UNIVERSIDAD DE BARCELONA
DIVISIÓN DE CIENCIAS JURÍDICAS, ECONÓMICAS Y SOCIALES
FACULTAD DE CIENCIAS ECONÓMICAS Y EMPRESARIALES
DEPARTAMENTO DE MATEMÁTICA ECONÓMICA, FINANCIERA Y ACTUARIAL

DINÁMICA DE LA ESTRUCTURA TEMPORAL DE TIPOS DE INTERÉS: MODELO DE TRES FACTORES

- Tesis Doctoral presentada por Mercedes Galisteo Rodríguez para optar al título de Doctora en Ciencias Económicas y Empresariales.
- Directora de la Tesis: Dra. Hortènsia Fontanals Albiol.
- Programa de doctorado: Metódos Matemáticos en Economía Financiera. Bienio: 92-94

Junio de 2000

ANEXO I

En la figura I se incluyen los gráficos de las series del primer *spread*, segundo *spread* y tipo de interés a largo plazo para la muestra 02-01-1991/02-01-1999. Para poder efectuar una comparación directa entre los gráficos de las series analizadas, éstos se presentan con la misma escala. Sin embargo y para la serie del segundo *spread*, se incluye un nuevo gráfico más ilustrativo, ya que aunque cambia de escala en el eje de ordenadas, da una idea más clara de la evolución de la serie.

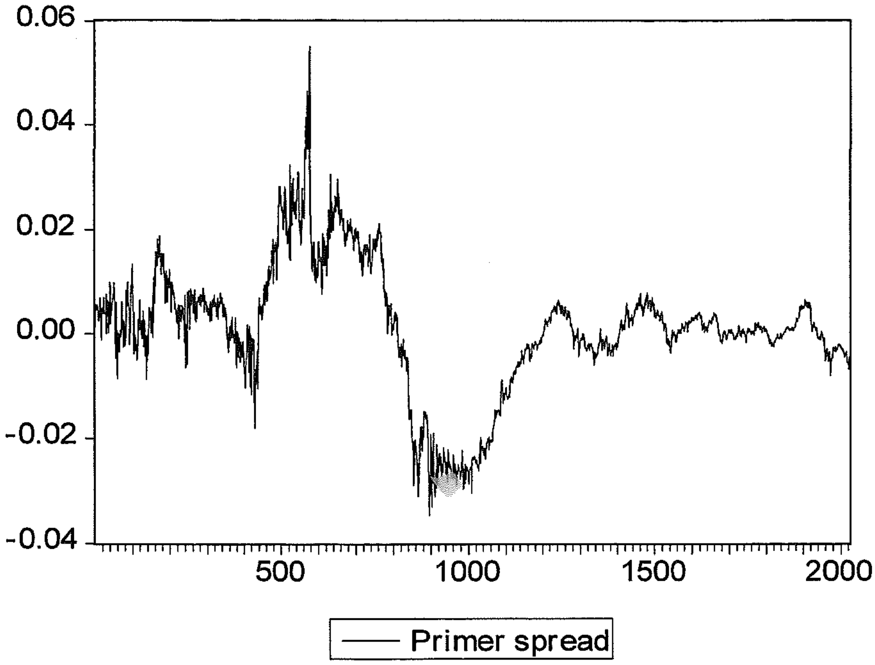
En la figura II, se incluyen las series de las diferencias del primer *spread*, diferencias del segundo *spread* y diferencias del tipo de interés al contado a largo plazo para la muestra 02-01-1991/02-01-1999. Los tres primeros gráficos tienen la misma escala y, al igual que antes, a continuación se presentan las dos últimas series a una escala más natural, que permite apreciar mejor su evolución en el tiempo.

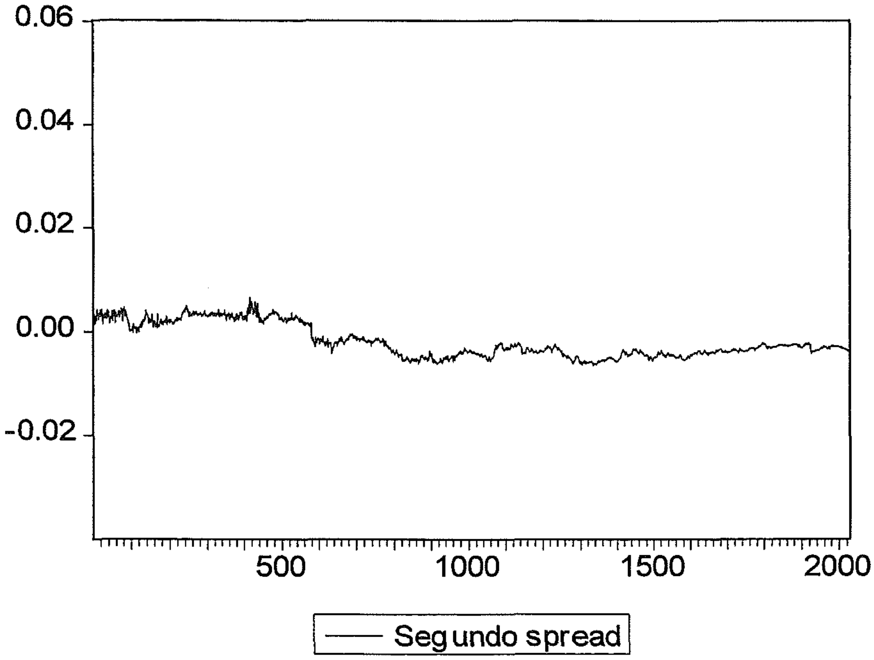
En la figura III se presentan los gráficos de las series de las diferencias del primer *spread*, diferencias del segundo *spread* y diferencias del tipo de interés al contado a largo plazo, para la muestra de 02-01-1995/09-03-1999. A continuación, también se resumen los principales estadísticos de los factores de esta segunda muestra, así como de las series diferencias de las variables.

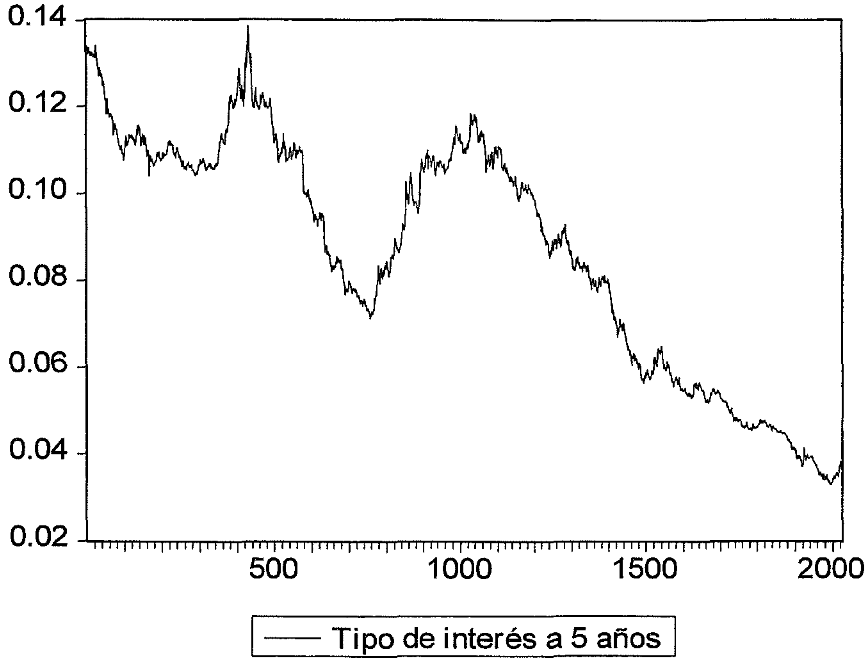
En la figura IV, se presentan los resultados del test de estacionariedad *Augmented Dickey-Fuller* (ADF), para las series de las diferencias del primer *spread*, segundo *spread* y tipo de interés a largo plazo y para los dos periodos considerados en la estimación del modelo.

Un valor del estadístico ADF alto y negativo rechaza la hipótesis nula de raíz unitaria y sugiere que la serie analizada es estacionaria. Si el estadístico ADF es mayor, en valor absoluto, que el correspondiente valor crítico, se rechaza la hipótesis nula de no estacionariedad. Así, los resultados sugieren estacionariedad en las series de las diferencias del primer *spread*, del segundo *spread* y del tanto de interés a largo plazo.

FIGURA I


0.06
0.04
0.02
0.00
-0.02
-0.04
500
1000
1500
2000
Primer spread


0.06
0.04
0.02
0.00
-0.02
500
1000
1500
2000
Segundo spread


0.14
0.12
0.10
0.08
0.06
0.04
0.02
500
1000
1500
2000
Tipo de interés a 5 años

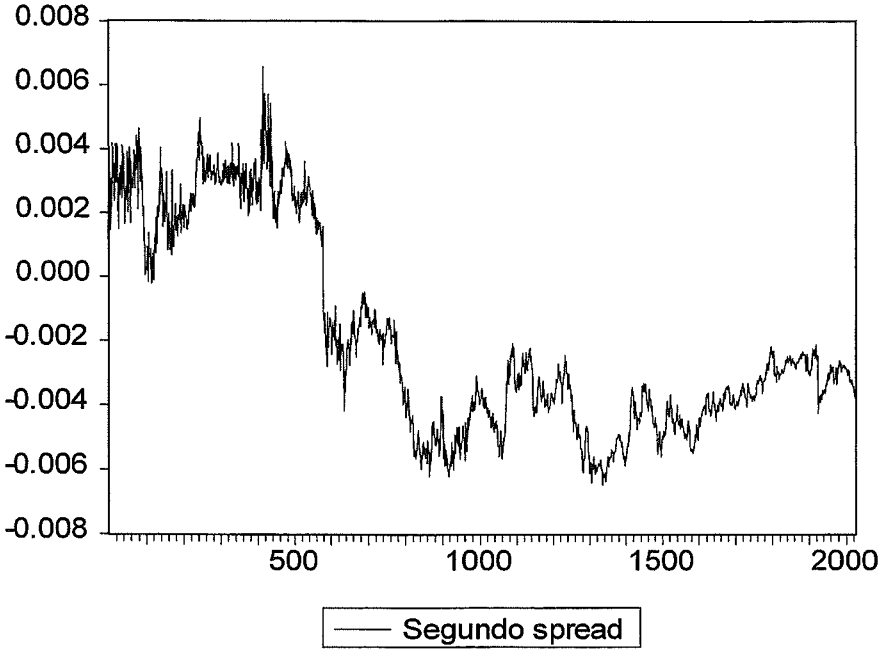
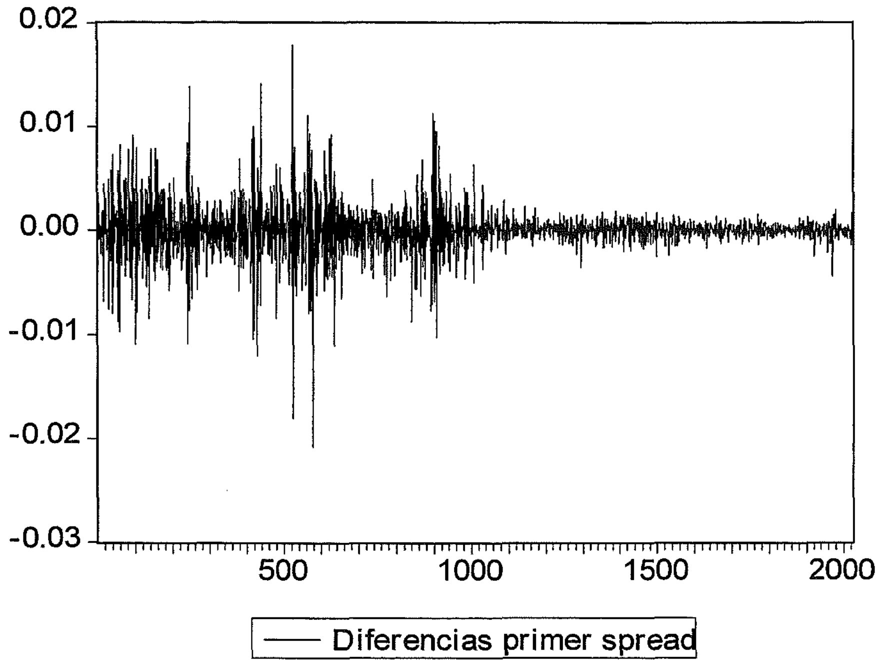
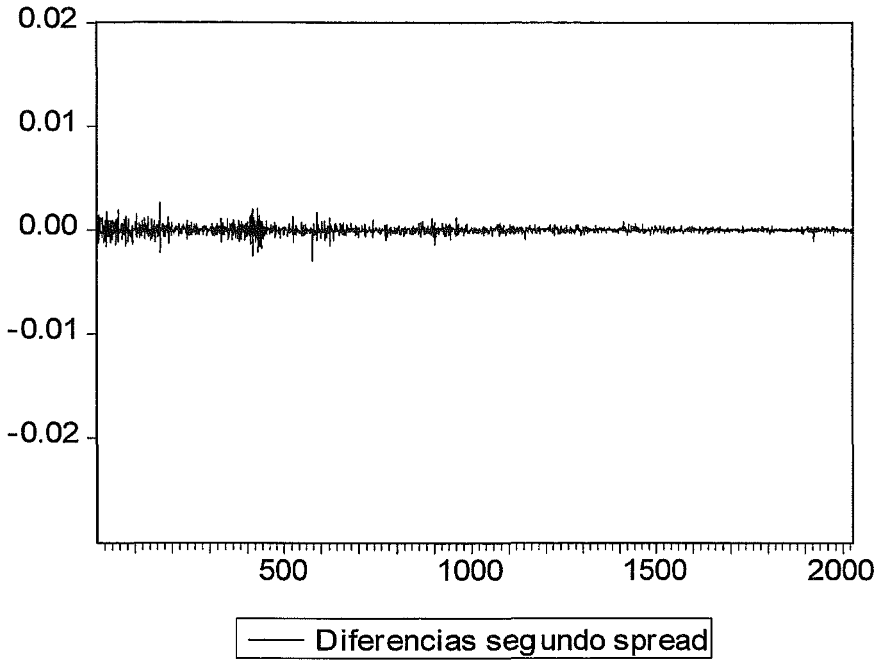
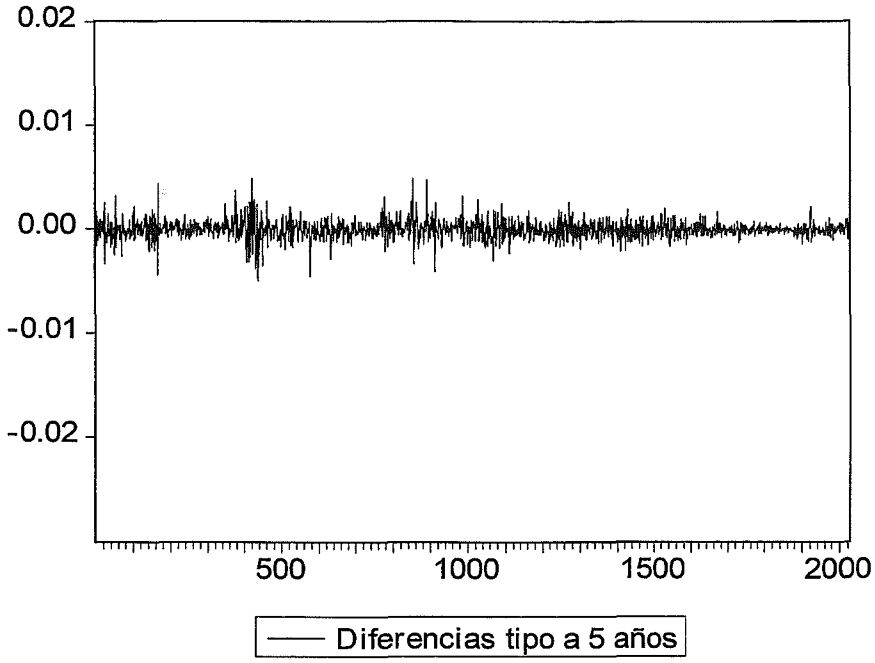
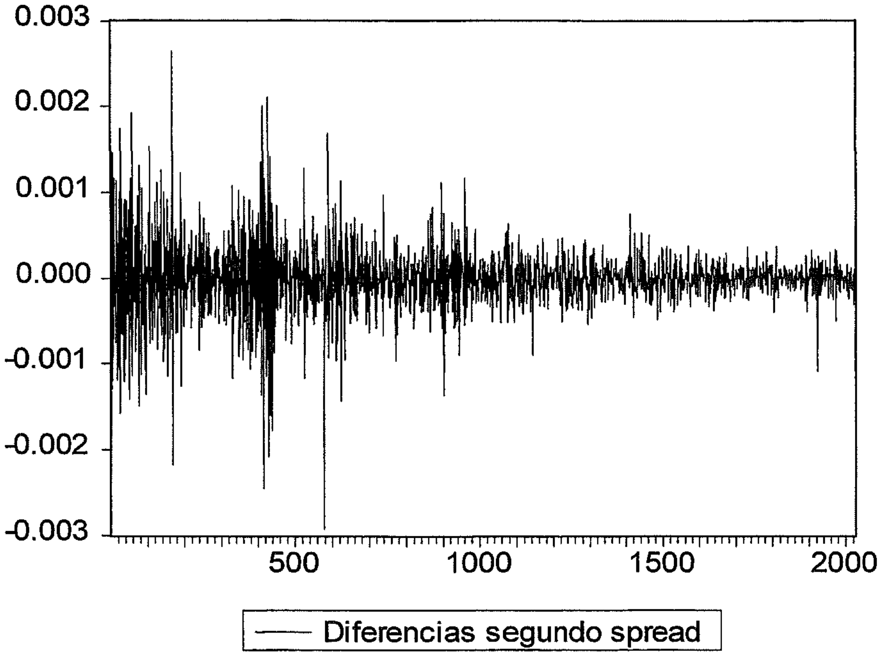

0.008
0.006
0.004
0.002
0.000
-0.002
-0.004
-0.006
-0.008
500
1000
1500
2000
Segundo spread

FIGURA II


0.02
0.01
0.00
-0.01
-0.02
-0.03
500
1000
1500
2000
Diferencias primer spread


0.02
0.01
0.00
-0.01
-0.02
500
1000
1500
2000
Diferencias segundo spread


0.02
0.01
0.00
-0.01
-0.02
500
1000
1500
2000
Diferencias tipo a 5 años


0.003
0.002
0.001
0.000
-0.001
-0.002
-0.003
500
1000
1500
2000
Diferencias segundo spread

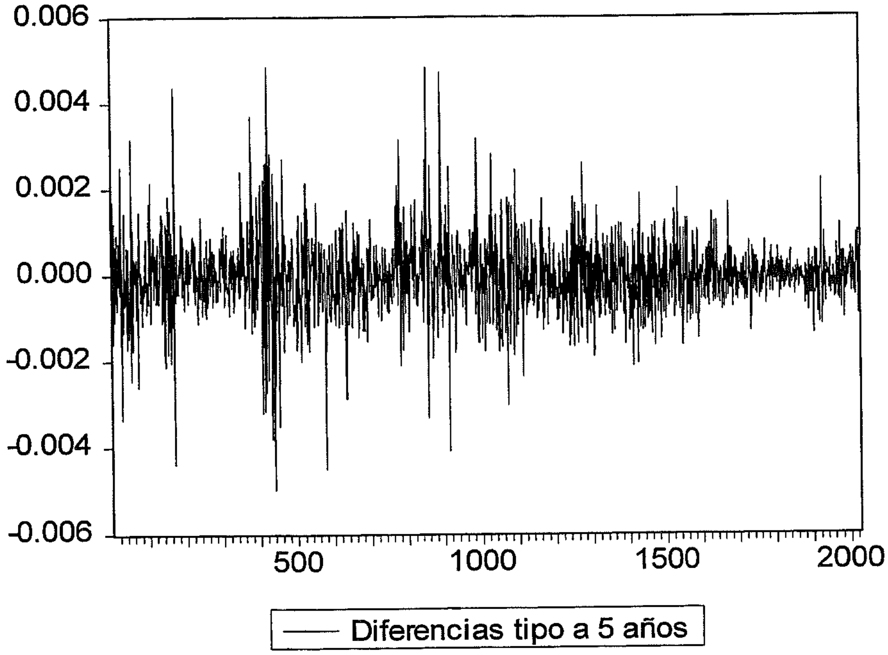
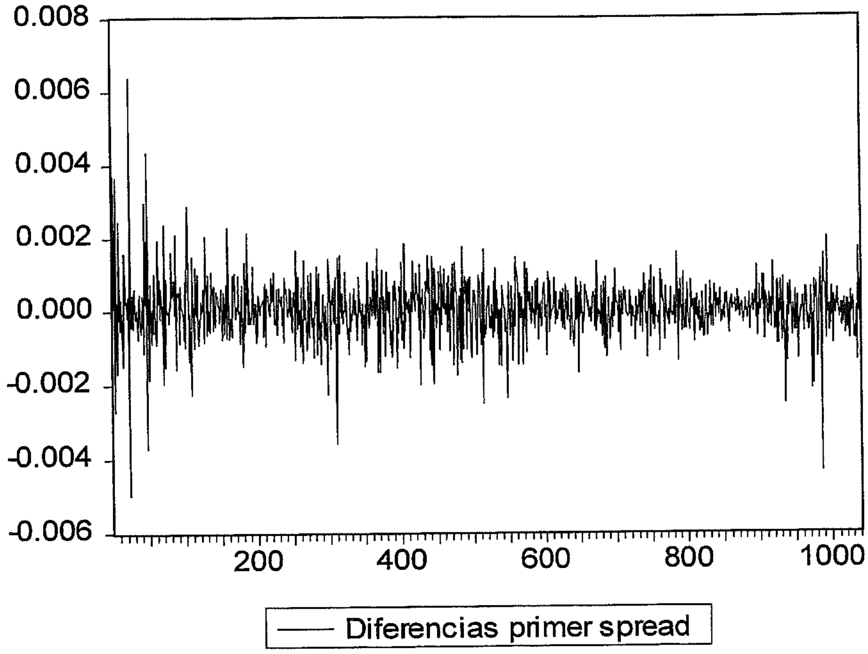
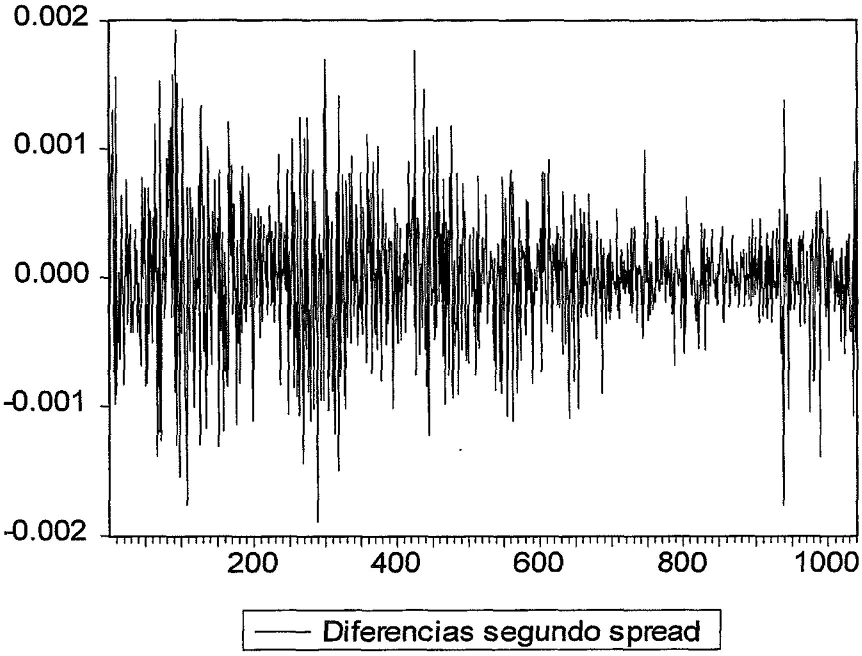
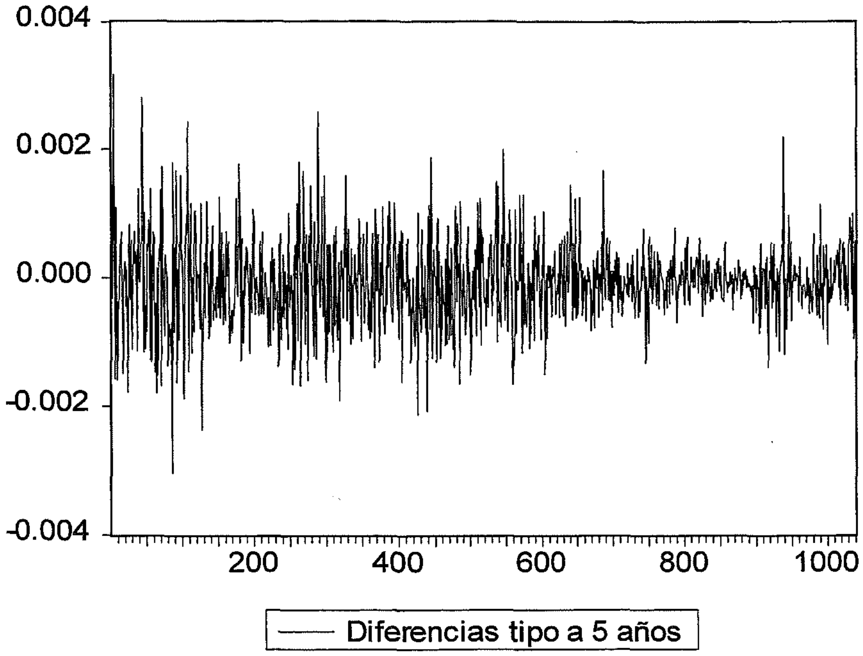

0.006
0.004
0.002
0.000
-0.002
-0.004
-0.006
500
1000
1500
2000
Diferencias tipo a 5 años

FIGURA III


0.008
0.006
0.004
0.002
0.000
-0.002
-0.004
-0.006
200
400
600
800
1000
Diferencias primer spread


0.002
0.001
0.000
-0.001
-0.002
200
400
600
800
1000
Diferencias segundo spread



0.004
0.002
0.000
-0.002
-0.004
200
400
600
800
1000
Diferencias tipo a 5 años

Estadísticos de las variables de estado

02-01-1995/09-03-1999

Variable	s_1	s_2	l
n	1040	1040	1040
Media	−0.002178	−0.003921	0.060548
Desv. Estándar	0.007572	0.000995	0.025247
Mediana	−0.000174	−0.003816	0.060548
Máximo	0.007916	−0.002090	0.118285
Mínimo	−0.030459	−0.006466	0.032837
Coef. Asimetría	−1.874820	−0.418515	0.404518
Coef. Curtosis	6.060345	2.365393	1.807380

Estadísticos de las diferencias de las variables de estado

02-01-1995/09-03-1999

Variable	ds_1	ds_2	dl
n	1039	1039	1039
Media	$3.38E-05$	$8.45E-06$	$-7.21E-05$
Desv. Estándar	0.000810	0.000497	0.000669
Mediana	$2.33E-05$	$1.67E-05$	$-6.65E-05$
Máximo	0.006385	0.001928	0.003168
Mínimo	−0.004967	−0.001894	−0.003038
Coef. Asimetría	0.196596	−0.051694	0.241814
Coef. Curtosis	10.91852	4.405727	4.912411

FIGURA IV

Augmented Dickey-Fuller Unit Root Test on DS1

ADF Test Statistic	-32.92799	1% Critical Value*	-2.5668
		5% Critical Value	-1.9395
		10% Critical Value	-1.6157

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
LS // Dependent Variable is D(DS1)
Date: 03/31/00 Time: 12:21
Sample(adjusted): 4 2024
Included observations: 2021 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DS1(-1)	-1.584799	0.048129	-32.92799	0.0000
D(DS1(-1))	0.247033	0.036726	6.726336	0.0000
D(DS1(-2))	0.070022	0.022198	3.154381	0.0016

R-squared	0.651888	Mean dependent var	2.50E-07
Adjusted R-squared	0.651543	S.D. dependent var	0.003772
S.E. of regression	0.002227	Akaike info criterion	-12.21292
Sum squared resid	0.010006	Schwarz criterion	-12.20459
Log likelihood	9476.478	F-statistic	1889.491
Durbin-Watson stat	2.006504	Prob(F-statistic)	0.000000

Serie diferencias primer spread (02-01-91/09-03-99)

Augmented Dickey-Fuller Unit Root Test on DS2

ADF Test Statistic	-33.88661	1% Critical Value*	-2.5668
		5% Critical Value	-1.9395
		10% Critical Value	-1.6157

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
LS // Dependent Variable is D(DS2)
Date: 03/31/00 Time: 12:21
Sample(adjusted): 4 2024
Included observations: 2021 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DS2(-1)	-1.649048	0.048664	-33.88661	0.0000
D(DS2(-1))	0.258705	0.037445	6.908930	0.0000
D(DS2(-2))	0.112038	0.021991	5.094717	0.0000

R-squared	0.680231	Mean dependent var	-7.95E-07
Adjusted R-squared	0.679915	S.D. dependent var	0.000624
S.E. of regression	0.000353	Akaike info criterion	-15.89736
Sum squared resid	0.000251	Schwarz criterion	-15.88903
Log likelihood	13199.61	F-statistic	2146.408
Durbin-Watson stat	1.996257	Prob(F-statistic)	0.000000

Serie diferencias segundo spread (02-01-91/09-03-99)

Augmented Dickey-Fuller Unit Root Test on DL

ADF Test Statistic	-24.81389	1% Critical Value*	-2.5668
		5% Critical Value	-1.9395
		10% Critical Value	-1.6157

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
LS // Dependent Variable is D(DL)
Date: 03/31/00 Time: 12:22
Sample(adjusted): 4 2024
Included observations: 2021 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DL(-1)	-0.903915	0.036428	-24.81389	0.0000
D(DL(-1))	-0.014454	0.030208	-0.478475	0.6324
D(DL(-2))	-0.015235	0.022226	-0.685468	0.4931

R-squared	0.459674	Mean dependent var	1.07E-06
Adjusted R-squared	0.459139	S.D. dependent var	0.001133
S.E. of regression	0.000833	Akaike info criterion	-14.17892
Sum squared resid	0.001401	Schwarz criterion	-14.17059
Log likelihood	11463.13	F-statistic	858.3926
Durbin-Watson stat	1.997281	Prob(F-statistic)	0.000000

Serie diferencias tipo de interés a largo plazo (02-01-91/09-03-99)

Augmented Dickey-Fuller Unit Root Test on DS1B

ADF Test Statistic	-22.16648	1% Critical Value*	-2.5677
		5% Critical Value	-1.9397
		10% Critical Value	-1.6158

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
LS // Dependent Variable is D(DS1B)
Date: 03/31/00 Time: 12:23
Sample(adjusted): 4 1039
Included observations: 1036 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DS1B(-1)	-1.362801	0.061480	-22.16648	0.0000
D(DS1B(-1))	0.176514	0.047465	3.718835	0.0002
D(DS1B(-2))	0.052354	0.030824	1.698479	0.0897

R-squared	0.587567	Mean dependent var	-2.10E-07
Adjusted R-squared	0.586769	S.D. dependent var	0.001224
S.E. of regression	0.000787	Akaike info criterion	-14.29188
Sum squared resid	0.000640	Schwarz criterion	-14.27756
Log likelihood	5936.172	F-statistic	735.8257
Durbin-Watson stat	1.986831	Prob(F-statistic)	0.000000

Serie diferencias primer *spread* (02-01-95/09-03-99)

Augmented Dickey-Fuller Unit Root Test on DS2B

ADF Test Statistic	-20.04975	1% Critical Value*	-2.5677
		5% Critical Value	-1.9397
		10% Critical Value	-1.6158

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
LS // Dependent Variable is D(DS2B)
Date: 03/31/00 Time: 12:23
Sample(adjusted): 4 1039
Included observations: 1036 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DS2B(-1)	-1.134819	0.056600	-20.04975	0.0000
D(DS2B(-1))	0.076421	0.045164	1.692054	0.0909
D(DS2B(-2))	0.024489	0.031100	0.787431	0.4312

R-squared	0.529432	Mean dependent var	-1.41E-06
Adjusted R-squared	0.528521	S.D. dependent var	0.000722
S.E. of regression	0.000495	Akaike info criterion	-15.21721
Sum squared resid	0.000254	Schwarz criterion	-15.20290
Log likelihood	6415.495	F-statistic	581.1088
Durbin-Watson stat	2.003318	Prob(F-statistic)	0.000000

Serie diferencias segundo *spread* (02-01-95/09-03-99)

Augmented Dickey-Fuller Unit Root Test on DLB

ADF Test Statistic	-18.68600	1% Critical Value*	-2.5677
		5% Critical Value	-1.9397
		10% Critical Value	-1.6158

*MacKinnon critical values for rejection of hypothesis of a unit root.

Augmented Dickey-Fuller Test Equation
LS // Dependent Variable is D(DLB)
Date: 03/31/00 Time: 12:23
Sample(adjusted): 4 1039
Included observations: 1036 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DLB(-1)	-1.000983	0.053569	-18.68600	0.0000
D(DLB(-1))	0.031793	0.043356	0.733304	0.4635
D(DLB(-2))	-0.011927	0.031135	-0.383074	0.7017

R-squared	0.486881	Mean dependent var	-4.09E-07
Adjusted R-squared	0.485887	S.D. dependent var	0.000938
S.E. of regression	0.000672	Akaike info criterion	-14.60620
Sum squared resid	0.000467	Schwarz criterion	-14.59189
Log likelihood	6098.993	F-statistic	490.0884
Durbin-Watson stat	1.992340	Prob(F-statistic)	0.000000

Serie diferencias tipo de interés a largo plazo (02-01-95/09-03-99)

ANEXO II

$S_1 := datos2_2 \quad S_2 := datos2_3 \quad l := datos2_1$

$$f1(\alpha_1,q_1,\tau) := e^{\left[\left(\frac{-0.000703^2}{4\cdot q_1}\right)\cdot\left(\frac{1-e^{-q_1\cdot\tau}}{q_1}\right)^2+\left(\frac{\alpha_1}{q_1}-\frac{0.000703^2}{2\cdot q_1^2}\right)\cdot\left(\frac{1-e^{-q_1\cdot\tau}}{q_1}-\tau\right)\right]}$$

$$f2(\alpha_2,q_2,\tau) := e^{\left[\left(\frac{-0.000176^2}{4\cdot q_2}\right)\cdot\left(\frac{1-e^{-q_2\cdot\tau}}{q_2}\right)^2+\left(\frac{\alpha_2}{q_2}-\frac{0.000176^2}{2\cdot q_2^2}\right)\cdot\left(\frac{1-e^{-q_2\cdot\tau}}{q_2}-\tau\right)\right]}$$

$$f3(q_3,g,\tau) := \frac{2\cdot g\cdot e^{(q_3+g)\cdot\frac{\tau}{2}}}{(q_3+g)\cdot\left(e^{g\cdot\tau}-1\right)+2\cdot g}$$

$$f4(q_1,q_2,q_3,g,\tau) := e^{-\frac{1-e^{-q_1\cdot\tau}}{q_1}\cdot S_1-\frac{1-e^{-q_2\cdot\tau}}{q_2}\cdot S_2-\left[\frac{2\cdot\left(e^{g\cdot\tau}-1\right)}{\left[(q_3+g)\cdot\left(e^{g\cdot\tau}-1\right)+2\cdot g\right]}\right]\cdot l}$$

$$f(\alpha_1,\alpha_2,q_1,q_2,q_3,g,\tau) := f1(\alpha_1,q_1,\tau)\cdot f2(\alpha_2,q_2,\tau)\cdot f3(q_3,g,\tau)\cdot f4(q_1,q_2,q_3,g,\tau)$$

$m := last\left(datos1^{\langle 1\rangle}\right) \quad m = 32 \quad \tau := datos1^{\langle 1\rangle} \quad R := datos1^{\langle 2\rangle}$

ORIGIN $\equiv$ 1

$$h(\alpha_1,\alpha_2,q_1,q_2,q_3,g) := \sum_{i=1}^{m}\left(R_i-f(\alpha_1,\alpha_2,q_1,q_2,q_3,g,\tau_i)\right)^2$$

$\alpha_1 := 0 \quad \alpha_2 := 0.0002 \quad q_1 := 0.004 \quad q_2 := 0.013 \quad g := 0.003 \quad q_3 := 0.13$

Given

$g = \left(q_3^2+0.000005938\right)^{\frac{1}{2}} \quad a := Minimize(h,\alpha_1,\alpha_2,q_1,q_2,q_3,g) \quad resultado^{\langle 1\rangle} := a$

$\alpha_1 := 0.0003 \quad \alpha_2 := 0.0003 \quad q_1 := 0.014 \quad q_2 := 0.013 \quad g := 0.013 \quad q_3 := 0.013$

Given

$g = \left(q_3^2+0.000005938\right)^{\frac{1}{2}} \quad b := Minimize(h,\alpha_1,\alpha_2,q_1,q_2,q_3,g) \quad resultado^{\langle 2\rangle} := b$

$\alpha_1 := 0$ $\alpha_2 := 0.0003$ $q_1 := 0.004$ $q_2 := 0.003$ $g := 0.01$ $q_3 := 0.003$

Given

$g = \left(q_3^{\,2} + 0.000005938\right)^{\frac{1}{2}}$ $c := \mathrm{Minimize}(h, \alpha_1, \alpha_2, q_1, q_2, q_3, g)$ $\mathrm{resultado}^{\langle 3 \rangle} := c$

$\alpha_1 := 0.0004$ $\alpha_2 := -0.0011$ $q_1 := 0.013$ $q_2 := 0.016$ $q_3 := -0.034$ $g := 0.034$

Given

$g = \left(q_3^{\,2} + 0.000005938\right)^{\frac{1}{2}}$ $d := \mathrm{Minimize}(h, \alpha_1, \alpha_2, q_1, q_2, q_3, g)$ $\mathrm{resultado}^{\langle 4 \rangle} := d$

$$\mathrm{resultado} = \begin{pmatrix} 0.002278 & 0.001331 & 0.002251 & 0.00051 \\ 0.0002 & 0.001245 & 0.0003 & -0.000205 \\ 0.004 & 0.074208 & 0.004 & 0.014953 \\ 0.013 & 0.065665 & 0.003 & 0.017901 \\ 0.001864 & 0.012549 & 0.003406 & -0.011493 \\ 0.003068 & 0.065292 & 0.004188 & 0.103645 \end{pmatrix} \qquad r := \mathrm{resultado}$$

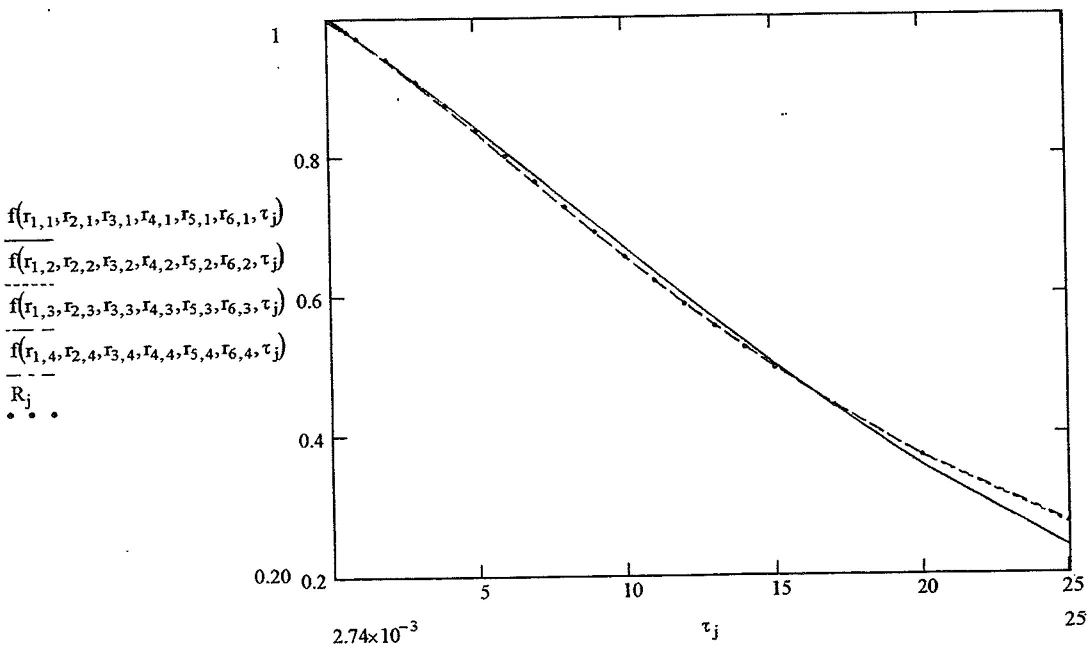
$i := 1..4$ $h_i := h(r_{1,i}, r_{2,i}, r_{3,i}, r_{4,i}, r_{5,i}, r_{6,i})$

$h_i =$

0.00275484779
0.00008134344
0.00282702723
0.00005012437

$\min(h) = 0.000050124374$

i := 1 .. m



f(r1,1, r2,1, r3,1, r4,1, r5,1, r6,1, τj)
f(r1,2, r2,2, r3,2, r4,2, r5,2, r6,2, τj)
f(r1,3, r2,3, r3,3, r4,3, r5,3, r6,3, τj)
f(r1,4, r2,4, r3,4, r4,4, r5,4, r6,4, τj)
Rj
1
0.8
0.6
0.4
0.20
0.2
5
10
15
20
25
25
2.74×10^-3
τj

Vencimientos (años)		Función de descuento (18-02-1999)
1 día	0.002740	0.999924
1 semana	0.019178	0.999467
2 semanas	0.041667	0.998933
3 semamas	0.062500	0.998400
1 mes	0.083333	0.997687
1 mes y medio	0.125000	0.996617
2 meses	0.166667	0.995366
2 meses y medio	0.208333	0.994292
3 meses	0.250000	0.993036
3 meses y medio	0.291667	0.991957
4 meses	0.333333	0.990696
4 meses y medio	0.375000	0.989613
6 meses	0.500000	0.985987
8 meses	0.666667	0.981233
1 año	1.000000	0.971580
2 años	2.000000	0.941311
3 años	3.000000	0.909012
4 años	4.000000	0.874896
5 años	5.000000	0.839381
6 años	6.000000	0.802963
7 años	7.000000	0.766138
8 años	8.000000	0.729356
9 años	9.000000	0.693000
10 años	10.000000	0.657384
11 años	11.000000	0.622747
12 años	12.000000	0.589267
13 años	13.000000	0.557067
14 años	14.000000	0.526224
15 años	15.000000	0.496780
17 años	17.000000	0.442114
20 años	20.000000	0.370320
25 años	25.000000	0.274777

Variables de estado (18-02-1999)	
s1	-0.00401055
s2	-0.00321906
l	0.035018114

Base de datos Servicio de estudios del Banco de España.
Soledad Nuñez (1995)
Método de ajuste de Svensson (1994)

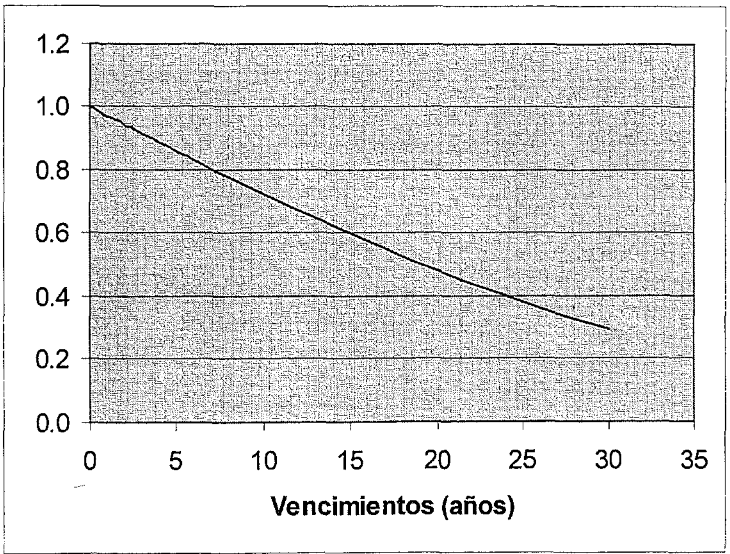
ANEXO III

	1er. Spread	2o. Spread	tipo a largo
valor	-0.004010	-0.003219	0.035018
k	0.009872	0.015401	0.001703
μ	0.000009	-0.003883	0.022196
σ	0.000703	0.000176	0.002437
a, c, λ^*	-0.725334	0.825000	-0.013196
b, d	7.227596	14.204545	

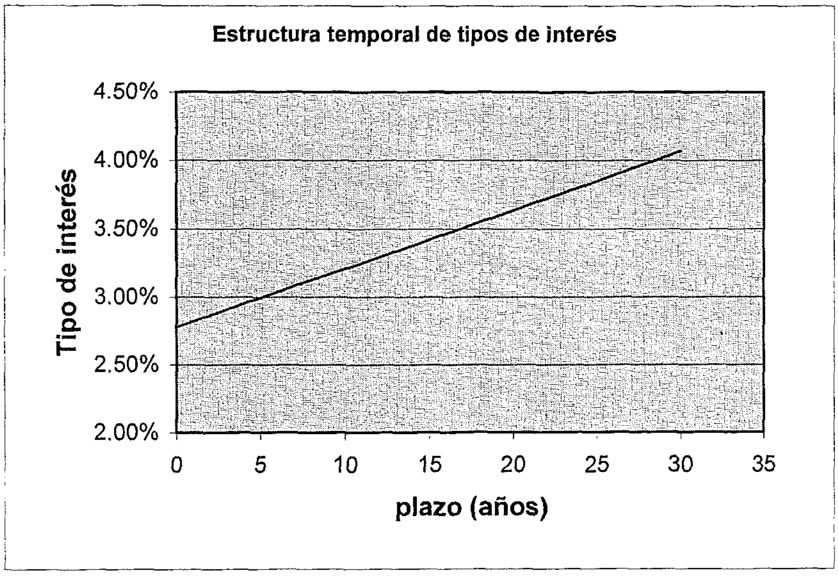
$A_1(\tau)$	$A_2(\tau)$	$A_3(\tau)$	$B(\tau)$	$C(\tau)$	$D(\tau)$
1	1	1	0	0	0
1	1	1	0.041654	0.041651	0.041677
0.999998	1.000001	1	0.083281	0.083271	0.083373
0.999993	1.000003	0.999999	0.166459	0.166418	0.166826
0.999972	1.000011	0.999998	0.332504	0.332341	0.333973
0.999936	1.000026	0.999995	0.498136	0.497769	0.501439
0.999887	1.000045	0.999992	0.663355	0.662704	0.669227
0.999824	1.000071	0.999987	0.828163	0.827149	0.837336
0.999746	1.000102	0.999981	0.992561	0.991103	1.005768
0.999604	1.000159	0.999970	1.238390	1.236119	1.259020
0.999431	1.000229	0.999957	1.483303	1.480040	1.513001
0.999227	1.000311	0.999942	1.727302	1.722873	1.767712
0.998991	1.000405	0.999924	1.970390	1.964621	2.023155
0.998725	1.000512	0.999903	2.212571	2.205290	2.279332
0.998428	1.000631	0.999881	2.453849	2.444885	2.536246
0.997744	1.000907	0.999828	2.933706	2.920868	3.052290
0.996938	1.001231	0.999765	3.409990	3.392611	3.571304
0.996013	1.001603	0.999693	3.882726	3.860150	4.093303
0.993810	1.002491	0.999518	4.817660	4.782767	5.146323
0.991145	1.003569	0.999304	5.738718	5.689015	6.211483
0.984472	1.006280	0.998753	7.540027	7.453561	8.378756
0.976089	1.009716	0.998037	9.288263	9.156051	10.596201
0.966094	1.013860	0.997153	10.984991	10.798666	12.864921
0.954590	1.018696	0.996097	12.631727	12.383512	15.186039
0.934735	1.027217	0.994182	15.011309	14.656891	18.768472
0.912088	1.037223	0.991861	17.286503	16.811402	22.475337
0.887015	1.048682	0.989122	19.461890	18.853262	26.310702
0.859881	1.061569	0.985955	21.541848	20.788359	30.278750
0.821108	1.080939	0.981046	24.173889	23.212022	35.783327

plazo (años)	función de descuento
0	**1**
0.041667	**0.998842**
0.083333	**0.997684**
0.166667	**0.995367**
0.333333	**0.990732**
0.500000	**0.986095**
0.666667	**0.981456**
0.833333	**0.976816**
1	**0.972174**
1.250000	**0.965210**
1.500000	**0.958243**
1.750000	**0.951276**
2	**0.944309**
2.250000	**0.937342**
2.500000	**0.930377**
3	**0.916454**
3.500000	**0.902545**
4	**0.888657**
5	**0.860964**
6	**0.833420**
8	**0.778944**
10	**0.725543**
12	**0.673496**
14	**0.623049**
17	**0.550828**
20	**0.483236**
23	**0.420667**
26	**0.363361**
30	**0.295286**

Función de descuento (18-02-1999)



Tipos de interés al contado	plazo (años)
0.027797	0.019178
0.027807	0.041667
0.027825	0.083333
0.027861	0.166667
0.027933	0.333333
0.028005	0.5
0.028077	0.666667
0.028148	0.833333
0.028220	1.000000
0.028328	1.25
0.028436	1.50
0.028543	1.75
0.028651	2
0.028758	2.25
0.028866	2.50
0.029081	3
0.029296	3.50
0.029511	4
0.029940	5
0.030370	6
0.031227	8
0.032083	10
0.032939	12
0.033795	14
0.035078	17
0.036363	20
0.037648	23
0.038937	26
0.040660	30



	1er. Spread	2o. Spread	tipo a largo
valor	-0.004010	-0.003219	0.035018
k	0.009872	0.015401	0.001703
μ	0.000009	-0.003883	0.022196
σ	0.000703	0.000576	0.002437
a, c, λ^*	-0.725334	0.825000	-0.013196
b, d	7.227596	14.204545	

$A_1(\tau)$	$A_2(\tau)$	$A_3(\tau)$	$B(\tau)$	$C(\tau)$	$D(\tau)$
1	1	1	0	0	0
1.000000	1.000000	1.000000	0.041654	0.041646	0.041677
0.999998	1.000002	1.000000	0.083281	0.083252	0.083373
0.999993	1.000007	0.999999	0.166459	0.166340	0.166826
0.999972	1.000030	0.999998	0.332504	0.332027	0.333973
0.999936	1.000067	0.999995	0.498136	0.497064	0.501439
0.999887	1.000118	0.999992	0.663355	0.661453	0.669227
0.999824	1.000185	0.999987	0.828163	0.825198	0.837336
0.999746	1.000265	0.999981	0.992561	0.988301	1.005768
0.999604	1.000414	0.999970	1.238390	1.231756	1.259020
0.999431	1.000595	0.999957	1.483303	1.473779	1.513001
0.999227	1.000809	0.999942	1.727302	1.714380	1.767712
0.998991	1.001054	0.999924	1.970390	1.953567	2.023155
0.998725	1.001332	0.999903	2.212571	2.191348	2.279332
0.998428	1.001642	0.999881	2.453849	2.427731	2.536246
0.997744	1.002356	0.999828	2.933706	2.896336	3.052290
0.996938	1.003196	0.999765	3.409990	3.359449	3.571304
0.996013	1.004160	0.999693	3.882726	3.817132	4.093303
0.993810	1.006459	0.999518	4.817660	4.716468	5.146323
0.991145	1.009245	0.999304	5.738718	5.594842	6.211483
0.984472	1.016247	0.998753	7.540027	7.290651	8.378756
0.976089	1.025122	0.998037	9.288263	8.908334	10.596201
0.966094	1.035834	0.997153	10.98499	10.451489	12.864921
0.954590	1.048359	0.996097	12.63173	11.923549	15.186039
0.934735	1.070512	0.994182	15.01131	14.005462	18.768472
0.912088	1.096697	0.991861	17.28650	15.945173	22.475337
0.887015	1.126938	0.989122	19.46189	17.752394	26.310702
0.859882	1.161295	0.985955	21.54185	19.436174	30.278750
0.821109	1.213677	0.981046	24.17389	21.503687	35.783327

plazo	f.descuento
0	**1**
0.041667	0.998842
0.083333	0.997685
0.166667	0.995372
0.333333	0.990749
0.500000	0.986134
0.666667	0.981524
0.833333	0.976921
1	0.972325
1.25	0.965442
1.5	0.958575
1.8	0.951724
2	0.944888
2.25	0.938069
2.50	0.931265
3	0.917708
3.50	0.904220
4	0.890803
5	0.864188
6	0.837880
8	0.786248
10	0.736026
12	0.687325
14	0.640242
17	0.572842
20	0.509521
23	0.450459
26	0.395770
30	0.329729

El precio de la obligación cupón cero a su vencimiento es igual a la unidad.

	1er. Spread	2o. Spread	tipo a largo
valor	-0.004010	-0.003219	0.035018
k	0.009872	0.015401	0.001703
μ	0.000009	-0.003883	0.022196
σ	0.000703	0.000576	0.002437
a, c, λ^*	-0.725334	0.825000	-0.013196
b, d	7.227596	14.204545	

$A_1(\tau)$	$A_2(\tau)$	$A_3(\tau)$	$B(\tau)$	$C(\tau)$	$D(\tau)$
1	1	1	0	0	0
1.000000	1.000000	1.000000	0.041654	0.041646	0.041677
0.999998	1.000002	1.000000	0.083281	0.083252	0.083373
0.999993	1.000007	0.999999	0.166459	0.166340	0.166826
0.999972	1.000030	0.999998	0.332504	0.332027	0.333973
0.999936	1.000067	0.999995	0.498136	0.497064	0.501439
0.999887	1.000118	0.999992	0.663355	0.661453	0.669227
0.999824	1.000185	0.999987	0.828163	0.825198	0.837336
0.999746	1.000265	0.999981	0.992561	0.988301	1.005768
0.999604	1.000414	0.999970	1.238390	1.231756	1.259020
0.999431	1.000595	0.999957	1.483303	1.473779	1.513001
0.999227	1.000809	0.999942	1.727302	1.714380	1.767712
0.998991	1.001054	0.999924	1.970390	1.953567	2.023155
0.998725	1.001332	0.999903	2.212571	2.191348	2.279332
0.998428	1.001642	0.999881	2.453849	2.427731	2.536246
0.997744	1.002356	0.999828	2.933706	2.896336	3.052290
0.996938	1.003196	0.999765	3.409990	3.359449	3.571304
0.996013	1.004160	0.999693	3.882726	3.817132	4.093303
0.993810	1.006459	0.999518	4.817660	4.716468	5.146323
0.991145	1.009245	0.999304	5.738718	5.594842	6.211483
0.984472	1.016247	0.998753	7.540027	7.290651	8.378756
0.976089	1.025122	0.998037	9.288263	8.908334	10.596201
0.966094	1.035834	0.997153	10.984991	10.451489	12.864921
0.954590	1.048359	0.996097	12.631727	11.923549	15.186039
0.934735	1.070512	0.994182	15.011309	14.005462	18.768472
0.912088	1.096697	0.991861	17.286503	15.945173	22.475337
0.887015	1.126938	0.989122	19.461890	17.752394	26.310702
0.859882	1.161295	0.985955	21.541848	19.436174	30.278750
0.821109	1.213677	0.981046	24.173889	21.503687	35.783327

plazo	f.descuento
0	1
0.041667	**0**
0.083333	**0**
0.166667	**0**
0.333333	**0**
0.50	**0**
0.666667	**0**
0.833333	**0**
1	**0**
1.25	**0**
1.50	**0**
1.75	**0**
2	**0**
2.25	**0**
2.50	**0**
3	**0**
3.50	**0**
4	**0**
5	**0**
6	**0**
8	**0**
10	**0**
12	**0**
14	**0**
17	**0**
20	**0**
23	**0**
26	**0**
30	**0**

La función de descuento se anula para valores muy elevados de las variables del modelo.

	1er. Spread	2o. Spread	tipo a largo
valor	-0.004010	-0.003219	0.035018
k	0.009872	0.015401	0.001703
μ	0.000009	-0.003883	0.022196
σ	0.000703	0.000576	0.002437
a, c, λ^*	-0.725334	0.825000	-0.013196
b, d	7.227596	14.204545	

$A_1(\tau)$	$A_2(\tau)$	$A_3(\tau)$	$B(\tau)$	$C(\tau)$	$D(\tau)$
1	1	1	0	0	0
1.000000	1.000000	1.000000	0.041654	0.041646	0.041677
0.999998	1.000002	1.000000	0.083281	0.083252	0.083373
0.999993	1.000007	0.999999	0.166459	0.166340	0.166826
0.999972	1.000030	0.999998	0.332504	0.332027	0.333973
0.999936	1.000067	0.999995	0.498136	0.497064	0.501439
0.999887	1.000118	0.999992	0.663355	0.661453	0.669227
0.999824	1.000185	0.999987	0.828163	0.825198	0.837336
0.999746	1.000265	0.999981	0.992561	0.988301	1.005768
0.999604	1.000414	0.999970	1.238390	1.231756	1.259020
0.999431	1.000595	0.999957	1.483303	1.473779	1.513001
0.999227	1.000809	0.999942	1.727302	1.714380	1.767712
0.998991	1.001054	0.999924	1.970390	1.953567	2.023155
0.998725	1.001332	0.999903	2.212571	2.191348	2.279332
0.998428	1.001642	0.999881	2.453849	2.427731	2.536246
0.997744	1.002356	0.999828	2.933706	2.896336	3.052290
0.996938	1.003196	0.999765	3.409990	3.359449	3.571304
0.996013	1.004160	0.999693	3.882726	3.817132	4.093303
0.993810	1.006459	0.999518	4.817660	4.716468	5.146323
0.991145	1.009245	0.999304	5.738718	5.594842	6.211483
0.984472	1.016247	0.998753	7.540027	7.290651	8.378756
0.976089	1.025122	0.998037	9.288263	8.908334	10.596201
0.966094	1.035834	0.997153	10.98499	10.451489	12.864921
0.954590	1.048359	0.996097	12.63173	11.923549	15.186039
0.934735	1.070512	0.994182	15.01131	14.005462	18.768472
0.912088	1.096697	0.991861	17.28650	15.945173	22.475337
0.887015	1.126938	0.989122	19.46189	17.752394	26.310702
0.859882	1.161295	0.985955	21.54185	19.436174	30.278750
4.2E-143	2.5E+99	0	66.876212	42.40375	3955.505

plazo	f.descuento
0	1
0.041667	0.998842
0.083333	0.997685
0.166667	0.995372
0.333333	0.990749
0.50	0.986134
0.666667	0.981524
0.833333	0.976921
1	0.972325
1.25	0.965442
1.50	0.958575
1.75	0.951724
2	0.944888
2.25	0.938069
2.50	0.931265
3	0.917708
3.50	0.904220
4	0.890803
5	0.864188
6	0.837880
8	0.786248
10	0.736026
12	0.687325
14	0.640242
17	0.572842
20	0.509521
23	0.450459
26	0.395770
10000	**0**

La función de descuento se anula para vencimientos muy elevados.

	1er. Spread	2o. Spread	tipo a largo
valor	-0.004010	-0.003219	0.035018
k	0.009872	0.015401	0.001703
μ	0.000009	-0.003883	0.022196
σ	0.000703	0.000576	0.002437
a, c, λ^*	-0.725334	0.825000	-0.013196
b, d	7.227596	14.204545	

$A_1(\tau)$	$A_2(\tau)$	$A_3(\tau)$	$B(\tau)$	$C(\tau)$	$D(\tau)$
1	1	1	**0.002740**	**0.002740**	**0.002740**
1.000000	1.000000	1.000000	**0.041654**	**0.041646**	**0.041677**
0.999998	1.000002	1.000000	**0.083281**	**0.083252**	**0.083373**
0.999993	1.000007	0.999999	**0.166459**	**0.166340**	**0.166826**
0.999972	1.000030	0.999998	**0.332504**	**0.332027**	**0.333973**
0.999936	1.000067	0.999995	**0.498136**	**0.497064**	**0.501439**
0.999887	1.000118	0.999992	**0.663355**	**0.661453**	**0.669227**
0.999824	1.000185	0.999987	**0.828163**	**0.825198**	**0.837336**
0.999746	1.000265	0.999981	**0.992561**	**0.988301**	**1.005768**
0.999604	1.000414	0.999970	**1.238390**	**1.231756**	**1.259020**
0.999431	1.000595	0.999957	**1.483303**	**1.473779**	**1.513001**
0.999227	1.000809	0.999942	**1.727302**	**1.714380**	**1.767712**
0.998991	1.001054	0.999924	**1.970390**	**1.953567**	**2.023155**
0.998725	1.001332	0.999903	**2.212571**	**2.191348**	**2.279332**
0.998428	1.001642	0.999881	**2.453849**	**2.427731**	**2.536246**
0.997744	1.002356	0.999828	**2.933706**	**2.896336**	**3.052290**
0.996938	1.003196	0.999765	**3.409990**	**3.359449**	**3.571304**
0.996013	1.004160	0.999693	**3.882726**	**3.817132**	**4.093303**
0.993810	1.006459	0.999518	**4.817660**	**4.716468**	**5.146323**
0.991145	1.009245	0.999304	**5.738718**	**5.594842**	**6.211483**
0.984472	1.016247	0.998753	**7.540027**	**7.290651**	**8.378756**
0.976089	1.025122	0.998037	**9.288263**	**8.908334**	**10.596201**
0.966094	1.035834	0.997153	**10.984991**	**10.451489**	**12.864921**
0.954590	1.048359	0.996097	**12.631727**	**11.923549**	**15.186039**
0.934735	1.070512	0.994182	**15.011309**	**14.005462**	**18.768472**
0.912088	1.096697	0.991861	**17.286503**	**15.945173**	**22.475337**
0.887015	1.126938	0.989122	**19.461890**	**17.752394**	**26.310702**
0.859882	1.161295	0.985955	**21.541848**	**19.436174**	**30.278750**
0.821109	1.213677	0.981046	**24.173889**	**21.503687**	**35.783327**

plazo	f.descuento
0.002740	0.999924
0.041667	0.998842
0.083333	0.997685
0.166667	0.995372
0.333333	0.990749
0.50	0.986134
0.666667	0.981524
0.833333	0.976921
1	0.972325
1.25	0.965442
1.50	0.958575
1.75	0.951724
2	0.944888
2.25	0.938069
2.50	0.931265
3	0.917708
3.50	0.904220
4	0.890803
5	0.864188
6	0.837880
8	0.786248
10	0.736026
12	0.687325
14	0.640242
17	0.572842
20	0.509521
23	0.450459
26	0.395770
30	0.329729

Se quiere hacer notar, empíricamente, que los valores de las funciones B, C y D que son las que aparecen en el exponente, multiplicando a cada variable de estado, se aproximan al plazo considerado. En plazos cortos, el valor es más similar.

s1	-0.00401	-0.00401	-0.00801	-0.00401
s2	-0.003219	-0.003219	-0.003219	-0.008219
I	0.035018	0.038018	0.035018	0.035018
r = s1+s2+I	**0.027789**	**0.030789**	**0.023789**	**0.022789**
	0.027789	**0.030789**	**0.023789**	**0.022789**
	0.027807	0.030808	0.023808	0.022809
	0.027825	0.030826	0.023827	0.022829
	0.027861	0.030864	0.023866	0.022868
	0.027933	0.030939	0.023943	0.022948
	0.028005	0.031013	0.024020	0.023027
	0.028077	0.031088	0.024096	0.023106
	0.028148	0.031163	0.024173	0.023185
	0.028220	0.031237	0.024250	0.023265
	0.028328	0.031350	0.024365	0.023383
	0.028436	0.031462	0.024480	0.023502
	0.028543	0.031574	0.024595	0.023621
	0.028651	0.031686	0.024710	0.023739
	0.028758	0.031798	0.024825	0.023858
	0.028866	0.031910	0.024940	0.023976
	0.029081	0.032133	0.025169	0.024213
	0.029296	0.032357	0.025399	0.024449
	0.029511	0.032581	0.025628	0.024686
	0.029940	0.033028	0.026086	0.025158
	0.030370	0.033475	0.026544	0.025629
	0.031227	0.034369	0.027457	0.026568
	0.032083	0.035262	0.028368	0.027505
	0.032939	0.036156	0.029278	0.028440
	0.033795	0.037049	0.030186	0.029372
	0.035078	0.038390	0.031546	0.030767
	0.036363	0.039734	0.032905	0.032160
	0.037648	0.041080	0.034264	0.033550
	0.038937	0.042431	0.035623	0.034939
	0.040660	0.044239	0.037437	0.036792

El tanto de interés instantáneo para un plazo infinitesimal coincide con el tipo de interés instantáneo sin riesgo r(t), que es suma de los tres factores del modelo. De esta forma, la curva de tipos de interés empieza en el valor del tanto de interés instantáneo.