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Can we identify Walrasian allocations?

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Abstract. We consider a discrete time, pure exchange infinite horizon economy with $n \geq 2$ consumers and $\ell \geq 1$ consumption goods per period. Within the framework of decentralized mechanisms, we show that for any given consumption trade at any period of time, say at time one, the consumers will need in general an infinite dimensional (informational) space to identify such a trade as an intertemporal Walrasian one. However, we show a set of environments where the Walrasian trades at each period of time can be achieved as the equilibrium trades of a sequence of decentralized competitive mechanisms, using only both current prices and quantities to coordinate decisions.

JEL classification: D51, D91

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1 Introduction

In this paper we consider the question concerning the information that is needed by agents to identify an intertemporal Walrasian trade allocation at a given period of time . The question is answered within the theoretical framework of decentralized resource allocation mechanisms (or processes), following the pioneering work of Hurwicz (1960), and Mount and Reiter (1974).

The economy is a discrete time infinite horizon pure exchange economy, with $n \geq 2$ consumer agents, and $\ell \geq 1$ consumption goods at each period of time.

Each agent is characterized by a positive infinite sequence of initial endowments, and a utility function (i.e. the discounted sum of the one period utility function) which is defined on the set of all uniformly bounded infinite sequences of non-negative $\ell - tuples$ of real numbers (i.e. the consumption space)

For an economy with only two agents and one good at each time, we take as a social goal the Walrasian correspondence at time one, which assigns the Walrasian trade that the two agents make at time one. This means that the allocation space is just the Cartesian plane. For this set of simple economies we show that any informationally decentralized resource allocation process that realizes such a correspondence (i.e. identifies Walrasian trades at time one) must have at least a message space as large as (i.e. with dimension equivalent to) the set of all sequences taking values in the interval $[0, 1]$, which is an infinite dimensional space. This result is formally stated in Theorem 4.3 and it can be interpreted in the following way. Suppose that we observe two agents engaged in trading at a period of time, and suppose that each of them only knows his own characteristics (i.e. his endowment sequence and utility function), which is the basic assumption of informational decentralization. The previous result implies that, from a theoretical point of view, to verify whether or not such a trade is an intertemporal Walrasian trade, we need to have a model, together with an equilibrium concept, which requires a transmission of information between each agent which involves an infinite number of linearly independent variables. Another way to interpret that result, within the framework of the traditional intertemporal Arrow-Debreu paradigm, is that at the starting date of the economy all present and future markets must neccesarily be open for all commodities in order to reach the agents an equilibrium trade for any given particular period of time. In other words, all agents need to know all equilibrium trades and prices for all periods of time to be sure that they reach an equilibrium trade at some particular time.

When the social goal is the Pareto efficient correspondence, a similar result is obtained by Hurwicz and Weinberger (1990), where it is considered a Cass-Koopmans-Ramsey economy with one consumer, one producer and one produced good per period. There, it is shown that to identify, at time one, Pareto efficient allocations it is needed a non-finite dimensional informational space. We should notice, that in the Cass-Koopmans-Ramsey economies considered by Hurwicz and Weinberger there is a unique Pareto optimal allocation which coincides with the Walrasian one. Therefore, the Hurwicz and Weinberger result holds, equivalently, both for the intertemporal Walrasian and the Pareto efficient social goal at time one. Hence, we may also consider our result as an extension of the Hurwicz and Weinberger's to pure exchange economies for the Walrasian correspondence case. However, such results are no longer true for the Pareto correspondence case, for pure exchange economies, as it is shown in Manresa (1995). In effect, to verify whether or not a trade allocation is intertemporal Pareto efficient at some period of time, agents only need to transmit a finite number of variables at that time, that is prices and quantities, which in general do not constitute an intertemporal Walrasian equilibrium.

Although the previous results hold for convex standard environments, here we can show and characterize a particular class of environments where a sequence of informationally decentralized competitive mechanisms, defined on the corresponding current period environments, and acting at each time by using current prices and quantities, yields equilibrium outcomes which are fully intertemporal Walrasian trades. Such a result allows us to identify those economies for which a lifetime budget constraint for any consumer offers the same restriction as a sequence of those constraints, each for a period of time. The result can be found in Theorem 4.4 and its Corollary, and it is similar in spirit to that obtained by Bala et al. (1991) for aggregate intertemporal production economies, with one consumer living forever and one firm at each period of time. In that paper, a sequence of mechanisms, which constitute what is called an evolutionary one, yields at each time productive efficient allocations of consumption and stocks of capital which maximizes the long run average of one period utilities from consumption, but not the discounted sum of utilities, which is the social goal of the present and Hurwicz and Weinrberger's paper.

The paper is organized as follows. In the next section we describe the economy, and we introduce basic definitions an assumptions. In Sect. 3 we present some standard concepts and results from the theory of decentralized mechanisms, while Sect. 4 contains the main results of the paper (Theorems 4.3, 4.4 and, 4.6). An Appendix gathers the proof of the results.

2 The economy, pareto optimal, individually rational, and Walrasian allocations

We consider an infinite horizon pure exchange economy starting at $t = 1$, with $\ell \geq 1$ non-storable consumption goods at each period of time, $x_t \in R^{\ell}_{+}$. There are n agents, $i = 1, 2, ..., n$, each of them characterized by an initial endowment sequence $w^i = (w^i_1, w^i_2, ..., w^i_t, ...)$, $w^i_t \in R^{\ell}_{+}$ and by a utility function

$$V^i(x^i) = \sum_{t=1}^{\infty} a^i_t \ u^i(x^i_t), \quad i = 1, 2, ..., n$$

where $x^i = (x^i_1, x^i_2, ..., x^i_t, ...)$ is a non-negative sequence of consumption goods for the i^{th} agent, $a^i = (a^i_1, a^i_2, ..., a^i_t, ...)$ is a positive summable sequence of real numbers, called discount factors, and $u^i : R^{\ell}_{+} \to R$ is his time period utility or felicity function.

The commodity space is the set of all sequences of ℓ-tuples of real numbers, denoted by $R^{\ell w}$, and the consumption space is the set of all non-negative elements of $R^{\ell w}$, denoted by $R^{\ell w}_{+}$. Both spaces are endowed with the product topology, inherited from the usual Euclidean topology on R^{ℓ}. Thus, each agent can be identified with the triple of characteristics,

$$e^i = (a^i, u^i, w^i) \in E^i, \quad i = 1, 2, ..., n$$

and E^i is considered the set of all possible characteristics for i. The set of all possible economies is $E = E^1 \times ... \times E^n$, and an economy is denoted by $e = (e^1, e^2, ..., e^n) \in E$. With each economy e we associate the *set of all feasible trade allocations* for e,

$$F(e) = \{z = (z^1, ..., z^n) \in R^{\ell w} \times \cdots \times R^{\ell w} : z^1 + ... + z^n = \widetilde{0}, \text{ and } z^i = x^i - w^i, \textit{ for some } x^i \in R_+^{\ell w}, i = 1, 2, ..., n\}$$

where $z^i = (z_1^i, z_2^i, ..., z_t^i, ...)$ is an infinite sequence of trades for the $i-th$ agent, and $\widetilde{0} = (\widetilde{0}_1, \widetilde{0}_2, ..., \widetilde{0}_t, ...)$, denotes the infinite zero sequence[1]. We will denote by $F(E)$ the set of all possible feasible net trades for some economy $e \in E$. The *Pareto interior correspondence*, denoted by $P^+ : E \to F(E)$, is defined, as usual, as:

$$P^+(e) = \{\overline{z} \in F(e) : V^i(z^i + w^i) \geq V^i(\overline{z}^i + w^i), \quad i = 1, 2, ..., n, \text{ and with strict inequality for some } i, \text{ imply } \; z = (z^1, ..., z^n) \notin F(e); \text{ and } \overline{z}^i + w^i \gg \widetilde{0}\}.$$

A trade allocation $\widehat{z} = (\widehat{z}^1, ..., \widehat{z}^n) \in F(e)$ is called *individually rational* whenever

$$V^i(\widehat{z}^i + w^i) \geq V^i(w^i), \qquad i = 1, 2, ..., n.$$

We make the following assumptions on the environments:

$(A.1)$We write $w_t^1 + ... + w_t^n = w_t, \quad t = 1, 2, ...,$ and we assume that for each i, $a < \left|w_t^i\right| < b$, where $0 < a < b < \infty, \; t = 1, 2, ...$

$(A.2)$ For each i, u^i is a continuous utility function on R_+^ℓ, twice continuously differentiable in R_{++}^ℓ, and strictly concave. Moreover, we assume that its partial derivatives[2] are strictly positive (i.e. for $x \in R_{++}^\ell$, $D_k u^i(x) > 0)$, $D_k u^i(x_t) \to \infty$ as $x_{tk} \to 0$, and $\sum_{k=1}^{\ell} D_k u^i(x) x_k < c$, for $x_k \in (0, d), \quad c, d < +\infty$.

Without changing notation we still let E be the set of economies satisfying $(A.1)$ and $(A.2)$. Under this assumptions it is known (see Kehoe and Levine 1985) that $\overline{z} = (\overline{z}^1, ..., \overline{z}^n) \in P^+(e)$ if and only if there exist some real numbers $\alpha^i > 0$, $i = 1, 2, ..., n$ such that $\{\overline{z}^1 + w^1, ..., \overline{z}^n + w^n\}$ solves the program:

$$\begin{aligned} & Max\{\alpha^1 V^1(z^1 + w^1) + ... + \alpha^n V^n(z^n + w^n)\} \\ s.t. \quad & (i) \quad z_t^i + w_t^i \geq 0, \quad i = 1, 2, ...n; \quad t = 1, 2, ..., \\ & (ii) \quad z_t^1 + ... + z_t^n = \widetilde{0}_t, \quad t = 1, 2, ... \end{aligned}$$

[1] Given $x, x' \in R^{\ell w}$, we define as ussual, $x'' = x + x'$ if and only if $x_t'' = x_t + x_t'$ for all t. We also define the following inequalities: $x \geq x' \Leftrightarrow x_t \geq x_t'$ for all t,

$x > x' \Leftrightarrow x_t \geq x_t'$ and $x \neq x'$.

$x \gg x' \Leftrightarrow x_t > x_t'$ all t.

[2] The partial derivative of $u(\cdot)$ at x with respect to the $k - variable$ is denoted by $D_k u(x)$, while the vector of partial derivatives of $u(\cdot)$ at x is denoted by $Du(x)$.

Furthermore, a set of necessary and sufficient conditions satisfied by any interior solution to the previous program is:

$(P.1)$ $\alpha^i a_t^i Du^i(z_t^i + w_t^i) = q_t$, for some $q_t > 0, t = 1, 2, ..; i = 1, ..., n$;
and
$(P.2)$ (i) and (ii).

Given $e \in E$, we define an *interior Walrasian Competitive Equilibrium with perfect foresight* for e as the pair of prices and net trade allocations $(\overline{p}, \overline{z})$, where $\overline{p} = (\overline{p}_1, \overline{p}_2, ..., \overline{p}_t, ...) > \widetilde{0}$, and $\overline{z} = (\overline{z}^1, ..., \overline{z}^n)$, such that:

$[D.1]$ For each $i = 1, 2, ...n$, $\overline{z}^i + w^i$ solves the following program:

$$\begin{array}{rc} & \max \quad V^i(z^i + w^i) \\ s.t. & \sum_{t=1}^{\infty} \overline{p}_t(z_t^i + w_t^i) = \sum_{t=1}^{\infty} \overline{p}_t w_t^i \\ & z_t^i + w_t^i \geq 0, \quad t = 1, 2, ..., \end{array}$$

$[D.2]$ Feasibility condition:

$$\sum_{i=1}^{n} \overline{z}_t^i = \widetilde{0}_t, \quad t = 1, 2, ...,$$

Alternatively, we may say that $(\overline{p}, \overline{z})$ constitutes an interior Walrasian Competitive Equilibrium with perfect foresight for e whenever $\overline{z} \in P^+(e)$, and

$(P.3)$ $\sum_{t=1}^{\infty} \overline{p}_t \overline{z}_t^i = 0 \quad i = 1, ..., n$.

Notice that if we identify $q_t = p_t$, conditions $(P.1)$ and $(P.3)$ are the usual first order conditions derived from the maximization of the $i-th$ consumer utility function under his budget constraint, and $1/\alpha^i$ is the Lagrange multiplier of such a maximization program. Conditions (i) and (ii) are respectively the non-negativity condition for consumption allocations and market clearing at each time. We denote by

$$W : E \to F(E)$$

the Walrasian correspondence, which associates to each economy e the set of Walrasian trade allocations for e, $W(e) \subseteq F(e)$. Furthermore, we denote by

$$W_t : E \to F_t(E)$$

the Walrasian trade allocations at time $t = 1, 2, ...,$ where $F_t(E) \subseteq R^{\ell}$ is the set of all feasible allocations at time $t = 1, 2, ...,$ for some economy $e \in E$.

3 Informationally decentralized mechanisms

In this section we state, for the case of two agents[3], some well known definitions and results from the classical theory of decentralization (see Hurwicz, 1986, or Aizpurua and Manresa, 1993, for a proof of the Theorem 3.2 and Lemma 3.1 stated in this section).

Following the Mount and Reiter 1984 framework, we define a *mechanism*, or a *process*, on E as a triple $\pi =< M, \mu, h >$ where M is the *message set*, $\mu : E \to M$ is the *equilibrium correspondence*, and $h : M \to F(E)$ is the *outcome function.*

The property of informational decentralization for a mechanism is a very important one for this theory. It says that every agent only knows *a priori* her/his own characteristics. So, agents should make decisions based only on the information transmitted by other agents, through the message set, and the knowledge they have about their own characteristics. A formalization of this property is given by the following definition.

We say that π is an *informationally decentralized mechanism* if there exist some individual equilibrium correspondences, $\mu^i : E^i \to M$, such that

$$\mu(e) = \mu^1(e^1) \cap \mu^2(e^2) \text{ for all } e \in E.$$

Let $H : E \to F(E)$ be a performance correspondence. We say that the mechanism π *realizes* H over E whenever $\phi \neq h(\mu(e)) \subseteq H(e)$ for all $e \in E$, where ϕ is the empty set.

Given a performance standard, a traditional question that this theory has been asking is to determine the minimal informational size of the message set of any decentralized mechanism that realizes such a performance correspondence.

The theory defines a partial order among all possible message sets of a mechanism, as a relative measure of their informational size, by adopting the concept of the *Fréchet ordering* for topological spaces (defined below), which corresponds to the dimensionality of the space, when such dimension exists. Theorem 3.1 gives us a general statement in order to find a lower bound for the size of the message set of a mechanism that realizes a given performance. It turns out that a critical step to apply such a Theorem to a particular case consist, first in finding a certain subset of the set of economies (called *with the uniqueness property*), where the performance function behaves in some particular way, and then use the result called the *single valuedness lemma*. In what follows we give precise statements for our previous informal discussion of the theory.

We say that $E^* \subseteq E$ *has the uniqueness property* with respect to H, whenever $\overline{e}, \widetilde{e} \in E^*$,

$$H(\overline{e}) \cap H(\overline{e}^1, \widetilde{e}^2) \cap H(\widetilde{e}^1, \overline{e}^2) \cap H(\widetilde{e}) \neq \phi, \text{ imply } \overline{e} = \widetilde{e}.$$

Lemma 3.1. *(The single-valuedness lemma).*

[3] For the case of n agents, see Hurwicz(1986).

Let $\pi =< M, \mu, h >$ be an informationally decentralized mechanism, which realizes H over E. Let $E^* \subseteq E$ have the uniqueness property with respect to H. Then, the restriction of μ to E^*, that is $\mu|_{E^*}$, is an *injective correspondence* (i.e. $\mu|_{E^*}(\overline{e}) \cap \mu|_{E^*}(\widetilde{e}) = \phi$ for all $\overline{e} \neq \widetilde{e}, \quad \widetilde{e},\overline{e} \in E^*$).

Let E and M be topological spaces. A *mechanism* π *is called regular* if $\mu : E' \to M$ is a *spot threaded correspondence* (i.e. there is some open set $E' \subseteq E$ and some continuous function $f : E' \to M$ such that $f(e) \in \mu(e)$ for all $e \in E'$).

A *topological space* T is said to have the *weak local similarity property* at an open subset U of T whenever there exist some $V \subseteq U$ which, in the relative topology, is homeomorphic to T. We say that T has the *strong local similarity property at* U whenever every open subset $U' \subseteq U$ has a subset $V' \subseteq U'$ which, in the relative topology, is homeomorphic to T.

Let M_1 and M_2 be *two topological spaces*. We say that M_1 *has as much information as* M_2 whenever there exists a subspace M_1' of M_1 which is homeomorphic to M_2. This definition corresponds to the so called *Fréchet ordering* for topological spaces, written $M_1 \geq_F M_2$.

Theorem 3.2. *Let E and M be topological spaces, and let $\pi = < M, \mu, h >$ be an informationally decentralized mechanism realizing $H : E \to A$ over E.Let $E^* \subseteq E$ be a subspace of E having the uniqueness property with respect to H. Let μ be spot-threaded with spot domain at the open subset $U \subseteq E^*$. Then $M \geq_F E^*$ if either of the following two conditions is satisfied:*

(a) *M and E^* are Hausdorff spaces, E^* is locally compact and has the strong local similarity property at U.*
(b) *$[\mu|_{E^*}]^{-1}$ is continuous on $\mu(U)$ and E^* has the weak local similarity property at U.*

4 Results

In this section we present the results of the paper. We consider first a particular class of economies satisfying $(A.1)$ and $(A.2)$, with two agents and one consumer good at each period. We investigate some properties of the Walrasian correspondence, the Pareto efficient, and the individually rational correspondences at time one. That is the allocation that each agent receives at the first period of time. This means that we restrict the allocation space of those correspondences to the plane. Formally, we let[4]

$$W_1 : E^1 \times E^2 \to F_1(E)$$

denote the Walrasian correspondence at period one, where $W_1(e)$ is the Walrasian allocation for e at time one, and $F_1(E)$ denotes the set of all feasible trade allocations at time one.

[4] See definition of F_t and W_t on p. 61.

The particular class of economies that we consider are those satisfying (A.1) and (A.2), where

$$u^i(x) = \ln x, \quad i = 1,2; \quad \text{and } w_t^1 + w_t^2 = 1, \quad t = 1,2,\ldots$$

This means that each agent $i = 1,2$ is now characterized only by the pair $e^i = (a^i, w^i)$.We now define a class of environments for which the unique Pareto optimal and individually rational allocation is the no trade allocation sequence, that is $z = (\widetilde{0}, \widetilde{0})$.

Let $\overline{\alpha}^1 > 0$, and $\overline{\alpha}^2 > 0$ be given, and for each $e^1 = (a^1, w^1) \in E^1$ consider the mapping

$$\Lambda : E^1 \to E^2 \qquad \text{defined by}$$

$$\Lambda(e^1) = \begin{cases} \frac{\overline{\alpha}^1}{\overline{\alpha}^2} \frac{(1-w_t^1)}{w_t^1} a_t^1 = a_t^2, & t = 1,2,\ldots \\ 1 - w_t^1 = w_t^2, & t = 1,2,\ldots \end{cases}$$

We define $\overline{E} = \overline{E}^1 \times \overline{E}^2 = \{(e^1, e^2) \in E^1 \times E^2 : e^2 = \Lambda(e^1)\}$.

Lemma 4.1. *The unique allocation that is Pareto optimal and individually rational for each $e \in \overline{E}$ is the zero net trade.*

Proof. (See Appendix).

We now define a subclass of environments which has the uniqueness property with respect to W_1. Let a^* be a given sequence of discount factors (i.e., $a_t^* > 0$, and $\sum a_t^* < +\infty$), and let $\overline{w} \in R_+$ be such that $0 < \overline{w} < 1$. Let E^* be defined as :

$$E^* = \left\{(e^1, e^2) \in \overline{E} : a_t^1 = a_t^*, \quad t = 1,2,\ldots, \text{ and } w_t^1 = \overline{w} \text{ for } t = 1\right\} .$$

Hence, we are taking those economies in $\overline{E}$ where the first agent has a fixed sequence of discount factors, and a fixed endowment only at time $t = 1$. It is very easy to see that E^* can be taken as the set of all sequences with values in the interval $[a, b]$, with their first element being the constant $\overline{w}$.

Lemma 4.2. *The set E^* has the uniqueness property with respect to the performance correspondence W_1.*

Proof. (See Appendix).

We now state the first result of this paper, Theorem 4.3, which can be interpreted in the following way. Consider a pure exchange economy with two agents living forever and one good per period. Suppose that at some period of time, say $t = 1$, they make some exchanges. What our next result shows is that to identify such a trade as an intertemporal Walrasian trade, for the two agents, the information that they will have to transmit involves an infinite number of linearly independent variables, provided that a priori each agent only knows his own characteristics.

Theorem 4.3. *Let $\pi =< M, \mu, h >$ be a regular informationally decentralized mechanism, which realizes $W_1 : E^1 \times E^2 \to F_1(E)$. If μ has spot domain at an open subset $U \subseteq E^*$, and either: (a) M is a Hausdorff space, or (b) $[\mu|_{E^*}]^{-1}$ is a continuous function on $\mu[U]$, then $M \geq_F [a, b]^{z+}$, ($[a, b]^{z+}$ is the space of all real sequences taking values in $[a, b]$, with the product topology).*

Proof. (See Appendix).

Consider the subset of economies $\widehat{E} \subseteq E$ where each agent $i = 1, 2, ..., n$, has characteristics $\widehat{e} = (\widehat{a}^i, \widehat{u}^i, \widehat{w}^i) \in \widehat{E}^i$ with the following properties:

$(A.3)$ $\widehat{u}^i : R^\ell_+ \to R$ satisfies that for each $x \in R^\ell_{++}$ we have $\sum_{k=1}^{\ell} D_k \widehat{u}^i(x) \cdot x_k = c^i$ for some $c^i > 0$.

$(A.4)$ Given any bounded sequence of total endowments for the economy, $\widehat{w} = (\widehat{w}_1, \widehat{w}_2, ..., \widehat{w}_t, ...)$ with $\widehat{w}_t \in R^\ell_{++}$, we assume that for each $t = 1, 2, ...,$ agents' endowments satisfies:

$$\widehat{w}^i_t = s^i w_t, \text{ with } 0 < s^i < 1 \text{ and } \sum_{i=1}^{n} s^i = 1 .$$

$(A.5)$ Discount factors are normalized so that $\sum_{t=1}^{\infty} \hat{a}^i_t = 1$.

The last assumption, $(A.5)$, is really innocuous, since it does not change agents' preferences at all. The assumption $(A.4)$ says that every agent has a constant share of the total endowment of the economy at every period of time. Condition $(A.3)$ is satisfied in general by the following family of utility functions:

$$\widehat{u}^i(x) = c^i \ln x_1 + \Psi^i(\ln \frac{x_1}{x_2}, \ln \frac{x_1}{x_3}, ..., \ln \frac{x_1}{x_\ell}) + const.$$

where $\Psi^i : R^{\ell-1}_+ \to R$ is an arbitrary twice continuously differentiable function. It should be noted that when there is only one good at each period of time, the unique, up to a constant, utility function that satisfies condition $(A.3)$ is the ln function. As we will see at the end of this section, both $(A.3)$ and $(A.4)$ are key conditions for the next Theorem 4.4 be true.

For the set of economies, called $\widehat{E}$, that we have just described, we can state the following result.

Theorem 4.4. *Let $\widehat{e} = (\widehat{e}^1, ..., \widehat{e}^n) \in \widehat{E}^1 \times \cdots \times \widehat{E}^n$ be an economy satisfying $(A.1) - (A.5)$. Suppose that for each $t = 1, 2, ...,$ we have that $(\overline{p}_t, \overline{z}_t)$, $\overline{p}_t \in R^\ell_{++}$ and $\overline{z}_t = (\overline{z}^1_t, ..., \overline{z}^n_t) \in R^{\ell n}$ satisfies the following conditions:*

$[D_t.1]$ for each i,$(\overline{z}^i_t + \widehat{w}^i_t) \geq 0$ solves the program:

$$\begin{array}{ll} \max & \widehat{a}^i_t\, \widehat{u}^i(x^i_t) \\ s.t. & \overline{p}_t x^i_t = \widehat{a}^i_t s^i \\ x^i_t \geq 0, & t = 1, 2, ..., \end{array}$$

[D.2] Feasibility condition:

$$\sum_{i=1}^{n} \overline{z}_t^i = 0, \quad t = 1, 2, ...,$$

Then $(\overline{p}_t, \overline{z}_t)_{t=1}^{\infty} \equiv (\overline{p}, \overline{z})$ constitutes a *Walrasian Competitive Equilibrium with perfect foresight* for the economy $\widehat{e}$.

Proof. (See Appendix).

The previous result tells us that it is possible to achieve a *Walrasian Competitive Equilibrium with perfect foresight* by means of period by period competitive equilibrium as long as we restrict our environment to the subset $\widehat{E}$ of economies. Within these economies now every agent can identify a Walrasian trade allocation at each time t, say $\overline{z}_t$, by looking at the feasibility condition [D.2], and by checking if his t consumption allocation, $(\overline{z}_t^i + \widehat{w}_t^i)$, solves his utility maximization problem at t, that is condition [D_t.1]. Such a condition says that agent i chooses his $t-$consumption bundle at time t by maximizing his $t-$utility function, $\widehat{a}_t u^i(x_t^i)$, subject to his budget constraint at time t.The income that the i^{th} agent spent at each time is a fraction (by assumption (A.5)), $\widehat{a}_t^i$,of the total income that the agents has along his entire life, which is, by assumption (A.4), s^i,since prices are normalized so that $\sum_{t=1}^{\infty} p_t \widehat{w}_t = 1$.

It should also be pointed out that the consumers problem at t only depends upon each agents characteristics at time t. Hence, we may say that the period by period competitive price mechanism, just described, realizes the Walrasian correspondence. We can formalize this statement within the mechanism design framework by using some definitions.

For each t, let $\pi_t = < M_t, \mu_t, h_t >$ be the $t-$ *period competitive mechanism*, defined as follows.

The message space is:

$M_t = \{((z_t^1, z_t^2, ..., z_t^n),\ p_t) \in R^{\ell n} \times R_{++}^{\ell} : \sum_{i=1}^{n} z_t^i = 0\}$, and so messages are $m_t = (z_t, p_t) \in M_t$.

Let the *equilibrium correspondence* $\mu_t : E_t \to M_t$ be $\mu_t(e) = \bigcap_{i=1}^{n} \mu_t^i(e_t^i)$, where:

$\mu_t^i(e_t^i) = \{m_t \in M_t : (\overline{z}_t^i + \overline{w}_t^i)$ is a solution to the $i-th$ consumer problem [D_t.1] at prices $p_t\}$.

For each $\overline{m} \in \mu_t(e)$, which is an equilibrium message for the mechanism, we let $h_t(\overline{m}) = \overline{z}_t$.

Once we have defined π_t we state without proving the following result, which is a corollary to the previous theorem.

Corollary 4.5. *The informationally decentralized mechanism* $\pi_t =< M_t, \mu_t, h_t >$, $t = 1, 2, ...,$ *realizes the Walrasian correspondence at each time* t :

$$W_t : E \to F_t(E)$$

on the environments $E = \widehat{E}$. *In other words, for each* $\widehat{e} \in \widehat{E}$,

$$h_t(\mu_t(\widehat{e}_t)) \in W_t(\widehat{e}), \quad t = 1, 2, ...,$$

Our next result tell us how important are our assumptions $(A.3)$ *and* $(A.4)$ *in Theorem 4.4.*

Theorem 4.6. *Let* $\widehat{e} = (\widehat{e}^1, \widehat{e}^2, ..., \widehat{e}^n) \in E^1 \times E^2 \times ... \times E^n$ *be an economy, satisfying* $(A.2)$ *and* $(A.5)$,*and let* $(\overline{p}_t, \overline{z}_t)_{t=1}^{\infty}$*be an interior Walrasian Competitive Equilibrium with perfect foresight for* e.

Then, we have that for each $i = 1, 2, ..., n$ and $t = 1, 2, ...,$ $\overline{x}_t^i = (\overline{z}_t^i + w_t^i) \geq 0$ is a solution to the program $[\overline{D}_t.1]$:

$$\begin{array}{c} Max \quad \widehat{a}_t^i \ \widehat{u}^i(x_t^i) \\ s.t. \quad \overline{p}_t x_t^i = \overline{I}_t^i \\ x_t^i \geq 0, \quad t = 1, 2, ..., \end{array}$$

where $(A.4')$ $\overline{I}_t^i = \widehat{a}_t^i \ \overline{s}^i$and $\overline{s}^i = (\sum_{t=1}^{\infty} \overline{p}_t w_t^i)/(\sum_{t=1}^{\infty} \overline{p}_t w_t)$,

if and only if:

$(A.3')$ $\sum_{k=1}^{\ell} D_k \widehat{u}^i(\overline{x}_t^i) \cdot \overline{x}_t^i = \overline{c}^i$ for all $t = 1, 2, 3, ...$and some $\overline{c}^i > 0$.

Proof. (See Appendix).

The "only if" part of the Theorem 4.6 says that if any given Walrasian Competitive Equilibrium for a given economy can be achieved through a period by period consumers maximization program like $[D_t.1]$ in Theorem 4.4, then it has to be in economies where $(A.3)$ is satisfied at equilibrium, that is what condition $(A.3')$ in Theorem 4.6 says. We should say that $\overline{c}^i$ may vary from one equilibrium to another in economies with more than one. From this point of view we say that $(A.3)$ characterizes the economies where Theorem 4.4 holds.

The "if" part of the result can be seen as the converse of Theorem 4.4, and it says that any Walrasian Competitive Equilibrium for an economy satisfying $(A.3')$ must necessarily solve a period by period consumer program like $[D_t.1]$. We should point out that $\overline{s}^i$,in Theorem 4.6, is the total amount of income that agent i receives at the Walrasian Competitive Equilibrium, once we normalize to one unit the equilibrium value of the total income of the economy. We should here stress the fact that in general any consumer i will need to know the entire infinite sequence of equilibrium prices in order to know the value of $\overline{s}^i$. However in economic environments, where $(A.4)$ is satisfied, the value of $\overline{s}^i$ is always known at any period of time and it is independent of the Walrasian equilibrium prices for the economy. This is why assumption (A.4) is important for Theorem 4.4.

It is also important to note that the class of economies that we consider in Theorem 4.2 satisfy (A.3) but they don't satisfy (A.4) when we consider the "cross-economies" $(\overline{e}, \widetilde{e})$.This also illustrates the importance of assumption (A.4).

5 Conclusions

In this paper we have shown, under the hypothesis of informational decentralization, that for infinite horizon pure exchange economies it is not possible, except for certain sets of economies that we characterize, to identify a walrasian equilibrium trade at a given period of time unless the agents have an infinite dimensional information set.

The interpretation of this result, in the classical intertemporal framework of an Arrow-Debreu economy, is that all agents need to know that all future markets are at equilibrium, or equivalently they must have perfect foresight about all equilibrium prices, in order to be sure that any particular present trading is an equilibrium trade.

An implication of this informational result is just to point out the lack of realism of such an equilibrium concept from the positive point of view of a theory that aims at explaining some stylized facts. In fact this and other kind of criticisms of that concept (see for instance Starr 1987) is what justifies, among others reasons, the importance of other types of equilibrium notions such as, for instance, that of Temporary equilibrium, developed by Grandmont and, from a more dynamic perspective, the Radner equilibrium. Such equilibrium concepts lower the information requirements of agents, at any point in time, to a finite set of variables but from a normative point of view it is difficult to recommend those concepts. One way to proceed from this situation is, as we do here, by setting a performance standard social welfare correspondence and look for a decentralized mechanism which realizes such an objective in a set of environments and with a finite number of variables at each equilibrium point of time. That is the kind of result that we obtain in Theorems 4.4 and 4.6, where we show a set of environments and a decentralized mechanism which realizes intertemporal Walrasian outcomes by means of a sequence of equilibria corresponding to a sequence of economies and where agents only use a finite number of variables. I believe that this approach can give us some method and discipline to evaluate dynamic equilibrium notions both from a normative point of view and also from a positive one.

Appendix

Proof of Lemma 4.1. Let $e \in \overline{E}$ be given. Let $\widehat{z} = (\widehat{z}^1, \widehat{z}^2) \in P^+(e)$ and individually rational. Hence, it follows from condition (P.1) that

$$\frac{\widehat{\alpha}^1 a_t^1}{w_t^1 + \widehat{z}_t^1} = \frac{\widehat{\alpha}^2 a_t^2}{w_t^2 + \widehat{z}_t^2}, \quad t = 1, 2, 3, \ldots \tag{1}$$

for some $\widehat{\alpha}^1, \widehat{\alpha}^2 > 0$. Let $\beta > 0$ be such that $\widehat{\alpha} = \widehat{\alpha}^2/\widehat{\alpha}^1 = \overline{\alpha} \cdot \beta$, *where* $\overline{\alpha} = \overline{\alpha}^2/\overline{\alpha}^1$ is the ratio of $\overline{\alpha}^1 > 0$, and $\overline{\alpha}^2 > 0$,which we used in the definition of $\overline{E}$.Then (1) becomes:

$$\frac{\dfrac{a_t^1}{w_t^1 + \widehat{z}_t^1}}{\dfrac{a_t^2}{w_t^2 + \widehat{z}_t^2}} = \overline{\alpha} \cdot \beta = \beta \cdot \frac{\dfrac{a_t^1}{w_t^1}}{\dfrac{a_t^2}{w_t^2}},$$

therefore, we obtain:

$$\frac{w_t^2 + \widehat{z}_t^2}{w_t^1 + \widehat{z}_t^1} = \beta \frac{w_t^2}{w_t^1},$$

and substituting $\widehat{z}_t^2 = -\widehat{z}_t^1$ in the last expression, we obtain that

$$\widehat{z}_t^1 = \frac{w_t^2 \cdot w_t^1 (1 - \beta)}{(\beta w_t^2 + w_t^1)} \ . \tag{2}$$

If $\beta < 1$, we have from (2) that $\widehat{z}_t^1 > 0$ for all $t = 1, 2, \ldots.$, and the individually rational condition is violated. Similarly, if $\beta > 1$, then $\widehat{z}_t^1 < 0$, leading condition (1) to a contradiction. Therefore, $\beta = 1$, which implies that $\widehat{z}_t^1 = \widehat{z}_t^2 = 0$ for all t. Clearly, $\widehat{z} = (\widetilde{0}, \widetilde{0})$ is individually rational, and it is a Pareto Optimal allocation for e, since we may take

$$q_t = \frac{\overline{\alpha}^1 a_t^1}{w_t^1} = \frac{\overline{\alpha}^2 a_t^2}{w_t^2}$$

□

Proof of Lemma 4.2. Let $\overline{e} = (\overline{e}^1, \overline{e}^2) \in E^*$, and let $\widetilde{e} = (\widetilde{e}^1, \widetilde{e}^2) \in E^*$. Assume that there exists some $z_1 = (z_1^1, z_1^2)$ such that

$$z_1 \in W_1(\overline{e}^1, \overline{e}^2) \tag{3}$$

$$z_1 \in W_1(\widetilde{e}^1, \widetilde{e}^2) \tag{4}$$

$$z_1 \in W_1(\overline{e}^1, \widetilde{e}^2) \tag{5}$$

$$z_1 \in W_1(\widetilde{e}^1, \overline{e}^2) \ . \tag{6}$$

We have to show that $\widetilde{e} = \overline{e}$. Since every walrasian allocation is Pareto Optimal and individually rational, by Lemma 4.1 we have that $z_1 = (0, 0)$, since $\widetilde{e}, \overline{e} \in \overline{E}$.

For any given economy $e \in E$, the walrasian allocation for each t is given by the following expressions

$$z_t^1 = \left[\frac{1}{1 + \dfrac{\alpha^2 a_t^2}{\alpha^1 a_t^1}} \right] - w_t^1 \tag{7}$$

$$z_t^2 = \left[\frac{1}{1 + \dfrac{\alpha^2 a_t^1}{\alpha^1 a_t^2}}\right] - w_t^2 \tag{8}$$

where $\alpha^2/\alpha^1 = \sum_t w_t^1 a_t^2 / \sum_t w_t^2 a_t^1$.

In particular, we consider (7) at $t = 1$, and for the particular economies we are dealing with, and taking into account that $z_1 = (0, 0)$, we obtain the following expressions from (3) through (6) respectively,

$$\frac{\overline{a}_1^1}{\overline{w}_1^1} = \left[\frac{\sum_t \overline{w}_t^2 \overline{a}_t^1}{\sum_t \overline{w}_t^1 \overline{a}_t^2}\right] \cdot \frac{\overline{a}_1^2}{\overline{w}_1^2} \tag{9}$$

$$\frac{\widetilde{a}_1^1}{\widetilde{w}_1^1} = \left[\frac{\sum_t \widetilde{w}_t^2 \widetilde{a}_t^1}{\sum_t \widetilde{w}_t^1 \widetilde{a}_t^2}\right] \cdot \frac{\widetilde{a}_1^2}{\widetilde{w}_1^2} \tag{10}$$

$$\frac{\overline{a}_1^1}{\widetilde{w}_1^1} = \left[\frac{\sum_t \widetilde{w}_t^2 \overline{a}_t^1}{\sum_t \overline{w}_t^1 \widetilde{a}_t^2}\right] \cdot \frac{\widetilde{a}_1^2}{\widetilde{w}_1^2} \tag{11}$$

$$\frac{\overline{a}_1^1}{\overline{w}_1^1} = \left[\frac{\sum_t \widetilde{w}_t^2 \overline{a}_t^1}{\sum_t \overline{w}_t^1 \widetilde{a}_t^2}\right] \cdot \frac{\widetilde{a}_1^2}{\widetilde{w}_1^2} \ . \tag{12}$$

Since $\overline{\alpha} = \overline{\alpha}^2/\overline{\alpha}^1$ is fixed and given by the definition of E^*, and $\overline{a}_1^1 = a_1^*$, and $w_1^1 = \overline{w}$,we have that $\widetilde{a}_1^2 = \overline{a}_1^2$, and $\widetilde{w}_1^2 = \overline{w}_1^2$. Therefore, from (9) and (10) we obtain:

$$\frac{\sum_t \overline{w}_t^2 \overline{a}_t^1}{\sum_t \overline{w}_t^1 \overline{a}_t^2} = \frac{\sum_t \widetilde{w}_t^2 \widetilde{a}_t^1}{\sum_t \widetilde{w}_t^1 \widetilde{a}_t^2} \ , \tag{13}$$

and from (9) and (11) we obtain:

$$\frac{\sum_t \overline{w}_t^2 \overline{a}_t^1}{\sum_t \overline{w}_t^1 \overline{a}_t^2} = \frac{\sum_t \widetilde{w}_t^2 \overline{a}_t^1}{\sum_t \overline{w}_t^1 \widetilde{a}_t^2} \ , \tag{14}$$

and from (10) and (11) we obtain:

$$\frac{\sum_t \widetilde{w}_t^2 \widetilde{a}_t^1}{\sum_t \widetilde{w}_t^1 \widetilde{a}_t^2} = \frac{\sum_t \overline{w}_t^2 \widetilde{a}_t^1}{\sum_t \widetilde{w}_t^1 \overline{a}_t^2} \ . \tag{15}$$

Taking into account that $\overline{a}_t^1 = \widetilde{a}_t^1 = a_t^*$ for all $t = 1, 2, \ldots$, it follows from (13) and (14) that

$$\sum_t \overline{w}_t^1 \widetilde{a}_t^2 = \sum_t \widetilde{w}_t^1 \widetilde{a}_t^2 \ , \tag{16}$$

and combining (13) and (15) we also obtain:

$$\sum_t \overline{w}_t^1 \overline{a}_t^2 = \sum_t \widetilde{w}_t^1 \overline{a}_t^2 \ . \tag{17}$$

From (16) and (17) we can derive:

$$\sum_t \overline{w}_t^1 (\widetilde{a}_t^2 - \overline{a}_t^2) = \sum_t \widetilde{w}_t^1 (\widetilde{a}_t^2 - \overline{a}_t^2)$$

from which we can obtain

$$\sum_t (\overline{w}_t^1 - \widetilde{w}_t^1)(\widetilde{a}_t^2 - \overline{a}_t^2) = 0\ . \tag{18}$$

Since,

$$\widetilde{a}_t^2 = 1/\overline{\alpha} \cdot (\widetilde{w}_t^2/\widetilde{w}_t^1)a_t^*, \quad \text{and} \tag{19}$$

$$\overline{a}_t^2 = 1/\overline{\alpha} \cdot (\overline{w}_t^2/\overline{w}_t^1)a_t^*, \tag{20}$$

we compute

$$\widetilde{a}_t^2 - \overline{a}_t^2 = \frac{1}{\overline{\alpha}} \frac{1}{\widetilde{w}_t^1 \overline{w}_t^1} (\overline{w}_t^1 - \widetilde{w}_t^1) a_t^*$$

which can be substituted into (18) to obtain:

$$\sum_t [\overline{w}_t^1 - \widetilde{w}_t^1]^2 \cdot \frac{a_t^*}{\overline{\alpha}\widetilde{w}_t^1 \overline{w}_t^1} = 0$$

which implies $\overline{w}_t^1 = \widetilde{w}_t^1$ for all t, and consequently $\overline{w}_t^2 = \widetilde{w}_t^2$. From (13) and (14) we conclude that $\overline{a}_t^2 = \widetilde{a}_t^2$ for all t. □

Proof of Theorem 4.3. We apply Theorem 3.1. Since $\overline{E}^*$ has the uniqueness property with respect W_1 and π is informationally decentralized and realizes W_1, we have by the *single valuedness lemma* that $\mu|_{\overline{E}^*} : \overline{E}^* \to M$ is an injective correspondence, and hence, $[\mu|_{\overline{E}^*}]^{-1} : \mu(\overline{E}^*) \to \overline{E}^*$ is a function. By Proposition 1, in Aizpurua and Manresa 1993, we have that $[a, b]^{Z_+}$, that is the set of all infinite sequences taking values in $[a, b]$, is a *Hausdorff space,* is locally compact and it has the similarity property. A simple application of Theorem 3.1, allows us to conclude the proof of this one. □

Proof of Theorem 4.4. For any given $i = 1, 2, \ldots, n$, and for each $t = 1, 2, 3, \ldots$ *let* $\overline{x}_t^i = \overline{z}_t^i + \widehat{w}_t^i$ be a solution to the program $[D_t.1]$:

$$\begin{array}{ll} & \max \quad \widehat{a}_t^i\, \widehat{u}^i(x_t^i) \\ s.t. & \overline{p}_t x_t^i = s^i \widehat{a}_t^i \quad t = 1, 2, \ldots, \\ & x_t^i \geq 0 \end{array}$$

we only have to show that $\{\overline{x}_t^i\}_{t=1}^{\infty}$ is a solution to the program $[D.1]$:

$$\begin{array}{ll} & \max \quad \sum_{t=1}^{\infty} \widehat{a}_t^i\, \widehat{u}^i(x_t^i) \\ s.t. & \sum_{t=1}^{\infty} \overline{p}_t x_t^i = \sum_{t=1}^{\infty} \overline{p}_t s^i w_t \\ & x_t^i \geq 0 \quad t = 1, 2, \ldots, \end{array}$$

First we notice, using $(A.5)$ that from program $[D_t.1]$:

$$\sum_{t=1}^{\infty} \overline{p}_t \overline{x}_t^i = s^i \sum_{t=1}^{\infty} \widehat{a}_t^i = s^i$$

and so,

$$\sum_{i=1}^{n}(\sum_{t=1}^{\infty}\overline{p}_t\overline{x}_t^i) = \sum_{i=1}^{n} s^i = 1.$$

Therefore,

$$\sum_{t=1}^{\infty}\overline{p}_t\widehat{w}_t = \sum_{t=1}^{\infty}\overline{p}_t(\sum_{i=1}^{n}\overline{x}_t^i) = 1.$$

Hence, if $\{\overline{x}_t^i\}_{t=1}^{\infty}$ satisfies the constraint of program $[D_t.1]$ for every $t = 1, 2, ...,$ it also satisfies the constraint of program $[D.1]$. From the first order necessary conditions for program $[D_t.1]$ we have that:

$$\widehat{a}_t^i D\widehat{u}(\overline{x}_t^i) = \lambda_t^i\ \overline{p}_t \quad \text{ for some } \lambda_t^i \geq 0,\ t = 1, 2, 3, ...$$

But, by $(A.3)$

$$\sum_k D_k\widehat{u}(\overline{x}_t^i)\cdot\overline{x}_k^i = c^i, \text{for some } c^i > 0,$$

and we have that $\lambda_t^i = \frac{c^i}{s^i} > 0$ for all $t = 1, 2, ...,$. Therefore, letting $\lambda^i = \frac{c^i}{s^i}$ we have that $\overline{x}_t^i$ satisfies:

$$\widehat{a}_t^i D\widehat{u}(\overline{x}_t^i) = \lambda^i\ \overline{p}_t \text{ for all } t.$$

Hence by the of concavity $\widehat{u}^i(\cdot)$, $\{\overline{x}_t^i\}_{t=1}^{\infty}$ is a solution to program $[D.1]$. □

Proof of Theorem 4.6. Let $(\overline{p}_t, \overline{z}_t)$ be an interior Walrasian Competitive Equilibrium satisfying the assumptions of the statement. Then without loss of generality we may assume that $\sum_{t=1}^{\infty}\overline{p}_t w_t = 1$. Hence for each i, $(\overline{x}_t^i)$ is a solution to the program:

$$\begin{array}{ll} Max & \sum_{t=1}^{\infty}\widehat{a}_t^i\ \widehat{u}^i(x_t^i) \\ s.t. & \sum_{t=1}^{\infty}\overline{p}_t x_t^i = \overline{s}^i \\ x_t^i \geq 0, & t = 1, 2, ... \end{array}$$

And, by the first order conditions, there exists some $\lambda^i \geq 0$ such that:

$$\widehat{a}_t^i\ D\widehat{u}^i(\overline{x}_t^i) = \lambda^i\overline{p}_t \quad \forall t = 1, 2, ..., \tag{21}$$

and by $(A.2)$ follows that $\lambda^i > 0$.

Now, let $\overline{x}_t^i$ be a solution to the program $[\overline{D}_t.1]$. By the first order conditions there exists some $\lambda_t^i \geq 0$ such that:

$$\widehat{a}_t^i\ D\widehat{u}^i(\overline{x}_t^i) = \lambda_t^i\overline{p}_t \quad \forall t = 1, 2, ..., \tag{22}$$

Hence, combining (21) and (22), it follows that $\lambda_t^i = \lambda^i > 0$ for all $t = 1, 2, ...,$

By post multiplying both sides of the equality (22) by $\overline{x}_t^i$, and using the constraint of program $[\overline{D}_t.1]$ we have that:

$$\widehat{a}_t^i\ D\widehat{u}^i(\overline{x}_t^i)\cdot\overline{x}_t^i = \lambda^i\overline{p}_t\overline{x}_t^i = \lambda^i\widehat{a}_t^i\overline{s}^i$$

which implies that:

$$\overline{c}^i \equiv D\widehat{u}^i(\overline{x}_t^i)\cdot\overline{x}_t^i = \lambda^i\overline{s}^i$$

for all $t = 1, 2, ...,$ which shows the "if" part of the result.

Let's assume now that $(A.3')$ holds. Then, we take $\lambda_t^i = \lambda^i > 0$ for all t, and so the condition (22)is satisfied.

Now, put $\widehat{I}_t^i \equiv \overline{p}_t \overline{x}_t$ for $t = 1, 2, ...,$. Then it follows that:

$$\widehat{a}_t^i \; D\widehat{u}^i(\overline{x}_t^i) \cdot \overline{x}_t^i = \lambda^i \overline{p}_t \overline{x}_t^i = \lambda^i \widehat{I}_t^i \tag{23}$$

adding the previous expression with respect to t up to infinity:

$$\sum_{t=1}^{\infty} \widehat{a}_t^i \; D\widehat{u}^i(\overline{x}_t^i) \cdot \overline{x}_t^i = \lambda^i \sum_{t=1}^{\infty} \overline{p}_t \overline{x}_t^i = \lambda^i \sum_{t=1}^{\infty} \widehat{I}_t^i$$

and by $(A.5), (A.3')$,and since $\sum_{t=1}^{\infty} \widehat{I}_t^i = \overline{s}_i$,we obtain:

$$\widehat{c}^i = \lambda^i \overline{s}^i > 0.$$

Taking this expression into account, and $(A.3')$, it follows from (23) that:

$$\widehat{a}_t^i \overline{s}^i \lambda^i = \lambda^i \widehat{I}_t^i ,$$

which implies that $\widehat{I}_t^i = \overline{I}_t^i = \widehat{a}_t^i \overline{s}^i$ for all $t = 1, 2, ...,$and all $i = 1, 2, ..., n$. □

References

1. Aizpurua, J., Manresa,A. (1993) An infinite dimensional extension of the theory of decentralized mechanism. *Mathematical Social Sciences* 26: 157–173
2. Bala, V., Majumdar, M., Mitra,T. (1991) Decentralized evolutionary mechanisms for intertemporal economies: a possibility result. *Journal of Economics* 53: 1–29
3. Hurwicz,L. (1960) Optimality and informational efficiency in resource allocation processes. In: Arrow, K., Karlin, S., Suppes, P. (eds.) *Mathematical methods in the social sciences.* Stanford University Press, Stanford
4. Hurwicz,L. (1986) On informational decentralization and efficiency in resource allocation mechanism. In: Reiter, S. (ed.) Studies in Mathematical Economics, Vol. 25. The Mathematical Association of America
5. Hurwicz, L., Weinberger, H (1990) A necessary condition for decentralization and an application to intertemporal allocation. *Journal of Economic Theory* 51: 313–345
6. Kehoe, T. Levine, D. (1985) Comparative statics and perfect foresight in infinite horizon economies: *Econometrica* 53: 433–453
7. Manresa, A. (1995) Evolutionary mechanisms in infinite horizon pure exchange economies. Manuscript. Universitat de Barcelona
8. Mount, K., Reiter, S. (1974) The informational size of message spaces. *Journal of Economic Theory* 8: 161–192
9. Starr, R. (1987) Sequence economies. In: Eatwell, J., Milgate, M., Newman, P. (eds.) The New Palgrave, vol 4. Macmillan, New York