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Parents' Investments and Education Returns

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Abstract: This paper analyses the relation between parents' earnings and their children's education. In a context of perfect altruism, the model describes parents' decisions on how much to consume and how much to invest in their children's education. The model predicts that returns on education in terms of wages should be linear. Using this model in a competitive economy, we show how the outcome depends on government subsidies or taxes on education. The usual tradeoff equality-efficiency arises in this context. Finally, the model provides some insights into the relation between education and productivity.

JEL Classification: E24, I22, I28, I32, J26

Keywords: Intergenerational altruism, education returns

Resum: Aquest article analitza la relació entre els ingressos dels pares i l'educació dels seus fills. En un context d'altruïsm perfecte, el model descriu les decisions dels pares sobre quant consumir i quant invertir en l'educació dels seus fills. El model prediu que els rendiments de l'educació en termes de sous haurien de ser lineals. Usant aquest model en una economia competitiva, es mostra com el resultat depèn dels subsidis o impostos del govern sobre l'educació. El compromís habitual igualtat-eficiència apareix en aquest context. Finalment, el model dona intuïcions sobre la relació entre educació i productivitat.

Paraules clau: Altruïsm intergeneracional, rendiments de l'educació.

1 Introduction

Broadly speaking, the principal economic decision that a person takes during his or her life is how much to consume and how much to leave on bequest. Once we establish our expectations of our lifetime wealth, we decide our lifetime consumption, and the rest is left to the next generation.

Kotlikoff and Summers (1981) report the high importance of the bequest motive in the wealth accumulation of individuals. The literature has proposed various explanations for voluntary bequests. For instance, Bernheim et al. (1986) model the possibility of strategic bequests, with individuals conditioning bequests to the decisions of the beneficiaries. The other main rationale behind the bequests is that parents receive utility not only from their own consumption, but from bequeathing wealth to their children. Laitner and Juster (1996) provide empirical evidence in this direction. Our model adopts this second approach.

This article centers on an important form of bequest: investing in one's children's human capital. The importance of human capital in modern economies is well known, and a huge share of production is devoted to its formation. As an example, Kendrick (1976) estimates that 1969 over a half of the total capital stock in United States was human capital.² A great part of this human capital investment is not provided or decided by individuals themselves, but by their parents or relatives. The fact that expenditure on one's own schooling is not decided individually but by one's parents is particularly clear in primary and secondary school.

Although we focus on this particular form of bequest, most of the results presented are also valid for more general kinds of intergenerational transfers. Human capital is usually related to education, but the decision of how much human capital

²For other seminal papers of empirical analysis in this area, look at Barro (1991) and Mankiw et al. (1992).

to provide for one's children is closely related to many other important investments. The opportunity cost of rearing children is high in general, because of the forgone earnings both of parents and the children while attending school. Moreover, many other expenditures are linked to the decision to have children. For example, Durlauf (1996) models the importance of choosing a suitable neighborhood for the quality of the education and the future income of the children. All these related expenditures make investment in the children's human capital one of the largest that parents make during the their lives.

Poorer parents have more restrictive budget constraints on deciding how to share their wealth between their own consumption and their children's human capital than parents that are richer. So human capital investment is likely to be correlated with the wealth of the parents. Becker and Tomes (1979) and Loury (1981) introduce models relating parental income to their children's human capital. The difficulty of borrowing to invest in children's human capital increases this correlation, even more especially because the returns on the human capital are earned by the children, and not by the parents.

We present a perfect altruism model where education is not paid for by the recipients themselves, but by their parents. In this model we assume that the wage of a job is related to the education required to be eligible for this job. Parents, taking their own education as given, decide their own consumption and the investment in the education of their children. So, parents face a trade off when deciding how to share their earnings. On the one hand they want to increase their own consumption. On the other, they want to increase their children's utility, which we will see, can be achieved by investing in their human capital.

Imposing stability in the job market, the model predicts that returns on education should be linear, at least for the first years of schooling. we use this result in a

classic macroeconomic model to explore how it can affect the efficiency of the production process. Introducing government taxes and subsidies, the model suggests an optimal policy for improving the outcome of the economy. This policy is subject to the classic tradeoff between efficiency and equality. Finally, the model stresses a possible channel for explaining the correlation between wages and education.

After this introduction, Section II presents the benchmark model. Section III analyses the results of using the benchmark model in a competitive economy where the production factors are different types of labor. The benchmark model is generalized in Section IV to a more realistic model, an economy with a continuum of possible educational investments and wages. Section V concludes.

2 A simple model: only two wages

In this first section we develop a simple model of parents choosing their children's education. We assume that there is intergenerational altruism, that is, parents' utility depends positively on their own consumption but also on their children's utility. We consider bequests as simple transfers to increase their children's utility and, eventually, their own. We assume that every parent has inelastically exactly only one child.

Let's consider an economy with only two kinds of jobs, which receive two different wages. We assume that these two kinds of job require different skills, which can be acquired through education. For the sake of simplicity we assume that there are two types of workers, low-skilled and high-skilled. Their type depends exclusively on the education investment they received from their parents. We denote the educational investments required to be a low-skilled worker or a high-skilled worker by x_L and x_H respectively³. We assume that $x_H > x_L$. We assume that the low-skilled jobs

³Throughout this paper, the wage, the consumption and the capital investment refer to lifetime

pay a wage equal to w_L , and high-skilled jobs w_H . We denote $w(x_L) \equiv w_L$ and $w(x_H) \equiv w_H$.

In our model the wage is the only constraint parameter that may be different across individuals. Two parents with same wage have identical choices and receive the same utility from each of them. Thus, because they solve the same maximization problem, the utility can be written as a function of the wage, $u(w)$. Parents take as given their own education level, and therefore their wage; they only decide the level of education that they will invest in their children. We assume the following general form for the utility:

$$u(w_p) = \max_{x_p \in \{x_L, x_H\}} \left\{ f[w_p - x_p, u(w(x_p))] \right\}, \quad (2.1)$$

where w_p is the parents' wage and x_p is their investment in their children's education. Parents decide considering their consumption, which is given by the net wage $w_p - x_p$, and the utility of the children $u(w(x_p))$, since $w(x_p)$ is the wage earned by them. Then, the maximization problem (2.1) defines the utility function in terms of the wage and the utility function itself. It turns out to be a general type of Bellman equation, which allows us to endogenously determine the utility function u . Unfortunately, the equation (2.1) cannot be solved analytically for a general function f .

We assume that the function f has positive but decreasing marginal returns in both arguments. We consider a general form for this function, and in particular it could not be separable into two additive parts. This is in line with the utility functions proposed by Epstein and Zin (1991) for the utility regarding the decision between consumption and leisure. We also assume that education is a normal good. This assumption comes from the evidence provided by Charles and Hurst (2003) of the positive correlation between parent's investments in their children's human values. We can interpret them simply as their present values.

capital and their wealth.

The variations in the portions of skilled and unskilled workers in our economy depend on the decisions of individuals regarding the education of their children. Provided that every parent has only one child, number of workers of each type can change from generation to generation only if there are parents who provide a level of education to their children that is different from their own. The assumption that education is a normal good ensures that low-skilled parents will not invest more in the education of their children than high-skilled parents. Then, shares will only remain constant when each worker chooses to provide her children with the same educational investment as the one she received. In this case, instead of the usual golden rule, parents follow the wise folk saying “Pay your children the debts you owe to your parents.”

The steady state, in which shares remain constant, is only attainable when the job with the higher required amount of investment in education provides a higher wage, $w_H > w_L$. Otherwise, the choice of providing a low level of education for their children is strictly preferred by all the parents, because it provides more disposable income and high utility. This positive relation between level of education and income has long been documented, especially since the seminal work of Mincer (1958). Then, the steady state implies the following two conditions:

$$u(w_L) = f(w_L - x_L, u(w_L)) \geq f(w_L - x_H, u(w_H)) \quad \text{and} \quad (2.2)$$

$$u(w_H) = f(w_H - x_H, u(w_H)) \geq f(w_H - x_L, u(w_L)) . \quad (2.3)$$

Note that these two equations are the same except for the swapping of subindexes. Considering only the equality relations, we see that w_i appears either as the variable of the utility function, $u(w_i)$, or in the net wage $w_i - x_i$. This implies that in equilibrium $u(w_i)$ can be written as function of the net wage.

Let's now consider a particular form for the function $f(\cdot, \cdot)$ in order to obtain

analytical results and interpret them. We assume a Cobb-Douglas utility, with the following form for the parents' utility

$$u(w_p) = \max_{x_p \in \{x_L, x_H\}} \{C \cdot (w_p - x_p)^\beta \cdot u(w(x_p))^\alpha\} , \quad (2.4)$$

where $0 < \alpha < 1$, $0 < \beta < 1$ and $C > 0$ are constants. When the economy is in the steady state the system (2.2)-(2.3) can be easily solved. The resulting utility function is the following:

$$u(w_i) = C^{\frac{1}{1-\alpha}} \cdot (w_i - x_i)^{\frac{\beta}{1-\alpha}} , \quad i = H, L . \quad (2.5)$$

We see that, as we previously noted, the utility can indeed be written as a function of the net wage. We recover one of the oft-used functions of the utility of consumption, the isoelastic utility function. Using these results we can rewrite the steady state conditions (2.2) and (2.3). Arranging these formulae we have

$$(w_L - x_L) \geq (w_H - x_H)^\alpha \cdot (w_L - x_H)^{1-\alpha} \quad \text{and} \quad (2.6)$$

$$(w_H - x_H) \geq (w_L - x_L)^\alpha \cdot (w_H - x_L)^{1-\alpha} . \quad (2.7)$$

These relations are displayed in Figure 1. In this figure we make the assumption that $x_L = 0$ and $x_H = x > 0$. This assumption can be made without losing generality due to the particular form of the restrictions (2.6) and (2.7). In these restrictions we can define $x = x_H - x_L$, $w'_L = w_L - x_L$ and $w'_H = w_H - x_L$. Then, considering $x_L > 0$ only shifts the graph, without changing neither the shape of the functions nor the conclusions we can extract from them.

Figure 1 shows that the space of wages is divided into three parts, determined by the two steady state conditions, (2.6) and (2.7). The set of points between these two curves and the w_H axis comprises the steady states of the economy (shaded area). For the states in the upper set, above the (2.7) curve, the wage of the low-skilled workers is sufficiently high to induce the high-skilled parents to invest little

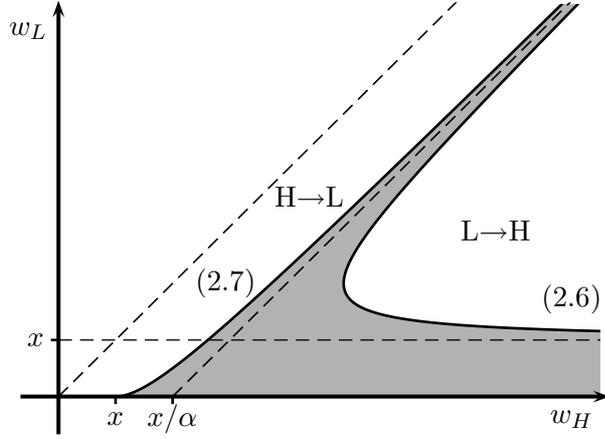


Figure 1: This figure shows the pairs (w_H, w_L) that lead the economy to the steady state when $x_H = x > 0$ and $x_L = 0$ (shadowed area).

in their children education (H→L). The states on the right side of the (2.6) curve are states where the high-skilled workers wage is high enough to provide incentives to the low-skilled workers to make high investments in their children’s education (L→H).

Let’s calculate the steady state conditions at the limit when w_H and w_L are much higher than the price of a high quality education, x . In this case, the steady state conditions converge to the following condition:

$$w_H = w_L + \frac{x}{\alpha} . \quad (2.8)$$

The convergence to this straight line can be observed easily in Figure 1. In fact, when the education cost is sufficiently low relative to the wages, the returns of the education measured as increase of wages divided by the cost of education are $\alpha^{-1} > 1$.

This model, in which parents take full consideration at the utility of their children, is included in the “perfect altruism” models. The great majority of macroeconomic dynamic models assume intergenerational transfers of resources between members of the same family. The most paradigmatic example of this kind of model

is the Ramsey (1928) model, interpreted by Barro (1974) as infinite dynasties. In this model individuals have preferences regarding the utility of the whole dynasty, not only regarding their consumption through their lives. Although the Diamond (1965) model does not include the bequest motive for saving, bequests have also been included in overlapping generation models such as Burbidge (1983). Like some of these models, our model exhibits an intertemporal inefficiency. In fact, parents apply an “selfish” discount factor to their children’s utility, which in our case is given by α . The lower the value of α , the lower the sensitivity of the parents’ utility to their children’s utility, and the higher the rewards have to be for parents who invest in their children education. This factor, together with labor market stability considerations, determines the returns on education.

3 Productive process

Until now we have assumed that wages are given exogenously. We now endogenize wages, introducing a productive process and a market economy. We assume that high-educated workers and low-educated workers are now production factors producing a homogeneous good. In a competitive market economy, all the workers will be paid their marginal productions.

Most of the macroeconomic literature on human capital as a production factor focuses on models in which it enters the production function at an aggregate level, regardless of its distribution among workers. Implicitly, it is assumed that the elasticity of substitution between skilled and unskilled workers is infinite, that is, that they are perfect substitutes. Nevertheless, many empirical surveys, like those of Hamermesh (1993) or Katz and Autor (1999), report values of this elasticity between 1 and 2, certainly lower than infinite. So, it may be worth analyzing the implications of our model when the production function has imperfect substitutability between

low-skilled and high-skilled labor.

Let's consider an economy that produces a unique homogeneous good. This good is produced by using two production factors, high-educated labor and low-educated labor, and can either be consumed or invested in children's education. All individuals are born unskilled and only if their parents pay the education cost, denoted by x , do they become skilled. All individuals live one period, and supply inelastically one unit of labor during this period. We denote the total number of low-educated workers in our economy by N_L and the total number of high-educated workers by N_H .

Although Katz and Murphy (1992) estimate a value for the elasticity of substitution near to 1.4, we consider a Cobb-Douglas production function, with elasticity of substitution 1. This assumption, which leaves the elasticity in the range of reasonable elasticities [1, 2], allows us to obtain analytical results that are also valid for other elasticities. Then, the overall production is given by

$$Y = A.N_L^\gamma.N_H^{1-\gamma} ,$$

where $A > 0$ and $\gamma \in (0, 1)$ are constants.

We consider that the economy is in a steady state, that is, that N_H and N_L remain constant. In this case we can define the net production as the total production minus the total cost of the education. The net production per capita can be expressed in the following way:

$$y^n = \frac{Y^n}{N_L + N_H} = \frac{Y - x.N_H}{N_L + N_H} = \frac{A.m^{1-\gamma} - x.m}{1 + m} , \quad (3.1)$$

where $m \equiv N_H/N_L$ is the relative number of high educated workers with respect to low educated workers. Assuming a competitive economy where markets clear, every worker will earn his marginal production. So, the wages will be given by

$$w_L = A.\gamma.m^{1-\gamma} \quad \text{and} \quad w_H = A.(1 - \gamma).m^{-\gamma} .$$

Let's consider that individuals have a utility function equal to (2.4). For simplicity, we assume that the steady states of the economy verify equation (2.8). Then, we have the following relation between x and the number of workers in our economy

$$\frac{x}{\alpha} = w_H - w_L = A.(1 - \gamma).m^{-\gamma} - A.\gamma.m^{1-\gamma} . \quad (3.2)$$

The right hand side of this equation is a strictly decreasing equation that ranges from ∞ to $-\infty$ when $m \in (0, \infty)$. Then, given a price of education equal to x , we can obtain the only relative ratio m that leaves the economy in the steady state. Unfortunately, for general values of the parameter γ this equation cannot be solved analytically.

The net production is only maximized when the marginal production of a low-skilled worker, given by w_L , equals the net marginal production of a high-skilled worker, given by $w_H - x$. Then, the value of m obtained from (3.2) only maximizes the production per capita (3.1) when $\alpha = 1$. Subsequently, the market is unable to maximize the outcome of the economy for general values of α . This is because there is an externality in our problem, given by the fact that investment in education is not decided by the recipients. In fact, because parents' utility has a discount coefficient on their children's utility, given by $\alpha < 1$, the education investment provided is lower than that desired by their children.

Through taxes and subsidies for education, the government can influence the decisions of the agents, and try to improve the outcome of the economy. Consider an income tax, with constant tax rate τ , that is used by the government to subsidize the education. We allow the possibility of $\tau < 0$, interpreting in this case τ as the income subsidy rate using the government's revenue from the taxation of the education. We denote the price paid by parents' to provide education for their children by x . We denote the real cost of being a high educated worker by z .

We assume that the government runs a balanced budget. Imposing then that

the government inflows and outflows coincide, we have

$$(z - x) \cdot \frac{m}{1 + m} = \tau \cdot \frac{A \cdot m^{1-\gamma}}{1 + m} . \quad (3.3)$$

The left hand side of this equation is the government subsidy/tax for education per capita. It is given by the difference between the real cost and the subsidized price multiplied by the portion of workers that receive education. The right hand side of this equation contains the revenue/expenditure of the government per capita coming from the tax/subsidy on income. It is given by the tax rate multiplied by the average income of the economy.

The model takes the real cost of education z as exogenously given. Then, equations (3.2) and (3.3) establish a two-equations system with three unknowns. These unknowns are the subsidized price of education x , the tax rate τ and the relative ratio of high educated workers with respect to low educated workers m . The solution of this system is the following:

$$x(m) = \frac{\alpha \cdot m^{-\gamma} \cdot (1 - \gamma \cdot (1 + m)) \cdot (1 - m^\gamma \cdot z)}{1 - \alpha \cdot (1 - \gamma \cdot (1 + m))} \quad \text{and} \quad (3.4)$$

$$\tau(m) = \frac{m^\gamma \cdot z - \alpha \cdot (1 - \gamma \cdot (1 + m))}{1 - \alpha \cdot (1 - \gamma \cdot (1 + m))} . \quad (3.5)$$

Using m as the independent variable instead of x or τ to determine the other variables may seem unnatural. In fact, this variable is in general endogenously derived from the endowments of the economy, their prices and the tax rates, and is not directly controllable by the government. Furthermore it is usually easy for the government to control the price of education and, in particular, the income tax rate. Unfortunately, the system can only be analytically solved expressing $x(m)$ and $\tau(m)$.

Now, utility depends on disposable income, that is, the income after taxes and subsidies. Then, the net wage $w_i - x_i$ in the utility (2.5) should be changed to include taxes and subsidies. For the low-skilled workers the net wage turns out to

be $w_L^n = (1 - \tau).w_L$, while for the high-skilled workers it is $w_H^n = (1 - \tau).w_H - x$. These two net wages are important when we make welfare considerations about the outcome of the economy.

In order to interpret the results, we obtain some interesting values for the variables resulting from equations (3.4) and (3.5). We first inquire about what tax rate implies $m = 0$, that is, an economy where all workers are low educated. Using (3.5) we obtain

$$\tau_0 \equiv \tau(0) = -\frac{1}{\frac{1}{\alpha \cdot (1-\gamma)} - 1}.$$

Since $0 < \alpha, \gamma < 1$ we have that $\tau_0 < 0$, that is, the education is taxed. In fact, as $\tau \rightarrow \tau_0$ the price of education tends to infinity. Secondly we inquire about the income tax rates that make education free. Solving $x(m) = 0$ in (3.4) we find two solutions for m . The first solution is $m = \frac{1-\gamma}{\gamma}$. This ratio maximizes the production per capita when the cost of the education is 0, and implies the same wages for all the workers in the economy. The tax rate in this case is $\tau_x \equiv \left(\frac{1-\gamma}{\gamma}\right)^\gamma .z$. The second solution is given by $m = z^{-\gamma}$. This solution implies $\tau = 1$, that is, all income is taxed. In this case, all wages after taxes are 0.

We use the tax rate τ as the variable to display the different variables in Figure 2. This election is motivated by the fact that this variable is restricted to a bounded interval, given by $\tau \in [\tau_0, 1]$. Moreover, this variable is directly controlled by the government. Note the variables displayed in this figure are net values.

Figure 2 shows that both the net production and the net wage of the low-skilled workers are 0 for $\tau = \tau_0$ and $\tau = 1$. For $\tau \rightarrow \tau_0$, as we have seen, $m \rightarrow 0$, and then the economy share of high-skilled workers tends to 0. Because the two production factors are essential for production, the net production per capita tends to 0. The relative abundance of low-skilled workers also depresses their net wages.⁴ When

⁴As we can see in Figure 2 in this case the net wage of the low-skilled workers tends to 0. This

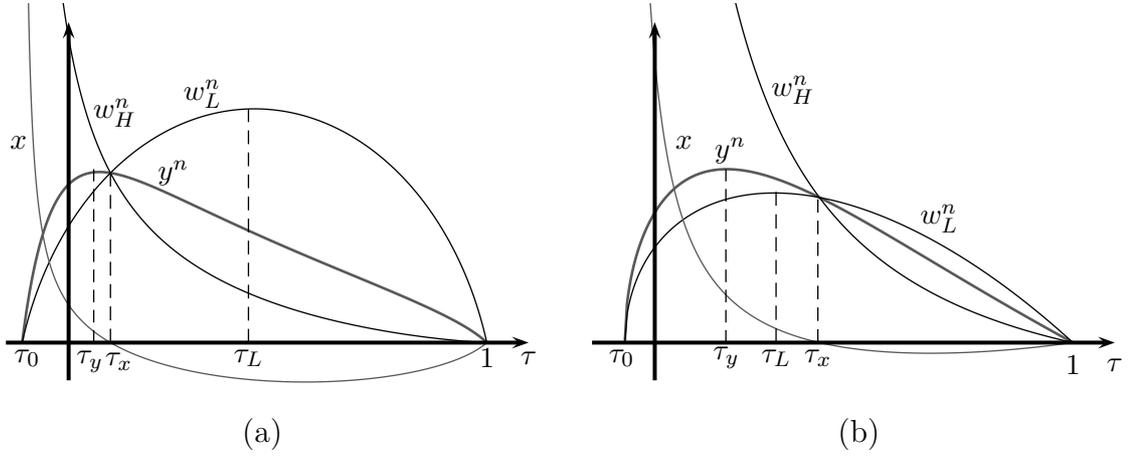


Figure 2: Pictures of the net wages, average net production and cost of the education for different parameters, as functions of the tax rate.

$\tau \rightarrow 1$ the whole wage is taxed, and then the net wage of all workers tends to 0. Since net wages tend to 0, nothing is left after spending on education, and so the net production is also 0 at this limit. So, it is easy to show that both the average net production and the low-skilled workers' net wage are hill shaped in the region $\tau \in [\tau_0, 1]$. We denote the tax rates that generate maxima in the net production and net low-skilled wages by τ_y and τ_L respectively.

For $\tau > \tau_L$ the net wages of both high-skilled and low-skilled workers are decreasing functions of τ . Because the utility (2.5) is an increasing function of the net wage, all individuals in the economy are better off when $\tau = \tau_L$ than when $\tau > \tau_L$. Then, the region $\tau > \tau_L$ is clearly Pareto inefficient. Inside the region $\tau_x < \tau < 1$ the education has a negative cost, $x < 0$. This region corresponds to a situation where people obtain money to be educated, but then high-skilled workers earn a lower net wage than the low-skilled workers. This situation seems unrealistic, especially since high-skilled workers can usually apply for low-skilled jobs, simply

occurs even though it is subsidized ($\tau_0 < 0$), due to an oversupply of low-skilled labor. A transfer from the rich (the only who pay the taxes of education) to the poor generates a huge inequality due to the high price of acquiring human capital.

hiding their academic background. We can assume as well that taxes that increase the wage inequality are not politically feasible unless they increase the average net production. Then, the government can only choose an income tax rate inside the region $\tau \in [\tau_y, \min\{\tau_x, \tau_L\}]$.

Inside the Pareto efficient and politically feasible region for the income tax rate there is a tradeoff between efficiency and equality. The income tax rate $\tau = \tau_y$ maximizes the net production of the economy, leaving the economy in a point of productive efficiency. Nevertheless, increasing τ , the net wages of the two types of workers converge, reducing the income inequality of the economy.

Note that a tax on income whose revenue is spent on subsidies in high education which only benefit the high income workers may seem a regressive fiscal policy. Indeed, from a static perspective, the low-skilled workers are worse off because their disposable income decreases, meanwhile those who are high-skilled are better off, since the reduction in the price of education is higher than the decrease in their income due to the taxes. Nevertheless, as we have seen, it may be a progressive fiscal policy from the dynamic point of view. In fact, once the economy reaches its new equilibrium, the low-skilled workers are better off, since they are fewer and their net wages are higher than before due to their relative scarcity.

The literature on human capital, pioneered by Becker (1962) and Schultz (1963), has generally related the high wages of the highly educated to an increase in productivity due to their human capital. Alternatively, Spence (1973, 1979) modeled the possibility that education may only be a signal for ex ante high-skilled workers. Our model obtains this relationship through a different channel. Parents only invest in their children's human capital if this investment is rewarded with a high wage. The relative scarcity of high-skilled workers should be such that it allows them to earn a wage that compensates the human capital investment. In fact, if

education is inexpensive, arbitrage between the two types of jobs will equal their wages, independently of the skills required to apply for this job.

This is particularly important when there is no perfect elasticity of substitution between the two types of jobs, as is suggested by the empirical data. In this case the meaning of more productive worker is not well defined but it depends on the relative scarcity of the two types. In fact, in this case the individual's marginal productivity depends on the actual endowments of the economy. Then, it is not education that directly makes a worker more productive, but the scarcity generated by the cost of acquiring new skills.

4 Many wages

We now generalize the benchmark model of section 2 and apply it to a more realistic case allowing many different jobs with many different wages. We will model it as a continuum of the possible levels of education that a person can achieve.

Let's assume that there is a wide range of jobs, each requiring a different level of skill, which depends on the education investment made by parents. Now parents' investment in education, x , can take values in a wide range of non negative curves, $X \subset \mathbb{R}_+$. We assume that the wage earned by a worker with an investment in education x is given by the function $\tilde{w}(x)$, that is, all the jobs that require the same investment in education pay the same wage. Moreover, since worker with the same wage have identical maximization problems, the utility should depend only on the wage. In this case the utility will be given by

$$u(w) = \max_{x \in X} f[w - x, u(\tilde{w}(x))] , \quad (4.1)$$

with the usual conditions on the partial derivatives, $f_1 > 0$, $f_2 > 0$, $f_{11} < 0$ and $f_{22} < 0$. This maximization problem is identical to its analogous problem in the discrete case (2.1) changing the subset of possible investments from discrete to continuous.

In general we have $u'(w) > 0$. As in the case of only two wages, this is because people with higher wages have strictly more utility than people with lower wages when they choose the same investment. In fact, when a parent with high wage invests the same quantity in their children's education as a parent with lower wage, the children of both parents has the same utility, but the parent with high income consumes more. Then, because parents are utility maximizers, $u(w)$ must be strictly increasing.

Let's now look at the first order condition (FOC) that is obtained from the problem (4.1). In order to maximize their own utility, the corresponding FOC that they have to solve is given by

$$-f_1[w - x, u(\tilde{w}(x))] + f_2[w - x, u(\tilde{w}(x))] \cdot u'(\tilde{w}(x)) \cdot \tilde{w}'(x) = 0 . \quad (4.2)$$

For any given functions $u(\cdot)$ and $w_f(\cdot)$ this equation establishes a relationship between the quantity invested in the children's education, x , and the wage earned, w , which we denote by $x(w)$.⁵ As before we assume that education is a normal good, what ensures that parents with high wages invest a higher quantity in their children's education than parents with low wages, and so $x'(w) > 0$. This is because when people earn more money they share the additional income between higher consumption (more disposable wage) and higher educational investment.

Let's finally impose the condition that the economy is in a steady state, that is, parents give their children the same education as the one they received. This means that, if parents earn a wage w , they invest an amount in their children's x in order to make their children earn a wage w . The equation that expresses this condition is

$$\tilde{w}(x(w)) = w . \quad (4.3)$$

This assumption is in line with the low intergenerational income mobility observed

⁵We do not explicit dependence of $x(w)$ on u and w_f to simplify the notation.

by Solon (1992). If we assume that the economy is in a steady state, we can replace $\tilde{w}(x)$ by w in all the equations we have just presented. Using this and for a given f , the equation (4.2) gives us $u(w)$.

In order to show how this model works and to obtain concrete results to interpret the equations we have found, we take a particular form for the function f . Consider that f has constant elasticity of substitution (CES) between its two arguments. Specifically, we assume that (4.1) takes the following form:

$$u(w) = \max_x C \cdot [(1 - \beta) \cdot (w - x)^\alpha + \beta \cdot u(\tilde{w}(x))^\alpha]^{\frac{1}{\alpha}} . \quad (4.4)$$

It is important to introduce the multiplying constant C because by definition the utility function depends on itself. In this kind of definition the cardinality of the utility function becomes important, and it is not just a rescaling of the utility function. To avoid the possibility of infinite utility we assume $C^\alpha \cdot (1 - \beta) < 1$ and $\beta \cdot C^\alpha < 1$. In the steady state the utility can be directly obtained from equation (4.4). We obtain the following result

$$u(w) = \left(\frac{(1 - \beta) \cdot C^\alpha}{1 - \beta \cdot C^\alpha} \right)^{\frac{1}{\alpha}} \cdot (w - x(w)) . \quad (4.5)$$

Note that utility depends linearly on the disposable wage $w - x(w)$ through a linear function. Comparing this equation with the analogous in the (2.5) we may infer that the elasticity of the utility with respect to the net income depends on the global returns to scale of the function f . In fact, in this part of the paper we assumed a CES with constant returns to scale of f in order to be able to obtain analytical results. If we impose constant returns to scale in the function f considered in the f function of the initial model in (2.4), $\beta = 1 - \alpha$, we recover exactly the same linear functional form for the utility.

Plugging (4.5) into the FOC (4.2) we can obtain the educational investment as

a function of the parents' wage, $x(w)$. Doing so we obtain the following relation

$$1 - \beta = \beta \cdot \frac{(1 - \beta) \cdot C^\alpha}{1 - \beta \cdot C^\alpha} \cdot (1 - x'(w)) \cdot \tilde{w}'(x) . \quad (4.6)$$

Differentiating the equation (4.3) we know that $x'(w) = 1/\tilde{w}'(x(w))$. Using this in the previous equation it becomes an ordinary differential equation. Solving this equation, the returns on the educational investment required to achieve the steady state take the following form:

$$\tilde{w}(x) = \frac{x}{\beta \cdot C^\alpha} + w_b , \quad (4.7)$$

where w_b is the integration constant. Note that this expression is very similar to (2.8) obtained in the case of two possible education investments. In fact, as in the simplified model, the returns on education are linear in the steady state. In this case, the slope of this dependence is $(\beta \cdot C^\alpha)^{-1} > 1$. Looking at the utility function (4.4), this term could also be interpreted as the discount factor of parents with respect to the utility of their children.

We can interpret w_b as the minimum wage of our economy, that is, the wage of workers that do not receive any educational investment from their parents. Note that in the continuous case returns on education are completely determined, while in the discrete case they were only constrained to be in a certain interval (see the shaded area in Figure 1). This is because in this case parents can decide between a continuum of values of the investment and can adjust the investment at the margin, but in the discrete case, as they can only choose between two values, the range of possible disadjustments is wide.

This model predicts that returns on education, measured as the wage earned with respect to educational investment, should be linear. Econometric studies, such as Patrinos and Psacharopoulos (2004), observe that returns on education are lower the later in life this education is received. These decreasing returns may be due to

the fact that the older an individual is, the more he or she decides about his or her education, and the greater the share of this education that he or she has to finance. Then, it is plausible to assume that the slope of the returns on education $(\beta.C^\alpha)^{-1}$ (or α in the discrete model) progressively tends to 1 when the years of education increase. This produces a reduction on the parents-children externality, and the premium for education diminishes.

Let's now apply the condition of stability (4.3) to the result for the returns on education (4.7) in order to quantify parent's investment in their children's education. This function is given by

$$x(w) = \beta.C^\alpha.(w - w_b) . \quad (4.8)$$

The quantity invested depends linearly on the difference of the wage with respect to the minimum wage. Using this result we finally obtain the utility with respect the wage. This utility is given by

$$u(w) = \left(\frac{(1 - \beta).C^\alpha}{1 - \beta.C^\alpha} \right)^{\frac{1}{\alpha}} \cdot ((1 - \beta.C^\alpha).w + \beta.C^\alpha.w_b) .$$

To shed light on the externality underlying these results we can consider the case when education investment is chosen by its recipients. Assume that the utility of an individual depends only on her net wage, so she maximizes $u(w(x) - x)$ with respect to x . Then, the first order condition is given by $w'(x) = 1$. This result contrasts with our finding that $w'_f(x) > 1$. The reason is that parents discount the utility of their children, and then require higher returns to invest in education.

Conclusions

The model presented in this paper provides different insights into the mechanisms that interrelate parents' earnings, children's earnings and cost of education. Since

many decisions and investments on human capital are mostly made by parents, intergenerational altruism helps to explain the incentives and the prices in the demand for education and labor market.

The first result that the model provides is that education returns should be higher than those on other investments. The literature usually relates this fact to imperfections in financial markets (no collateral to allow borrowing) and moral hazard. In this model this differential is explained by the fact that parents apply a discount rate on the utility of their children. In fact, education can be considered an intergenerational investment, and therefore subject to an additional interpersonal discount rate.

Returns on education, measured as the relation between wages and education, are found to be linear. This linearity does not arise from the increase in productivity caused by education, but is derived by imposing the stability of incomes throughout generation. Nevertheless, when education is decided at least in part by the recipients, the slope of the curve may flatter, and the curve becomes concave. Then, decreasing returns may be observed when tertiary education is considered.

The fact that individuals do not decide their level of education may be considered an externality on the decisions of their parents. In trying to solve this externality the government can subsidize education by taxing income. From the static point of view this tax is regressive. In our model wealthy parents are the only that provide education for their children, and then the only who benefit from the subsidy. Nevertheless, from a dynamic point of view, a subsidy on education may increase the next wage of the low-skilled workers in equilibrium by making them more scarce. In deciding on the optimal tax rate, the government faces the traditional tradeoff between efficiency and equality. The model predicts that for certain values of the parameters, a completely subsidized education system is not Pareto efficient.

Our model stresses the fact that high wages of high educated workers may not be linked to a gain in productivity. For general production functions, with imperfect substitutability between the different types of labor, the wage of a worker depends crucially on the scarcity of his type. Then, in our model, high-skilled workers earn more not because they are more productive, but because they are scarce and have higher marginal returns. When the price of education is subsidized, the wages of high and low educated workers converge.

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