so, we have introduced learning-by-hiring in a model of learning-by-doing, so that while sectors may have free and instant access to the knowledge developed within their own sector (learning-by-doing), they can only learn from other sectors by hiring external workers (learning-by-hiring). Hence, hiring decisions and thus worker mobility affect knowledge exploitation and thus the TFP. We have analyzed the equilibrium outcome of such an economy and obtained that labor mobility makes growth at least as high as that of Arrow's economy as long as the production function exhibits a sufficient degree of substitution between the different types of workers.

Moreover we have shown how other labor market inefficiencies may reinforce or even be the cause of mobility costs. In particular, we argue that mobility costs are increasing in labor income taxes and, therefore, labor income taxes might affect TFP. We also have introduced labor search in the economy and showed that the higher the labor market tightness the higher the hiring costs, so that search costs could take the role of mobility costs. The important implications of these two examples is that any other labor market inefficiency, as the existence of either trade unions or an insider-outsider bargaining power, may partially explain differences in TFP and then income per capita across countries, even when they have a similar level of capital per capita.

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Proof of proposition 5.1

First we show existence and unicity of the steady state. To this end, we combine (3.11) and (3.7) and we impose $\dot{c}=0$ to obtain $h(x)^{\frac{1}{c}}=b^{\frac{1}{c}}x$ where

$$b=\left(\frac{\delta+\rho}{1-\alpha}\right)\left(\frac{m}{q}\right)^{\frac{\alpha(1-\xi)}{\xi\sigma(\phi-1)}},$$

and

$$c=\frac{\alpha(1-\xi)(1-\sigma)}{\xi\sigma(\phi-1)}.$$

Note that

$$\frac{\partial h(x)^{\frac{1}{c}}}{\partial x}=\frac{\alpha S}{c}h(x)^{\frac{1}{c}}\left(\frac{(m-\sigma)+mSx(1-\sigma)}{\sigma(1+Sx)[1+Smx]}\right).$$

It follows that $\frac{\partial h(x)^{\frac{1}{c}}}{\partial x}>(<)\,0$ if and only if $\phi>(<)\,1$. Given that $h(0)=1>0$ and that $h(\infty)\to 0$ if $\phi<1$ and $h(\infty)\to\infty$ if $\phi>1$, there is a unique steady state when $\phi<1$.

If $\phi>1$ then there is the possibility of multiple steady states. Given that $h(0)>0$, this steady states satisfy the following condition $\frac{\partial h(x)^{\frac{1}{c}}}{\partial x}<(>)\,b^{\frac{1}{c}}$ if it is an odd (even) steady state.

Next, I proceed to show saddle path stability when $\phi<1$. Note first that $\frac{\partial\dot{c}}{\partial c}=0$, $\frac{\partial\dot{k}}{\partial c}=-1$ and $\frac{(1-\alpha)h(x)k^{\alpha(\xi-1)}-\delta-\rho}{\theta}$

$$\begin{aligned}\frac{\partial\dot{c}}{\partial k} &= c\frac{(1-\alpha)k^{\alpha(\xi-1)}}{\theta}\left[h'(x)\frac{\partial x}{\partial k}+\alpha(\xi-1)\frac{h(x)}{k}\right]\\ &= c\frac{(1-\alpha)h(x)k^{\alpha(\xi-1)-1}}{\theta}\left[\underbrace{Sx\left(\frac{(m-\sigma)+mSx(1-\sigma)}{(1+Sx)[1+Smx]}\right)\left(\frac{\xi(\phi-1)}{1-\sigma}\right)+(\xi-1)}_{Q(x)}\right]\end{aligned}$$

Note that $\frac{\partial\dot{c}}{\partial k}<0$ if $\phi<1$. This implies that the equilibrium is saddle path stable when $\phi<1$. When $\phi>1$ and there are multiple steady states, note that

$$\begin{aligned}Q(x) &= \frac{\partial h(x)^{\frac{1}{c}}}{\partial x}\frac{c}{\alpha}\frac{x}{h(x)^{\frac{1}{c}}}\left(\frac{\xi\sigma(\phi-1)}{(1-\sigma)}\right)+(\xi-1)=\\ &= \frac{\partial h(x)^{\frac{1}{c}}}{\partial x}\frac{c}{\alpha b^{\frac{1}{c}}}\left(\frac{\xi\sigma(\phi-1)}{(1-\sigma)}\right)+(\xi-1)\\ &= \left(\frac{1-\xi}{b^{\frac{1}{c}}}\right)\left[\frac{\partial h(x)^{\frac{1}{c}}}{\partial x}-b^{\frac{1}{c}}\right]\end{aligned}$$

which is negative in odd steady states and positive in even steady states. Implying that odd steady states exhibit saddle path stability and even steady states are either stable or unstable.