UNIVERSITAT DE BARCELONA
FACULTAT DE CIÈNCIES ECONÒMIQUES I EMPRESARIALS
DEPARTAMENT DE MATEMÀTICA ECONÒMICA, FINANCERA I ACTUARIAL

LA TEORIA DE LA CREDIBILITAT : EXTENSIONS I APLICACIONS

Lluís Bermúdez i Morata
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Apèndix C

Programes informàtics

Per poder comprovar i comparar les diferents aplicacions i extensions d'aquesta tesi, hem cregut interessant, i fins a cert punt necessari, fer els càlculs amb dades reals. Per fer aquests càlculs, hem programat els models que hem analitzat teòricament. El llenguatge informàtic emprat en aquest treball, ha estat l'APL. Hem escollit aquest llenguatge, principalment, per dues raons: en primer lloc, l'APL ja havia estat utilitzat per programar models de Teoria de la Credibilitat per Goovaerts, M.; en segon lloc, la gran dimensió de les matrius de dades que fem servir el fa aconsellable des d'un punt de vista informàtic. A continuació exposem els programes que hem elaborat per als models que hem estudiat al capítol 2, càlcul de primes amb recàrrec sobre la variància, al capítol 3 i 4, nous models de classificació creuada, i a la segona part d'aquesta tesi, dedicada al càlcul de reserves IBNR.

En primer lloc, presentem els programes de càlcul de primes de credibilitat amb recàrrec segons els diferents models vistos (seccions 2.2, 2.3 i 2.4):

Programa C.1 *Programa* BLOAD *de càlcul de prima de credibilitat amb recàrrec segons Bühlmann, H.*

```
      ∇BLOAD[⎕]∇
[0]   BLOAD X;K;T;H;XJW;XWW;WI;WC;S2;A;Z;XZW;E2;EE2;VE2;AD;E22;EE22;VE22;C;L
P2;P3;P;ZI;TPC;J
[1]     ⎕IO←1
[2]     W←(⍴X)⍴1
[3]     K←1↓⍴X
[4]     T←1↑⍴X
[5]     +H←0.25
[6]     +XJW←(÷WI←+⌿W)×+⌿(W×X)
[7]     +XWW←(÷WC←+/WI)×+/WI×XJW
[8]     +S2←(÷K×T-1)×+/+/W×(X-(⍴X)⍴XJW)*2
[9]     +A←0⌈(WC÷(WC*2)-+/WI*2)×(+/WI×(XJW-XWW)*2)-S2×(K-1)
[10]    +Z←(÷S2+A×WI)×A×WI
[11]    +XZW←(÷ZI←+/Z)×+/Z×XJW
[12]    +E2←(÷(T-1))×+⌿(X-(T⌿AD←(1,K)⍴XJW))*2
[13]    +EE2←(÷K)×+/E2
[14]    +VE2←(+/(E2-EE2)*2)÷(K-1)
[15]    +E22←((X[1;]-((X[1;]+X[2;])÷(2)))*2)+((X[2;]-((X[1;]+X[2;])÷(2)))*2)
[16]    +EE22←(÷K)×+/E22
[17]    +VE22←(+/(E22-EE22)*2)÷(K-1)
[18]    +C←((((T)-1)×VE2)-(VE22))÷(((T)-2)×VE2)
[19]    +P1←((1-Z)×XZW)+(Z×XJW)
[20]    +P2←((1-C)×S2)+(C×E2)
[21]    +P3←(1-Z)×A
[22]    +P←P1+L←H×(P2+P3)
[23]    ⍎L
[24]    +TPC←(L÷P1)×100
```

Programa C.2 *Programa* CLOAD *de càlcul de prima de credibilitat amb recàrrec segons Centeno, L.*

```
      ∇CLOAD[⎕]∇
[0]   W CLOAD X;H;K;T;XJW;XWW;WI;WC;S2;A;Z;XZW;E2;EE2;VE2;AD;ZI;L;VS2
P3;PL;TPC
[1]     ⎕IO←1
[2]     →(+/+/W≠1)⍴ET0
[3]     W←(⍴X)⍴1
[4]   ET0:
[5]     K←1↓⍴X
[6]     T←1↑⍴X
[7]     +H←0.25
[8]     +XJW←(÷WI←+⌿W)×+⌿(W×X)
[9]     +XWW←(÷WC←+/WI)×+/WI×XJW
[10]    +S2←(÷K×T-1)×+/+/W×(X-(⍴X)⍴XJW)*2
[11]    +A←0⌈(WC÷(WC*2)-+/WI*2)×(+/WI×(XJW-XWW)*2)-S2×(K-1)
[12]    +Z←(÷S2+A×WI)×A×WI
[13]    +XZW←(÷ZI←+/Z)×+/Z×XJW
[14]    +E2←(÷(T-1))×+⌿W×((X-(T⌿AD←(1,K)⍴XJW))*2)
[15]    +EE2←(÷K)×+/E2
[16]    +VE2←(+/((E2-EE2)*2))÷(K-1)
```

```
[17]   +L←(÷K)×+/(E2*2)
[18]   +VS2←VE2-((2×L)÷(T+1))
[19]   +C←(VS2)÷(VE2)
[20]   +WM←(WI)÷(T)
[21]   +P1←((1-Z)×XZW)+(Z×XJW)
[22]   +P2←((1-C)×EE2)+(C×E2)
[23]   +P3←(1-Z)×A
[24]   +P←P1+PL←H×((P2÷WM)+P3)
[25]   ⍒PL
[26]   +TPC←(PL÷P1)×100
```

Programa C.3 *Programa* LOAD *de càlcul de prima de credibilitat amb recàrrec segons Bauwenlickx, T. et al.*

```
       ∇LOAD[⎕]∇
[0]   W LOAD X;K;T;WI;WC;XJW;XZW;XWW;S2;A;AD;Z;ZI;P;P1;P2;P3;P4;PC
2;H;COB;COW;TPC;C
[1]    ⎕IO←1
[2]    →(+/+/W≠1)⍴ET0
[3]    W←(⍴X)⍴1
[4]   ET0:
[5]    K←1↓⍴X
[6]    T←1↑⍴X
[7]    +H←0.25
[8]    +XJW←(÷WI←+⌿W)×+⌿(W×X)
[9]    +XWW←(÷WC←+/WI)×+/WI×XJW
[10]   +S2←(÷K×T-1)×+/+/W×(X-(⍴X)⍴XJW)*2
[11]   +A←0⌈(WC÷(WC*2)-+/WI*2)×(+/WI×(XJW-XWW)*2)-S2×(K-1)
[12]   +Z←(÷S2+A×WI)×A×WI
[13]   +XZW←(÷ZI←+/Z)×+/Z×XJW
[14]   +P←((1-Z)×XZW)+(Z×XJW)
[15]   XJW2←(÷WI)×+⌿(W×X*2)
[16]   XZW2←(÷ZI)×+/(Z×XJW*2)
[17]   XWW2←(÷WC)×+/(WI×XJW*2)
[18]   COB←(÷WC)×+/(WI×(XJW-XWW)×(XJW2-XWW2))
[19]   COW←(÷WC)×+/+⌿(W×(X-(⍴X)⍴XJW)×((X*2)-(⍴X)⍴XJW2))
[20]   AD←COB-(K-1)×COW÷K×(T-1)
[21]   +P1←((1-Z)×XZW2)+(Z×XJW2)
[22]   +P2←((2×WC*2)÷((WC*2)-+/WI*2))×(Z÷A)×AD
[23]   +P3←(2×(Z*2)×WC×COW)÷(A×WI×K×(T-1))
[24]   +P4←((2×WC*2)÷((WC*2)-+/WI*2))×((Z*2)÷A)×AD
[25]   +PC←P+C←H×(P1+(((P2-P3)-P4)×(XJW-XZW))-(P*2))
[26]   ⍒C
[27]   +TPC←(C÷P)×100
```

En segon lloc, presentem els programes dels models de credibilitat de classificació creuada que vàrem veure als capítols 3 i 4:

Programa C.4 *Programa* KK *del model de credibilitat de classificació creuada "two way"*

```
      ∇KK[⎕]∇
[0]   W KK P;I;J
[1]    →(+/+/+/W≠0)⍴ET0
[2]    W←0∘.=(0∘.=P)
[3]   ET0:
[4]    D3←(+/D3←0∘.=(0∘.=P))
[5]    DIM3←2↓⍴P
[6]    +X←(+/(W×P))÷(O←(+/W))
[7]    DIM1←1↑⍴O
[8]    DIM2←1↓⍴O
[9]    +MI←(+/O×X)÷(WI←(+/O))
[10]   +MJ←(+⌿O×X)÷(WJ←(+⌿O))
[11]   +M←(+/+/O×X)÷(WT←(+/+/O))
[12]   ⍕O
[13]   +S2←(+/+/+/W×(P-(DIM3/H←(DIM1,DIM2,1)⍴,X))*2)÷(+/+/(D3-1))
[14]   C←3⍴0
[15]   HI←(DIM1,1)⍴MI
[16]   HJ←(1,DIM2)⍴MJ
[17]   C[2]←+/(÷DIM2)×((÷WJ)×(+⌿(O×(X-(DIM1⌿HJ))*2)))-((S2)×(DIM1-1)÷(WJ))
[18]   C[1]←+/(÷DIM1)×((÷WI)×(+/(O×(X-(DIM2/HI))*2)))-((S2)×(DIM2-1)÷(WI))
[19]   C[3]←(+/+/((O×(X-M)*2)÷(WT)))-(((DIM1×DIM2)-1)×(S2)÷WT)
[20]   ⍕C
[21]   D1←1-((+/((+/O*2)÷(WI*2)))÷DIM1)
[22]   D2←1-((+/((+⌿O*2)÷(WJ*2)))÷DIM2)
[23]   D3←1-(+/(WI÷WT)*2)
[24]   D4←1-(+/(WJ÷WT)*2)
[25]   D5←1-(+/+/(O÷WT)*2)
[26]   D←(3,3)⍴0,D1,D1,D2,0,D2,D3,D4,D5
[27]   ⍕D
[28]   +B←1E¯12⌈(C⌹D)
[29]   +Z←(B[3]×O)÷((B[3]×O)+S2)
[30]   +ZI←(B[1]×+/Z)÷((B[1]×+/Z)+B[3])
[31]   +ZJ←(B[2]×+⌿Z)÷((B[2]×+⌿Z)+B[3])
[32]   LJ←DIM2⍴1
[33]   LI←DIM1⍴1
[34]   LJC←DIM2⍴1
[35]   IT←0
[36]  BEGIN:
[37]   IT←IT+1
[38]   LJ←(÷2)×LJ+LJC
[39]   LI←(((÷+/Z)×(+/Z×(X-(DIM1⌿K1←(1,DIM2)⍴LJ))))-M)×ZI
[40]   LJC←(((÷+⌿Z)×(+⌿Z×(X-(DIM2/K2←(DIM1,1)⍴LI))))-M)×ZJ
[41]   →STOP×⍳IT≥1000
[42]   →BEGIN
[43]  STOP:LJ←LJC
[44]   MX←(DIM1,DIM2)⍴M
[45]   +LIJ←(((X-MX)-(⌽(⍴(⌽X))⍴LI))-(⊖(⍴X)⍴LJ))×Z
[46]   ⍕LI
[47]   ⍕LJ
[48]   +YI←(÷+/Z)×(+/(Z×(X-(DIM1⌿H1←(1,DIM2)⍴LJ))))
[49]   +YJ←(÷+⌿Z)×(+⌿(Z×(X-(DIM2/H2←(DIM1,1)⍴LI))))
[50]   ID←(DIM1,DIM2)⍴⍳1
[51]   Q2←(⍴O)⍴(ZJ×(YJ-M))
[52]   Q1←⌽((⍴(⌽O))⍴(ZI×(YI-M)))
[53]   +XE←MX+(Z×(X-MX))+((ID-Z)×Q1)+((ID-Z)×Q2)
```

Programa C.5 *Programa* KKA *del model de credibilitat de classificació creuada additiu "two way"*

```
      ∇KKA[⎕]∇
[0]   W KKA P;I;J
[1]    →(+/+/+/W≠0)⍴ET0
[2]    W←0∘.=(0∘.=P)
[3]   ET0:
[4]    D3←(+/D3←0∘.=(0∘.=P))
[5]    DIM3←2↓⍴P
[6]    +X←(+/(W×P))÷(O←(+/W))
[7]    DIM1←1↑⍴O
[8]    DIM2←1↓⍴O
[9]    +MI←(+/O×X)÷(WI←(+/O))
[10]   +MJ←(+⌿O×X)÷(WJ←(+⌿O))
[11]   +M←(+/+/O×X)÷(WT←(+/+/O))
[12]   ⍕O
[13]   +S2←(+/+/+/W×(P-(DIM3/H←(DIM1,DIM2,1)⍴,X))*2)÷(+/+/(D3-1))
[14]   C←2⍴0
[15]   HI←(DIM1,1)⍴MI
[16]   HJ←(1,DIM2)⍴MJ
[17]   C[2]←+/(÷DIM2)×((÷WJ)×(+⌿(O×(X-(DIM1⍴HJ))*2)))-((S2)×(DIM1-1)÷(WJ))
[18]   C[1]←+/(÷DIM1)×((÷WI)×(+/(O×(X-(DIM2⍴HI))*2)))-((S2)×(DIM2-1)÷(WI))
[19]   ⍕C
[20]   D1←1-((+/((+/O*2)÷(WI*2)))÷DIM1)
[21]   D2←1-((+/((+⌿O*2)÷(WJ*2)))÷DIM2)
[22]   D←(2,2)⍴0,D1,D2,0
[23]   ⍕D
[24]   +B←1E¯12⌈(C⍪D)
[25]   +ZI←(B[1])÷((B[1]+(S2÷WI)))
[26]   +ZJ←(B[2])÷((B[2]+(S2÷WJ)))
[27]   LJ←DIM2⍴1
[28]   LI←DIM1⍴1
[29]   LJC←DIM2⍴1
[30]   IT←0
[31]  BEGIN:
[32]   IT←IT+1
[33]   LJ←(÷2)×LJ+LJC
[34]   LI←(ZI×(MI-M))-((ZI÷WI)×(+/(O×(DIM1⍴K1←(1,DIM2)⍴LJ))))
[35]   LJC←(ZJ×(MJ-M))-((ZJ÷WJ)×(+⌿(O×(DIM2⍴K2←(DIM1,1)⍴LI))))
[36]   →STOP×⍳IT≥1500
[37]   →BEGIN
[38]  STOP:LJ←LJC
[39]   MX←(DIM1,DIM2)⍴M
[40]   ⍕LI
[41]   ⍕LJ
[42]   YI←MI-((÷WI)×(+/(O×(DIM1⍴K1←(1,DIM2)⍴LJ))))
[43]   YJ←MJ-((÷WJ)×(+⌿(O×(DIM2⍴K2←(DIM1,1)⍴LI))))
[44]   ⍕YI
[45]   ⍕YJ
[46]   ID←(DIM1,DIM2)⍴⍳1
[47]   Q2←(⍴O)⍴(ZJ×(YJ-M))
[48]   Q1←⍉((⍴(⍉O))⍴(ZI×(YI-M)))
[49]   +XE←MX+Q1+Q2
```

Programa C.6 *Programa* KKB *del model de credibilitat de classificació creuada de Bühlmann-Straub*

```
      ∇KKB[□]∇
[0]   W KKB P;DIM1;DIM2;DIM3;X;O;MI;MJ;M;S2;WI;WJ;WT;H;D;B;C;XE;MX
[1]    →(+/+/+/W≠0)⍴ET0
[2]    W←0∘.=(0∘.=P)
[3]   ET0:
[4]    D3←(+/D3←0∘.=(0∘.=P))
[5]    DIM3←2↓⍴P
[6]    +X←(+/(W×P))÷(O←(+/W))
[7]    DIM1←1↑⍴O
[8]    DIM2←1↓⍴O
[9]    +MI←(+/O×X)÷(WI←(+/O))
[10]   +MJ←(+⌿O×X)÷(WJ←(+⌿O))
[11]   +M←(+/+/O×X)÷(WT←(+/+/O))
[12]   +S2←(+/+/+/W×(P-(DIM3/H←(DIM1,DIM2,1)⍴,X))*2)÷(+/+/(D3-1))
[13]   +C←(+/+/((O×(X-M)*2)÷(WT)))-(((DIM1×DIM2)-1)×(S2)÷WT)
[14]   +D←1-(+/+/(O÷WT)*2)
[15]   +B←1E¯12⌈(C⌹D)
[16]   +Z←(B×O)÷((B×O)+S2)
[17]   MX←(DIM1,DIM2)⍴M
[18]   +XE←MX+Z×(X-MX)
```

Programa C.7 *Programa* KKJ1 *del model de credibilitat de classificació creuada de Jewell (segons l'expressió (3.122))*

```
      ∇KKJ1[□]∇
 [0]  W KKJ1 P;I;J
 [1]   →(+/+/+/W≠0)⍴ET0
 [2]   W←0∘.=(0∘.=P)
 [3]  ET0:
 [4]   D3←(+/D3←0∘.=(0∘.=P))
 [5]   DIM3←2↓⍴P
 [6]   +X←(+/(W×P))÷(O←(+/W))
 [7]   DIM1←1↑⍴O
 [8]   DIM2←1↓⍴O
 [9]   +MI←(+/O×X)÷(WI←(+/O))
[10]   +MJ←(+⌿O×X)÷(WJ←(+⌿O))
[11]   +M←(+/+/O×X)÷(WT←(+/+/O))
[12]   ⍕O
[13]   +S2←(+/+/+/W×(P-(DIM3/H←(DIM1,DIM2,1)⍴,X))*2)÷(+/+/(D3-1))
[14]   C←2⍴0
[15]   HI←(DIM1,1)⍴MI
[16]   HJ←(1,DIM2)⍴MJ
[17]   C[1]←+/(÷DIM2)×((÷WJ)×(+⌿(O×(X-(DIM1⍴HJ))*2)))-((S2)×(DIM1-1)÷(WJ))
[18]   C[2]←(+/+/((O×(X-M)*2)÷(WT)))-(((DIM1×DIM2)-1)×(S2)÷WT)
[19]   ⍕C
[20]   D2←1-((+/((+⌿O*2)÷(WJ*2)))÷DIM2)
[21]   D3←1-(+/(WI÷WT)*2)
[22]   D5←1-(+/+/(O÷WT)*2)
```

```
[23]  D←(2,2)⍴D2,D2,D3,D5
[24]  ⍕D
[25]  +B←1E¯12⌈(C⌹D)
[26]  +B←106.17084 2729.79953
[27]  +Z←(B[2]×O)÷((B[2]×O)+S2)
[28]  +ZI←(B[1]×+/Z)÷((B[2]+(B[1]×+/Z)))
[29]  MX←(DIM1,DIM2)⍴M
[30]  +YI←(÷+/Z)×(+/(Z×X))
[31]  +LI←ZI×(YI-M)
[32]  +LIJ←((X-MX)-(⍉(⍴(⍉X))⍴LI))×Z
[33]  ID←(DIM1,DIM2)⍴⍳1
[34]  Q1←⍉((⍴(⍉O))⍴(ZI×(YI-M)))
[35]  +XE←MX+(Z×(X-MX))+((ID-Z)×Q1)
```

Programa C.8 *Programa* KKJ3 *del model de credibilitat de classificació creuada de Jewell (segons l'expressió (3.133))*

```
      ∇KKJ3[⎕]∇
[0]   W KKJ3 P;I;J
[1]    →(+/+/+/W≠0)⍴ET0
[2]    W←0∘.=(0∘.=P)
[3]   ET0:
[4]    D3←(+/D3←0∘.=(0∘.=P))
[5]    DIM3←2↓⍴P
[6]    +X←(+/(W×P))÷(O←(+/W))
[7]    DIM1←1↑⍴O
[8]    DIM2←1↓⍴O
[9]    +MI←(+/O×X)÷(WI←(+/O))
[10]   +MJ←(+⌿O×X)÷(WJ←(+⌿O))
[11]   +M←(+/+/O×X)÷(WT←(+/+/O))
[12]   ⍕O
[13]   +S2←(+/+/+/W×(P-(DIM3/H←(DIM1,DIM2,1)⍴,X))*2)÷(+/+/(D3-1))
[14]   C←2⍴0
[15]   HI←(DIM1,1)⍴MI
[16]   HJ←(1,DIM2)⍴MJ
[17]   C[1]←+/(÷DIM1)×((÷WI)×(+/(O×(X-(DIM2/HI))*2)))-((S2)×(DIM2-1)÷(WI))
[18]   C[2]←(+/+/((O×(X-M)*2)÷(WT)))-(((DIM1×DIM2)-1)×(S2)÷WT)
[19]   ⍕C
[20]   D2←1-((+/((+⌿O*2)÷(WJ*2)))÷DIM2)
[21]   D3←1-(+/(WI÷WT)*2)
[22]   D5←1-(+/+/(O÷WT)*2)
[23]   D←(2,2)⍴D1,D1,D4,D5
[24]   ⍕D
[25]   +B←1E¯12⌈(C⌹D)
[26]   +Z←(B[2]×O)÷((B[2]×O)+S2)
[27]   +ZJ←(B[1]×+⌿Z)÷((B[2]+(B[1]×+⌿Z)))
[28]   MX←(DIM1,DIM2)⍴M
[29]   +YJ←(÷+⌿Z)×(+⌿(Z×X))
[30]   +LJ←ZJ×(YJ-M)
[31]   +LIJ←((X-MX)-(⊖(⍴X)⍴LJ))×Z
[32]   ID←(DIM1,DIM2)⍴⍳1
[33]   Q2←(⍴(O))⍴(ZJ×(YJ-M))
[34]   +XE←MX+(Z×(X-MX))+((ID-Z)×Q2)
```

Per últim, ens queden els programes destinats al càlcul de les reserves IBNR, que ocupen la segona part de la tesi doctoral. Tots els programes calculen la resta del triangle "run-off" a partir de les dades conegudes:

Programa C.9 *Programa* VYLD *de càlcul de reserves IBNR segons el model de De Vylder*

```
      ∇VYLD[□]∇
[0]   W VYLD X;DIM1;DIM2;HC;Y;H;B;S2;S2AL;IT;A;AC;J;Z;BC;LB;LY;XC;M
[1]    →(+/+/W≠0)⍴ET0
[2]    W←0∘.=(0∘.=X)
[3]   ET0:
[4]    □IO←1
[5]    DIM1←1↑⍴X
[6]    DIM2←1↓⍴X
[7]    HC←0∘.=(0∘.=X)
[8]    Q←X
[9]    +Y←(+⌿(X×W))÷(+⌿W)
[10]   H←HC×(⊖(⍴X)⍴Y)
[11]   PJ←W[;1]
[12]   +B←(+/H×X)÷(+/H*2)
[13]   +S2←(÷M←+/(+/HC)-DIM1⍴1)×(+/+/W×(X-((⊖(⍴X)⍴Y)×(⍉(⍴X)⍴B)))*2)
[14]   S2J←(S2)÷(W[;1])
[15]   S2AL←(S2J)÷(+/H*2)
[16]   IT←0
[17]   A←1
[18]   AC←1
[19]  BEGIN:J←1
[20]   IT←IT+1
[21]   A←(÷2)×A+AC
[22]   Z←(÷DIM1⍴A+S2AL)×A
[23]   AC←(÷DIM1)×+/Z×(B-DIM1⍴1)*2
[24]   →STOP×⍳(0.00001>|(A-AC)÷A)∨IT≥100
[25]   →BEGIN
[26]  STOP:BC←DIM1⍴0
[27]   +A←AC
[28]   +Z
[29]   +BC←(DIM1⍴1-Z)+Z×B
[30]   J←2
[31]  INI1:
[32]   S←2
[33]  INI2:
[34]   →((J+S)<(DIM1+2))⍴IC
[35]   X[J;S]←Y[S]×BC[J]
[36]  IC:
[37]   →(DIM1≥S←S+1)⍴INI2
[38]   →(DIM1≥J←J+1)⍴INI1
[39]   +X←X×(⍉(⍴W)⍴PJ)
```

Programa C.10 *Programa* MK *de càlcul de reserves IBNR segons el model de De Vylder-Mack*

```
      ∇MK[⎕]∇
[0]   W MK P;T;D;Z;A;ZM;S2;PV;IT;C;CC;ZC;J;S;ZE;PJ
[1]    ⎕IO←1
[2]    →(+/+/W≠0)⍴ET0
[3]    W←0∘.=(0∘.=P)
[4]   ET0:
[5]    T←1↑⍴P
[6]    HC←0∘.=(0∘.=P)
[7]    Q←P
[8]    +D←(+⌿(W×P))÷(+⌿W)
[9]    PJ←W[;1]
[10]   Z←P÷(⊖(⍴W)⍴D)
[11]   A←1
[12]   ZM←(+/(W×Z×(⊖(⍴W)⍴D)*A))÷PV←(+/(W×((⊖(⍴W)⍴D)*A)))
[13]   +S2←2×(÷(T×(T-1)))×+/((+/W×((⊖(⍴W)⍴D)*A)×(Z-(⍉(⍴W)⍴ZM))*2))
[14]   IT←0
[15]   C←1
[16]   CC←1
[17]  BEGIN:J←1
[18]   IT←IT+1
[19]   C←(÷2)×C+CC
[20]   ZC←PV÷(PV+(S2÷C))
[21]   CC←(÷T)×+/ZC×(ZM-1)*2
[22]   →STOP×⍳(0.00001>|(C-CC)-C)∨IT≥100
[23]   →BEGIN
[24]  STOP:
[25]   +C←CC
[26]   +ZC
[27]   +ZE←(ZC×ZM)+(1-ZC)
[28]   J←2
[29]  INI1:
[30]   S←2
[31]  INI2:
[32]   →((J+S)<(T+2))⍴IC
[33]   P[J;S]←D[S]×ZE[J]
[34]  IC:
[35]   →(T≥S←S+1)⍴INI2
[36]   →(T≥J←J+1)⍴INI1
[37]   +P←P×(⍉(⍴W)⍴PJ)
```

Programa C.11 *Programa* YY *de càlcul de reserves IBNR segons el model "two way"*

```
      ∇YY[⎕]∇
[0]   W YY P;D;U;UO;UD;UA;DD;QD;QO;QA;A;Q;BO;BD;S2;B;ZO;ZD;NO;ND;LO
;A2;A3;A4;DL;J;S;T;WA;WD;WO;WT;W2;PJ
[1]    ⎕IO←0
[2]    →(+/+/W≠0)⍴ET0
[3]    W←0∘.=(0∘.=P)
[4]   ET0:
[5]    T←1↑⍴P
[6]    HC←0∘.=(0∘.=P)
[7]    H←P
[8]    +D←(+⌿(W×P))÷(+⌿W)
```

```
[9]   U←P÷(⍷(⍴P)⍴D)
[10]  +PJ←W[;0]
[11]  C←1
[12]  W←W×(⍷(⍴P)⍴D)*(2-C)
[13]  +UO←(÷WO←+/W)×(+/W×U)
[14]  +UD←(+⌿(-⍳T)⌽(W×U))÷(WD←+⌿(-⍳T)⌽W)
[15]  A1←(WT←+/+/W)-(+/((+⌿(-⍳T)⌽W2←W*2)÷(WD)))
[16]  A2←T×(T-1)×(÷2)
[17]  A3←WT-(+/((÷WO)×+/W2))
[18]  A4←WT-(+/((÷WA←+⌿W)×+⌿W2))
[19]  +UA←(÷WA)×(+⌿W×U)
[20]  DD←(T,T)⍴0
[21]  J←0
[22] ETI1:
[23]  DD[J;]←(J↓UD),(J⍴0)
[24]  →((T-1)≥J←J+1)⍴ETI1
[25]  QD←+/(+⌿(-⍳T)⌽(W×(U-DD)*2))
[26]  QO←+/+/(W×(U-(⍉(⍴W)⍴UO))*2)
[27]  QA←+/+⌿(W×(U-(⍷(⍴W)⍴UA))*2)
[28]  +A←(3,3)⍴A1,0,A2,0,A3,A2,A4,A4,A2
[29]  +Q←3⍴QD,QO,QA
[30]  +B←Q⌹A
[31]  BO←B[0]
[32]  BD←B[1]
[33]  S2←B[2]
[34]  +ZO←(BO)÷(BO+(S2÷WO))
[35]  +ZD←(BD)÷(BD+(S2÷WD))
[36]  NO←ZO×(UO-1)
[37]  ND←ZD×(UD-1)
[38]  IT←0
[39]  LO←T⍴1
[40]  LD←T⍴1
[41]  LDC←T⍴1
[42]  DL←(⍴W)⍴0
[43] BEGIN:
[44]  IT←IT+1
[45]  →(IT≥400)⍴STOP
[46]  LD←(÷2)×LD+LDC
[47]  J←0
[48] ET3:
[49]  DL[J;]←(J↓LD),(J⍴0)
[50]  →((T-1)≥J←J+1)⍴ET3
[51]  LO←NO-(ZO×((÷WO)×(+/(DL×W))))
[52]  LDC←ND-(ZD×((÷WD)×(+⌿(-⍳T)⌽((⍉(⍴W)⍴LO)×W))))
[53]  →BEGIN
[54] STOP:
[55]  +LD←LDC
[56]  ⍎LO
[57]  J←1
[58] INI1:
[59]  S←1
[60] INI2:
[61]  →((J+S)<(T))⍴CIU
[62]  P[J;S]←D[S]×(1+LO[J])
[63] CIU:
[64]  →(T>S←S+1)⍴INI2
[65]  →(T>J←J+1)⍴INI1
[66]  +P1←P×(⍉(⍴W)⍴PJ)
```

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