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Near-relativistic electron events. Monte Carlo simulations of solar injection and interplanetary transport

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A Parker interplanetary magnetic field

In the ecliptic plane, the Parker interplanetary magnetic field can be expressed in polar coordinates, $\vec{B} = (B_r, B_\Phi)$, by

$$\begin{aligned} B_r &= B_0 \left(\frac{r_0}{r}\right)^2 \\ B_\Phi &= -B_r \left(\frac{\Omega}{u}\right) r \end{aligned} \tag{A.1}$$

where Ω is the sidereal solar rotation rate, u is the solar wind speed, r_0 is the radius at which the field is completely frozen into the solar wind, and $B_0 = B(r_0)$. This radius is greater than the conventional 'source surface' where it is assumed that $\vec{B}$ is purely radial. Thus the azimuthal component decreases with $1/r$ while the radial component decreases as $1/r^2$. The sign of B_0 determines the polarity.

The field strength of $\vec{B}$, $B = |\vec{B}|$, decreases with r as

$$B(r) = B_0 \left(\frac{r_0}{r}\right)^2 \sqrt{1 + \frac{r^2}{a^2}} \tag{A.2}$$

where $a = u/\Omega$. The angle ψ between the magnetic field direction and the radius vector from the Sun is given by

$$\sec\psi(r) = \sqrt{1 + \frac{r^2}{a^2}} \tag{A.3}$$

i.e. $\tan\psi = r\Omega/u$. Thus

$$dz = \sec\psi \, dr \tag{A.4}$$

An important propagation parameter is the focusing length, L, given by the parallel scale length of the fractional variation of the field,

$$\frac{1}{L} = -\frac{1}{B}\frac{dB}{dz} = \left(\frac{1}{r}\right) \frac{2 + r^2/a^2}{(1 + r^2/a^2)^{3/2}} \tag{A.5}$$

Thus $L \simeq r/2$ for $r^2/a^2 \ll 1$ but $L \simeq r^2/a$ for $r^2/a^2 \gg 1$.

A.1 Length of a field line

The length of a field line is the integral of the differential distance dz. A simple formula for the length of the field line is obtained by transforming the variable of integration

$$\frac{r}{a} = \sinh \varepsilon \implies \sqrt{1 + \frac{r^2}{a^2}} = \cosh \varepsilon \tag{A.6}$$

and

$$\frac{dr}{a} = \cosh \varepsilon \, d\varepsilon \tag{A.7}$$

Making use of Equation (A.7) to express Equation (A.4) in an integral form, we obtain

$$\frac{z}{a} = \int \frac{dr}{a} \sqrt{1 + \frac{r^2}{a^2}} = \int \cosh^2 \varepsilon \, d\varepsilon = \frac{1}{2} \int (1 + \cosh 2\varepsilon) \, d\varepsilon \tag{A.8}$$

where we have used the identity $\sinh 2u = 2 \sinh u \cosh u$. Integrating,

$$\frac{z}{a} = \frac{1}{2} (\varepsilon + \sinh \varepsilon \cosh \varepsilon) + \mathrm{C} \tag{A.9}$$

where C is the integration constant. Since $\varepsilon \to 0$ and $z \to 0$ as $r/a \to 0$, we can set the constant of integration to 0 and define the length of the field line "from the center of the Sun" ($r = 0$) as

$$z = \frac{a}{2} \left[\sinh^{-1} \frac{r}{a} + \frac{r}{a} \sqrt{1 + \frac{r^2}{a^2}} \right] \tag{A.10}$$

From the identity $\varepsilon = \ln(\cosh \varepsilon + \sinh \varepsilon)$, we can also write

$$\sinh^{-1} \left(\frac{r}{a} \right) = \ln \left[\sqrt{1 + \frac{r^2}{a^2}} + \frac{r}{a} \right] \tag{A.11}$$

Therefore the general expression for the distance along the field line from the center of the Sun is given by

$$z(r) = \frac{a}{2} \left[\ln \left(\sqrt{1 + \frac{r^2}{a^2}} + \frac{r}{a} \right) + \frac{r}{a} + \frac{r}{a} \sqrt{1 + \frac{r^2}{a^2}} \right] \tag{A.12}$$

Note that at small distances, $r^2/a^2 \ll 1$, $z \simeq r$, whereas in the limit $r^2/a^2 \gg 1$, we have

$z \simeq r^2/2a$. We noted above that for $r^2/a^2 \ll 1$, the scale distance of the field is given by $L \simeq r/2$; thus $L \simeq z/2$. For large distances, however, $L \simeq r^2/a$ which implies that $L \simeq 2z$.

A.2 Particle transverse kinetic energy change

In collisionless plasmas, the first adiabatic invariant, $\Gamma = p_\perp^2/2B$, remains constant in a slowly varying magnetic field. Thus, if the particle speed remains constant, the quantity $\sin^2\alpha/B$ is also constant; here α is the pitch-angle of the particle, i.e. the angle between the particle velocity and the magnetic field vector. Then, $(1-\mu^2)/B$ is the invariant of the motion, where $\mu = \cos\alpha$. Thus, if the particle is initially at position $r = r_0$ with pitch-angle cosine $\mu = \mu_0$, it will reach position r with pitch-angle cosine

$$\mu(r) = \pm\sqrt{1 - \frac{B(r)}{B(r_1)}(1-\mu_1^2)}. \tag{A.13}$$

Taking Equation (A.2) and substituting, we obtain

$$\mu(r) = \pm\left[1 - (1-\mu_0^2)\left(\frac{r_0^2}{r^2}\right)\frac{\sqrt{1+r^2/a^2}}{\sqrt{1+r_0^2/a^2}}\right] \tag{A.14}$$

The sign of $\mu(r)$ is the same as that of μ_0, unless $\mu_0 < 0$ and the particle has mirrored at $r_m < r$.

A.3 Transit time along the field line

The differential distance along the particle's full trajectory, ds, can be expressed as a function of the differential distance along the field line by

$$ds = \frac{1}{\mu}dz \tag{A.15}$$

or as a function of the differential time $ds = v\,dt$. Taking Equation (A.4) and substituting in Equation (A.15), we have

$$ds = \frac{dz}{\mu} = \frac{\sqrt{1+r^2/a^2}}{\mu}dr \tag{A.16}$$

for a particular choice of sign of the pitch-angle cosine. Then, substituting Equation (A.14) and integrating we obtain (E. Roelof; 2003, private communication)

$$\frac{s(r,\mu;r_0,\mu_0)}{a} = \frac{1+k^2}{2}\sinh\iota\cosh\iota + 2k\sqrt{1+k^2}\sinh\iota + \frac{\iota}{2}(1+3k^2) \tag{A.17}$$

where

$$2k = \frac{x_0^2(1-\mu_0^2)}{\sqrt{1+x_0^2}} \quad \text{where } x_0 = \frac{r_0}{a} \tag{A.18}$$

and

$$\iota = \ln\left(\frac{\sqrt{1+r^2/a^2} - k + \sqrt{r^2/a^2 - 2k\sqrt{1+r^2/a^2}}}{\sqrt{1+k^2}}\right) \tag{A.19}$$

Thus, the trajectory path length $\Delta s(r,\mu;r_0,\mu_0)$ between the initial position, r_0, (where the particle has pitch-angle cosine μ_0) and r is given by

$$\Delta s(r,\mu;r_0,\mu_0) = |s(r,\mu;r_0,\mu_0) - s(r_0,\mu_0;r_0,\mu_0)| \tag{A.20}$$

if $r > r_m$, and the time elapsed in the propagation is

$$\Delta t(r,\mu;r_0,\mu_0) = \frac{\Delta s(r,\mu;r_0,\mu_0)}{v} \tag{A.21}$$

A.4 Mirror point position

The mirror point position of a particle travelling along the magnetic field line with initial position r_0 and pitch-angle cosine μ_0 ($\mu_0 < 0$), is given by (E. Roelof; 2003, private communication)

$$r_m = a\sqrt{\left(k \pm \sqrt{1+k^2}\right)^2 - 1} \tag{A.22}$$