DOCUMENTS DE TREBALL

DE LA FACULTAT D'ECONOMIA I EMPRESA

Col·lecció d'Economia

E08/210

Inter-firm labor mobility and knowledge diffusion: a theoretical approach

Montserrat Vilalta-Bufi[1]

---

1 Departament de Teoria Econòmica and CAEPS (Universitat de Barcelona) and Departament d'Economia i Història Econòmica (Universitat Autònoma de Barcelona). Email: montsevilalta@ub.edu. Tel: +(34)934020107. Fax: +(34)934039082. I would like to thank Jordi Caballé, Bruno Cassiman and Danilo Guaitoli for helpful comments and suggestions. I also benefited from comments by Ramon Marimon, David Pérez-Castrillo. The author acknowledges financial support from the Spanish Ministry of Education through the grant SEJ2005-03196/ECON and from Generalitat de Catalunya through the grant 2005SGR-00460 and a FI grant.

Abstract

We analyze an economy with two main features: labor mobility goes together with knowledge transfer and firm productivity increases with the exchange of ideas. Each firm develops some specific knowledge that will be transmitted to the rest of the industry through the mobility of workers. We study two labor market settings and use comparative statics to derive the implications of the model. They reveal how labor mobility depends on the variety and level of knowledge, the presence of mobility costs, the institutional environment, the absorptive capacity of the firms and the size of the industry. Results are robust to different labor market settings.

Resum

Analitzem una economia amb dues característiques principals: la mobilitat dels treballadors implica transferència de coneixement i la productivitat de l'empresa augmenta amb l'intercanvi de coneixement. Cada empresa desenvolupa un tipus de coneixement que serà trasmès a la resta de la indústria mitjançant la mobilitat de treballadors. Estudiem dues estructures de mercat laboral i utilitzant un anàlisi comparatiu derivem les implicacions del model. Els resultats revelen com la mobilitat de treballadors depèn en la varietat i nivell del coneixement, la presència de costos de mobilitat, les institucions, la capacitat d'absorvir coneixement per part de les empreses i la mida de la indústria. Els resultats no depenen de l'estructura del mercat laboral.

Keywords: inter-firm labor mobility, knowledge diffusion, exchange of knowledge

JEL classification: J23, J61, O33

1. Introduction

Knowledge diffusion has long been recognized as very important for innovation and productivity. It is not enough that new ideas are created to increase productivity, but they need to be spread and used in the economy in order to obtain their benefit. In this paper we propose an alternative way to model knowledge diffusion through labor mobility. In contrast to the seminal papers on endogenous growth (Romer, 1986; Lucas, 1988, and Romer, 1990) which are based on exogenous knowledge spillovers, we explicitly set the mechanism through which knowledge gets diffused into the economy, and although in this paper we present a static analysis, we believe our approach can also be applied in a dynamic setting. There are several papers already dealing with knowledge diffusion through labor mobility from a theoretical approach, but they are not within the neoclassical framework.

In our model knowledge is transferred across firms via high-skilled labor mobility. Even where intellectual property rights are well-protected, tacit knowledge is embodied in the workers and can be used by any firm who hires them. We assume that there are several firms in the industry. Each of them has developed a particular type of knowledge, which is embodied in its workers. Then each firm can learn external knowledge by hiring workers from other firms. The exchange of different types of knowledge, and the subsequent positive effect on productivity and innovation, can only occur when the different types of workers work in the same firm. The model reveals new insights into the patterns of labor mobility within an industry.

Theoretical literature on workers' flows across firms is scarce. It is mostly within labor economics and industrial organization that this topic is developed. We can classify this research into three different strands. A first strand models labor mobility as the result of optimal investment decisions and it is very related to human capital theory. Young workers look for jobs with a high learning component. When they get older, they move with their acquired human capital towards better paid jobs and less learning opportunities. Rosen (1972), Jovanovic and Nyarko (1995) and Moen (2005) are in this line. The main issue in these papers is to disentangle who has incentives to pay for the general training.

The second group, which follows Pakes and Nitzan (1983), introduces an element of specificity in the learning-in-the-job that allows workers to get some rents. Competitors are willing to pay high wages to experienced workers to learn their embodied specific knowledge, so workers will either move to competitors or be better-paid by their current employer who wants to retain them. Fosfuri, Motta and Ronde (2001) use the latter argument to analyze whether a firm should export or go multinational. If it goes multinational, its technology may become spread into the local firms through labor mobility. In Franco and Filson (2005), workers evaluate whether to start up a new firm after having learned-in-the-job. Combes and Duranton (2006) are also in this strand of literature while considering a model of reciprocal poaching of labor. In their paper, clustering decision of firms depends on product market competition and workers heterogeneity in terms of knowledge transfer cost. As rivalry in product market competition intensifies, firms get more concerned about keeping information private, so they pay higher wages to their workers in order to prevent them from moving to competitors.

Still within this second group, but from an industrial organization approach, Fosfuri and Ronde (2004) and Gersbach and Schmutzler (2003) use labor mobility to transfer knowledge across firms. Fosfuri and Ronde (2004) have a two-firm two-period model with cumulative innovation where technology spillovers arise through labor mobility. According to their results, firms are more likely to cluster when the growth potential of an industry is high, competition in the product market is soft and probability of a firm to develop an innovation is neither very high nor very low. Gersbach and Schmutzler (2003) show that the effect of product market competition on incentives to innovate depends on spillovers being endogenous or exogenous. One of the main issues studied in this research line is the interaction between labor mobility and imperfect competition in the product market.

A third strand of literature uses matching models where labor mobility brings higher productivity in the economy through the reallocation of mismatched workers. In general, though, they do not assume any diffusion of knowledge with the mobility of workers. Jovanovic and Moffitt (1990) and Cooper (2001) are two examples of this literature.

Our work looks at labor mobility and knowledge diffusion from a new perspective within the neoclassical framework. Firstly, instead of focusing on the interaction between labor mobility and human capital, product market competition or labor market mismatches, we look at how several institutional and sectoral variables affect labor mobility. Secondly, we have a continuum of workers in each firm (as in Combes and Duranton, 2001, 2006), while in most of the literature there is only one worker per firm. With multiplicity of workers per firm, the ability to retain workers becomes less trivial. Furthermore, we enlarge the analysis to N firms, which until now had only been done in a matching model setup.

The paper is divided into four further sections. Next, we present the basic model in a framework of perfect competition and derive the equilibrium outcome. In section 3 we do a comparative static analysis of this equilibrium to see how labor mobility and wages are affected by different parameters. In section 4 we reconsider the model by relaxing the perfect competition assumption in the labor market, which may seem too strong for this model. In particular firms now set wages to experienced workers and we allow them to have an advantage over their rivals when offering wages to their own workers. We show that the main results of the basic model remain unchanged. In section 5 we derive the empirical implications of our model, which as we showed in section 4 do not depend on the assumption of perfect competition in the labor market. Finally we summarize the results.

2. The model

The model proposed here is a static one with labor mobility. We assume that knowledge is developed within each firm, but valuable to the whole industry. There are N firms in the economy. They are identical in everything, but in the specific knowledge they have, which is embodied in their experienced workers. Workers live for two periods only. Each period there is a continuum supply of young workers with measure $N\Lambda$, who do not have any labor experience ($\Lambda \in \Re_{++}$). Assume that in period 0, a measure Λ of young workers work in each firm. Without loss of generality and for the rest of the paper we will assume that $\Lambda = 1$, so that in total there is a measure N of young workers in the economy and each firm hires a measure 1 of these workers. We can think that each firm requires a fixed amount of research assistants or unskilled workers, independently of the type of research or production they are pursuing. By working in the firm they learn some specific knowledge without any cost (learning-by-doing), so that, at the beginning of period 1, there is a measure 1 of senior workers with the knowledge developed in each firm. We call them experienced workers.

In period 1 firms may hire their own experienced workers and external experienced workers. Denote by λ_i^j the amount of experienced workers from firm j that are hired by firm i, $j \neq i$.[1] As already stated above, they have embodied knowledge type j. We call them poached workers. Similarly, let λ_i^i be the amount of own experienced workers hired by the same firm i, which have knowledge type

1 Notice that since we normalized Λ to 1, λ_i^j also corresponds to the fraction of workers of firm j that are hired by firm i.

i. They are called retained workers.

The production function of each firm is $Y_i = H_i^\alpha L_i^{1-\alpha}$ where H_i is a measure of human capital and L_i is the total young employment of firm i $(i = 1,...N)$. For simplicity, we do not include physical capital in the model. The amount of young workers hired by each firm is assumed exogenous and with measure 1, so we can simplify further the production function to $Y_i = H_i^\alpha$. We define human capital as an asymmetric CES function on all types of experienced workers hired by the firm.

$$H_i = \left((\lambda_i^i k_i)^\alpha + p \sum_{j \neq i} (\lambda_i^j k_j)^\alpha \right)^{\frac{1}{\alpha}}, \tag{1}$$

where k_j is an indicator of the specific type of knowledge of firm j and p is a parameter which lies between 0 and 1 and measures how much a firm can access external knowledge. This parameter p includes three factors: one refers to the intrinsic characteristics of the knowledge in question (whether it is firm or industry-specific); the second factor is the degree of capacity of firms to acquire such external knowledge (concept of absorptive capacity of firms developed by Cohen and Levinthal (1990), and finally, the type of environment where firms develop their tasks (e.g. institutions, local legal system which may enforce or not clauses not-to-compete, strongly defend trade secrets, etc.).[2] Notice that the asymmetry appears because we assume that knowledge from own workers (λ_i^i) is fully accessible by the firm while knowledge from poached workers may be less accessible ($p \in [0,1]$).

Knowledge in our model has two dimensions: variety and level of

2 There is empirical evidence that shows how important differences in legal systems may be in determining the rate of labor mobility of a region. Hyde (1998), Gilson (1999) and Valetta (2002) argue that Silicon Valley was originated in California precisely because there clauses not-to-compete have weak enforceability. Almeida and Kogut (1999) point out at the importance of "*social institutions that support a viable flow of ideas within the spatial confines of regional*

knowledge. The subindex in k_i indicates the type of knowledge, while the level of knowledge is indicated by the particular value of k. As an illustration, imagine that each type of knowledge is a particular color, while the level of knowledge refers to the intensity of each color. We do not assume anything about the level of knowledge, that is, each firm may or may not have a different level of knowledge. In this way, the equilibrium outcome we obtain is a general one.

With such specifications, we obtain a functional form for output similar to the one derived in Romer (1990), but instead of different types of capital goods, here we have different types of human capital. In the conventional specification, total human capital is implicitly defined as being proportional to the sum of all the types of human capital, assuming perfect substitutability among them. The model here considers the case in which all types of human capital have additively separable effects on output. This means that each type of human capital does not affect the marginal productivity of the rest of human capital types.

$$Y_i = (\lambda_i^i k_i)^\alpha + p\sum_{j\neq i}(\lambda_i^j k_j)^\alpha .$$

The production function stresses the importance of variety of knowledge by its additive form. Notice that with such functional form it is not obvious *a priori* that there will be labor mobility in equilibrium since having one type of knowledge is sufficient for production.[3] We assume decreasing returns to all types of experienced workers ($0<\alpha<1$). It seems plausible that the amount of new knowledge that a worker adds to the firm is lower the more workers of his type are already hired by that firm. Moreover, we assume that without workers there is no access to the knowledge. Notice how productivity increases with the exchange of ideas. When the firm only hires workers with the same type of knowledge, the

economies" for creating the externalities that foster innovation (p.916).

3 In contrast, by assuming a standard Cobb-Douglas type function on all inputs, complementarities among types of knowledge would make of labor mobility a trivial result.

maximum productivity it can achieve is k_i^{α}, with i indicating the type of knowledge of the workers hired. On the other hand, when the firm hires the same amount of workers but with different knowledge, its productivity may get higher values thanks to the assumed concavity.

We assume perfect competition in the product market to be able to isolate the exchange of knowledge effect on the labor market.[4] To simplify we assume that all firms can sell all the product at a given price, which we normalize to 1. Hence, we do not consider here how imperfect product market competition may affect the results.

As mentioned above, at the beginning of the period there is a measure 1 of experienced workers with each type of knowledge in the industry. Moreover, there is a positive cost for workers to move from one firm to the other, which we denote by m. We can think of it as the cost of changing place of residence, for instance. We consider first the case of perfect competition in the labor market, so that firms take wages as given.[5] Let ω_j be the wage to experienced workers type j working in firm j. A worker will move to another firm if and only if he is paid for the mobility costs, that is, if he gets $\omega_j + m$. Then, an experienced worker poached by another firm will get $\omega_j + m$, while his colleague who is retained by firm j gets ω_j. Then, each firm i decides the amount λ_i^i of own experienced workers to retain and the amount λ_i^j of experienced workers to poach from each firm j $(j \neq i)$.

4 We can think that either owners of the firms consume the product produced themselves or they sell it to a large competitive market.
5 At this point, some reader may think that the firm that hires workers when young should have some power on deciding their wage when workers get experienced. In section 4 we present a set-up where firms offer a wage to their own workers before any other firm does. This gives them an advantage not considered in the perfect competition case. However, although it introduces some inefficiency, the main results remain the same.

The firm faces the following problem: [6]

$$\max_{\lambda_i^i, \{\lambda_i^j\}_{j\neq i}} (\lambda_i^i k_i)^\alpha + p\sum_{j\neq i}(\lambda_i^j k_j)^\alpha - \omega_i\lambda_i^i - \sum_{j\neq i}(\omega_j + m)\lambda_i^j$$

The first order conditions are the following:

$$\alpha\lambda_i^{i*\alpha-1}k_i^\alpha = \omega_i, \tag{2}$$

$$p\alpha(\lambda_i^{j*})^{\alpha-1}k_j^\alpha = \omega_j + m,\ j\neq i. \tag{3}$$

Notice that λ_i^{j*} is independent of i, meaning that firms that poach workers from a firm j will all poach the same amount of workers from this firm j. Then, $\lambda_i^{j*} = \lambda^{j*}$ for all i, j. Notice that since marginal productivity of poached workers at $\lambda_i^j = 0$ is infinity for all i, j and there is no cost of adapting variety of knowledge, all firms will poach workers from all the other firms in the industry to access to the whole range of knowledge in the economy.[7] Moreover, firms will always want to retain some of their own workers because the marginal productivity of retained workers when the firm retains zero workers is infinite. Nevertheless, these conditions are not necessary to obtain positive labor mobility in equilibrium. The necessary condition for positive labor mobility is that the marginal productivity of a type i worker when all workers of type i are working for firm i is lower than the marginal productivity of the first worker of type i that moves to any other firm. Similarly, the condition for having some retained workers in equilibrium is that the marginal productivity of the first retained worker is higher than the marginal productivity of this type of worker in any other firm when all workers of his type are working for that firm.

The market clearing conditions for this equilibrium are

6Notice that since the firm has an additively-separable production function, it faces N independent one-variable maximization problems.

7We could limit the number of firms from which to poach workers by introducing a cost of adaptation of external knowledge which increases with the variety of knowledge. This would complicate the analysis without giving any new insights into the model.

$$\lambda_i^{i*} + \sum_{j \neq i} \lambda_j^{i*} = 1 \;\; \forall i, \tag{4}$$

where the left-hand-side (from now on LHS) is demand for experienced workers of type i and the right-hand-side (RHS) is its supply. They can be rewritten as $\lambda_i^{i*} + (N-1)\lambda^{i*} = 1 \;\; \forall i$ because there are N firms in the industry and from equation (3) we know that $\lambda_j^{i*} = \lambda^{i*} \quad \forall j, j \neq i$. Note that the these equilibrium conditions are independent across types of labor. For each type i of labor, total demand depends only on ω_i.

The equilibrium is composed by the triplets λ_i^{i*}, λ^{i*} and ω_i^* for all i, which are determined by equations (2), (3) and (4). Substituting the first order conditions in the market clearing condition we characterize ω_i^*:

$$\left(\frac{\alpha k_i^{\alpha}}{\omega_i^*}\right)^{\frac{1}{1-\alpha}} + (N-1)\left(\frac{p\alpha k_i^{\alpha}}{\omega_i^* + m}\right)^{\frac{1}{1-\alpha}} = 1. \tag{5}$$

Putting together all the equilibrium conditions we obtain that the *net* marginal productivity of experienced workers (marginal productivity net of mobility costs) should be the same, independently whether they are retained or poached by another firm,

$$\alpha(1-(N-1)\lambda^{i*})^{\alpha-1} k_i^{\alpha} = p\alpha(\lambda^{i*})^{\alpha-1} k_i^{\alpha} - m. \tag{6}$$

This condition determines the equilibrium decision on the number of poached workers λ^{i*} by each firm. Market clearing condition ensures that λ^{i*} is an interior solution.[8] In fact, since marginal productivity of the first poached worker is infinite, and the same happens with retained workers, we obtain an interior

8 On one side, λ^{i*} must be non-negative (we can not poach a negative amount of workers), and on the other side, the total demand of workers can not exceed the supply of such workers, or in other words, the firm can not retain a negative amount of workers. Mathematically, $\sum_{i \neq j} \lambda_i^{j*} = (N-1)\lambda^{j*} \leq 1$. Hence, λ^{j*} must belong to the interval $\left[0, \frac{1}{N-1}\right]$ to have an interior equilibrium.

equilibrium in λ^{i*} (check the appendix for details). Notice also that the equilibrium wage and the labor mobility rate are a function of the level of knowledge of the firm: $\omega_i^*(k_i)$ and $\lambda^{i*}(k_i)$. Thus, in general the equilibrium is not symmetric. Only when the levels of knowledge are the same among firms, labor mobility and wages are symmetric in equilibrium. The proof of the following proposition is in the appendix.

Proposition 1 *In an economy with perfect competition in the labor market, there exists a unique equilibrium, given by the vector of wages and labor mobility* $(\{\omega_i\}_{i=1}^N, \{\lambda_i^i\}_{i=1}^N, \{\lambda_i^j\}_{i=1}^N)$ *satisfying equations (2), (3) and (4). This equilibrium displays positive labor mobility.*

Note that there is no friction in this economy; that is, markets are all perfectly competitive and there are no market failures. Hence, the equilibrium solution is efficient.

3. Comparative static analysis

We proceed now to check the comparative statics of the previous equilibrium. We are interested in analyzing how labor mobility and wages for experienced workers change with each parameter of the model. This analysis will allow us to better understand the functioning of the model.

We would expect that an increase in p (the level of transferability of knowledge) will result in higher labor mobility and higher wages, since it increases the value of poached workers for the hiring firm. Firms are able to extract more knowledge from every poached worker they hire, which translates into more value. Thus, they are willing to hire more poached workers and pay them better. And this is what happens in equilibrium. The marginal productivity of poached workers is higher when p increases, while the marginal productivity of retained workers remains the same. Thus, a firm is hiring more of poached workers to equalize back the two marginal productivities.[9] This effect increases the demand for each type of workers, which results in higher wages because supply is completely inelastic.[10]

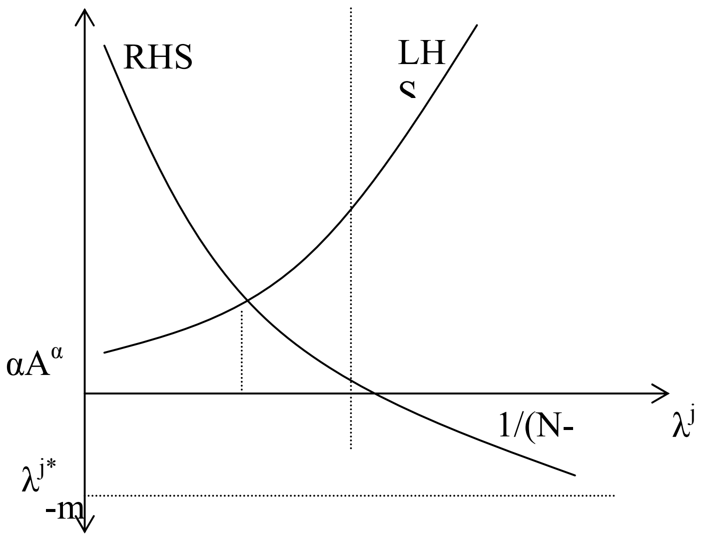

Figure 1. Determination of the level of labor mobility for the basic model (from equation (6)).

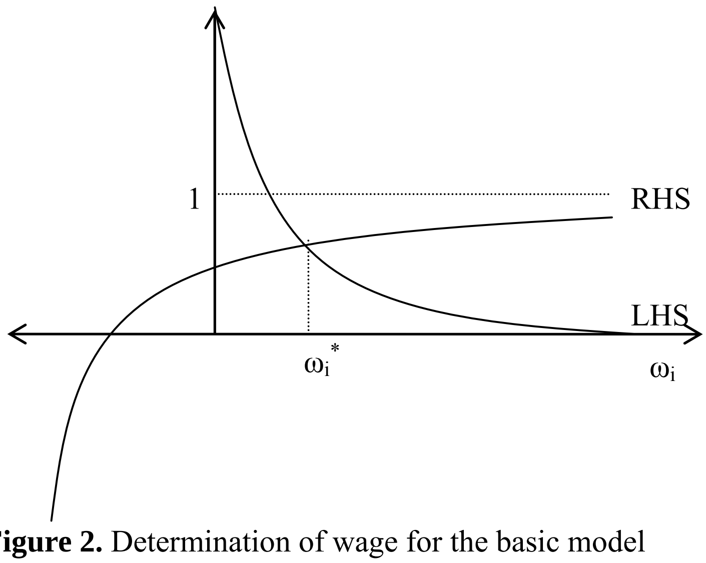

Figure 2. Determination of wage for the basic model (from equation (5')).

9 It can be illustrated in Figure 1. An increase in p shifts the RHS of equation (6) to the right, while the LHS remains the same. Thus, the new equilibrium has higher labor mobility.
10 Similarly as before, we can illustrate this result in Figure 2. The RHS of equation (5') shifts to the right, while the LHS remains the same. This results in a higher wage in equilibrium.

Mobility cost m is a real cost in this model. This means that higher mobility costs reduce the total final output of a firm. Thus, when mobility costs increase, firms prefer to substitute some of their poached workers for more retained workers who do not incur mobility costs, reducing labor mobility in equilibrium. At the same time this results in lower demand for each type of experienced worker, so wages also decline.[11] Notice however that while total wage for retained workers (ω_i^*) decreases, the total wage for poached workers ($\omega_i^* + m$) must increase so that poaching decreases.[12]

To analyze the effects of an increase in the level of knowledge on labor mobility we have to consider two cases. When mobility costs are zero, then the level of knowledge has no effect on labor mobility because it affects in the same way the marginal productivity of workers that stay in the firm and of those that are poached by other firms.[13] In contrast, when mobility costs are positive, then an increase in the level of knowledge type i increases labor mobility of this type of labor. In any case, an increase in the level of knowledge leads to an increase in the wage for the worker with this knowledge through a higher demand of such workers.[14]

A larger industry size (N) or, equivalently in our model a larger variety of knowledge, increases the demand for each type of experienced workers. This is because there are more firms interested in poaching them. The result of this

11 We can illustrate these results similarly as in the p analysis. However, now the RHS moves to the left in both figures, which reduces the equilibrium levels of labor mobility and wages.
12 The market clearing equation have to keep holding. Firms retain more workers and poach less external workers. It must be that wage for retained has decreased and wages for poached workers overall has increased.
13 This result needs of the assumption of separability between different types of experienced workers and it would not hold in the case of a production function with complementarities among them.
14 Take Figure 2 and see how an increase in k shifts both curves RHS and LHS to the right, giving a new equilibrium with a higher wage than before. To see the change in labor mobility

increase in demand, given the fact that supply is inelastic, is an increase in wages. And it is because of these higher wages that then firms poach less experienced workers from each firm in equilibrium. Notice, however, that the same higher wages makes firms retain less workers, so the total amount of labor mobility in the new equilibrium ($(N-1)\lambda^{j*}$) increases.

We summarize the previous results in the following proposition. The formal proof of these results can be found in the appendix.

Proposition 2. *1) An increase in p results in higher labor mobility and higher wages in equilibrium.*

2) An increase in m results in lower labor mobility and lower wages for retained workers in equilibrium. However, poached workers get a higher salary, $\omega_i^* + m$.

3) An increase in N results in lower λ^{i*} and higher wages. The total amount of labor mobility, $(N-1)\lambda^{i*}$, increases with N.

4) If $m = 0$, then an increase in k_i does not have any effect on the equilibrium level of labor mobility.

5) If $m > 0$, then an increase in k_i has a positive effect on the equilibrium level of labor mobility for type-i labor.

6) An increase in k_i unambiguously increases equilibrium wage ω_i^*.

4. The model with imperfect competition in the labor market

As in the previous model we have N firms, each of which hires a fixed amount of young workers. These young workers learn the specific knowledge developed in the firm as a by-product of production and they become experienced workers in the following period. It is as experienced workers that they can move to

check the appendix.

competitors or keep working for the same firm. Until now, we were assuming that firms can not influence the market price for these workers, so they took wages as given. However, one can argue that firms may try to agree on a wage with their own experienced workers before these go to check the external market. Firms may have easier access to their own workers, who are unique because they have embodied a specific type of knowledge. Would this affect the previous results? If so, in which direction? These are the questions we want to answer in this section. For that, we develop a sequential game for the previous economy.

At the beginning of the period there is a measure 1 of experienced workers with each type of knowledge in the industry. Then, the stages of the game within period 1 are the following:

Stage 1- Each firm i commits to a wage ω_i $(i=1,2,...N)$ to its own experienced workers, which are of type i.

Stage 2- Each firm i offers a wage r_i^j $(i, j=1,...,N$ and $i \neq j)$ to experienced workers of the other firms, where j refers to the type of experienced worker. If $r_i^j \geq \omega_j + m$, where m is a positive mobility cost, then firm i may choose how many workers to hire from firm j, which we denote by λ_i^j and refer to as poached workers. Otherwise, firm i can not poach any worker type- j.

Stage 3- Production takes place, products are sold at price 1 in an external market and firms get their profits.

We changed perfect competition in the labor market to allow firms to set wages to experienced workers in the following order: first, to retained workers, and then to poached workers. This is to give firms the first-mover advantage over their own workers commented above. Notice that now firms do not decide how many workers to retain, but this variable will come from the feasibility condition $\lambda_i^i = 1 - \sum_{j \neq i} \lambda_j^i$.

We use backward induction to solve the game. Stage 3 does not involve any decision from the firm. In **stage 2** firms decide first r_i^j and then λ_i^j taking as given the decisions ω_i for all i. The problem they face is the following:

$$\max_{r_i^j,\lambda_i^j}\Pi^i = (1-\sum_{j\neq i}\lambda_j^i)^\alpha k_i^\alpha + p\sum_{j\neq i}(\lambda_i^j k_j)^\alpha - \omega_i(1-\sum_{j\neq i}\lambda_j^i) - \sum_{j\neq i} r_i^j \lambda_i^j$$

$$s.t.\ \ r_i^j \geq \omega_j + m,$$

$$0 \leq \lambda_i^j \leq 1.$$

Since r_i^j enters as a cost only, the firm wants to minimize it as much as possible. Thus, given the first constraint, the optimal wage is

$$r_i^{j*} = \omega_j + m. \tag{7}$$

To determine the optimal number of workers to be poached from firm j we need to derive the first order condition of the problem, and after straightforward algebraic computations we obtain the interior solution of the problem,

$$\lambda_i^{j*} = \left(\frac{p\alpha k_j^\alpha}{\omega_j + m}\right)^{\frac{1}{1-\alpha}} \ i, j = 1,2,..N \ and \ i \neq j. \tag{8}$$

Note that λ_i^{j*} is independent of i, meaning that firms that poach workers from a firm j will all poach the same amount of workers from this firm j. Then, $\lambda_i^{j*} = \lambda^{j*}$ for all i, j. By second-order condition we know that the interior solution is a global maximum ($\frac{\partial^2\Pi}{\partial\lambda_i^{j2}} < 0$).

In **stage 1**, firms decide the wage of their experienced workers, taking into account how this decision will influence the poaching decision of the other firms. Firm i's problem is then to choose ω_i to maximize profits given the optimal response function for poaching. The general problem[15] is

15 Although concavity of the problem is not satisfied for all non-negative ω_i, it can be proved

$$\max_{\omega_i \geq 0}[(1-\sum_{j\neq i}\lambda_j^{i*}(\omega_i))^{\alpha} k_i^{\alpha} + p\sum_{j\neq i}(\lambda_i^{j*}(\omega_j)k_j)^{\alpha}] - \omega_i(1-\sum_{j\neq i}\lambda_j^{i*}(\omega_i)) - \sum_{j\neq i}(\omega_j + m)\lambda_i^{j*}(\omega_j),$$

which gives the following first order condition:

$$-\alpha(1-\sum_{j\neq i}\lambda_j^{i*})^{\alpha-1} k_i^{\alpha} \sum_{j\neq i}\frac{\partial \lambda_j^{i*}}{\partial \omega_i} = (1-\sum_{j\neq i}\lambda_j^{i*}) - \omega_i \sum_{j\neq i}\frac{\partial \lambda_j^{i*}}{\partial \omega_i}. \tag{9}$$

By increasing the wage ω_i the firm can retain more workers. Then the total gains from increasing marginally the wage is the increase in productivity due to this additional number of retained workers (LHS of equation (9)). The right-hand side accounts for the total cost increase, which comprises a higher bill due to the marginal increase in wage to be paid to all retained workers and the salaries to be paid to the additional amount of workers the firm manages to retain.

As we noticed previously $\lambda_i^{j*} = \lambda^{j*}$ for all i, j. Moreover, it is easy to derive from equation (8) that $\frac{\partial \lambda^{i*}}{\partial \omega_i} = \frac{-\lambda^i}{(1-\alpha)(\omega_i + m)}$. Then we can rewrite the previous equation as

$$\frac{\lambda^{i*2-\alpha}}{p(1-(N-1)\lambda^{i*})^{1-\alpha}} = \frac{1-\alpha}{(N-1)} + \alpha\lambda^{i*} - \frac{m\lambda^{i*2-\alpha}}{p\alpha k_i^{\alpha}}, i = 1,2,...N \tag{10}$$

where λ^{i*} comes from equation (8).

Definition 1 *A subgame perfect equilibrium for this economy consists of the policy functions* $\{\lambda_i^j(\omega_j)\}_{i,j=1,..N,i\neq j}$ *and a vector of non-negative wages* $(\{r_i^j\},\{\omega_i\})$ *such that: a) the policy functions* $\lambda_i^j(\omega_j)$ *and wage* r_i^j *solve the stage 2 problems and, b) the wage* ω_i *solve the problem of stage 1 taking into account the solutions of the 2nd stage of the game.*

---

that for the relevant range of ω_i all $\omega_i < \omega_i^*$ (where ω_i^* is such that $\frac{\partial \Pi_i}{\partial \omega_i}(\omega_i^*) = 0$) the profit function is strictly increasing, while for all $\omega_i > \omega_i^*$, it is strictly decreasing. This is enough to prove that ω_i^* is a global maximum. See appendix for proof.

Proposition 3 *With a strategic wage setting, there exists a unique subgame perfect equilibrium with positive labor mobility for each set of parameters. If all non-negativity constraints of wages are non-binding, then the equilibrium wages and levels of labor mobility are determined by equations (8) and (10). If some of the non-negativity constraints of wages are binding, then their corresponding equilibrium wages and labor poaching are* $\omega_i^* = 0$ *and* $\lambda^{i*} = \left(\frac{\alpha p k_i^{\alpha}}{m} \right)^{\frac{1}{1-\alpha}}$.

When all non-negativity constraints of wages are non-binding, we obtain an *interior equilibrium*. In contrast, an equilibrium with some of the non-negativity constraints binding is called *corner equilibrium*. The proof of the previous proposition is in the appendix.

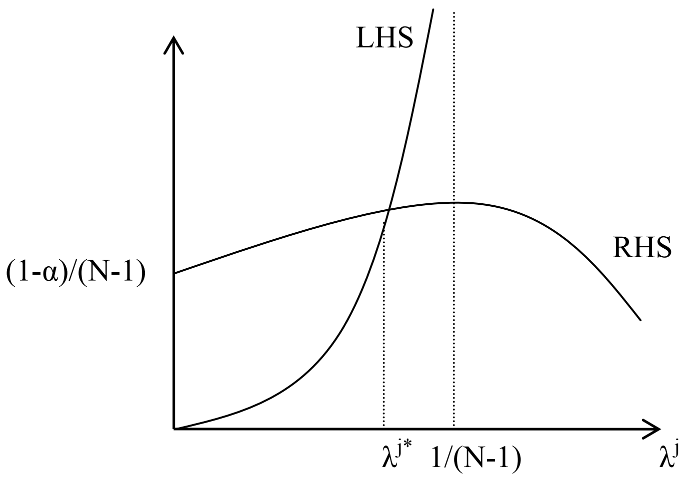

Figure 3. Determination of the level of labor mobility for the sequential game (from equation (10)).

Notice that the interior equilibrium level of labor mobility when mobility costs are zero is independent of the level of knowledge, so we obtain a symmetric equilibrium in the labor mobility (equation (10) is independent of k when $m = 0$). In contrast, with positive mobility costs and in the corner equilibria, the labor mobility rate is increasing with the level of knowledge.[16] These are the same results we obtained in the basic model with perfect competition.

The parameters p and m affect the results also in accordance to previous findings. An increase in labor mobility cost (m) decreases labor poaching, while more general knowledge or less protected property rights (higher p) translates into higher labor mobility. Moreover, when m is high, firms can prevent workers from moving with a lower wage, while higher p requires higher wage to retain workers. Also the number of firms in the industry (N) and the level of knowledge of the workers (k) make it more difficult to retain workers and require a higher wage.

The main difference with the basic model comes in terms of efficiency. While the basic model is efficient, the sequential game here analyzed presents some inefficiency due to the first-mover advantage. In particular we observe that the equilibrium wages are too low in the sequential game, which induces too much labor mobility in equilibrium. A complete efficiency analysis can be found in the appendix.

5. Testable implications

In this section we describe in detail the testable implications that come out from the previous theoretical analysis and relate them to recent studies. They are derived from the two labor market settings analyzed here since, as we showed, results are common to both of them.

Implication 1. *When workers have to incur a positive cost for moving across firms (change of residence, mobility costs,...), the level of knowledge of firms affects positively labor mobility. This has two further implications:*

a) Within an industry, we should observe higher mobility of those workers

16To illustrate this result see figure 3. An increase in k moves the RHS of equation 10 upwards and the LHS stays the same. Thus, the equilibrium level of labor mobility grows.

hired initially by the leading firm (the most innovative). Thus this firm will suffer from losing more experienced labor than followers.

b) Comparing across industries, we should observe a higher level of labor mobility on high-tech industries (which are associated to high level of knowledge), such as biotechnology, and less labor mobility on traditional type of industries (which in general require a lower level of knowledge).

In the case of firms spread across the territory (we can even think of firms within an industry located in different countries) the level of knowledge affects labor mobility. This has two implications. First, *ceteris paribus*, technological leaders within an industry will lose a higher proportion of experienced workers than firms lagging behind. There are some cases that suggest this idea, such as the move of qualified workers from Sony to Samsung. This finding is also consistent with the fact that many startups use learning-by-hiring to access the knowledge of the leading firm. There is not, however, to our best knowledge, any comprehensive analysis including mobility of workers from leader to follower firms and *vice versa*.

As for the analysis across industries, given two equally located industries, with similar parameters, we expect to observe higher labor mobility in the industry more technologically advanced. That is, we should observe higher labor mobility in high-tech industries than in traditional sectors after controlling for other factors. Empirical evidence is lacking again. However, the higher propensity to migrate for highly-skilled workers (engineers, technicians,...) as compared to average employees may suggest that this is the case.

Implication 2. *A high degree of generality of knowledge, a high level of absorptive capacity of firms and a proper institutional environment are essential features for having mobility of workers, even when firms are located together.*

There are three essential aspects to take into account when analyzing the level of labor mobility of an industry. First of all, the usability of knowledge across firms. When the knowledge is industry-specific (as opposite to firm-specific), then there is room for diffusion of knowledge through the mobility of workers. Secondly, to make use of the external knowledge firms need the capacity to absorb and assimilate this knowledge. In particular, firms need to integrate adequately the poached workers into their labor force and encourage them to share their knowledge and apply it in the production function. Some organizational structures are more successful in this enterprise, namely horizontal rather than hierarchical ones, etc. (Leonard and Sensiper, 1998). And finally, the environment must be favorable for such labor mobility (no institutional constraints in the broadest sense: legal, social, political,...). This is to say that there must be no external factor to the firms that prevents labor mobility from occurring. We already commented the importance of covenants not to compete in explaining the differences in performance between Silicon Valley and Route 128. In the Third Italy the presence of extensive families played an important role for risk-sharing and allowed for a flexible labor market. These examples reveal that extremely different institutions may lead to an environment prone to labor mobility and that there is no unique recipe for creating a successful industrial cluster. What draws clearly from this analysis is that without all of these three factors, any attempt to create a *learning region* will fail.

Implication 3. *In general, results show that an increase in mobility costs pulls down the level of labor mobility. Thus, we should observe lower labor mobility between firms that are located far from each other, other things equal.*

High mobility costs reduce the firms' incentives to poach experienced workers from other firms because then it is more costly to attract such workers. Higher marginal costs move the optimal level of poaching down. This seems a sensible result, consistent for instance with the higher labor mobility experienced in the U.S. as compared to Europe. Mobility costs in the former are thought to be lower due to language and cultural homogeneity across the territory. In contrast, within Europe, any worker is more likely to be more reticent about changing country of residence than moving within his/her own country. Notice that mobility costs may include the cost of adaptation to the new place of residence, the cost of learning the language, the emotional cost of moving, the cost of learning new job opportunities within the industry... apart from the monetary cost of the moving. Thus, there is room to reduce mobility costs by promoting the learning of foreign languages in the general population and reducing costs of searching for a job.

Implication 4. *Wages increase with the level of knowledge, the degree of generality of knowledge, the openness of the environment and the number of firms in the industry. Since most of these factors are associated with higher labor mobility too, it is expected to observe a positive correlation between the level of labor mobility and wages.*

Many authors have found that wages in industrial districts, where labor mobility is found to be higher than in the rest of the economy, are often above national averages (Triglia, 1992). With our model we show what may be driving this result. The higher level of knowledge is one important factor for observing high wages. Moreover, our model defines the real access firms have to this knowledge as another factor to take into account when analyzing wages.

6. Conclusions

We present an economy with two main features: labor mobility goes together with knowledge transfer and exchange of knowledge enhances productivity. We consider an industry with N firms and non-rivalry in the product market. Either with perfect competition in the labor market or with sequential wage-setting we find that, as long as the knowledge is industry-specific and regional laws do not enforce clauses `not-to-compete' too strongly (i.e. positive p in our model), then there will be labor mobility in equilibrium. Moreover, clustering (lower m in our model) increases labor mobility. These conditions will create higher productivity in the whole industry.

We find the following results: An increase in the ability to use external knowledge (p in our model) as well as in the level of external knowledge (k_j) translates into higher salaries to experienced workers. When firms cluster together (low m), then it is harder to retain workers and firms must pay experienced workers higher wages (ω_i). When the number of firms in the industry increases, then it is also more difficult to retain workers, so they are paid higher wage. Regarding labor mobility (λ^i), it increases with the ability to use external knowledge (p) and the level of such knowledge (k_i). Higher mobility costs (m) lead to lower labor mobility and an increase in the size of the industry has an ambiguous effect on the total labor mobility.

This labor mobility across firms could be the rationale to explain how the cooperation culture of some clusters was originated. Workers move but they keep in touch with some previous co-workers. This communication between former colleagues could be a substitute for labor mobility, or at least facilitate further the exchange of knowledge across firms. Agrawal, Cockburn and McHale (2006) give evidence in this direction.

As for policy analysis, this model is far too partial. However, it gives a clear view that locating firms together, although it enhances labor mobility, is not enough for getting high production. The type of industry-knowledge and the regional institutions, as well as the level of absorptive capacity of firms, will be critical for developing a successful industrial cluster.

The simplicity of our model makes it useful as a baseline to experiment with different hypotheses. One possibility is to introduce heterogeneity in the ability to learn of young workers. This would allow us to analyze under which assumptions more capable workers are more prone to move and compare findings with real data. Another possible extension is to construct a general equilibrium model with dynamics. This can lead to an endogenous growth model, which can be useful to check robustness of the results of endogenous growth models with exogenous spillovers. An obvious extension of this work is to contrast empirically the implications of the model.

References

Agrawal, A. K., Cockburn, I. M. and McHale, J. (2006). “Gone but not forgotten: labor ows, knowledge spillovers and enduring social capital”, Journal of Economic Geography 6(5): 571-591.

Almeida, P. and Kogut, B. (1999). “Localization of knowledge and the mobility of engineers in regional networks”, Management Science 45(7): 905-917.

Cohen, W. M. and Levinthal, D. A. (1990). “Absorptive capacity: a new perspective on learning and innovation”, Administrative Science Quarterly 35(1): 128-152.

Combes, P.-P. and Duranton, G. (2006). “Labour pooling, labour poaching and spatial clustering”, Regional Science and Urban Economics 36: 1-28.

Cooper, D. P. (2001). “Innovation and reciprocal externalities: information transmission via job mobility”, Journal of Economic Behavior and Organization 45: 403-425.

Fosfuri, A., Motta, M. and Ronde, T. (2001). “Foreign direct investment and spillovers through workers' mobility”, Journal of International Economics 53: 205-222.

Fosfuri, A. and Ronde, T. (2004). “High-tech clusters, technology spillovers and trade secret laws”, International Journal of Industrial Organization 22: 45-65.

Franco, A. M. and Filson, D. (2006). “Spin-outs: knowledge diffusion through employee mobility”, RAND Journal of Economics 37:4: 841 -860.
Gersbach, H. and Schmutzler, A. (2003). “Endogenous technological spillovers: causes and consequences”, Journal of Economics and Management Strategy 12(2): 179-205.
Gilson, R. J. (1999). “The legal infrastructure on high technology industrial districts: Silicon valley, route 128 and covenants not to compete”, New York University Law Review 74(3): 575-629.
Hyde, A. (1998). “Silicon valley's high-velocity labor market”, Bank of America, Journal of Applied Corporate Finance 11(2): 28 -37.
Jovanovic, B. and Mofitt, R. (1990). “An estimate of a sectoral model of labor mobility”, Journal of Political Economy 98(4): 827-852.
Jovanovic, B. and Nyarko, Y. (1995). “The transfer of human capital”, Journal of Economic Dynamics and Control 19: 1033 -1064.
Leonard, D. and Sensiper, S. (1998). “The role of tacit knowledge in group Innovation”, California Management Review 40(3): 112 -132.
Lucas, R. E. (1988). “On the mechanics of economic development”, Journal of Monetary Economics 22(1): 3 -42.
Moen, J. (2005). “Is mobility of technical personnel a source of r&d spillovers?”, Journal of Labor Economics 23(1): 81 -114.
Pakes, A. and Nitzan, S. (1983). “Optimum contracts for research personnel, research employment and the establishment of rival enterprises”, Journal of Labor Economics 1(4): 345 -365.
Romer, P. M. (1986). “Increasing returns and long-run growth”, The Journal of Political Economy 94(5): 1002 -1037.
Romer, P. M. (1990). “Endogenous technological change”, The Journal of Political Economy 98(5): 71 -102.

Rosen, S. (1972). “Learning and experience in the labor market”, The Journal of Human Resources 7: 326 -342.
Triglia, C. (1992). “Industrial districts and local economic regeneration”, Geneva: International Institute for Labour Studies, chapter Work and politics in the Third Italy's industrial districts.
Valetta, R. (2002). “On the move: California employment law and high-tech Development”. FRBSF Economic Letter, No.2002-24, Regional Report.

Appendix

Proof of Proposition 1

Here we prove that the economy with perfect competition in the labor market has a unique equilibrium and, moreover, it is interior.

Equation (6) determines the value that λ^{i*} takes in equilibrium:

$$\alpha(1-(N-1)\lambda^{i*})^{\alpha-1}k_i^{\alpha} = p\alpha(\lambda^{i*})^{\alpha-1}k_i^{\alpha} - m. \quad (6)$$

The LHS is increasing in λ^i with intercept αk_i^{α}. Moreover it goes to infinity at $\lambda^i = \frac{1}{N-1}$. On the other hand, the RHS is decreasing in λ^i and convex. It goes to infinity at $\lambda^i = 0$ and tends to $-m$ as λ^i goes to infinity. As shown in figure 1, the only point where these two lines cross corresponds to a positive value of λ^{i*} lower than $\frac{1}{N-1}$, that is, an interior solution.

Equation (5') specifies which is the equilibrium value for ω_i:

$$\left(\frac{\alpha k_i^{\alpha}}{\omega_i^*}\right)^{\frac{1}{1-\alpha}} = 1-(N-1)\left(\frac{p\alpha k_i^{\alpha}}{\omega_i^*+m}\right)^{\frac{1}{1-\alpha}}. \quad (5')$$

The LHS is decreasing and convex in ω_i. It has a positive asymptote at $\omega_i = 0$ and the limit as ω_i goes to infinity is zero. The RHS is increasing and concave in ω_i. It has a negative asymptote at $\omega_i = -m$ and it goes to 1 as ω_i grows to infinity.

As shown in figure 2, there is a unique solution, which corresponds to a positive wage ω_i.

Proof of Proposition 2

In order to check the effect of changes in parameters on labor mobility and wages we will use equations (6) and (5) respectively.

$$\alpha(1-(N-1)\lambda^{i*})^{\alpha-1}k_i^\alpha = p\alpha(\lambda^{i*})^{\alpha-1}k_i^\alpha - m, \quad (6)$$

$$\left(\frac{\alpha k_i^\alpha}{\omega_i^*}\right)^{\frac{1}{1-\alpha}} = 1-(N-1)\left(\frac{p\alpha k_i^\alpha}{\omega_i^*+m}\right)^{\frac{1}{1-\alpha}}. \quad (5')$$

Effects of *p* on labor mobility and wages

$$\frac{\partial LHS(6)}{\partial p} = 0,$$

$$\frac{\partial RHS(6)}{\partial p} = \alpha\lambda^{i\alpha-1}k_i^\alpha > 0.$$

An increase in p does not affect the marginal productivity of retained workers (LHS(6)), yet it increases the marginal productivity of poached workers (RHS(6)). Thus the LHS in figure 1 does not change, while the RHS shifts up. In the new equilibrium there is more labor mobility (λ^{i*}).

$$\frac{\partial LHS(5')}{\partial p} = 0,$$

$$\frac{\partial RHS(5')}{\partial p} = \frac{-(N-1)}{(1-\alpha)p}\left(\frac{p\alpha k_i^\alpha}{\omega_i+m}\right)^{\frac{1}{1-\alpha}} < 0.$$

At the same time, an increase in p shifts up the RHS in figure 2, which results in higher wage for type-i worker.

Effects of *m* on labor mobility and wages

$$\frac{\partial LHS(6)}{\partial m} = 0,$$

$$\frac{\partial RHS(6)}{\partial m} = -1.$$

An increase in m affects only the marginal productivity of poached workers (RHS(6)), negatively. Thus, in figure 1, the RHS shifts down and we obtain a lower labor mobility in the new equilibrium.

$$\frac{\partial LHS(5^{'})}{\partial m} = 0,$$

$$\frac{\partial RHS(5^{'})}{\partial m} = \frac{(N-1)}{(1-\alpha)(\omega_i + m)}\left(\frac{p\alpha k_i^{\alpha}}{\omega_i + m}\right)^{\frac{1}{1-\alpha}} > 0.$$

It also shifts up the RHS(5'), which implies a lower wages in the new equilibrium (figure 2).

Effects of *N* on labor mobility and wages

$$\frac{\partial LHS(6)}{\partial N} = \alpha(1-\alpha)(1-(N-1)\lambda^{i})^{\alpha-2}\lambda^{i}k_i^{\alpha} > 0,$$

$$\frac{\partial RHS(6)}{\partial N} = 0.$$

An increase in N affects positively the marginal productivity of retained workers (LHS(6)), while marginal productivity of poached workers (RHS(6)) remains the same. Thus, LHS shifts up in figure 1 and the new equilibrium has a lower level of labor mobility.

$$\frac{\partial LHS(5^{'})}{\partial N} = 0,$$

$$\frac{\partial RHS(5^{'})}{\partial N} = -\left(\frac{\alpha p k_i^{\alpha}}{\omega_i + m}\right)^{\frac{1}{1-\alpha}} < 0.$$

An increase in N affects only the RHS of equation (5'), shifting it down.

This gives higher wages in equilibrium.

Effects of *k* on labor mobility and wages

To analyze the effects of k on labor mobility we use equation (6). See figure 1 from the appendix to illustrate it. From equation (6) we know that an increase in k shifts the RHS to the right and the LHS to the left. Thus, in general, we should say that the total effect of this increase on the equilibrium level of labor mobility is ambiguous. Notice however that the first derivative of the RHS and the LHS with respect to k_j valued at λ^{j*} gives us by how much each curve shifts respectively at this point.

$$\left.\frac{\partial LHS}{\partial k_j}\right|_{\lambda^j=\lambda^{j*}} = \alpha^2(1-(N-1)\lambda^{j*})^{\alpha-1}k_j^{\alpha-1} > 0,$$

$$\left.\frac{\partial RHS}{\partial k_j}\right|_{\lambda^j=\lambda^{j*}} = \alpha^2(\lambda^{j*})^{\alpha-1}pk_j^{\alpha-1} > 0.$$

Thus, if $\left.\frac{\partial LHS}{\partial k_j}\right|_{\lambda^j=\lambda^{j*}} > \left.\frac{\partial RHS}{\partial k_j}\right|_{\lambda^j=\lambda^{j*}}$, then the new equilibrium level of labor mobility is lower than the original one, and *vice versa*, if $\left.\frac{\partial RHS}{\partial k_j}\right|_{\lambda^j=\lambda^{j*}} > \left.\frac{\partial LHS}{\partial k_j}\right|_{\lambda^j=\lambda^{j*}}$, then the new equilibrium level of labor mobility is higher.

Using equation (6) which holds in equilibrium, we can rewrite the derivative of the LHS with respect to k_j as

$$\left.\frac{\partial LHS}{\partial k_j}\right|_{\lambda^j=\lambda^{j*}} = \alpha k_j^{-1}\left(\alpha(\lambda^{j*})^{\alpha-1}pk_j^{\alpha} - m\right),$$

subtract it from the derivative of the RHS with respect to k_j and we obtain the necessary condition to have higher labor mobility when knowledge increases:

$$\alpha m > 0.$$

The previous condition holds whenever $m > 0$. When $m = 0$, then the level of knowledge does not affect labor mobility in equilibrium.

Next we study how k affects wages. The higher the level of knowledge, the higher the demand for experienced workers, so the higher the equilibrium wages. In equation (6) if k increases, the wage ω must increase too in order to recover the equilibrium.

Proof of Proposition 3

We want to prove here the existence and uniqueness of the sub-game perfect equilibrium of the strategic game stated in Proposition 3. We do this in three steps. First, we check that λ^{i*} exists and is unique. Second, we prove that the stage 1 problem is well-defined. Finally we check that the equilibrium wage is unique and positive.

Step 1:

From the maximization problem we obtain two equations defining the equilibrium:

$$\lambda^{i*} = \left(\frac{p\alpha k_i^{\alpha}}{\omega_i + m} \right)^{\frac{1}{1-\alpha}}, \ (8)$$

$$\frac{\lambda^{i*2-\alpha}}{(1-(N-1)\lambda^{i*})^{1-\alpha} p} = \frac{1-\alpha}{N-1} + \alpha\lambda^{i*} - \frac{m\lambda^{i*2-\alpha}}{\alpha p k_i^{\alpha}}. \ (10)$$

To check existence and uniqueness of equilibrium, first we want to analyze the LHS and the RHS of equation (10) and see that there is only one solution in the interval $\lambda^{i*} \in (0, \frac{1}{N-1})$. The range of λ^{i*} is easy to determine. To have an equilibrium we need that the total amount of workers poached from firm i is lower than supply of type i experienced workers. This means that $(N-1)\lambda^{i*} < 1$, which gives the upper-bound of the range. The lower-bound is obvious, since we can not poach a negative amount of workers.

The RHS is a concave function with intercept in $\frac{1-\alpha}{N-1}$. It starts with a positive slope at $\lambda^i = 0$, but slope is decreasing as λ^i increases, eventually getting negative.

$$\frac{\partial RHS}{\partial \lambda^i} = \alpha - \frac{(2-\alpha)m(\lambda^i)^{1-\alpha}}{\alpha p k_i^{\alpha}}.$$

$$\frac{\partial^2 RHS}{\partial \lambda^{i2}} = -\frac{(2-\alpha)(1-\alpha)m\lambda^{-\alpha}}{\alpha p k_i^{\alpha}} < 0.$$

The LHS is a convex strictly increasing function, it goes through the origin and the image at $\lambda_i^* = \frac{1}{N-1}$ is infinite. Moreover, the slope at $\lambda_i^* = 0$ is zero and at $\lambda_i^* = \frac{1}{N-1}$ is infinite.

$$\frac{\partial LHS}{\partial \lambda_i} = \frac{(2-\alpha)\lambda^{1-\alpha}(1-(N-1)\lambda_i) + (1-\alpha)(N-1)\lambda_i^{2-\alpha}}{p(1-(N-1)\lambda_i)^{2-\alpha}} > 0.$$

$$\frac{\partial^2 LHS}{\partial \lambda_i^2} = \frac{(1-\alpha)(2-\alpha)\lambda^{-\alpha}}{p(1-(N-1)\lambda)^{3-\alpha}} > 0.$$

Given this information we draw the LHS and the RHS in figure 3, which shows that they only cross once and the optimal λ_i^* belongs to the required interval.

Step 2:

Now we prove that the maximization problem in the stage 1 from the sequential wage setting model is well-defined in the relevant range of ω_i. We need to prove that the solution to the problem is a global maximum.

Let us first define the relevant range for ω_i. From equation (8) we can translate the range of λ^{i*} in terms of ω_i^*. The upper-bound for λ^i becomes a lower-bound for ω_i, while there is no upper-bound to ω_i. In particular, the relevant range for ω_i is $((N-1)^{1-\alpha}\alpha p k_i^{\alpha} - m, +\infty)$.

The first order condition, once we introduce that $\lambda_i^j = \lambda^{j*}(\omega_j)$ for all j, is

$$\frac{\partial \Pi^i}{\partial \omega_i} = \frac{\alpha\left(1-(N-1)\lambda^{i*}(\omega_i)\right)^{\alpha-1} k_i^\alpha (N-1)\lambda^{i*}(\omega_i)}{(1-\alpha)(\omega_i+m)} - 1 + (N-1)\lambda^{i*}(\omega_i) - \frac{\omega_i (N-1)\lambda^{i*}(\omega_i)}{(1-\alpha)(\omega_i+m)} = 0.$$

Let's divide this expression in two parts. We define $\frac{\partial \Pi^i}{\partial \omega_i} = LHS - RHS$, where

$$LHS = [\alpha\left(1-(N-1)\lambda^{i*}(\omega_i)\right)^{\alpha-1} k_i^\alpha - \omega_i]\frac{(N-1)\lambda^{i*}(\omega_i)}{(1-\alpha)(\omega_i+m)},$$

$$RHS = 1-(N-1)\lambda^{i*}(\omega_i).$$

The RHS is increasing and concave. It has an asymptote at $\omega = -m$ to minus infinity and it goes to 1 as ω goes to infinity.

$$\frac{\partial RHS}{\partial \omega_i} = \frac{(N-1)}{1-\alpha}\frac{\alpha p k_i^\alpha \lambda^{i*}}{(\omega_i+m)^2} > 0,$$

$$\frac{\partial^2 RHS}{\partial \omega_i^2} = \frac{-(N-1)(2-\alpha)\lambda^{i*}}{(1-\alpha)^2(\omega_i+m)^2} < 0.$$

The LHS has two asymptotes: one at $\omega = -m$ and another at $\omega = (N-1)^{1-\alpha}\alpha p k^\alpha - m$. We study only the function in the relevant range of ω, that is, on the right side of the second asymptote. For low enough values of ω the LHS is decreasing, however, as ω gets sufficiently large, the function increases. The value of the LHS at $\omega = (N-1)^{1-\alpha}\alpha p k^\alpha - m$ is $+\infty$, and the function goes to zero as ω grows to infinity. Moreover, the LHS crosses the x-axis only once.

$$\frac{\partial LHS}{\partial \omega_i} = \frac{(N-1)\lambda^{i*}[\omega_i + m(\alpha-1) + k^\alpha(1-(N-1)\lambda^{i*})^{\alpha-2}\alpha(-2+\alpha+(N-1)\lambda^{i*}]}{(m+\omega_i)^2(\alpha-1)^2}.$$

Given these characteristics, Figure 4 shows that there is only one equilibrium. Moreover, since the LHS is strictly above the RHS on the left of ω^* and strictly below the RHS on the right of this value, we can ensure that $\frac{\partial \Pi^i}{\partial \omega_i} > 0$ for all $\omega_i < \omega_i^*$ and $\frac{\partial \Pi^i}{\partial \omega_i} < 0$ for all $\omega_i < \omega_i^*$, which proves that the profit function is quasiconcave in the wage and that ω^* is the unique global maximum.

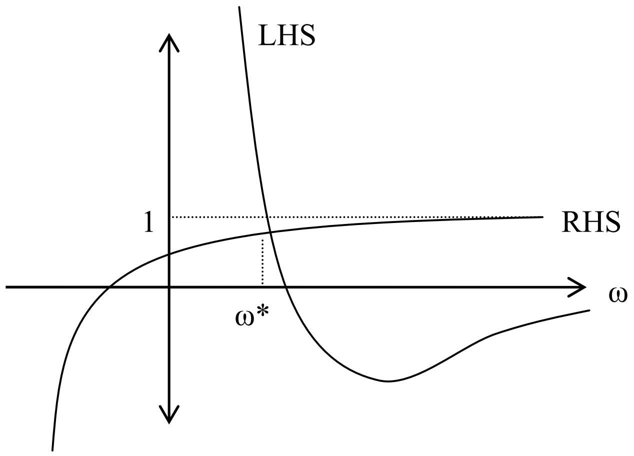

Figure 4A. Case with non-binding non-negativity of ω

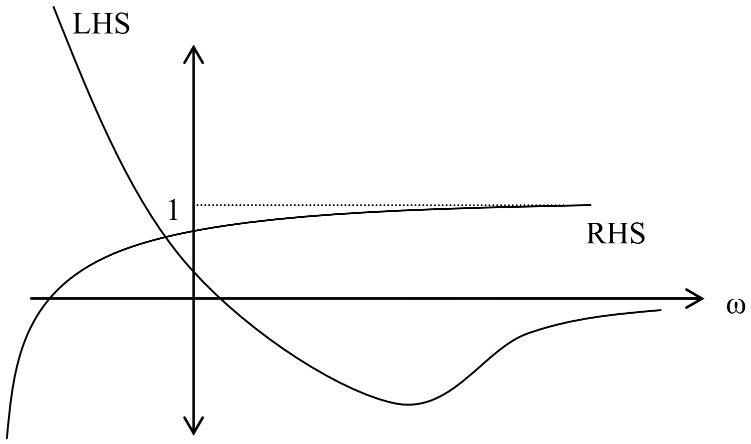

Figure 4B. Case with non-negativity of wage binding.

Figure 4. Determination of wage for the sequential game (from equations (12) and (13)).

Step 3:

Now it is only left to check that the optimal wage is non-negative.

When $\omega = (N-1)^{1-\alpha}\alpha pk^{\alpha} - m \geq 0$, then we have always a positive wage in equilibrium (Figure 4A).

When the last condition is not satisfied, we need that the $LHS(0) \geq RHS(0)$ in equation (10), that is, that the intercept of the LHS must be higher than the intercept of the RHS in order to have a non-negative wage.

$$\frac{\left(\frac{\alpha pk_i^{\alpha}}{m}\right)^{\frac{2-\alpha}{1-\alpha}}}{p\left(1-(N-1)\left(\frac{\alpha pk_i^{\alpha}}{m}\right)^{\frac{1}{1-\alpha}}\right)^{1-\alpha}} \geq \frac{1-\alpha}{N-1} + \alpha\left(\frac{\alpha pk_i^{\alpha}}{m}\right)^{\frac{1}{1-\alpha}} - \frac{m\left(\frac{\alpha pk_i^{\alpha}}{m}\right)^{\frac{2-\alpha}{1-\alpha}}}{\alpha pk_i^{\alpha}}.$$

When the non-negativity condition for wage is binding for some firm i, then the equilibrium outcome is such that $\omega_i = 0$ and $\lambda^i = \left(\frac{\alpha pk_i^{\alpha}}{m}\right)^{\frac{1}{1-\alpha}}$ (Figure 4B).

To sum up, we can find two types of equilibria:

- interior equilibrium with positive labor mobility and non-negative wage.

- corner equilibrium, where the non-negativity constraint of wage is binding for some firms. In this case, the equilibrium wage for these firms is zero, while for the other firms the optimal wage is the same as in the interior equilibrium.

Welfare analysis

In this section we investigate if labor flows occur at the efficient level in both equilibria we found. In order to address this issue, we first determine the conditions that ensure efficiency in labor mobility, which will be then compared to the equilibrium conditions. For the former, we need to solve the following problem where a social planner maximizes the total production net of mobility costs. This will determine the efficient conditions because we assumed perfect competition in the product market. In this section, we are not interested about how rents are distributed, but we want to characterize the allocation of labor that maximizes *net* production. It can be checked that the maximization problem is well-defined.

$$\max_{\lambda_i^j} \sum_{i=1}^{N} [(1-\sum_{j\neq i} \lambda_j^i)^\alpha k_i^\alpha + p\sum_{j\neq i} (\lambda_i^j k_j)^\alpha] - m\sum_{j\neq i}\sum_{i\neq j} \lambda_i^j.$$

The first order conditions for the social planner's problem are the following:

$$\alpha \lambda_i^{j*\alpha-1} pk_j^\alpha - m = \alpha(1-\sum_{s\neq j} \lambda_s^{j*})^{\alpha-1} k_j^\alpha \forall i, j. \quad (11)$$

They say that, in an efficient equilibrium, *net* marginal productivity of each type of worker must be equalized across firms. Thus, an additional worker specialized in knowledge k_i should produce m units more when he changes firms in order to cover the mobility costs. It can be proved that there is symmetry in terms of i. That is, all firms poaching from firm j will poach the same amount λ^j. Thus we can rewrite the equation as

$$\alpha \lambda^{j*\alpha-1} pk_j^\alpha - m = \alpha(1-(N-1)\lambda^{j*})^{\alpha-1} k_j^\alpha \forall j.$$

Next we present the comparison between the first order conditions from the planner's problem and the solutions from the previous models in order to check for efficiency.

Basic model (with perfect competition in the labor market)

In the case of perfect competition in the labor market, we obtain an efficient equilibrium. It is straightforward to check it since equations (6) and (14) are identical.

Sequential game model

To check if efficiency conditions hold in equilibrium, we use equations (8) and (9) from the sequential game equilibrium to get

$$\alpha \lambda^{i*\alpha-1} pk_i^\alpha = \omega_i + m,$$

$$\alpha(1-(N-1)\lambda^{i*})^{\alpha-1}k_i^{\alpha} = \omega_i - \frac{1-(N-1)\lambda^{i*}}{(N-1)\dfrac{\partial\lambda^{i*}}{\partial\omega_i}},$$

which combined give

$$\alpha(1-(N-1)\lambda^{i*})^{\alpha-1}k_i^{\alpha} = \alpha\lambda^{i*\alpha-1}pk_i^{\alpha} - m - \frac{1-(N-1)\lambda^{i*}}{(N-1)\dfrac{\partial\lambda^{i*}}{\partial\omega_i}}. \quad (12)$$

Comparing equation (14) to (15), we can say that, in general, equilibrium outcome is not efficient. Recall that $\alpha\lambda^{i\alpha-1}pk_i^{\alpha} - m$ is the net marginal productivity of a worker type i working in another firm, whereas $\alpha(1-(N-1)\lambda^{i})^{\alpha-1}k_i^{\alpha}$ is the marginal productivity of a worker type i hired by firm i. Equation (15) shows that in the sequential game equilibrium $\alpha(1-(N-1)\lambda^{i})^{\alpha-1}k_i^{\alpha} > \alpha\lambda_j^{\alpha-1}pk_i^{\alpha} - m$ because $\frac{\partial\lambda_j^*}{\partial\omega_i}$ is negative. In other words, the marginal productivity of workers type i is lower in any other firm than in firm i. Thus, we could increase total production by having a lower λ^{i*}, or what is the same, lower mobility of workers. This means that there is too much labor mobility in equilibrium. The inefficiency appears because firms have some market power on their workers since they set wages first. The result is, as in any other case of market power, that the firms set lower wages, which implies too much labor mobility.