BEHAVIOR OF PROPAGATING AND EVANESCENT COMPONENTS IN AZIMUTHALLY-POLARIZED NONPARAXIAL FIELDS

Rosario Martínez-Herrero,¹ Pedro M. Mejías,¹ Ignasi Juvells,² and Artur Carnicer²

¹ Departamento de Óptica, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, 28040 Madrid, Spain

² Departament de Física Aplicada i Òptica, Facultat de Física, Universitat de Barcelona (UB), 08028 Barcelona, Spain

Corresponding author E-mail: artur.carnicer@ub.edu

Abstract

The contribution of the propagating and the evanescent waves associated with freely propagating nonparaxial light fields whose transverse component is azimuthally polarized at some plane is investigated. Analytic expressions are derived for describing both the spatial shape and the relative weight of the propagating and the evanescent components integrated over the transverse plane. The analysis is carried out within the framework of the plane-wave angular spectrum approach. These results are used to illustrate the behavior of a kind of doughnut-like beams with transverse azimuthal polarization at some plane.

1. Introduction

In recent years, strongly focused light beams have attracted the attention of multiple researchers. Applications of such fields range from particle trapping or high resolution microscopy to material processing, among others. In particular, when the beam is focused in a spot smaller than the wavelength, the paraxial approach cannot be used and a nonparaxial treatment is required [1-9].

Among the different formulations for vectorial nonparaxial electromagnetic fields available in the literature [10-14], the description of such beams using the plane-wave angular spectrum framework has been proved particularly suitable [5,15-17], since this approach enables us to separate the propagating and evanescent contributions of the electromagnetic field [1, 5,18-20]. Moreover, the propagating and evanescent waves can be described as the sum of two terms: a first one, transverse to the direction of propagation z , and a second one that is a non-zero longitudinal component along the z axis.

In this paper, attention is focused on beams whose transverse component at some plane is azimuthally polarized. The objective of this article is to develop analytical expressions for computing both the spatial shape of transverse azimuthally-polarized (TAP) fields and the ratio between the different components; the analysis is carried out within the framework of the plane-wave angular spectrum approach.

This paper is organized as follows. In section 2, the formalism and the key definitions are introduced. General equations for the propagating and evanescent waves are provided. In section 3, TAP fields are introduced and in section 4 an illustrative example of such beams

is discussed. In particular, the relative contribution of the transverse and longitudinal terms of both the propagating and the evanescent parts is considered. Finally, the conclusions are presented in Section 5.

2. Formalism and basic definitions

Let us consider a monochromatic electromagnetic beam whose propagation is described by the Maxwell equations (for simplicity, we will consider free-space propagation). As is well known, the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{H}$, can be expressed in terms of their plane-wave angular spectrum [17],

$$\mathbf{E}(x, y, z) = \int_{-\infty}^{+\infty} \tilde{\mathbf{E}}(u, v, z) \exp ik(xu + yv) du dv \quad (1.a)$$

$$\mathbf{H}(x, y, z) = \int_{-\infty}^{+\infty} \tilde{\mathbf{H}}(u, v, z) \exp ik(xu + yv) du dv, \quad (1.b)$$

where k is the wavenumber, and, for simplicity, the time-dependent factor $\exp(-i\omega t)$ has been omitted. In these equations, $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ denote the spatial Fourier transform of $\mathbf{E}$ and $\mathbf{H}$, respectively. For simplicity, we will choose z as the direction of propagation of the beam. For the sake of convenience, we will next use cylindrical coordinates R , θ and z , i.e. $x=R\cos\theta$ and $y=R\sin\theta$, along with polar coordinates, ρ and ϕ , related to the transverse Cartesian Fourier-transform variables u , v by the expressions $u=\rho\cos\phi$ and $v=\rho\sin\phi$. In terms of these variables, a general solution of the Maxwell equations in the Fourier space can formally be written as follows

$$\tilde{\mathbf{E}}(\rho, \phi, z) = \tilde{\mathbf{E}}_0(\rho, \phi) \exp(ikz\xi), \quad (2)$$

with

$$\begin{aligned} \xi &= (1 - \rho^2)^{1/2} & \rho \leq 1 \\ \xi &= \pm i(\rho^2 - 1)^{1/2} & \rho > 1 \end{aligned} \quad (3)$$

where the signs $+$ and $-$ correspond, respectively, to positive ($z > 0$) and negative ($z < 0$) values of the variable z . In the present work, attention will be focused on the region $z \geq 0$. Since the electric field should obey the Maxwell equation $\nabla \cdot \mathbf{E} = 0$, the Cartesian components of $\tilde{\mathbf{E}}_0$ should satisfy the condition

$$\tilde{E}_{0x}u + \tilde{E}_{0y}v + \tilde{E}_{0z}\xi = 0. \quad (4)$$

The vector $\tilde{\mathbf{H}}$ is then obtained from $\tilde{\mathbf{E}}_0$ in the form

$$\tilde{\mathbf{H}}(\rho, \phi, z) = (\boldsymbol{\sigma} \times \tilde{\mathbf{E}}_0) \exp(ikz\xi) \quad (5)$$

Where the so-called Gaussian system of units is used and

$$\boldsymbol{\sigma} = (u, v, \xi) = (\rho\cos\phi, \rho\sin\phi, \xi), \quad (6)$$

We see from Eq. (6) that the third component of vector $\boldsymbol{\sigma}$ could be a complex number whose presence is closely related with the appearance of evanescent waves (see below). On

the basis of the plane-wave spectrum given by Eq. (2), the electric field solution of the Maxwell equations at any transverse plane z reads

$$\mathbf{E}(R, \theta, z) = \int_0^\infty \int_0^{2\pi} \tilde{\mathbf{E}}_0(\rho, \phi) \exp[ik\rho R \cos(\theta - \phi)] \exp(ikz\xi) \rho d\rho d\phi, \quad (7)$$

where $\tilde{\mathbf{E}}_0$ should fulfill Eq. (4). For our purposes, let us now split this field $\mathbf{E}(R, \theta, z)$ as the sum of two terms [19,20]:

$$\mathbf{E}(R, q, z) = \mathbf{E}_{pr}(R, q, z) + \mathbf{E}_{ev}(R, q, z), \quad (8)$$

where

$$\mathbf{E}_{pr}(R, \theta, z) = \int_0^1 \int_0^{2\pi} \tilde{\mathbf{E}}_0(\rho, \phi) \exp[ik\rho R \cos(\theta - \phi)] \exp(ikz\xi) \rho d\rho d\phi, \quad (9.a)$$

$$\mathbf{E}_{ev}(R, \theta, z) = \int_1^\infty \int_0^{2\pi} \tilde{\mathbf{E}}_0(\rho, \phi) \exp[ik\rho R \cos(\theta - \phi)] \exp(ikz\xi) \rho d\rho d\phi. \quad (9.b)$$

The first term, $\mathbf{E}_{pr}$, involves a superposition of plane waves, and represents the contribution of the propagating waves. The second term of Eq. (8), $\mathbf{E}_{ev}$, should be understood as a superposition of inhomogeneous waves that decay at different rates along the propagation axis.

Let us now choose a reference system formed by the orthogonal unitary vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, namely [17]

$$\mathbf{e}_1 = (\sin\phi, -\cos\phi, 0) \quad (10.a)$$

$$\mathbf{e}_2 = (\sqrt{1-\rho^2}\cos\phi, \sqrt{1-\rho^2}\sin\phi, -\rho) \quad (10.b)$$

The propagating field can then be written in the form

$$\mathbf{E}_{pr}(\mathbf{r}) = \int_0^1 \int_0^{2\pi} a(\rho, \phi) \mathbf{e}_1(\phi) \exp(iks \cdot \mathbf{r}) \rho d\rho d\phi + \int_0^1 \int_0^{2\pi} b(\rho, \phi) \mathbf{e}_2(\rho, \phi) \exp(iks \cdot \mathbf{r}) \rho d\rho d\phi \quad (11)$$

Where $\mathbf{r}$ denotes the position vector $\mathbf{r} = (x, y, z)$, functions $a(\rho, \phi)$ and $b(\rho, \phi)$ give the projection of $\tilde{\mathbf{E}}_0$ onto the vectors $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively, i.e., $a(\rho, \phi) = \tilde{\mathbf{E}}_0 \cdot \mathbf{e}_1$; $b(\rho, \phi) = \tilde{\mathbf{E}}_0 \cdot \mathbf{e}_2$, the dot symbolizing the inner product. It is interesting to remark that, for the propagating field $\mathbf{E}_{pr}$, condition $\nabla \cdot \mathbf{E} = 0$ reduces to $\tilde{\mathbf{E}}_0(\rho, \phi) \cdot \mathbf{s}(\rho, \phi) = 0$, where $\mathbf{s}(\rho, \phi) = (\rho \cos\phi, \rho \sin\phi, \sqrt{1-\rho^2})$, with $\rho \in [0, 1]$, is a unitary vector associated to each plan-wave. Thus, the triad $\mathbf{s}, \mathbf{e}_1$ and $\mathbf{e}_2$ constitute a mutually orthogonal system of unit vectors.

Let us finally consider the field solution associated with the evanescent term (cf. Eq. (8) and (9.b)). In a similar way to that used for the propagating term, we can formally write the evanescent part of the field in the form [19,20]

$$\mathbf{E}_{ev} = \int_1^\infty \int_0^{2\pi} \left[a(\rho, \phi) \mathbf{e}_1(\phi) + \hat{b}(\rho, \phi) \mathbf{e}_{ev}(\rho, \phi) \right] \exp(ik(\mathbf{s}_{ev} \cdot \mathbf{r})) \rho d\rho d\phi, \quad (12)$$

where the unit vector $\mathbf{e}_1$ was defined before,

$$\mathbf{e}_{ev} = \frac{1}{\sqrt{2\rho^2 - 1}} (\pm i(\rho^2 - 1)^{1/2} \cos\phi, \pm i(\rho^2 - 1)^{1/2} \sin\phi, -\rho) \quad (13.a)$$

$$\mathbf{s}_{ev} = (\rho \cos\phi, \rho \sin\phi, \pm i\sqrt{\rho^2 - 1}), \text{ with, } \rho \in [0, 1] \quad (13.b)$$

are vectors with complex components (again, the double sign + and - refers to $z > 0$ and $z < 0$, respectively) and

$$\hat{b}(\rho, \phi) = \tilde{\mathbf{E}}_0 \cdot \mathbf{e}_{ev}. \quad (14)$$

In Eq. (14) the product $\mathbf{a} \cdot \mathbf{b} \equiv a_x b_x^* + a_y b_y^* + a_z b_z^*$. Thus, note that $\mathbf{e}_{ev} \cdot \mathbf{s}_{ev} = 0$ for any z . In addition, the field $\mathbf{E}_{ev}$ fulfills condition (4) when $\rho \in [1, \infty]$. It should be remarked that the separate contribution of the evanescent part allows one to compare the relative weight of this term with regard to the propagating field by calculating the respective square modulus, integrated throughout the beam profile.

Note also that this evanescent part can be thought of as a superposition of inhomogeneous waves whose constant phase surfaces are planes orthogonal to the (non-unitary) transverse vector $\mathbf{s}_0$, defined in the form $\mathbf{s}_0 = (\rho \cos\phi, \rho \sin\phi, 0)$, and whose constant-amplitude planes are perpendicular to the propagation axis z .

The global field $\mathbf{E}(R, \theta, z)$ would finally read (see Eq. (8))

$$\begin{aligned} \mathbf{E}(R, \theta, z) = & \int_0^1 \int_0^{2\pi} a(\rho, \phi) \mathbf{e}_I(\phi) \exp(ik\mathbf{s} \cdot \mathbf{r}) \rho d\rho d\phi \\ & + \int_0^1 \int_0^{2\pi} b(\rho, \phi) \mathbf{e}_2(\rho, \phi) \exp(ik\mathbf{s} \cdot \mathbf{r}) \rho d\rho d\phi \\ & + \int_1^\infty \int_0^{2\pi} \left[a(\rho, \phi) \mathbf{e}_I(\phi) + \hat{b}(\rho, \phi) \mathbf{e}_{ev}(\rho, \phi) \right] \exp(ik\mathbf{s}_{ev} \cdot \mathbf{r}) \rho d\rho d\phi, \end{aligned} \quad (15)$$

where the angular spectrum $\tilde{\mathbf{E}}_0$ at some initial plane would characterize each particular beam.

In this work we will focus our attention in the radial and azimuthal components, E_R and E_θ

$$E_R = \cos\theta E_x(R, \theta, z) + \sin\theta E_y(R, \theta, z) \quad (16.a)$$

$$E_\theta = -\sin\theta E_x(R, \theta, z) + \cos\theta E_y(R, \theta, z) \quad (16.b)$$

By using Eqs. (11) and (12), general expressions for the radial and azimuthal components of propagating and evanescent field can be obtained, namely

$$E_R(R, \theta, z) = \int_0^1 \int_0^{2\pi} \left[-a(\rho, \phi) \sin(\theta - \phi) + \sqrt{1 - \rho^2} b(\rho, \phi) \cos(\theta - \phi) \right] \exp(iks \cdot \mathbf{r}) \rho d\rho d\phi \\ + \int_1^\infty \int_0^{2\pi} \left[-a(\rho, \phi) \sin(\theta - \phi) + \hat{b}(\rho, \phi) \frac{i\sqrt{\rho^2 - 1}}{\sqrt{2\rho^2 - 1}} \cos(\theta - \phi) \right] \exp(iks_{ev} \cdot \mathbf{r}) \rho d\rho d\phi, \quad (17.a)$$

$$E_\theta(R, \theta, z) = - \int_0^1 \int_0^{2\pi} \left[a(\rho, \phi) \cos(\theta - \phi) + \sqrt{1 - \rho^2} b(\rho, \phi) \sin(\theta - \phi) \right] \exp(iks \cdot \mathbf{r}) \rho d\rho d\phi \\ - \int_1^\infty \int_0^{2\pi} \left[a(\rho, \phi) \cos(\theta - \phi) + \frac{i\sqrt{\rho^2 - 1}}{\sqrt{2\rho^2 - 1}} \hat{b}(\rho, \phi) \sin(\theta - \phi) \right] \exp(iks_{ev} \cdot \mathbf{r}) \rho d\rho d\phi, \quad (17.b)$$

Azimuthally polarized fields at the transversal plane $z=0$ are obtained imposing that the radial component $E_R(R, \theta, 0)$ should vanish, i.e.

$$E_R(R, \theta, 0) = \int_0^1 \int_0^{2\pi} \left[-a(\rho, \phi) \sin(\theta - \phi) + \sqrt{1 - \rho^2} b(\rho, \phi) \cos(\theta - \phi) \right] \exp(ikR\rho \cos(\theta - \phi)) \rho d\rho d\phi \\ + \int_1^\infty \int_0^{2\pi} \left[-a(\rho, \phi) \sin(\theta - \phi) + \hat{b}(\rho, \phi) \frac{i\sqrt{\rho^2 - 1}}{\sqrt{2\rho^2 - 1}} \cos(\theta - \phi) \right] \exp(ikR\rho \cos(\theta - \phi)) \rho d\rho d\phi = 0. \quad (18)$$

Eq. (18) leads to the condition

$$\frac{1}{R} \frac{\partial A}{\partial \theta} + \frac{\partial B}{\partial R} = 0, \quad (19)$$

where

$$A(R, \theta) = \int_0^\infty \int_0^{2\pi} a(\rho, \phi) e^{ikR\rho \cos(\theta - \phi)} d\rho d\phi \quad (20.a)$$

and

$$B(R, \theta) = \int_0^1 \int_0^{2\pi} \sqrt{1 - \rho^2} b(\rho, \phi) e^{ikR\rho \cos(\theta - \phi)} d\rho d\phi + \int_1^\infty \int_0^{2\pi} \frac{i\sqrt{\rho^2 - 1}}{\sqrt{2\rho^2 - 1}} \hat{b}(\rho, \phi) e^{ikR\rho \cos(\theta - \phi)} d\rho d\phi \quad (20.b)$$

3. Transverse azimuthally-polarized fields

Let us now consider a nonparaxial field whose transverse component at plane $z = 0$ is azimuthally polarized. By applying the condition presented in Eq. (19), the Fourier series expansion (spiral spectrum) of functions $a(\rho, \phi)$, $b(\rho, \phi)$ and $\hat{b}(\rho, \phi)$ can be expressed as

$$a(\rho, \phi) = \begin{cases} a_0(\rho) + \sum_{m \neq 0} \frac{-i}{m} \frac{d}{d\rho} \left[\rho \sqrt{1 - \rho^2} b_m(\rho) \right] \exp(im\phi), & \rho \leq 1 \\ a_0(\rho) + \sum_{m \neq 0} \frac{-i}{m} \frac{d}{d\rho} \left[\frac{i\rho \sqrt{\rho^2 - 1}}{\sqrt{2\rho^2 - 1}} \hat{b}_m(\rho) \right] \exp(im\phi), & \rho > 1 \end{cases} \quad (21.a)$$

$$b(\rho, \phi) = \sum_{m \neq 0} b_m(\rho) \exp(im\phi), \quad \rho \leq 1 \quad (21.b)$$

$$\hat{b}(\rho, \phi) = \sum_{m \neq 0} \hat{b}_m(\rho) \exp(im\phi), \quad \rho > 1 \quad (21.c)$$

These relations are the main result of this work (see the Appendix for a more detailed discussion). We thus conclude that both, the propagating and the evanescent waves of a TAP field, are given as a sum of two parts:

- One of them remains azimuthally polarized at transverse planes z other than the initial one $z = 0$, and its longitudinal component is zero. This corresponds to the plane-wave angular spectrum without topological charge ($m = 0$).
- The other part (whose angular spectrum exhibits topological charge, $m \neq 0$) is no longer azimuthally polarized upon free propagation, and its longitudinal component differs from zero even at the plane $z = 0$.

The significance of the evanescent waves can be evaluated by means of the ratio I_e/I , being I_e and I the integrated squared modulus of the evanescent and total electric fields respectively

$$I_e / I = \frac{\int_0^\infty \int_0^{2\pi} |E_{ev}|^2 R dR d\theta}{\int_0^\infty \int_0^{2\pi} |E_{pr}|^2 R dR d\theta + \int_0^\infty \int_0^{2\pi} |E_{ev}|^2 R dR d\theta} \quad (22)$$

Let us now focus our attention on the longitudinal component of TAP fields. Associated with the propagating and evanescent waves, the z -component is analytically determined by the following expressions:

$$(E_{pr})_z(R, \theta, z) = - \sum_{m \neq 0} i^m 2\pi \exp(im\phi) \int_0^1 b_m(\rho) J_m(kR\rho) \exp(ikz\sqrt{1-\rho^2}) \rho d\rho \quad (23.a)$$

$$(E_{ev})_z(R, \theta, z) = - \sum_{m \neq 0} i^m 2\pi \exp(im\phi) \int_1^\infty \frac{\rho}{\sqrt{2\rho^2 - 1}} \hat{b}_m(\rho) J_m(kR\rho) \exp(-kz\sqrt{\rho^2 - 1}) \rho d\rho \quad (23.b)$$

where J_m is the Bessel functions of order m . It is important to note that $(E_{pr})_z(0, \theta, z) = (E_{ev})_z(0, \theta, z) = 0$, for any z .

The relative contributions of the transverse and longitudinal terms of the propagating field can be compared to each other through the ratio

$$\eta_{pr} = \frac{\int_0^\infty \int_0^{2\pi} |(E_{pr})_z|^2 R dR d\theta}{\int_0^\infty \int_0^{2\pi} |(E_{pr})_T|^2 R dR d\theta}, \quad (24)$$

where the subscripts T and z refer to the transverse and longitudinal components of E_{pr} , respectively, and the integrals involve the squared modulus of such components, integrated over the whole transverse plane. Since the components of E_{pr} are written in terms of their plane-wave spectrum, it follows from the application of the Parseval theorem, that they do

not change upon free propagation. Consequently, it would suffice to calculate the above ratio at the initial plane $z=0$.

With regard to the evanescent wave associated with TAP fields, the relative weight of its longitudinal and transverse components can be compared at $z = 0$ by using the ratio

$$\eta_{ev} = \frac{\int_0^\infty \int_0^{2\pi} |(E_{ev})_z|^2 R dR d\theta}{\int_0^\infty \int_0^{2\pi} |(E_{ev})_T|^2 R dR d\theta}, \quad (25)$$

where again the integrals represent the squared modulus (integrated over the whole plane) of the longitudinal and transverse components associated with the evanescent part of the field. Finally, in a similar way, the relative importance of the total longitudinal component I_z/I can be evaluated from the ratio

$$I_z/I = \frac{\int_0^\infty \int_0^{2\pi} |(E_{pr})_z|^2 R dR d\theta + \int_0^\infty \int_0^{2\pi} |(E_{ev})_z|^2 R dR d\theta}{\int_0^\infty \int_0^{2\pi} |E_{pr}|^2 R dR d\theta + \int_0^\infty \int_0^{2\pi} |E_{ev}|^2 R dR d\theta}. \quad (26)$$

It is apparent from Eqs. (21) that the Fourier spectrum of a, b and $\hat{b}$ play a fundamental role in the behaviour of the preceding parameters. In the next section we will apply these parameters, namely, η_{pr} , η_{ev} , I_z/I , I_e/I , to an example.

4. Example

To illustrate the behaviour of the propagating and evanescent component of a TAP field with respect to the topological charge, let us consider a kind of doughnut-shape field characterized by the following angular spectrum (see Eqs. (15) and (21)):

$$a(\rho, \phi) = \frac{-i}{m} \frac{d}{d\rho} \left[\rho^{n+1} (1 - \rho^2) \exp\left(-\frac{\rho^2}{D^2}\right) \right] \exp(im\phi), \quad \rho \leq 1 \quad (27.a)$$

$$a(\rho, \phi) = \frac{i}{m} \frac{d}{d\rho} \left[\frac{\rho^{n+1} (\rho^2 - 1)}{\sqrt{2\rho^2 - 1}} \exp\left(-\frac{\rho^2}{D^2}\right) \right] \exp(im\phi), \quad \rho > 1$$

$$b(\rho, \phi) = \rho^n \sqrt{1 - \rho^2} \exp\left(-\frac{\rho^2}{D^2}\right) \exp(im\phi), \quad \rho \leq 1 \quad (27.b)$$

$$\hat{b}(\rho, \phi) = i\rho^n \sqrt{\rho^2 - 1} \exp\left(-\frac{\rho^2}{D^2}\right) \exp(im\phi), \quad \rho > 1 \quad (27.c)$$

where $m, n = 1, 2, 3, \dots$, and $D = 1/kw_0$. Note that here w_0 denotes a parameter closely connected with the transverse size of the field at the beam waist. Moreover, this field exhibits a topological charge m ; the variation of n provides information about the relative importance of the evanescent component.

Fig. 1 plots the parameters η_{pr} , η_{ev} , I_z/I and I_e/I at plane $z = 0$ in terms of m and n for $w_0=0.2\lambda$. We observe from the figures that, η_{pr} , η_{ev} and I_z/I are increasing functions of topological charge m (for fixed n). For η_{pr} and I_z/I the growth is more pronounced for higher n , opposite occurs for η_{ev} . Moreover, the longitudinal component as well as the evanescent wave is particularly significant for $n \geq 4$. Fig. 2 shows the same parameters at the plane $z = 0$ in terms of m and k for $w_0=0.5\lambda$. The behavior of the parameters η_{pr} , η_{ev} , I_z/I and I_e/I is similar to the previous case, but now η_{ev} increases with n for fixed m . A significant decrease of the evanescent component is another consequence of w_0 enlargement. Finally, Fig. 3 represents, at the plane $z = 0$, the global (propagating + evanescent) contributions (squared modulus of the electric-field) of the transverse and longitudinal components of the beams considered in the example, for $w_0=0.2\lambda$ and $w_0=0.5\lambda$. In all cases, the figures exhibit doughnut-shape profiles, and are equal to zero on the propagation axis z ($x = y = 0$), as expected.

Fig. 1. Parameters η_{pr} , η_{ev} , I_e/I and I_z/I in terms of m and different values of k for $w_0=0.2\lambda$

Fig. 2. Parameters η_{pr} , η_{ev} , I_e/I and I_z/I in terms of m and different values of k for $w_0=0.5\lambda$.

Fig. 3. Squared modulus (at $z = 0$) of the electric-field associated to the global (propagating + evanescent) contributions of the total (transverse + longitudinal), the transverse and the longitudinal components of the beams considered in the example for $m=k=1$ and $m=k=10$. The left and right columns show the results obtained for $w_0=0.2\lambda$ and $w_0=0.5\lambda$ respectively. All the curves have been normalized to the global case.

Finally, to provide a more visual presentation of the information contained in the previous Figure, Fig. 4 displays the 2d- distributions of the square module of global field distributions at $z=0$. It is apparent that the beams tend to be less focused for larger values of m and k .

Fig. 4. 2d representation of the total field at $z=0$ (see Fig.3, first row). The images are presented in false color using the jet colormap and normalized to its maximum value.

5. Conclusions

In terms of the plane-wave angular spectrum of a light beam, the analytical structure is shown for both the propagating and the evanescent parts of nonparaxial fields whose transverse component at some plane is azimuthally polarized (TAP fields). Such beams have been split into transverse and longitudinal components. The analytical expressions allow us to compute both the spatial shape of such components and the ratio between their respective contributions, integrated over the whole transverse plane. In the examples, a kind of doughnut-like TAP fields has been investigated in terms of their transverse beam size for different values of their topological charge. The example shows that in the non-paraxial regime the contribution of both the longitudinal and evanescent components of these TAP fields can be significant.

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References

1. A. Ciattoni, B. Crosignani, P. Di Porto, "Vectorial analytical description of propagation of a highly nonparaxial beam", *Opt. Commun.* **202**, 17-20 (2002).
2. R. Dorn, S. Quabis, G. Lench, "Sharper focus for a radially polarized light beam", *Phys. Rev. Lett.* **91**, 233901 (1-4) (2003).
3. J. Lekner, "Polarization of tightly focused laser beams", *J. Opt. A: Pure Appl. Opt.* **5**, 6-14 (2003).
4. D. M. Deng, "Nonparaxial propagation of radially polarized light beams", *J. Opt. Soc. Am. B* **23**, 1228-1234 (2006).
5. P. C. Chaumet, "Fully vectorial highly nonparaxial beam close to the waist", *J. Opt. Soc. Am. A* **23**, 3197-3202 (2006).
6. A. April, "Nonparaxial TM and TE beams in free space", *Opt. Lett.* **33**, 1563-1565 (2008).
7. Z. R. Mei and D. M. Zhao, "Nonparaxial propagation of controllable dark-hollow beams", *J. Opt. Soc. Am. A* **25**, 537-542 (2008).
8. G. Q. Zhou, "The analytical vectorial structure of nonparaxial Gaussian beam close to the source", *Opt. Express* **16**, 3504-3514 (2008).
9. S. R. Seshadri, "Fundamental electromagnetic Gaussian beam beyond the paraxial approximation", *J. Opt. Soc. Am. A* **25**, 2156-2164 (2008).
10. P. Varga and P. Török, "Exact and approximate solutions of Maxwell's equation and the validity of the scalar wave approximation", *Opt. Lett.* **21**, 1523-1525 (1996).
11. P. Varga and P. Török, "The Gaussian wave solution of Maxwell's equations and the validity of the scalar wave approximation", *Opt. Commun.* **152**, 108-118 (1998).
12. C. J. R. Sheppard and S. Saghaei, "Electromagnetic Gaussian beams beyond the paraxial approximation", *J. Opt. Soc. Am. A* **16**, 1381-1386 (1999).
13. C. J. R. Sheppard, "Polarization of almost-planes waves", *J. Opt. Soc. Am. A* **17**, 335-341 (2000).
14. A. Ciattoni, B. Crosignani, P. Porto, and A. Yariv, "Azimuthally Polarized Spatial Dark Solitons: Exact Solutions of Maxwell's Equations in a Kerr Medium", *Phys. Rev. Lett.* **94**, 073902-4 (2005).
15. H. M. Guo, J. B. Chen, and S. L. Zhuang, "Vector plane wave spectrum of an arbitrary polarized electromagnetic wave", *Opt. Express* **14**, 2095-2100 (2006).
16. R. Martínez-Herrero and P. M. Mejías, "Propagation of light fields with radial or azimuthal polarization distribution at a transverse plane", *Opt. Express* **16**, 9021-9033 (2008).
17. R. Martínez-Herrero, P. M. Mejías, S. Bosch, and A. Carnicer, "Vectorial structure of nonparaxial electromagnetic beams", *J. Opt. Soc. Am. A* **18**, 1678-1680 (2001).
18. N. I. Petrov, "Nonparaxial focusing of wave beams in a graded-index medium", *Quantum Electron.* **29**, 249-255 (1999).
19. R. Martínez-Herrero, P. M. Mejías, and A. Carnicer, "Evanescent field of vectorial highly nonparaxial beams", *Opt. Express* **16**, 2845-2858 (2008).
20. R. Martínez-Herrero, P. M. Mejías, I. Juvells, and A. Carnicer, "Transverse and longitudinal components of the propagating and evanescent waves associated to radially polarized nonparaxial fields", *Appl. Phys. B: Laser and Optics* **106**, 151-159 (2012).

Appendix

To go further into the analysis of condition presented in Eq. (18) let us consider the Fourier series expansion (spiral spectrum) of functions a, b and $\hat{b}$

$$\begin{aligned} a(\rho, \phi) &= \sum_m a_m(\rho) e^{im\phi} \\ b(\rho, \phi) &= \sum_m b_m(\rho) e^{im\phi} \\ \hat{b}(\rho, \phi) &= \sum_m \hat{b}_m(\rho) e^{im\phi} \end{aligned} \quad (\text{A1})$$

By using these Fourier expansions, it can be shown that, after some algebra, condition (18)

$\frac{1}{R} \frac{\partial A}{\partial \theta} + \frac{\partial B}{\partial R} = 0$, can be expressed as follows

$$\sum_m i^m e^{im\phi} D_m(R) = 0 \quad (\text{A2})$$

where

$$\begin{aligned} D_m(R) &= \frac{1}{R} \int_0^1 \left[i m a_m(\rho) - \frac{d}{d\rho} \left(\rho \sqrt{1-\rho^2} b_m(\rho) \right) \right] J_m(kR\rho) d\rho \\ &+ \frac{1}{R} \int_1^\infty \left[i m a_m(\rho) - \frac{d}{d\rho} \left(\frac{i \rho \sqrt{\rho^2-1}}{\sqrt{2\rho^2-1}} \hat{b}_m(\rho) \right) \right] J_m(kR\rho) d\rho. \end{aligned} \quad (\text{A3})$$

Condition (A2) would be fulfilled provided that

$$D_m(R) = 0 \quad \forall m, \quad (\text{A4})$$

and, therefore, functions $a_m(\rho), b_m(\rho)$ and $\hat{b}_m(\rho)$ must satisfy

$$a_m(\rho) = \frac{-i}{m} \frac{d}{d\rho} \left[\rho \sqrt{1-\rho^2} b_m(\rho) \right], \quad \rho \leq 1 \text{ and } m \neq 0 \quad (\text{A5.a})$$

$$a_m(\rho) = \frac{-i}{m} \frac{d}{d\rho} \left[\frac{i \rho \sqrt{\rho^2-1}}{\sqrt{2\rho^2-1}} \hat{b}_m(\rho) \right], \quad \rho > 1 \text{ and } m \neq 0 \quad (\text{A5.b})$$