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Growth Models with Exogenous Saving Rates, Unemployment and Wage Inertia*

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Resum

El propòsit d'aquest article és introduir una mercat de treball no competitiu i atur en el model de creixement amb taxes d'estalvi exògenes que es pot trobar en els llibres de text de creixement (Sala-i-Martín, 2000; Barro and Sala-i-Martín, 2003; Romer, 2006). Primer, derivem un marc general amb una funció de producció neoclàssica per analitzar la relació entre creixement i ocupació. Utilitzem aquest marc per estudiar les dinàmiques conjuntes del creixement i l'ocupació sota diferents regles de fixació salarial.

Abstract

The purpose of this paper is to introduce a non-competitive labor market and unemployment into the growth models with exogenous saving rates found in economic growth textbooks (Sala-i-Martín, 2000; Barro and Sala-i-Martín, 2003; Romer, 2006). We first derive a general framework with a neoclassical production function to analyze the relationship between growth and employment. We use this framework to study the joint dynamics of growth and employment when different wage setting rules are considered.

Key words: Unemployment, growth, wage inertia.
JEL numbers: E24, O41.

1. Introduction

Most textbooks when dealing with economic growth assume full-employment, with the obvious implication that the relationship between employment and growth cannot be analyzed. The purpose of this paper is primary pedagogical as it seeks to extend growth models with exogenous saving rates and a competitive labor market, found largely in advanced growth textbooks (Sala-i-Martín, 2000; Barro and Sala-i-Martín, 2003; Romer, 2007), to the case of non-competitive labor markets. In these models, the equilibrium exhibits unemployment and, thus, we can study the relationship between growth and employment both in the short and in the long run.

Aricó (2003) classifies models of growth and unemployment according to how unemployment is generated. On the one hand, we have models with non-frictional unemployment, where unemployment occurs because the wage set by some economic agent causes excess supply in a labor market without friction. This economic agent may be the unions and, so, we have the monopoly union model (McDonald and Solow 1981), or it may be the firms and, so, we have the efficiency wage model (Solow 1979). On the other hand, we have models with frictional unemployment (Pissarides 1990) where, in addition to wage setting, frictions arise in the labor market due to matching problems. In these models, it is the unemployment rate that equilibrates flows in and out of the labor market. The models with non-frictional unemployment can also be referred to as models with disequilibrium unemployment because labor demand is lower than labor supply, whereas the models with frictional unemployment can be referred to as models with equilibrium unemployment because the unemployment rate specifically equilibrates flows in and out of the labor market. In this paper, we present models without frictions because it is the simplest framework to illustrate the effects of wage setting on employment, capital accumulation and growth. Moreover, this framework allows a direct comparison with the growth models with full-employment that can be found in the aforementioned textbooks.

For teaching and educational purposes, we assume an exogenous and constant saving rate. As is well-known, the growth model with a constant saving rate, neoclassical production function and full-employment is the Solow model. Therefore, in this paper we extend the Solow model by introducing unemployment. We first present a general framework that illustrates with a neoclassical production function the relationship between growth and employment and its dependence on wages. Having presented this general framework, in Section 3 we analyze the joint dynamics of growth and employment when three different wage setting rules are considered. These rules are differentiated by the intensity of wage inertia (persistence). We show that if wages are completely rigid, which is an extreme case of inertia, then the growth of capital is constant, whereas the employment rate either grows or falls depending on the wage level. In contrast, if wages are flexible (i.e., there is a complete absence of inertia), then the employment rate is constant and affects the economic growth rate. We show that, in this case, fiscal policy and labor market imperfections that reduce the employment rate imply a lower growth rate during the transition and a smaller long-run per capita income. After studying these two extreme wage settings, we assume that wages exhibit inertia and we study the time paths of both capital and employment. In this case, we show how the initial value of wages affects the joint dynamics of employment and growth. In particular, by using a simple phase

diagram, we show that two economies with the same initial capital stock and different initial wages at first exhibit opposite time paths of growth and employment: employment increases and capital accumulates in the economy with low wages initially, whereas both employment and capital decrease in the economy with high wages initially.

From a comparison of these three cases, we conclude that the intensity of wage inertia determines the time path of both employment and growth and that it also drives the short- and long-run effects on economic variables of technological and fiscal policy shocks. We also conclude that cross-country differences in the dynamics of employment and growth can be explained as the result of cross-country differences in labor market imperfections that imply differences in either the intensity of wage inertia or the initial level of wages.

2. Model

In this section, we describe the general framework. We first introduce the neoclassical production function and then we characterize the accumulation of capital. In this general analysis, we assume that firms are price takers and that there is no technological progress. However, this analysis could easily be extended to consider imperfect product markets or exogenous technological progress.

2.1. Firms

We assume the following neoclassical production function:

$$Y_t = F(K_t, L_t),$$

with constant returns to scale, $F_K > 0$, $F_L > 0$, $F_{KK} < 0$, and $F_{LL} < 0$. The production function in intensive form is

$$\hat{y}_t = f(\hat{k}_t),$$

where $\hat{y}_t = Y_t/L_t$ is the output per worker, $\hat{k}_t = K_t/L_t$ is the capital per worker and the production function satisfies $f' > 0$ and $f' < 0$. We assume a price taking firm that maximizes profits

$$F(K_t, L_t) - w_t L_t - (r_t + \delta)K_t,$$

where w_t is the real wage, r_t is the interest rate and $\delta \in (0, 1)$ is the constant depreciation rate. The first order conditions are

$$w_t = F_L(K_t, L_t),$$

and

$$r_t + \delta = F_K(K_t, L_t),$$

which can be rewritten in intensive form as

$$w_t = f(\hat{k}_t) - \hat{k}_t f'(\hat{k}_t), \tag{2.1}$$

and

$$r_t + \delta = f'(\hat{k}_t). \tag{2.2}$$

We assume that the labor supply is equal to population, N_t, and that it grows at the constant growth rate n. In the models with growth and unemployment, we also rewrite the production function in terms of output per capita $y_t = Y_t/N_t$, capital per capita $k_t = K_t/N_t$ and the employment rate $l_t = L_t/N_t$. As we assume constant returns to scale, we can rewrite the production function as

$$\frac{Y_t}{N_t} = F\left(\frac{K_t}{N_t}, \frac{L_t}{N_t}\right) = F(k_t, l_t).$$

The analysis conducted so far is identical to that conducted in any textbook introducing the neoclassical growth model except that, because of unemployment, we distinguish between per-capita and per-worker variables. As mentioned in the introduction, unemployment is a consequence of wages that are set above the Walrasian wage. Therefore, in the remainder of this section, in order to clarify the role of wages to students, we show that the main model variables can be rewritten as functions of wages. First, we use (2.1) to obtain that capital per worker depends on wages according to the following function:

$$\hat{k}_t = \tilde{k}(w_t), \tag{2.3}$$

where $\tilde{k}' > 0$. It is interesting to note that this derivative implies that the higher the wage, the higher is the capital per worker ratio demanded by the firm. The student obtains this property applying the implicit function theorem, where the assumption that $f''(\hat{k}_t) < 0$ is fundamental. This assumption implies, in fact, that there are many techniques of production available and, then, as the wage increases firms use a more capital intensive technique. It is important to explain to students that the substitution between capital and labor drives the relationship between growth and employment. Indeed, it is straightforward to show that this relationship disappears when the production function has constant coefficients (the Leontief production function) and substitution between inputs is not possible.

Next, by combining (2.2) and (2.3), we obtain the interest rate

$$r_t = \tilde{r}(w_t) = f'(\tilde{k}(w_t)) - \delta, \tag{2.4}$$

where $\tilde{r}' < 0$, implying that if the wage increases, the interest rate must decrease in order to have zero profits. Of course, if the interest rate is higher than the rate given by this function, we have negative profits and the capital and labor demands are zero. If it is lower, we have positive profits and the capital and labor demands are infinite.

It is also interesting to show how the product per unit of capital depends on the wage, as this ratio will be key in the growth equation. By combining the definition of product per worker and (2.3), we obtain the following relations:

$$\frac{Y_t}{K_t} = \frac{y_t}{k_t} = \frac{F(k_t, l_t)}{k_t}.$$

Using the constant returns to scale assumption, we can rewrite the previous relations as follows

$$\frac{Y_t}{K_t} = F\left(1, \frac{l_t}{k_t}\right) = \tilde{f}\left(\frac{1}{\hat{k}_t}\right) = \tilde{f}\left(\frac{1}{\tilde{k}(w_t)}\right) = h(w_t), \tag{2.5}$$

where $h' < 0$. Thus, a higher wage lowers output per unit of capital. Obviously, this is a consequence of the fact that higher wages reduce employment and in turn reduce the amount of output that can be produced with one unit of capital.

From (2.3) and the definition of capital per worker, we obtain the labor demand function

$$L_t^d = \tilde{L}(w_t, K_t) = \frac{K_t}{\tilde{k}(w_t)}, \tag{2.6}$$

where $\tilde{L}_\omega < 0$ and $\tilde{L}_{K^d} > 0$. In wage setting models there is no equilibrium in all markets if the wage is different from the Walrasian wage. Either there is an excess supply of labor (unemployment) or there is an excess supply of capital. In this paper, we assume that the wage is larger than the Walrasian wage and that there is unemployment. In this case, the amount of employment is constrained by the labor demand and thus

$$L_t = L_t^d = \tilde{L}(w_t, K_t) = \frac{K_t}{\tilde{k}(w_t)}. \tag{2.7}$$

Using (2.7) and dividing by N_t, we obtain the employment rate

$$l_t = \frac{k_t}{\tilde{k}(w_t)}. \tag{2.8}$$

Note that the employment rate depends positively on the stock of capital and negatively on the wage. We interpret the first relationship saying that employment depends on growth. Karanassou et al. (2008) provide empirical support for this positive relationship between capital and employment. The negative relationship between employment and wages is a consequence of firms substituting labor with capital when wages increase.

2.2. Families and Equilibrium in the Capital Market

We assume that families save a constant fraction $s \in (0, 1)$ of their total income. In this stylized economy without government or external sectors, savings are totally devoted to gross investment, I_t. Thus,

$$I_t = sF(K_t, L_t)$$

and gross investment is devoted to the accumulation of new capital and to compensating the depreciation of the existing stock.[1] Therefore, $I_t = \dot{K}_t + \delta K_t$ and

$$\dot{K}_t = sF(K_t, L_t) - \delta K_t.$$

[1] As there is unemployment, it may be that some members of the family are unemployed and receive unemployment benefits. In this case, total income does not change if the government has a balanced budget constraint.

In per capita terms, we obtain that the accumulation of capital per capita satisfies

$$\dot{k}_t = sF\left(k_t, l_t\right) - \left(\delta + n\right) k_t. \tag{2.9}$$

This equation shows that an increase in the employment rate increases the accumulation of capital per capita. Indeed, the difference between this equation and the fundamental equation with a competitive labor market and full employment (i.e., the Solow model) is the employment rate. Dividing by k_t, we obtain the growth rate of capital per capita

$$\gamma_k \equiv \frac{\dot{k}_t}{k_t} = \frac{sF(k_t, l_t)}{k_t} - (n + \delta). \tag{2.10}$$

Using (2.5), the growth rate of capital per capita is obtained as a function of wages

$$\gamma_k = sh(w_t) - (n + \delta). \tag{2.11}$$

As follows from this equation, an increase in the wage reduces the growth rate of capital per capita because it decreases output per unit of capital.

3. Joint Dynamics of Growth and Employment

In the previous section we have shown that the accumulation of capital is obtained from equation (2.11) and that the time path of employment is determined by equation (2.8). These two equations depend on the time path of wages. As follows from these two equations, an increase in wages reduces both the employment rate and the accumulation of capital, implying a reduction in the GDP growth rate. Therefore, in order to close the model, we need to introduce a wage equation that determines the time path of wages. In what follows, we study the joint dynamics of growth and employment when three different wage setting rules are considered. These wage setting rules are differentiated by the dependence of current wages on past wages, known as wage inertia. In Subsection 3.1, we assume that wages are completely rigid implying that they are identical to past wages. In Subsection 3.2, we consider that wages are flexible, meaning that they do not depend on the past. Finally, in the last section, we make an intermediate assumption and we consider that wages exhibit inertia, which implies that the change in wages depends on the value of past wages. By comparing between these three subsections, we conclude that the dynamics of growth and employment crucially depend on the intensity of wage inertia.

3.1. Rigid Real Wages. A Constant Growth Rate of Capital

We first assume that current wages are identical to past wages and, therefore, they are constant.[2] Let $w_t = \overline{w}$ be this constant wage.[3] With a constant real wage, the growth rate of capital per

[2] This case is interesting because having a constant real wage is frequently recommended by governments, central banks and other economic institutions to ensure stability.

[3] Benassy (1997) presents a model that generates a constant real wage.

capita is constant as follows from (2.11):

$$\gamma_k = sh(\overline{w}) - (n + \delta).$$

Note that a larger real wage implies a lower growth rate. In order to understand the dynamics implied by this equation, suppose that that there exists a $\overline{w}^*$ such that $\gamma_k = 0$. If $\overline{w} < \overline{w}^*$, then $\gamma_k > 0$ and the employment rate increases until full-employment is achieved. If $\overline{w} > \overline{w}^*$, then $\gamma_k < 0$ and the employment rate decreases until zero. We conclude that in this case there is a long-run relationship between growth and employment in the sense that a positive growth rate implies full-employment in the long run.

3.2. Flexible Wages. A Constant Employment Rate

In this section, we study the time path of growth and employment when the wage is flexible in the sense that it does not depend on the past. In this case, an increase in labor productivity (due to either capital accumulation or technological progress) translates fully into wage increases and, thus, employment is not affected. This implies that the employment rate is constant throughout the transition. This constant employment rate is denoted as structural employment because it is independent of both capital and technology. The class of wage equations implying a constant employment rate requires a linear relationship between the wage and a reservation (reference) wage or alternative income; i.e.

$$w_t = m_w z_t, \tag{3.1}$$

where $m_w \in (0, 1)$ is the constant wage mark-up and z_t is the reservation wage or alternative income. In Appendix A, we obtain this equation from static wage setting models.[4]

We assume that the reservation wage is the unemployment benefit and that it is financed with a constant tax rate, τ, on the wage income of employed workers. In this case, the reservation wage is

$$z_t = \frac{\tau w_t L_t}{N_t - L_t}.$$

Combining both equations, we obtain

$$w_t = m_w \frac{\tau w_t L_t}{N_t - L_t},$$

and, by using the definition of the employment rate, we obtain the following constant employment rate

$$l_t = l^* = \frac{1}{1 + \tau m_w}.$$

Note that $l^* \leq 1$ if $\tau \geq 0$. Note also that the employment rate is constant throughout the

[4] Obtaining the wage equation from dynamic models with growth is more difficult because the wage now affects capital accumulation via equation (2.11) and, then, future employment. To avoid this problem in models with unions we have to assume that for some reason "the union does not internalize the dynamic consequences of setting the wage over capital accumulation and future employment" (Maffezzoli, 2001 p.869). This may occur if the union only cares about present employment or if it is sufficiently small for thinking that modifying the wage will affect aggregate income, aggregate savings, the aggregate stock of capital and then future employment. Galí (1996) and Van der Ploeg (1987) present models when these dynamic consequences are taken into account.

transition and decreases with the tax rate and the value of the parameter m_w. In Appendix A, we show that this parameter depends on either the elasticity of the labor demand, income taxes, the bargaining power of unions or the degree of market power. Thus, an increase in either the income tax rate or the other parameters characterizing the degree of imperfection in the labor market causes a reduction in this constant employment rate. In contrast, this structural employment rate does not depend on the stock of capital nor on the technology. As mentioned, this is a consequence of the fact that any increase in labor productivity causes a larger wage that completely crowds out any positive effect on employment.[5]

When the employment rate is constant, the dynamics of the model with unemployment is similar to the dynamics of the model with full employment. To see this, we divide (2.9) by k_t to obtain

$$\frac{\dot{k}_t}{k_t} = \frac{sF(k_t, l^*)}{k_t} - (n + \delta). \tag{3.2}$$

Figure 1 illustrates the transitional dynamics implied by this differential equation by comparing the transitional dynamics of two economies that are differentiated only by the employment rate: economy A exhibits full employment, while economy B exhibits unemployment. From the analysis of this figure, we obtain the following results. First, regardless of the value of the employment rate, there is global convergence to the unique steady state, as in the Solow model with full-employment. Second, the growth rate of capital per capita is lower for a given level of k_t in a model with unemployment. This implies that employment affects growth. The figure illustrates this by comparing economies A and B. Both economies have the same initial capital stock but a different growth rate. In economy A, because of full-employment, the growth rate is initially positive and capital increases until it converges to the steady state. In economy B, the growth rate is negative, implying that capital decreases until it converges to the steady state. Note that the differences in the employment rate imply differences in the growth rate throughout the transition and in the long-run levels of per capita capital. Thus, the long-run per capita capital stock and, hence, the long-run per capita income are smaller when there is unemployment. Therefore, this model implies a long-run relationship between employment and the long-run levels of capital and income per capita. Finally, increases in income taxes or changes in labor market regulations that reduce the employment rate will reduce both the economic growth rate during the transition and the long-run level of per capita income.

3.3. Wage Setting Rules with Inertia

Empirical evidence shows that current real wages change depending on the value of past real wages (see Blanchard and Katz, 1997 and 1999; and Hogan, 2004). This is known as wage persistence or wage inertia. In this section, we analyze the consequences of wage inertia on both employment and growth. As in the previous subsection, we assume that the wage equation is $w_t = m_w z_t$, where z_t is the reservation wage. We introduce wage inertia by assuming that the

[5] Layard, Nickell and Jackman (1991), Pissarides (1990) and Raurich, Sala and Sorolla (2006) assume that the reservation wage is the average labor income. In this case, the employment rate is also constant.

reservation wage is a weighted average of the actual unemployment benefit and past wages, i.e.

$$z_t = \lambda w_{t-1} + (1-\lambda) b_t,$$

where $\lambda \in [0,1]$ and b_t is the unemployment benefit.[6] Combining with the wage equation, we obtain

$$w_t = m_w(\lambda w_{t-1} + (1-\lambda) b_t).$$

On the one hand, if $\lambda = 1$ and $m_w = 1$ then $w_t = w_{t-1}$ implying that wages are rigid as in Subsection 3.1. On the other hand, if $\lambda = 0$ then $w_t = m_w b_t$, which coincides with the wage equation in Subsection 3.2. Therefore, the parameter λ provides a measure of wage inertia: the larger λ is, the more rigid wages become. Thus, the wage equation in this subsection generalizes both situations, allowing us to characterize the relationship between employment and growth when wages exhibit inertia.

We also assume that the unemployment benefit is financed with a tax on wage income implying that

$$b_t = \frac{\tau w_t L_t}{N_t - L_t} = \frac{\tau w_t l_t}{1 - l_t}.$$

Therefore, we obtain

$$w_t = \lambda m_w w_{t-1} + (1-\lambda) m_w \frac{\tau w_t l_t}{1 - l_t}, \tag{3.3}$$

which can be rewritten as the following differential equation:

$$\frac{w_t - w_{t-1}}{w_{t-1}} \simeq \frac{\dot{w}_t}{w_t} = \frac{(\lambda m_w - 1)(1 - l_t) + (1-\lambda) m_w \tau l_t}{1 - l_t - (1-\lambda) m_w \tau l_t}. \tag{3.4}$$

It is straightforward to show from this equation that wages grow faster when the employment rate is higher and that they decrease (increase) when the employment rate is below (above) its long run value. This equation together with the law of motion of capital per capita, (2.11), and the employment rate, $l_t = k_t / \tilde{k}(w_t)$, determine the joint dynamics of capital, wages and employment. We proceed to characterize this joint dynamics.

We first obtain the long-run equilibrium or steady state. From (3.4) and assuming that $\dot{w}_t = 0$, we obtain that the steady state value of the employment rate is

$$l^* = \frac{1 - \lambda m_w}{1 - \lambda m_w + (1-\lambda)\tau m_w}.$$

From (2.11) and assuming that $\dot{k}_t = 0$, we obtain that the steady state value of the wages satisfies

$$h(w^*) = \frac{n + \delta}{s}.$$

[6] Blanchard and Katz (1997) justify wage inertia in the reservation wage by claiming that "models based on fairness suggest that the reservation wage may depend on factors such as the level and the rate of growth of wages in the past, if workers have come to consider such wage increases as fair. Perhaps a better word than *reservation* wage in that context is an *aspiration* wage (p.54)". Collard and de la Croix (2000), de la Croix et al., (2000) Danthine and Kurmann (2004) and Raurich and Sorolla (2011) introduce wage inertia in an efficiency wage model where workers' disutility depends on the comparison between current and past wages.

Finally, the steady state value of capital is $k^* = l^* \tilde{k}(w^*)$.

Note that if $\lambda = 0$ and there is no wage inertia then $l^* = 1/(1 + \tau m_w)$, as in Subsection 3.2. In contrast, when $\lambda = 1$ wages are rigid and $l^* = 1$, as in Subsection 3.1. Note also that an increase in the savings rate increases wages and capital, but does not affect the long-run employment rate. In contrast, an increase in either labor market imperfections, m_w, or the tax rate, τ, decreases both the long-run employment rate and the long-run capital stock, but does not affect the long-run level of wages.

The transitional dynamics are obtained from using (2.11) and (3.4) and they are illustrated in the phase diagram in Figure 2. This figure shows that the steady state is stable.[7] Given that both capital and wages are state variables, this means that, given the initial values of both capital and wages, there is a unique equilibrium that converges to the steady state.[8] As shown in this figure, two economies with the same initial capital stock and different initial wages will exhibit very different transitional dynamics. Economy A, with initially high wages and high capital stock, will monotonically reduce both capital, wages and employment during the transition until convergence. In contrast, economy B, with initially low wages and high capital stock will at first increase both capital, wages and employment. After these initial periods, capital decreases while wages continue to increase. Eventually, capital, wages and employment decrease until they converge. Note that these two economies with the same initial capital stock will exhibit different transitional dynamics due to different initial wages. Thus, the time path of the variables throughout the transition depends on the initial values of both capital and wages.

4. Conclusions

We have derived the general structure of growth models with exogenous and constant saving rates and a non-competitive labor market. We have used this general structure to study the relationship between growth and employment and we have shown that this relationship depends on the intensity of wage inertia. When wages are rigid, the accumulation of capital is constant and determined by these rigid wages. In this case, employment either converges to full-employment or diverges to zero employment. When wages are flexible, employment is constant and does not depend on growth. In contrast, capital and income exhibit a non-constant time path that converges to a steady state. We show that larger labor market imperfections or larger income tax rates that reduce the employment rate cause a smaller growth rate during the transition and a smaller per capita income in the long-run. Therefore, the flexible wage model implies that cross-country differences in the labor market explain cross-country differences in income per capita. Finally, when wages exhibit inertia, both the growth rate and the employment rate exhibit transition and the time path of these two variables depends on the initial value of both capital and wages. Therefore, two economies with the same initial capital stock and different initial wages may exhibit very different time paths of employment and growth.

[7] Appendix B provides a formal proof of stability.

[8] Wages are a state variable when wage inertia is introduced into the model. In contrast, when wages are flexible, as in Subsection 3.2, they are directly determined by the capital stock. In this case, the unique state variable is the capital stock.

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A. Derivation of the wage equation

In this appendix we obtain the wage equation (3.1) for two different wage setting models. First, in a monopolistic union model, where the wage is set by unions and, second, in an efficiency wage model, where the wage is set by firms.

Monopolistic union wage setting

We assume that there is a national union that chooses the wage in order to maximize the average labor income

$$W = \tilde{L}_t^d(w_t)w_t + (N_t - \tilde{L}_t^d(w_t))z_t$$

where $\tilde{L}_t^d(w_t)$ is the labor demand and z_t is the alternative income, which, in this case, is equal to the unemployment benefit. The first order condition for the optimal wage is

$$\tilde{L}_t^d(w_t) + (w_t - z_t)\frac{d\tilde{L}^d(w_t)}{dw_t} = 0,$$

which rewritten in terms of the elasticity of the labor demand with respect to the wage, ε_t, simplifies to

$$\varepsilon_t \equiv -\frac{d\tilde{L}^d(w)}{dw}\frac{w}{\tilde{L}^d(w)} = \frac{w_t}{w_t - z_t}.$$

This equation can be rewritten as $w_t = m_w z_t$, where $m_w = \varepsilon_t / (\varepsilon_t - 1)$ is the wage mark-up. This mark-up is constant when the labor demand elasticity is constant, which occurs when the production function is Cobb-Douglas, $Y_t = L_t^\alpha$. In this case, the elasticity of the labor demand is $1/(1-\alpha)$, where $w_t = m_w z_t$ and $m_w = 1/\alpha$.

An obvious extension is to introduce taxes on labor income. In this case, the objective function of the union is

$$W = \tilde{L}_t^d(w_t)(1-\tau)w_t + (N_t - \tilde{L}_t^d(w_t))z_t$$

and the optimal wage is given by

$$w_t = \frac{\varepsilon_t}{(1-\tau)(\varepsilon_t - 1)}z_t.$$

A further obvious extension is to assume that wages are bargained between unions and firms. If we assume that the objective of the unions is to maximize the difference between wages and the reservation wage of workers, whereas the objective of firms is to maximize profits, the Nash product is

$$W = \left[\tilde{L}_t^d(w_t)(w_t - z_t)\right]^\beta \left[F(\tilde{L}^d(w_t)) - w_t\tilde{L}^d(w_t)\right]^{1-\beta}$$

where β measures the power of unions in the bargaining process. Assuming a Cobb-Douglas production function, we obtain that the wage is $w_t = m_w z_t$ where $m_w = \frac{\beta}{\alpha} + (1-\beta)$. Note that the mark-up depends positively on the power of unions in the bargaining process, β.

Efficiency wages

We assume that the production function of the firm is $Y_t = F(e_t L_t)$, where e_t is effort. We also assume that effort depends on the wage according to the following effort function: $e_t = \tilde{e}(w_t)$, where $\tilde{e}' > 0$. The problem of the firm is to choose w_t and L_t in order to maximize profits taking into account the effort function. Therefore, the objective function of the firm is

$$\Pi(w_t, L_t) = F(\tilde{e}(w_t)L_t) - w_t L_t.$$

The first order conditions are

$$\begin{aligned} F'(.)\tilde{e}'(w_t) &= 1, \\ F'(.)\tilde{e}(w_t) &= w_t. \end{aligned}$$

From these two equations, we obtain that the optimal wage is such that the elasticity of the effort function is one, which implies that $\tilde{e}'(w_t)w_t/\tilde{e}(w_t) = 1$. Assuming that the effort function satisfies[9]

$$e_t = \begin{cases} \left(\frac{w_t - z_t}{z_t}\right)^{\beta} \text{ if } w_t > z_t \\ 0 \text{ if } w_t \leq z_t \end{cases}$$

we obtain that $w_t = m_w z_t$, where $m_w = \frac{1}{1-\beta}$.

B. Stability of the dynamic equilibrium

In order to prove the stability of the equilibrium when wages exhibit inertia, we obtain the Jacobian matrix evaluated at the steady state from the system of differential equations (2.11) and (3.4). The second of these differential equations can be rewritten as

$$\dot{w}_t = w_t \left(\frac{\lambda m_w (1 - l_t)}{1 - l_t - (1-\lambda) m_w \tau l_t} - 1 \right) = w_t g(l_t)$$

and, using the definition of the employment rate, we obtain

$$\dot{w}_t = w_t g\left(\frac{k_t}{\tilde{k}(w_t)}\right).$$

The Jacobian matrix is

$$J = \begin{pmatrix} \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial w} \\ \frac{\partial \dot{w}}{\partial k} & \frac{\partial \dot{w}}{\partial w} \end{pmatrix} = \begin{pmatrix} 0 & k^* s h'(w^*) \\ \frac{w^* g'(l^*)}{\tilde{k}(w^*)} & -w^* g'(l^*) l^* \frac{\tilde{k}'(w^*)}{\tilde{k}(w^*)} \end{pmatrix}.$$

The determinant of the Jacobian matrix is

$$Det(J) = -s h'(w^*) w^* g'(l^*) l^*$$

[9] This effort function can be found in Romer (2006), section 9.3.

and the trace is

$$Tr\left(J\right) = -w^{*}g'\left(l^{*}\right)l^{*}\frac{\tilde{k}'(w^{*})}{\tilde{k}(w^{*})}$$

We have shown that $h'\left(w^{*}\right) < 0$ and $\tilde{k}'(w^{*}) > 0$, and

$$g'\left(l^{*}\right) = \frac{\lambda m_{w}(1-\lambda)m_{w}\tau}{\left(1-l_{t}-(1-\lambda)m_{w}\tau l_{t}\right)^{2}} > 0.$$

Using the sign of these derivatives, we obtain that the determinant is positive and the trace is negative. As a result, the characteristic roots of the dynamic system are negative implying that the steady state is stable. Moreover, as both wages and capital are state variables, there is a unique equilibrium converging to this steady state, given initial conditions for both capital and wages.

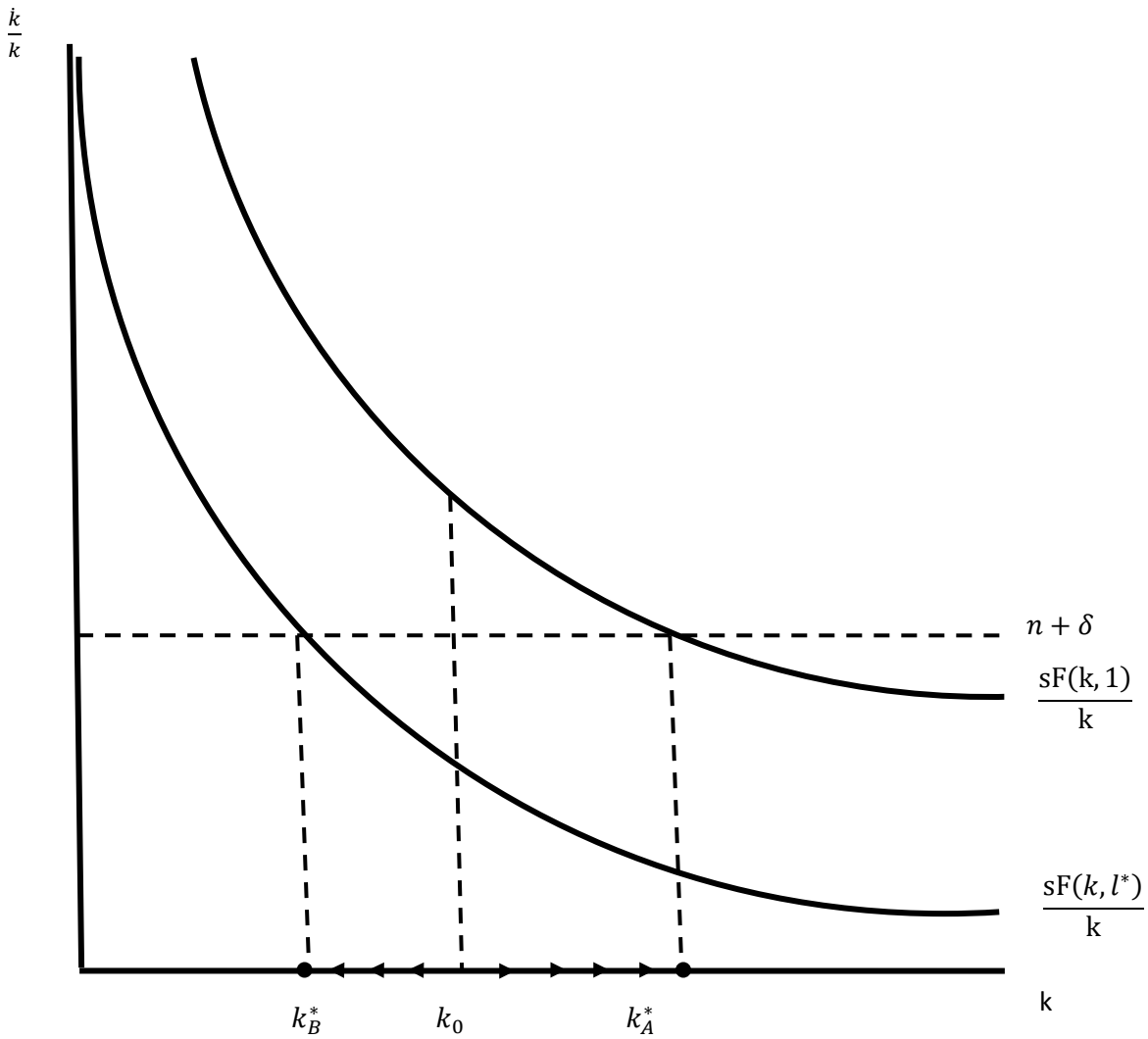

Figure 1. Transitional dynamics of capital in an economy with flexible wages.

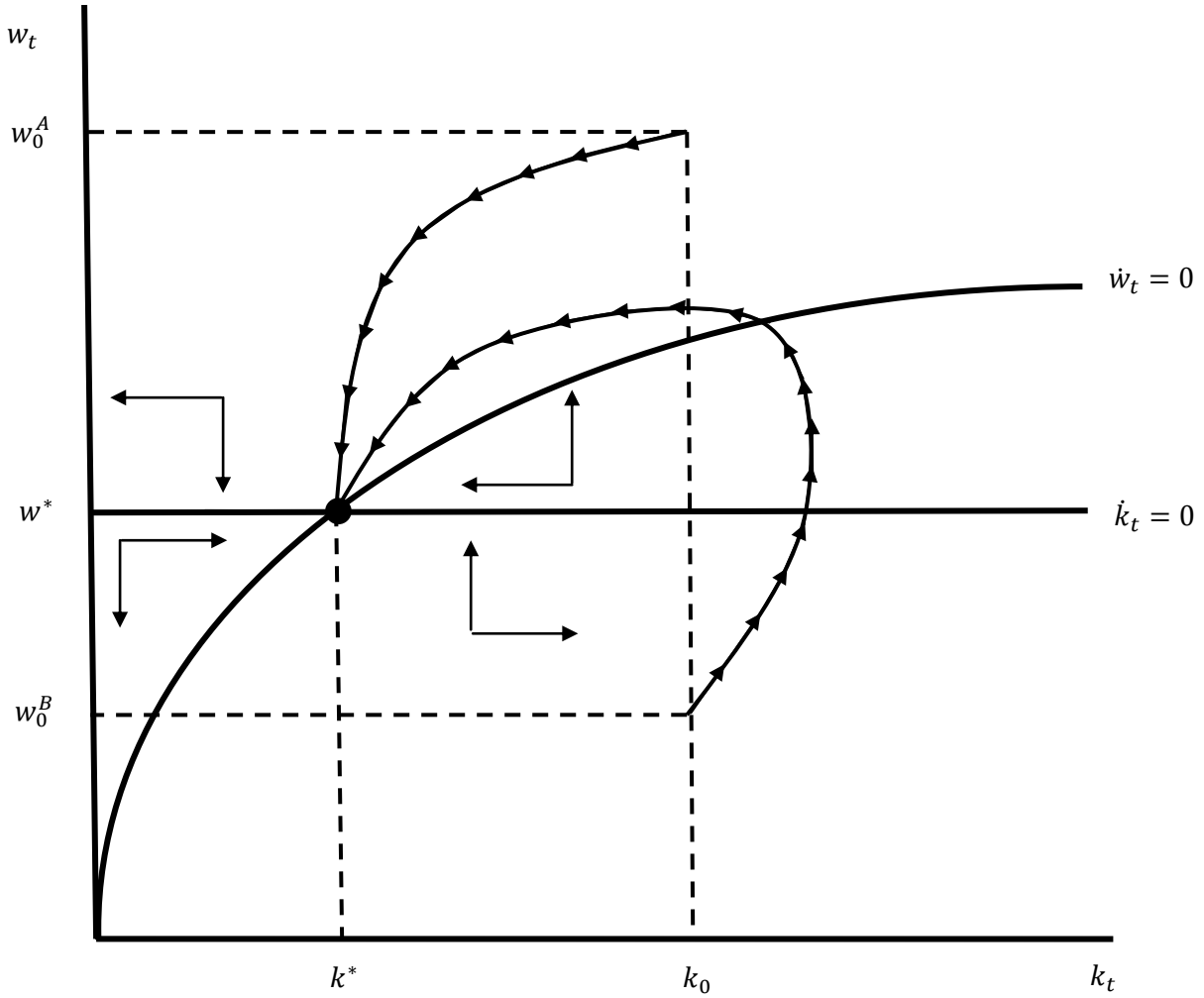

Figure 2. Transitional dynamics of capital and wages in an economy with wage inertia.