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Measuring and making decisions for social reciprocity

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Authors note

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Abstract

Social reciprocity may explain certain emerging psychological processes, which are likely to be founded on dyadic relations. Although some indices and statistics have been proposed to measure and make statistical decisions regarding social reciprocity in groups, these were generally developed to identify association patterns rather than to quantify the discrepancies between what each individual addresses to his/her partners and what is received from them in return. Additionally, social researchers are not only interested in measuring groups at the global level, since dyadic and individual measurements are also necessary for a proper description of social interactions. This study is concerned with a new statistic for measuring social reciprocity at the global level and with decomposing it in order to identify those dyads and individuals which account for a significant part of asymmetry in social interactions. In addition to a set of indices some exact analytical results are derived and a way of making statistical decisions is proposed.

Social psychology research has been mainly focussed on the individualistic approach, even though social phenomena involve two or more individuals. The individualistic approach ignores the social context within which individuals are embedded, a feature that may be its most important drawback. This probably explains why dyadic analysis has increasingly been applied to measure and to analyse groups (Kenny, 1994; Kenny, Kashy, & Cook, 2006). Furthermore, dyadic analysis enables social researchers to determine interaction effects between individuals in a dyad, a main effect that cannot be known by using the individualistic approach since individualistic research methods are not able to reveal patterns of mutual influence and interdependence (Bond, Horn, & Kenny, 1997; Campbell & Kashy, 2002; Gonzalez & Griffin, 1999; Griffin & Gonzalez, 1995).

Asymmetric social relationships in pairs of individuals are not uncommon. A simple example might be the number of times individuals help each other. Square matrices are useful to represent this and other similar examples of asymmetric relationships, where rows and columns correspond, respectively, to a set of individuals in one mode and the same individuals but in a different mode. In general, rows represent individuals as actors or initiators of some kind of behaviour, while columns correspond to them as partners or receivers of actors' behaviours. These sociomatrices are usually asymmetrical and this lack of symmetry renders some established statistical methods inappropriate. However, since asymmetry embodies some important information contained in the data, asymmetric data should be analysed (Saito & Yadohisa, 2005). Hence, although one could carry out mathematical transformations and thus obtain symmetric matrices, this would ignore departures from symmetry that may be informative.

A number of studies have already dealt with asymmetrical sociomatrices. Regarding dominance hierarchy, two related linearity indices have been developed to measure this attribute in many groups, and statistical methods were also proposed to test whether the linearity is stronger than expected by chance (Landau, 1951; Kendall & Babington Smith, 1940). More recent research has proposed an improved method for those cases in which measurements include tied and/or unknown relationships (de Vries, 1995). Other statistical methods have been developed to rank individuals as a function of the outcomes of dyadic dominance encounters (de Vries, 1998; de Vries & Appleby, 2000; de Vries, Stevens, & Vervaecke, 2006). As for reciprocity, interchange and other social interaction patterns, some statistical methods, which consist of computing association between two matrices, have been recommended to analyse interaction data and, in order to avoid distorting effects due to mutual dependency, permutation tests should be applied (Hemelrijk, 1990a, 1990b).

In the context of social psychology any sociomatrix could be decomposed into its variance components (Warner, Kenny, & Stoto, 1979). This model, called the Social Relations Model (SRM; Kenny & La Voie, 1984), is based on round-robin designs to gather interaction data. Actor, partner and relationship effects can be estimated and a random-effects *ANOVA* is used to partition variance into components. This statistical model also enables social researchers to estimate dyadic and generalized reciprocity by means of correlation coefficient values, although it does not allow social reciprocity to be measured at the global level.

In general, the statistical methods mentioned above use association indices to estimate global reciprocity in social systems. Although these indices undoubtedly enable social researchers to identify the specific social relations that emerge in social interactions, association indices do not measure the correspondence between what an individual gives others and what is received from them in return. The directional consistency index (van Hooff & Wensing, 1987), which is based on absolute differences between what each member of a pair gives to the other and what is received from her/him, enables researchers to obtain a measure of global reciprocity in which the magnitude of the behaviour is taken into account. Recently, another statistic for measuring global reciprocity has been developed to quantify the discrepancy between what is addressed to others and what is received in return (Solanas, Salafranca, Riba, Sierra, & Leiva, 2006). This reciprocity statistic can be partitioned in such a way that people who contribute more to the lack of reciprocity can be identified and dyadic and generalized reciprocity may also be measured. This reciprocity statistic, called the skew-symmetry statistic, can also be tested for statistical significance (Leiva, Solanas, & Salafranca, 2008). The skew-symmetry statistic ranges from *0* to *.5* and the random variable is located at the denominator. Hence, it would be better to develop a statistic whose values range from *0* to *1* and, more importantly, one for which exact analytical results could be obtained. Furthermore, in order to make comparisons between different studies, the reciprocity statistic should be normalized for minimum and maximum values. Regarding statistical inference, there is also a need for a statistical method capable of making statistical decisions.

The main aim of the present study is to propose a new statistic to quantify social reciprocity and a corresponding statistical method for making decisions regarding social reciprocity in groups. Although we assume that social phenomena depend on dyadic interactions and, hence, that the statistic should be based on dyadic data, we propose new indices of social reciprocity at global, dyadic and individual levels. Our study could be useful for those researchers who are interested in family interactions, play relationships, cooperative learning, agonistic behaviours, and other topics.

A new index for measuring social reciprocity

Consider n individuals who are labelled i = 1, 2, …, n in an experiment involving dyadic interactions. Let $c_{ij} = c_{ji}$ be the number of observed interactions between the individuals i and j, and let x_{ij} be the number of times i is recorded to address a specific behaviour to j. Note that $c_{ij} = x_{ij} + x_{ji}$, since x_{ij} and x_{ji}, respectively, represent the number of behaviours each individual of the dyad addresses to the other. It is assumed that the probability that an individual i addresses behaviour to j in each single interaction remains steady during the period of time the observations are made. The parameter π_{ij} denotes the probability that individual i addresses the behaviour of interest to j. It should be noted that $\pi_{ij} + \pi_{ji} = 1$, as it is assumed that only one individual of each dyad addresses the behaviour to the other in each social interaction. Thus, social reciprocity in a group can be represented in a matrix **Π** as follows:

$$\mathbf{\Pi} = \begin{pmatrix} 0 & \pi_{12} & \pi_{13} & \cdots & \pi_{1n} \\ 1-\pi_{12} & 0 & \pi_{23} & \cdots & \pi_{2n} \\ 1-\pi_{13} & 1-\pi_{23} & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1-\pi_{1n} & 1-\pi_{2n} & \cdots & \cdots & 0 \end{pmatrix}$$

The parameters of the matrix **Π** can be used to define a new measurement of social reciprocity, since these parameters contain the essential information to quantify dyadic reciprocity among all pairs of individuals. Note that the values of π_{ij} are unknown since they are parameters, and also note that social reciprocity can be described by means of *n(n−1)/2* independent parameters. It should be stressed that independence among dyads is assumed.

In order to define the new index the trace of the product matrix **Π'Π** is obtained as follows:

$$tr\left(\mathbf{\Pi}'\mathbf{\Pi}\right)=\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\pi_{ij}^{2}=\sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\pi_{ij}^{2}+\left(1-\pi_{ij}\right)^{2}\right)=\frac{n\left(n-1\right)}{2}+2\sum_{i=1}^{n}\sum_{j=i+1}^{n}\pi_{ij}^{2}-2\sum_{i=1}^{n}\sum_{j=i+1}^{n}\pi_{ij}$$

Note that tr(**Π'Π**) will take its maximum value if $\pi_{ij} = 0$ or $\pi_{ij} = 1$ for all *i* and *j*. Thus, the maximum value of tr(**Π'Π**) is equal to

$$\max\left(tr\left(\mathbf{\Pi}'\mathbf{\Pi}\right)\right)=\frac{n\left(n-1\right)}{2}$$

Thus, the maximum value only depends on the number of individuals in a group and corresponds to those cases in which there is a complete lack of reciprocity in every dyad.

The minimum value of tr(**Π'Π**) corresponds to $\pi_{ij} = 1/2$ for all *i* and *j*. Thus, we can obtain the minimum value of tr(**Π'Π**) as follows:

$$\min\left(tr\left(\mathbf{\Pi}'\mathbf{\Pi}\right)\right)=\frac{n\left(n-1\right)}{2}+2\frac{n\left(n-1\right)}{2}\left(\frac{1}{2}\right)^{2}-2\frac{n\left(n-1\right)}{2}\left(\frac{1}{2}\right)=\frac{n\left(n-1\right)}{4}$$

The minimum value of tr(**Π'Π**) once again depends only on the number of individuals in a group.

We can now define a new index to measure overall social reciprocity in groups, taking into account that it should be bounded between *0* and *1*, as follows:

$$\Phi_r = \frac{tr\left(\mathbf{\Pi}^{'}\mathbf{\Pi}\right) - \min\left(tr\left(\mathbf{\Pi}^{'}\mathbf{\Pi}\right)\right)}{\max\left(tr\left(\mathbf{\Pi}^{'}\mathbf{\Pi}\right)\right) - \min\left(tr\left(\mathbf{\Pi}^{'}\mathbf{\Pi}\right)\right)}, \quad 0 \le \Phi_r \le 1$$

This index takes its minimum value if all π_{ij} are equal to *.5*, while the maximum value corresponds to those cases in which $\pi_{ij} = 1$ and $\pi_{ji} = 0$ or $\pi_{ij} = 0$ and $\pi_{ji} = 1$ for every dyad. If the index value equals zero, it indicates that individuals are completely reciprocal with respect to others. On the other hand, the group will show a large lack of reciprocity if Φ_r is close to *1*. In other words, if $\Phi_r = 0$, it denotes symmetrical relationships among all individuals regarding the behaviour of interest, while $\Phi_r = 1$ corresponds to the maximum level of asymmetry.

It can be shown, by means of some algebraic operations, that the index Φ_r can also be obtained from the following equation:

$$\Phi_r = \frac{4tr\left(\mathbf{\Pi}^{'}\mathbf{\Pi}\right) - n(n-1)}{n(n-1)} = \frac{8\sum_{i=1}^{n}\sum_{j=i+1}^{n}\pi_{ij}^2 - 8\sum_{i=1}^{n}\sum_{j=i+1}^{n}\pi_{ij} + n(n-1)}{n(n-1)}$$
$$= 1 - \frac{8\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n}\pi_{ij} - \sum_{i=1}^{n}\sum_{j=i+1}^{n}\pi_{ij}^2\right)}{n(n-1)}$$

Note in the last expression that if $\pi_{ij} = \pi$ for all $i < j$, the mathematical expression can be written in the following way:

$$\Phi_r = 1 - \frac{8\left(\frac{n(n-1)}{2}\pi - \frac{n(n-1)}{2}\pi^2\right)}{n(n-1)} = 1 - 4\left(\pi - \pi^2\right)$$

This means that if $\pi_{ij} = \pi$ for all $i < j$, the value of the index does not depend on the number of individuals, but is only a function of π. Figure 1 shows how the value of the index Φ_r increases as a function of the value π.

INSERT FIGURE 1 ABOUT HERE

As an example, suppose that the matrix **Π** of a group is as follows:

$$\mathbf{\Pi} = \begin{pmatrix} 0 & .8 & .8 & .8 \\ .2 & 0 & .8 & .8 \\ .2 & .2 & 0 & .8 \\ .2 & .2 & .2 & 0 \end{pmatrix}$$

Then the index Φ_r is equal to

$$\Phi_r = 1 - 4\left(\pi - \pi^2\right) = 1 - 4\left(.8 - .8^2\right) = .36$$

Now we can define the index Ψ_r as follows:

$$\Psi_r = 1 - \Phi_r, \quad 0 \le \Psi_r \le 1$$

Conversely to Φ_r, the index Ψ_r takes a value close to *0* if dyadic interactions within a group are asymmetrical, while it is equal to *1* if there is complete reciprocation.

Estimating asymmetry in social relations

Social researchers do not know the value of Φ_r since they collect empirical data. Thus, an estimator of asymmetry in social relations is required to obtain some information about social reciprocity. An estimator of the index Φ_r can be defined as follows:

$$\hat{\Phi}_r = \frac{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2 - \min\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2\right)}{\max\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2\right) - \min\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq 1}}^{n} x_{ij}^2 / c_{ij}^2\right)}, \quad 0 \leq \hat{\Phi}_r \leq 1$$

In order to standardize $\hat{\Phi}_r$ values the following maximum value should be obtained

$$\max\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2\right) = \sum_{i=1}^{n}\sum_{j=i+1}^{n} c_{ij}^2 / c_{ij}^2 = n\left(n-1\right)/2$$

Three cases should be distinguished to obtain the minimum value. Firstly, if all c_{ij} are even, the minimum value is obtained from the following expression

$$\min\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2\right) = \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\left(c_{ij}/2\right)^2}{c_{ij}^2} = \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{c_{ij}^2}{4c_{ij}^2} = \frac{n(n-1)}{4}$$

Secondly, if all c_{ij} are odd, the minimum value is equal to

$$\min\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2\right) = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\frac{\left(c_{ij}-1\right)^2}{4c_{ij}^2} + \frac{\left(c_{ij}+1\right)^2}{4c_{ij}^2}\right) = \frac{n(n-1)}{4} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{1}{c_{ij}^2}$$

Thirdly, in the general case, if there are p and $n(n-1)/2 - p$ dyads in which the values c_{ij} are even and odd, respectively, the minimum value equals

$$\begin{aligned}\min\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2\right) &= \frac{2p}{4} + \frac{2\left(\frac{n(n-1)}{2} - p\right)}{4} + \frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=i+1\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}\\ &= \frac{n(n-1)}{4} + \frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=i+1\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}\end{aligned}$$

We denote this minimum value by m in what follows.

We have previously supposed that $c_{ij} \neq 0$ for all i and j, where $i \neq j$. If there are dyads for which $c_{ij} = 0$, the maximum is as follows:

$$z = \max\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i\\ c_{ij}\neq 0}}^{n} x_{ij}^2 / c_{ij}^2\right) = card\left\{c_{ij} \neq 0\right\};\ for\ i < j$$

where *card* denotes the number of different dyads for which c_{ij} is not equal to *0*.

As regards the minimum

$$m = \min\left(\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i \\ c_{ij} \neq 0}}^{n} x_{ij}^2 / c_{ij}^2 \right) = \frac{z}{2} + \frac{1}{2} \sum_{i=1}^{n} \sum_{\substack{j=i+1 \\ c_{ij}\ odd}}^{n} \frac{1}{c_{ij}^2}$$

Now, the expression corresponding to the statistic $\hat{\Phi}_r$ can be written as follows:

$$\hat{\Phi}_r = \frac{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ij}^2 / c_{ij}^2 - m}{n(n-1)/2 - m}$$

If there are some $c_{ij} = 0$, the statistic equals

$$\hat{\Phi}_r = \frac{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i \\ c_{ij} \neq 0}}^{n} x_{ij}^2 / c_{ij}^2 - m}{z - m}$$

Decomposing individuals' contribution to social asymmetry

Note that the following expression enables us to know each individual's contribution to asymmetry in social interactions

$$\hat{\Phi}_r = \frac{\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i\\ c_{ij}\neq 0}}^{n} x_{ij}^2 / c_{ij}^2 - m}{z-m} = \sum_{i=1}^{n}\frac{\sum_{\substack{j=1\\ j\neq i\\ c_{ij}\neq 0}}^{n}\left(x_{ij}^2 / c_{ij}^2 - \frac{1}{4}\right) - \frac{1}{2}\sum_{\substack{j=i+1\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}}{z-m} = \sum_{i=1}^{n}\hat{\phi}_{ia}$$

where $\hat{\phi}_{ia}$ denotes the contribution of individual i to asymmetry as an actor. Note that this quantity is a measure at the individual level. Positive values for $\hat{\phi}_{ia}$ denote that individuals address more behaviour to others than they receive in return. On the other hand, negative values correspond to those individuals who address less behaviour to others than they receive in return. In addition, the following expression provides the contribution of individual j to asymmetry as a partner

$$\hat{\Phi}_r = \sum_{j=1}^{n}\frac{\sum_{\substack{i=1\\ i\neq j\\ c_{ij}\neq 0}}^{n}\left(x_{ji}^2 / c_{ij}^2 - \frac{1}{4}\right) - \frac{1}{2}\sum_{\substack{i=j+1\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}}{z-m} = \sum_{j=1}^{n}\hat{\phi}_{pj}$$

A positive value of this measurement means that the individual receives more behaviour from others than she/he gives in return, while negative values indicate the opposite.

Now, note that the expression for $\hat{\Phi}_r$ can be written as follows:

$$\hat{\Phi}_r = \sum_{i=1}^{n}\frac{\sum_{\substack{j=1\\ j\neq i\\ c_{ij}\neq 0}}^{n}\left(x_{ij}^2 / c_{ij}^2 - \frac{1}{4}\right) - \frac{1}{2}\sum_{\substack{j=i+1\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}}{z-m} = \sum_{i=1}^{n}\frac{\sum_{\substack{j=1\\ j\neq i\\ c_{ij}\neq 0}}^{n}\left(4x_{ij}^2 / c_{ij}^2 - 1\right) - 2\sum_{\substack{j=i+1\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}}{4z-4m}$$

$$= \sum_{i=1}^{n}\frac{\sum_{\substack{j=1\\ j\neq i\\ c_{ij}\neq 0}}^{n}\left(4x_{ij}^2 / c_{ij}^2 - 1\right) - \sum_{\substack{j=1\\ j\neq i\\ c_{ij}\ odd}}^{n}\frac{1}{c_{ij}^2}}{4z-4m} = \sum_{i=1}^{n}\hat{\phi}_{ia} = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\hat{\phi}_{ij} + \hat{\phi}_{ji}\right) = \sum_{i=1}^{n}\sum_{j=i+1}^{n}\hat{\Phi}_{ij}$$

where $\hat{\Phi}_{ij}$ corresponds to each dyad's contribution to asymmetry. This measure is a dyadic one, and, in particular, it quantifies the contribution of each dyad to the overall asymmetry in groups. Additionally, $\hat{\phi}_{ij}$ corresponds to a directional dyadic measure. Specifically, it measures the contribution of individual i to asymmetry that is due to individual j.

These dyadic and individual contributions to asymmetry can be useful to identify those individuals and dyads that are associated with larger differences between what is given and received from others.

Mathematical expectancy of $\hat{\Phi}_r$

We denote the expected value of $\hat{\Phi}_r$ by $E\left[\hat{\Phi}_r\right]$. Note that we can write the expression for $\hat{\Phi}_r$ as follows:

$$\hat{\Phi}_r = \frac{2\sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2 - 2m}{n\left(n-1\right) - 2m}$$

The mathematical expectancy of the estimator under a specific null hypothesis is (see Appendix A)

$$E\left[\hat{\Phi}_r\right] = \frac{2\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n} 1/c_{ij} + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{\pi_{ij}^2\left(c_{ij}-1\right)}{c_{ij}} - m\right)}{n\left(n-1\right)-2m}$$

If $\pi_{ij} = .5$ for all *i* and *j*, the expected value equals

$$E\left[\hat{\Phi}_r\right] = \frac{2\sum_{i=1}^{n}\sum_{j=i+1}^{n} 1/c_{ij} + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{c_{ij}-1}{2c_{ij}} - 2m}{n\left(n-1\right)-2m}$$

Furthermore, if $\pi_{ij} = .5$ for each *i* and *j* individual and $c_{ij} = c$ for all dyads,

$$E\left[\hat{\Phi}_r\right] = \frac{n\left(n-1\right)\left(c+1\right)-4cm}{2n\left(n-1\right)c-4cm}$$

Knowing the expected value for the proposed whole statistic allows social researchers to make proper comparisons and suitable decisions since the estimator is biased. In other words, if complete reciprocation is supposed, note that the expected value is not equal to *0*.

Standard error of $\hat{\Phi}_r$

The variance of $\hat{\Phi}_r$ can be expressed as follows:

$$\sigma^2\left(\hat{\Phi}_r\right) = E\left[\hat{\Phi}_r^2\right] - E^2\left[\hat{\Phi}_r\right] = E\left[\left(\frac{2\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2 - 2m}{n\left(n-1\right)-2m}\right)^2\right] - E^2\left[\hat{\Phi}_r\right]$$

$$= 4\left(n\left(n-1\right)-2m\right)^{-2} E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2 - m\right)^2\right] - E^2\left[\hat{\Phi}_r\right]$$

Note that $E^2\left[\hat{\Phi}_r\right]$ is known since the expected value for the estimator has already been derived. Now, the other term of the previous expression is decomposed as follows:

$$E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2 - m\right)^2\right] = E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2\right)^2\right] - 2mE\left[\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2\right] + m^2$$

where

$$2E\left[\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2\right] = 2\sum_{i=1}^{n}\sum_{j=1}^{n} E\left[x_{ij}^2\right] / c_{ij}^2 = E\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right) + 2m$$

Then,

$$\sigma^2\left(\hat{\Phi}_r\right) = \frac{4\left(E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2\right)^2\right] - m\left(E\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)+2m\right)+m^2\right)}{\left(n\left(n-1\right)-2m\right)^2} - E^2\left[\hat{\Phi}_r\right]$$

$$= \frac{4\left(E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2\right)^2\right] - mE\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)-m^2\right)}{\left(n\left(n-1\right)-2m\right)^2} - E^2\left[\hat{\Phi}_r\right]$$

It thus only remains to solve the following term in order to obtain the expected variance for the estimator under a specific null hypothesis (see Appendix B)

$$E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^2 / c_{ij}^2\right)^2\right]$$

Finally, its standard error is equal to

$$\sigma\left(\hat{\Phi}_r\right) = \frac{2\sqrt{\sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{q_{ij} + q_{ji} + 2s_{ij}}{c_{ij}^4}}}{n\left(n-1\right) - 2m}$$

$$\begin{aligned} q_{ij} + q_{ji} + 2s_{ij} = {} & 4c_{ij}\left(\pi_{ij} - 7\pi_{ij}^2 + 12\pi_{ij}^3 - 6\pi_{ij}^4\right) + \\ & 8c_{ij}^2\left(-\pi_{ij} + 6\pi_{ij}^2 - 10\pi_{ij}^3 + 5\pi_{ij}^4\right) + \\ & 4c_{ij}^3\left(\pi_{ij} - 5\pi_{ij}^2 + 8\pi_{ij}^3 - 4\pi_{ij}^4\right) \end{aligned}$$

Note that the estimator is consistent since its standard error tends to zero as *n* increases.

If $\pi_{ij} = .5$ for all *i* and *j*, the standard error can be computed by means of the following formula

$$\sigma\left(\hat{\Phi}_r\right) = \frac{\sqrt{2\sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{c_{ij} - 1}{c_{ij}^3}}}{n\left(n-1\right) - 2m}$$

Social researchers can be interested in comparing $\hat{\Phi}_r$ with other measures of social reciprocity (e.g., directional consistency; van Hooff and Wensing, 1987). This comparison can be carried out by means of the mean square error (*MSE*), which is computed as follows for an estimator $\hat{\theta}$:

$$MSE\left(\hat{\theta}\right) = E^2\left[\hat{\theta} - \theta\right] + \sigma^2\left(\hat{\theta}\right) = Bias^2\left(\hat{\theta}\right) + \sigma^2\left(\hat{\theta}\right)$$

Then, the *MSE* for $\hat{\Phi}_r$ equals

$$MSE\left(\hat{\Phi}_r\right) = E^2\left[\hat{\Phi}_r - \Phi_r\right] + \sigma^2\left(\hat{\Phi}_r\right) = Bias^2\left(\hat{\Phi}_r\right) + \sigma^2\left(\hat{\Phi}_r\right)$$

The null hypothesis

Social researchers are often interested in testing whether social relations among individuals in a group are symmetrical, as they often test dyadic and generalized reciprocity. In this regard the null hypothesis for complete reciprocation can be expressed as follows:

$$H_0 : \Phi_r = 0$$

This statistical hypothesis is equivalent to $\pi_{ij} = \pi_{ji} = .5$ for all *i* and *j*. Thus, if the null hypothesis is rejected, we can conclude that social interactions within a group are partially asymmetrical, at least as regards the behaviour of interest. In other words, individuals do not completely reciprocate one another. Note that rejecting the null hypothesis does not tell us anything about the degree of asymmetrical relations since we only have some evidence that Φ_r is different from zero. At all events, the values of $\hat{\Phi}_r$ are point estimates of the actual degree of asymmetry in groups, and researchers should take into account the estimator's bias when interpreting their results. On the other hand, if the null hypothesis were accepted, this would still not allow social researchers to conclude that individuals within a group completely reciprocate, as statistical tests do not completely control Type II error.

Social relations within a group will be asymmetrical if $\pi_{ij} \neq \pi_{ji}$ for any *i* and *j* and, therefore, we should conclude that there is at least one dyad for which $\pi_{ij} \neq \pi_{ji}$ if the null hypothesis were rejected.

Statistical significance

For any value of the test statistic it is useful to know its exact statistical significance. In order to obtain exact statistical significance it is necessary to establish all possible c_{ij} values for a given n and to compute the test statistic for all admissible x_{ij} results. However, there are such a number of possibilities that this task is not practical, a difficulty that has been encountered in other dyadic methods (Rapoport, 1949; Landau, 1951). To assess statistical significance we propose employing Monte Carlo sampling. This statistical method becomes particularly useful if the population distribution is known but the sampling distribution of the estimator of interest has not been analytically derived (Noreen, 1989). Note that the sampling distribution of the test statistic can be estimated since all x_{ij} are supposed to be binomially distributed.

A computer code has been developed for testing asymmetry in groups, specifically SAS/IML and R functions which are delivered on request. To carry out the simulation the program needs the original matrix **X** to compute the values c_{ij} from x_{ij} and x_{ji} and the number of individuals n. Additionally, the null hypothesis must also be specified, that is, the values π_{ij}. The program then computes $\hat{\Phi}_r$, its mathematical expectancy and standard error, as well as the dyadic and individual indices.

Probabilities $\pi_{ij} = .5$ are used to obtain simulated sociomatrices if the complete reciprocation hypothesis is tested. For each dyad the value x_{ij} is drawn at random and x_{ji} is computed by subtracting x_{ij} from c_{ij}. The simulated interaction matrix is then used to compute the test statistic. This process is iterated *NS* times, where *NS* denotes the number of simulated sociomatrices. The one-tailed statistical significance of the test equals p = (NOS+1)/(NS+1), where *NOS* denotes the number of significant samples. That is, *NOS* is equal to the number of simulated sociomatrices for which the value of the test statistic is at least as large as the observed value in the original sociomatrix. This Monte Carlo procedure included in the computer program allows social researchers to obtain statistical significance for the global, dyadic and individual statistics under any null hypothesis regarding the degree of social reciprocity.

An example

The following matrix concerns aggressive behaviour in children and represents the number of aggressive acts that each child has addressed to others during play interaction (Kenny et al., 2007. Printed with permission), with rows and columns representing individuals as actors and partners, respectively:

$$X = \begin{pmatrix} 0 & 17 & 12 & 57 & 11 & 14 \\ 15 & 0 & 6 & 95 & 18 & 128 \\ 20 & 59 & 0 & 89 & 19 & 59 \\ 30 & 38 & 47 & 0 & 83 & 294 \\ 25 & 8 & 4 & 140 & 0 & 36 \\ 6 & 87 & 11 & 272 & 31 & 0 \end{pmatrix}$$

The value of the statistic for the matrix **X** is approximately equal to *.1712* and the expected value for the estimator equals *.0201* under the null hypothesis of complete reciprocation. This shows that the empirical value is clearly larger than the expected one. Regarding the variance and the standard error of the estimator, these are equal, respectively, to *.000082* and *.009038* under the null hypothesis of social reciprocity in the group. Note that the difference between the statistic's value and the expected value is large enough in comparison with the standard error. As an initial descriptive analysis these results suggest that the individuals of the group are nonreciprocal at the global level as regards aggressive behaviour.

Although descriptive analysis is illustrative, a statistical decision regarding the null hypothesis that establishes the complete reciprocation between individuals is also needed. In order to make a statistical decision with respect to the null hypothesis of complete reciprocation the statistical significance for the value of the asymmetry statistic was estimated by Monte Carlo simulation, in which we established *99,999* iterations. The obtained p value was less than *.0001*, which clearly suggests that the null hypothesis is unlikely. As a conclusion, at least one pair of children is responsible for the asymmetry in these dyadic relations. That is, there is at least one child in the group who does not reciprocate in aggression.

We know that the lack of social reciprocity in the group at the individual level is mainly explained by individual *3* as an actor, but, as a partner, individuals *3* and *6* account for the main part of asymmetry (see Table 1). That is, individual *3* is the child who addresses the most aggression and receives less from others (positive value of $\hat{\phi}_{ia}$ and negative value of $\hat{\phi}_{pj}$, respectively). Additionally, individual *6* receives more aggressive behaviour from partners than she/he addresses to them.

At the dyadic level the dyad *2-3* makes the largest contribution to asymmetry (see Table 2). However, all dyads show very low values of contribution to asymmetry. When a Monte Carlo procedure is carried out for testing the null hypothesis of complete reciprocation, six of the fifteen dyads in the group appear not to make a significant contribution to asymmetry in the dyadic relationship ($\hat{\Phi}_{ij}$), specifically, dyads *1-2*, *1-3*, *1-6*, *2-5*, *4-6* and *5-6*. These results are also confirmed when a Monte Carlo sampling is carried out for the directional dyadic contributions to asymmetry (see $\hat{\phi}_{ij}$ measures in Table 1) with the exception of the dyad *2-5*. Thus, individual *1*'s asymmetry addressed towards individuals *2*, *3* and *6* (and vice versa) and individual *6*'s asymmetry addressed to individual *5* (and vice versa) are not significantly different from what is expected under the null hypothesis, which states complete reciprocation in aggressive behaviour. In contrast, the directional dyadic contributions to the lack of balance in aggression for individuals *2* and *5*, both $\hat{\phi}_{25}$ and $\hat{\phi}_{52}$, are significant at 5% the level.

INSERT TABLES 1 AND 2 ABOUT HERE

Conclusion

The present research is intended for use by researchers interested in measuring and testing asymmetric relationships in groups. The proposed statistic is founded on values

x_{ij} of sociomatrices **X**. This statistic enables social researchers to assess social asymmetry since discrepancies between what each individual gives others and what is received in return is taken into account. Note that the product-moment correlation coefficient remains unchanged under location and scale transformations. This characteristic of the correlation coefficient is adequate if social researchers are interested in estimating generalized reciprocity, but it does not lead them to a precise assessment of absolute dyadic reciprocity, as the product-moment correlation coefficient does not take into account differences between what individuals give others and what they receive from them in return.

The proposed statistic $\hat{\Phi}_r$ is bounded between *0* and *1*, which refer, respectively, to the maximum and minimum of social reciprocity. Thus, if the statistic is equal to *0*, there will be a complete reciprocation in groups, while a value close to *1* denotes asymmetric relationships (i.e., the larger the value of the index the lower the dyadic reciprocity among individuals). Hence, the statistic allows social researchers to measure the degree of social reciprocity at the group level. Additionally, the statistic has been decomposed into components to determine the contribution of each individual and dyad to asymmetry.

Regarding statistical inference, we have demonstrated that the estimator $\hat{\Phi}_r$ is biased and this kind of statistical error should be considered when interpreting estimated values of social reciprocity in groups. Since mathematical expectancy has been analytically derived for this estimator, social researchers are able to determine whether the test statistic takes a value larger than the expected value under the null hypothesis. In other words, researchers should not expect the test statistic to be equal to *0* in samples if there is actually complete reciprocation in populations.

We have also analytically derived the standard error for the sampling distribution of the estimator. Since the bias and the standard error of the estimator are known, it is feasible to compare this test statistic with others. Thus, the mean square error can be computed and used to select the best estimator. Unfortunately, we cannot propose an exact statistical test as the sampling distribution of the estimator is not known. Further work is thus needed to determine the exact sampling distribution for the estimator. Since the exact sampling distribution of the estimator has not been derived we proposed using the Monte Carlo method to estimate statistical significance. Briefly, the method consists of randomly sampling sociomatrices from a population whose parameters are in accordance with the null hypothesis and where the number of individuals is equal to that of the original sample. All test statistic values of simulated samples are arranged in ascending order and the original value is located in this distribution. In other words, the algorithm computes the number of simulated test statistic values that are at least as large as the test statistic for the original sociomatrix. This estimate of statistical significance enables social researchers to make statistical decisions regarding the null hypothesis. We suggest specifying a large number of simulated sociomatrices in the algorithm in order to improve statistical significance estimates. Note that the reliability of these statistical significance estimates increases with the number of simulated sociomatrices.

From an applied point of view, we can envision several applications of the social reciprocity statistic for a variety of topics in social psychology and social ethology, for instance, interpersonal perception (Kenny, 1994), social conflict and reconciliation in gregarious species (Cooper, Bernstein, & Hemelrijk, 2005), and agonistic dominance (Vervaecke, de Vries, & van Elsacker, 1999).

To sum up, the present research describes a new procedure to measure and make statistical decisions regarding social reciprocity at global, dyadic, and individual levels.

While other indices and statistics are based on association coefficients or do not enable social researchers to obtain dyadic and individual effects, the proposed statistical method allows them not only to quantify the lack of social reciprocity as a function between what is given to others and what is received in return but also to estimate dyadic and individual effects. Additionally, a Monte Carlo method is proposed to obtain approximate statistical significance. We highlight that the estimator's values can also be obtained for sociomatrices with incomplete data.

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Appendix A

The expected value for $\hat{\Phi}_r$ is equal to

$$E\left[\hat{\Phi}_r\right] = E\left[\frac{2\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n} x_{ij}^2 / c_{ij}^2 - 2m}{n(n-1)-2m}\right] = \frac{2\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n} E\left[x_{ij}^2\right] / c_{ij}^2 - 2m}{n(n-1)-2m}$$

If x_{ij} follows a binomial distribution, $E\left[x_{ij}^2\right]$, which is the second moment about zero, can be written as follows (Johnson, Kotz, & Kemp, 1992),

$$E\left[x_{ij}^2\right] = c_{ij}\pi_{ij} + c_{ij}\left(c_{ij}-1\right)\pi_{ij}^2$$

Thus,

$$E\left[\hat{\Phi}_r\right] = \frac{2\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\left(c_{ij}\pi_{ij} + c_{ij}\left(c_{ij}-1\right)\pi_{ij}^2\right)/c_{ij}^2 - 2m}{n(n-1)-2m} = \frac{2\sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\left(c_{ij}\pi_{ij} + c_{ij}^2\pi_{ij}^2 - c_{ij}\pi_{ij}^2\right)/c_{ij}^2 - 2m}{n(n-1)-2m}$$

$$= \frac{2\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n} 1/c_{ij} + \sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\pi_{ij}^2 - \sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\pi_{ij}^2/c_{ij} - m\right)}{n(n-1)-2m}$$

$$= \frac{2\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n} 1/c_{ij} + \sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}\frac{\pi_{ij}^2\left(c_{ij}-1\right)}{c_{ij}} - m\right)}{n(n-1)-2m}$$

If there are dyads for which c_{ij} equals 0, mathematical expectancy can be obtained as follows:

$$E\left[\hat{\Phi}_r\right] = \frac{2\left(\sum_{i=1}^{n}\sum_{\substack{j=i+1\\ c_{ij}\neq 0}}^{n} 1/c_{ij} + \sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i\\ c_{ij}\neq 0}}^{n} \frac{\pi_{ij}^{2}\left(c_{ij}-1\right)}{c_{ij}} - m\right)}{2z-2m}$$

Appendix B

The variance of the estimator for a specific null hypothesis equals

$$\sigma^2\left(\hat{\Phi}_r\right)=\frac{4\left(E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n}x_{ij}^2/c_{ij}^2\right)^2\right]-m\left(E\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)+2m\right)+m^2\right)}{\left(n\left(n-1\right)-2m\right)^2}-E^2\left[\hat{\Phi}_r\right]$$

$$=\frac{4\left(E\left[\left(\sum_{i=1}^{n}\sum_{j=1}^{n}x_{ij}^2/c_{ij}^2\right)^2\right]-mE\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)-m^2\right)}{\left(n\left(n-1\right)-2m\right)^2}-E^2\left[\hat{\Phi}_r\right]$$

Note that

$$E\left[\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}x_{ij}^2/c_{ij}^2\right)^2\right]=E^2\left[\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}x_{ij}^2/c_{ij}^2\right]+Var\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}x_{ij}^2/c_{ij}^2\right)$$

$$=E^2\left[\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}x_{ij}^2/c_{ij}^2\right]+\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}Var\left(x_{ij}^2\right)/c_{ij}^4+$$

$$2\sum_{i=1}^{n}\sum_{j=i+1}^{n}Cov\left(x_{ij}^2/c_{ij}^2,x_{ji}^2/c_{ij}^2\right)$$

where

$$E^2\left[\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}x_{ij}^2/c_{ij}^2\right]=\left(\frac{E\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)+2m}{2}\right)^2$$

Now, the variance for x_{ij}^2 has to be calculated. Thus,

$$Var\left(x_{ij}^2\right) = E\left[x_{ij}^4\right] - E^2\left[x_{ij}^2\right] = c_{ij}\pi_{ij} + 7c_{ij}\left(c_{ij}-1\right)\pi_{ij}^2 + 6c_{ij}\left(c_{ij}-1\right)\left(c_{ij}-2\right)\pi_{ij}^3 + c_{ij}\left(c_{ij}-1\right)\left(c_{ij}-2\right)\left(c_{ij}-3\right)\pi_{ij}^4 - \left(c_{ij}\pi_{ij} + \left(c_{ij}^2 - c_{ij}\right)\pi_{ij}^2\right)^2$$

since the fourth moment about zero for a binomially distributed variable is equal to (Johnson, Kotz, & Kemp, 1992)

$$E\left[x_{ij}^4\right] = c_{ij}\pi_{ij} + 7c_{ij}\left(c_{ij}-1\right)\pi_{ij}^2 + 6c_{ij}\left(c_{ij}-1\right)\left(c_{ij}-2\right)\pi_{ij}^3 + c_{ij}\left(c_{ij}-1\right)\left(c_{ij}-2\right)\left(c_{ij}-3\right)\pi_{ij}^4$$

After some algebraic operations, we obtain

$$\begin{aligned} Var\left(x_{ij}^2\right) &= c_{ij}\pi_{ij} + 7\left(c_{ij}^2 - c_{ij}\right)\pi_{ij}^2 + 6\left(c_{ij}^3 - 3c_{ij}^2 + 2c_{ij}\right)\pi_{ij}^3 + \left(c_{ij}^4 - 6c_{ij}^3 + 11c_{ij}^2 - 6c_{ij}\right)\pi_{ij}^4 - \\ &\quad c_{ij}^2\pi_{ij}^2 + 2c_{ij}^2\pi_{ij}^3 - 2c_{ij}^3\pi_{ij}^3 - c_{ij}^2\pi_{ij}^4 + 2c_{ij}^3\pi_{ij}^4 - c_{ij}^4\pi_{ij}^4 \\ &= c_{ij}\pi_{ij} - 7c_{ij}\pi_{ij}^2 + 6c_{ij}^2\pi_{ij}^2 + 12c_{ij}\pi_{ij}^3 - 16c_{ij}^2\pi_{ij}^3 + 4c_{ij}^3\pi_{ij}^3 - 6c_{ij}\pi_{ij}^4 + 10c_{ij}^2\pi_{ij}^4 - 4c_{ij}^3\pi_{ij}^4 \\ &= c_{ij}\left(\pi_{ij} - 7\pi_{ij}^2 + 12\pi_{ij}^3 - 6\pi_{ij}^4\right) + c_{ij}^2\left(6\pi_{ij}^2 - 16\pi_{ij}^3 + 10\pi_{ij}^4\right) + c_{ij}^3\left(4\pi_{ij}^3 - 4\pi_{ij}^4\right) = q_{ij} \end{aligned}$$

It can also be shown that

$$Var\left(x_{ji}^2\right) = c_{ij}\left(\pi_{ij} - 7\pi_{ij}^2 + 12\pi_{ij}^3 - 6\pi_{ij}^4\right) + c_{ij}^2\left(-4\pi_{ij} + 18\pi_{ij}^2 - 24\pi_{ij}^3 + 10\pi_{ij}^4\right) + c_{ij}^3\left(4\pi_{ij} - 12\pi_{ij}^2 + 12\pi_{ij}^3 - 4\pi_{ij}^4\right) = q_{ji}$$

Thus,

$$q_{ij} + q_{ji} = 2c_{ij}\left(\pi_{ij} - 7\pi_{ij}^2 + 12\pi_{ij}^3 - 6\pi_{ij}^4\right) + c_{ij}^2\left(-4\pi_{ij} + 24\pi_{ij}^2 - 40\pi_{ij}^3 + 20\pi_{ij}^4\right) + c_{ij}^3\left(4\pi_{ij} - 12\pi_{ij}^2 + 16\pi_{ij}^3 - 8\pi_{ij}^4\right)$$

and we can write

$$\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{Var\left(x_{ij}^{2}\right)}{c_{ij}^{4}}=\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{q_{ij}+q_{ji}}{c_{ij}^{4}}$$

Now we will solve the covariances,

$$\begin{aligned}\sum_{i=1}^{n}\sum_{j=i+1}^{n}Cov\left(x_{ij}^{2}/c_{ij}^{2},x_{ji}^{2}/c_{ij}^{2}\right)&=\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{Cov\left(x_{ij}^{2},x_{ji}^{2}\right)}{c_{ij}^{4}}\\&=\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{E\left[\left(x_{ij}^{2}-E\left[x_{ij}^{2}\right]\right)\left(x_{ji}^{2}-E\left[x_{ji}^{2}\right]\right)\right]}{c_{ij}^{4}}\\&=\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{E\left[x_{ij}^{2}x_{ji}^{2}\right]-E\left[x_{ij}^{2}\right]E\left[x_{ji}^{2}\right]}{c_{ij}^{4}}\end{aligned}$$

Given that:

$$E\left[x_{ij}^{2}x_{ji}^{2}\right]=E\left[x_{ij}^{2}\left(c_{ij}-x_{ij}\right)^{2}\right]=c_{ij}^{2}E\left[x_{ij}^{2}\right]-2c_{ij}E\left[x_{ij}^{3}\right]+E\left[x_{ij}^{4}\right]$$

$$E\left[x_{ij}^{2}\right]E\left[x_{ji}^{2}\right]=E\left[x_{ij}^{2}\right]E\left[\left(c_{ij}-x_{ij}\right)^{2}\right]=c_{ij}^{2}E\left[x_{ij}^{2}\right]-2c_{ij}E\left[x_{ij}\right]E\left[x_{ij}^{2}\right]+E^{2}\left[x_{ij}^{2}\right]$$

Then,

$$
\begin{aligned}
Cov\left(x_{ij}^2, x_{ji}^2\right) &= E\left[x_{ij}^4\right] - 2c_{ij}E\left[x_{ij}^3\right] + 2c_{ij}E\left[x_{ij}\right]E\left[x_{ij}^2\right] - E^2\left[x_{ij}^2\right] \\
&= c_{ij}\left(\pi_{ij} - 7\pi_{ij}^2 + 12\pi_{ij}^3 - 6\pi_{ij}^4\right) + c_{ij}^2\left(-2\pi_{ij} + 12\pi_{ij}^2 - 20\pi_{ij}^3 + 10\pi_{ij}^4\right) + \\
&\quad c_{ij}^3\left(-4\pi_{ij}^2 + 8\pi_{ij}^3 - 4\pi_{ij}^4\right) = s_{ij} = s_{ji}
\end{aligned}
$$

since the third moment about zero for a binomially distributed variable equals (Johnson, Kotz, & Kemp, 1992)

$$
E\left[x_{ij}^3\right] = c_{ij}\pi_{ij} + 3c_{ij}\left(c_{ij}-1\right)\pi_{ij}^2 + c_{ij}\left(c_{ij}-1\right)\left(c_{ij}-2\right)\pi_{ij}^3
$$

It can be shown that

$$
\begin{aligned}
q_{ij} + q_{ji} + 2s_{ij} &= 4c_{ij}\left(\pi_{ij} - 7\pi_{ij}^2 + 12\pi_{ij}^3 - 6\pi_{ij}^4\right) + 8c_{ij}^2\left(-\pi_{ij} + 6\pi_{ij}^2 - 10\pi_{ij}^3 + 5\pi_{ij}^4\right) + \\
&\quad 4c_{ij}^3\left(\pi_{ij} - 5\pi_{ij}^2 + 8\pi_{ij}^3 - 4\pi_{ij}^4\right)
\end{aligned}
$$

Now we can write,

$$
\begin{aligned}
E\left[\left(\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} x_{ij}^2 / c_{ij}^2\right)^2\right] &= \left(\frac{E\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)+2m}{2}\right)^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{q_{ij}+q_{ji}}{c_{ij}^4} + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{s_{ij}}{c_{ij}^4} \\
&= \left(\frac{E\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)+2m}{2}\right)^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{q_{ij}+q_{ji}+2s_{ij}}{c_{ij}^4}
\end{aligned}
$$

And finally,

$$\sigma^2\left(\hat{\Phi}_r\right)=\frac{\left(E\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)+2m\right)^2}{\left(n\left(n-1\right)-2m\right)^2}+$$

$$\frac{4\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{q_{ij}+q_{ji}+2s_{ij}}{c_{ij}^4}-mE\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)-m^2\right)}{\left(n\left(n-1\right)-2m\right)^2}-E^2\left[\hat{\Phi}_r\right]$$

$$=\frac{4mE\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)+4m^2}{\left(n\left(n-1\right)-2m\right)^2}+$$

$$\frac{4\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{q_{ij}+q_{ji}+2s_{ij}}{c_{ij}^4}-mE\left[\hat{\Phi}_r\right]\left(n\left(n-1\right)-2m\right)-m^2\right)}{\left(n\left(n-1\right)-2m\right)^2}$$

$$=\frac{4\sum_{i=1}^{n}\sum_{j=i+1}^{n}\frac{q_{ij}+q_{ji}+2s_{ij}}{c_{ij}^4}}{\left(n\left(n-1\right)-2m\right)^2}$$

Again, when there are dyads for which $c_{ij} = 0$, the variance can be obtained as follows:

$$\sigma^2\left(\hat{\Phi}_r\right)=\frac{4\sum_{i=1}^{n}\sum_{\substack{j=i+1\\c_{ij}\neq 0}}^{n}\frac{q_{ij}+q_{ji}+2s_{ij}}{c_{ij}^4}}{\left(2z-2m\right)^2}$$

Tables

Table 1. $\hat{\phi}_{ia}$ and $\hat{\phi}_{pj}$ denote individuals' social asymmetry contribution as actors and partners, respectively, while $\hat{\phi}_{ij}$ corresponds to their contribution associated with each partner.

	Individual						
$\hat{\phi}_{ij}$	1	2	3	4	5	6	$\hat{\phi}_{ia}$
1	0	$.004298^{NS}$	$-.014586^{NS}$	$.023900^{**}$	$-.020888^{*}$	$.032006^{NS}$	$.024730^{NS}$
2	$-.004037^{NS}$	0	$-.032211^{**}$	$.034698^{**}$	$.030577^{*}$	$.013927^{**}$	$.042954^{*}$
3	$.018753^{NS}$	$.076526^{**}$	0	$.023771^{**}$	$.057603^{**}$	$.061398^{**}$	$.238051^{**}$
4	$-.017487^{**}$	$-.022455^{**}$	$-.017412^{**}$	0	$-.014866^{**}$	$.002642^{NS}$	$-.069578^{**}$
5	$.030972^{*}$	$-.020714^{*}$	$-.029369^{**}$	$.019221^{**}$	0	$.005154^{NS}$	$.005264^{NS}$
6	$-.021337^{NS}$	$-.011504^{**}$	$-.030046^{**}$	$-.002541^{NS}$	$-.004798^{NS}$	0	$-.070226^{**}$
$\hat{\phi}_{pj}$	$.006864^{NS}$	$.026151^{NS}$	$-.123624^{**}$	$.099049^{**}$	$.047628^{*}$	$.115127^{**}$	.1712

NS = non significant; * = *p* value < .05; ** = *p* value < .01. *P* values were estimated by means of a Monte Carlo procedure with *99,999* simulated matrices under the null hypothesis of complete reciprocation.

Table 2. $\hat{\Phi}_{ij}$ denotes the contribution to overall asymmetry for the dyad *i-j*.

	Individual					
$\hat{\Phi}_{ij}$	1	2	3	4	5	6
1	0	$.00026^{NS}$	$.004167^{NS}$	$.006413^{**}$	$.010084^{*}$	$.010669^{NS}$
2	.00026	0	$.044315^{**}$	$.012243^{**}$	$.009864^{NS}$	$.002423^{**}$
3	.004167	.044315	0	$.006359^{**}$	$.028234^{**}$	$.031352^{**}$
4	.006413	.012243	.006359	0	$.004355^{**}$	$.000101^{NS}$
5	.010084	.009864	.028234	.004355	0	$.000356^{NS}$
6	.010669	.002423	.031352	.000101	.000356	0

NS = non significant; * p value < .05; ** p value < .01. P values were estimated by means of a Monte Carlo procedure with *99,999* simulated matrices under the null hypothesis of complete reciprocation. P-value results are only shown above the main diagonal since matrix is symmetrical.

Figure Captions

Figure 1. The graph shows the values of index Φ_r for different parameter values in which $\pi_{ij} = \pi$ for all $i < j$.

Figure 1.