

## Neutron Skin of $^{208}\text{Pb}$ , Nuclear Symmetry Energy, and the Parity Radius Experiment

X. Roca-Maza,<sup>1,2</sup> M. Centelles,<sup>1</sup> X. Viñas,<sup>1</sup> and M. Warda<sup>3</sup>

<sup>1</sup>*Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Facultat de Física, Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain*

<sup>2</sup>*INFN, sezione di Milano, via Celoria 16, I-20133 Milano, Italy*

<sup>3</sup>*Katedra Fizyki Teoretycznej, Uniwersytet Marii Curie-Skłodowskiej, ul. Radziszewskiego 10, 20-031 Lublin, Poland*

(Received 7 March 2011; published 21 June 2011)

A precise determination of the neutron skin  $\Delta r_{np}$  of a heavy nucleus sets a basic constraint on the nuclear symmetry energy ( $\Delta r_{np}$  is the difference of the neutron and proton rms radii of the nucleus). The parity radius experiment (PREX) may achieve it by electroweak parity-violating electron scattering (PVES) on  $^{208}\text{Pb}$ . We investigate PVES in nuclear mean field approach to allow the accurate extraction of  $\Delta r_{np}$  of  $^{208}\text{Pb}$  from the parity-violating asymmetry  $A_{PV}$  probed in the experiment. We demonstrate a high linear correlation between  $A_{PV}$  and  $\Delta r_{np}$  in successful mean field forces as the best means to constrain the neutron skin of  $^{208}\text{Pb}$  from PREX, without assumptions on the neutron density shape. Continuation of the experiment with higher precision in  $A_{PV}$  is motivated since the present method can support it to constrain the density slope of the nuclear symmetry energy to new accuracy.

DOI: 10.1103/PhysRevLett.106.252501

PACS numbers: 21.10.Gv, 21.65.Ef, 21.30.Fe, 25.30.Bf

New interest in masses and density distributions of nuclei is being prompted by the production of rare isotopes in radioactive beam facilities [1]. Exciting phenomena discovered in these isotopes, such as thick skins, halos, and new shell closures, urge better understanding of neutrons in nuclei. Yet, our knowledge of neutron density distributions is limited even in the stable nuclei. As neutrons are uncharged, neutron densities have been probed mostly by nucleon scattering [2,3],  $\alpha$  scattering [4], and nuclear effects in exotic atoms [5,6]. Even if some of these experiments reach small errors, all hadronic probes require model assumptions to deal with the strong force introducing possible systematic uncertainties.

Parity-violating electron scattering was suggested as a model-independent probe of neutron densities [7]. An electroweak probe is not hindered by the complexity of the strong force, and the reaction mechanism with the nucleus need not be modeled [7–9], similarly to clean electron scattering for nuclear charge densities. The novel parity radius experiment (PREX) at the Jefferson Lab [9,10] aims to measure the parity-violating asymmetry  $A_{PV}$  in polarized electron scattering on  $^{208}\text{Pb}$  to 3% accuracy. This accuracy is estimated to constrain the neutron rms radius  $r_n$  of  $^{208}\text{Pb}$  to 1% [9,10]. Currently,  $r_n$  of  $^{208}\text{Pb}$  is uncertain by  $\sim 2\%$  and data may be model dependent [2–6,11]; in contrast, the charge radius of  $^{208}\text{Pb}$  is accurately known as  $r_{ch} = 5.5010(9)$  fm [12]. In recent years it has been established that the neutron skin thickness  $\Delta r_{np} = r_n - r_p$  (difference of the neutron and proton rms radii) of  $^{208}\text{Pb}$  is strongly correlated with the density dependence of the nuclear symmetry energy around saturation [13–17]. Knowledge of the density dependence of the nuclear symmetry energy is a cornerstone for drip lines, masses, densities, and collective excitations of

neutron-rich nuclei [13–19], flows and multifragmentation in heavy-ion collisions [20,21], and for astrophysical phenomena like supernovae, neutrino emission, and neutron stars [15,22–24]. A constraint from PREX on  $\Delta r_{np}$  of  $^{208}\text{Pb}$  is thus regarded as a landmark for isospin physics. In addition to being important for its own sake, it has broad implications for different communities of nuclear physics and astrophysics. Fostered by the seminal study of Ref. [9], PREX completed an initial run in 2010. First analyses [10] show the validity of the experimental technique, the adequacy of instruments, and that systematic errors are under control. Additional beam time is now under request to attain the planned 3% accuracy in the parity-violating asymmetry  $A_{PV}$  [10].

The direct output of PREX is the value of the asymmetry  $A_{PV}$  at a single scattering angle [9,10]. The neutron rms radius  $r_n$  of the nucleus may be deduced only if a shape for the neutron density such as a two-parameter Fermi function [9] is assumed. A systematic uncertainty in the analysis is unavoidable in this way. Here, we provide a different and accurate strategy to deduce  $r_n$  and  $\Delta r_{np}$  from PREX that removes this problem. By study of parity-violating electron scattering on  $^{208}\text{Pb}$  in successful nuclear mean field (MF) forces of wide use in nuclear research and astrophysical applications, we reveal a high linear relation between  $\Delta r_{np}$  and  $A_{PV}$  that allows one to extract  $r_n$  and  $\Delta r_{np}$  from  $A_{PV}$  model and shape independently. Moreover, our approach unifies the extraction of  $\Delta r_{np}$  from  $A_{PV}$  with the same framework where  $\Delta r_{np}$  is correlated to the symmetry energy. We show that the present method can support PREX to narrow down the value of the density slope of the nuclear symmetry energy to novel accuracy. This result provides a new and important motivation to continue the experiment to increased precision.

Electrons interact with nuclei by exchanging photons and  $Z^0$  bosons. The former mainly couple to protons and the latter to neutrons because, opposite to the nucleon electric charges, the neutron weak charge  $Q_W^n = -1$  is much larger than the proton weak charge  $Q_W^p = 1 - 4\sin^2\theta_W \approx 0.075$  ( $\theta_W$  being the Weinberg angle). Therefore, electron scattering can probe both the electric and the weak charge distributions in a nucleus [7–9]. PREX measures the elastic differential cross sections  $d\sigma_{\pm}/d\Omega$  for incident electrons of positive or negative helicity. The parity-violating asymmetry,

$$A_{PV} = \left( \frac{d\sigma_+}{d\Omega} - \frac{d\sigma_-}{d\Omega} \right) / \left( \frac{d\sigma_+}{d\Omega} + \frac{d\sigma_-}{d\Omega} \right) \quad (1)$$

for massless electrons (it is  $m_e/p_e \approx 0.0005$  at PREX energy), is sensitive to the parity-violating term induced by the weak interaction in the scattering amplitude. According to their helicity, electrons interact with a potential  $V_{\text{Coulomb}}(r) \pm G_F \rho_W(r)/2^{3/2}$ , with  $G_F$  the Fermi constant and  $\rho_W$  the weak density of the target [7–9]. We solve the associated Dirac equation via the exact phase-shift analysis in distorted-wave Born approximation (DWBA) [11] to compute  $A_{PV}$ . Our benchmarks are the pointlike densities of protons  $\rho_p(r)$  and neutrons  $\rho_n(r)$  calculated self-consistently in MF models. We fold  $\rho_p(r)$  and  $\rho_n(r)$  with electromagnetic proton and neutron form factors to obtain the charge density [11], and with electric form factors for the coupling to a  $Z^0$  to obtain the weak density [9,11,25]:  $\rho_W(r) = \int d\mathbf{r}' \{ 4G_n^{Z^0}(r') N \rho_n(|\mathbf{r} - \mathbf{r}'|) + 4G_p^{Z^0}(r') Z \rho_p(|\mathbf{r} - \mathbf{r}'|) \}$ .

Though not useful for realistic calculations, it is worth recalling the Born approximation (BA) to  $A_{PV}$  [7,9],

$$A_{PV}^{\text{BA}} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}} \left[ 4\sin^2\theta_W - 1 + \frac{F_n(q)}{F_p(q)} \right], \quad (2)$$

as it nicely illustrates that  $A_{PV}$  relates to the neutron and proton nuclear form factors  $F_{n,p}(q)$ . Furnstahl [14] showed that  $F_n(q) = (4\pi)^{-1} \int d^3r j_0(qr) \rho_n(r)$  is at low momentum transfer  $q$  strongly correlated with  $r_n$  of  $^{208}\text{Pb}$  in nuclear MF models, evidencing that PREX would directly constrain the neutron radius and the symmetry energy. Realistic DWBA calculations of  $A_{PV}$  in MF models can be found in [8,11,25,26].

At the optimal kinematics of PREX the electron beam energy is 1.06 GeV and the scattering angle is  $5^\circ$  ( $q_{\text{lab}} \approx 0.47 \text{ fm}^{-1}$ ) [10]. We compute  $A_{PV}$  in DWBA at this kinematics in a comprehensive large sample of 47 nuclear MF interactions. We display the results in Fig. 1 as a function of the neutron rms radius of  $^{208}\text{Pb}$ . To prevent eventual biases in our study, we avoid including more than two models of the same kind fitted by the same authors and protocol. We also avoid models yielding a charge radius of  $^{208}\text{Pb}$  away from experiment [12] by more than 1% (same level as the 1% pursued by PREX in  $r_n$ ). The considered

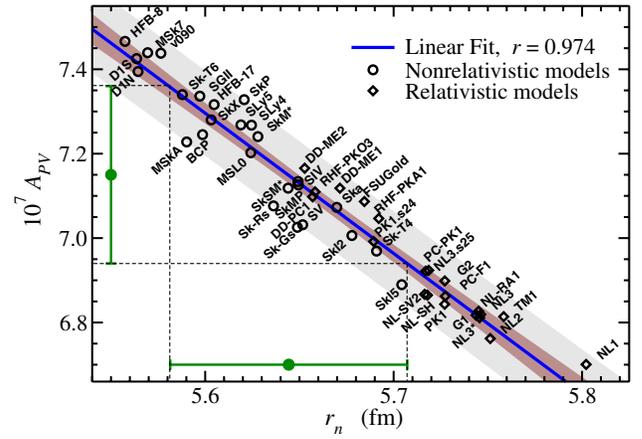


FIG. 1 (color online). Parity-violating asymmetry for  $^{208}\text{Pb}$  at the kinematics of PREX against the neutron radius of  $^{208}\text{Pb}$  in nuclear models. The linear fit is  $10^7 A_{PV} = 25.83 - 3.31 r_n$ . The inner (outer) colored regions depict the loci of the 95% confidence (prediction) bands of the regression (see, e.g., Chap. 3 of [27]). An assumed sample measurement  $A_{PV} = 0.715$  ppm of 3% accuracy and its projection on the  $r_n$  axis are also drawn.

models rest on very different theoretical grounds, from *nonrelativistic* models of zero range (models HFB, v090, and those starting with S or M) or finite range (D1S, D1N, BCP), to *relativistic* models with meson self-interactions (NL and PK models, FSUGold, G1, G2, TM1), density-dependent vertices (DD-ME, RHF-PK) or point couplings (DD-PC1, PC-PK1, PC-PF1) [11,17–19]. (NL3.s25 and PK1.s24 are variants of NL3 and PK1 giving  $\Delta r_{np} = 0.25$  and  $0.24$  fm in  $^{208}\text{Pb}$ .) All such models accurately describe general properties of nuclei such as binding energies and charge radii along the periodic table. However, one readily sees in Fig. 1 that the predicted  $r_n$  of  $^{208}\text{Pb}$  varies largely, from 5.55 to 5.8 fm, as the isovector channel of the nuclear models is little constrained by current phenomenology. The models with softer (stiffer) symmetry energy at saturation density [11] yield smaller (larger)  $r_n$  and larger (smaller)  $A_{PV}$ . One notes that the information encoded in the models implies a value of about 0.67–0.75 ppm for  $A_{PV}$  at PREX kinematics. A significant linear trend is found between  $A_{PV}$  and  $r_n$  (the correlation coefficient is  $r = 0.974$ ).

As the experimental value of  $A_{PV}$  is not yet available, we have chosen for study a plausible test value  $A_{PV} = 0.715$  ppm of 3% accuracy, depicted in Fig. 1. The assumed sample measurement of  $A_{PV}$  determines through the linear fit shown in Fig. 1 a fiducial neutron rms radius  $r_n = 5.644 \pm 0.065$  fm, within typical values deduced from hadronic probes [2–6]. Note that a 3% accuracy in  $A_{PV}$  does lead to  $\sim 1\%$  accuracy in  $r_n$ , thereby supporting the expectations of PREX. It is to be pointed out that the analysis described in this Letter is actually independent of the exact value of the parity-violating asymmetry. Thus, once the experimental value of PREX is known, one can

repeat the same type of analysis using the actual  $A_{PV}$  instead of our test value. We also plot in Fig. 1 the confidence band of the regression (boundary of the possible straight lines) and the so-called prediction band (the wider band that basically coincides with the envelope of the models in the figure) at 95% confidence level [27].

While one first thinks of using a PREX extraction of  $r_n$  to constrain  $\Delta r_{np}$  of  $^{208}\text{Pb}$ , we show in Fig. 2 that  $A_{PV}$  and  $\Delta r_{np}$  have themselves a very high linear dependence (the correlation coefficient is 0.995). The small fluctuation of  $A_{PV}$  with the charge density is more effectively removed by analyzing  $A_{PV}$  vs  $r_n - r_p$ . Actually, the correlation of  $A_{PV}$  and  $\Delta r_{np}$  is implicit in the BA. That is, expanding Eq. (2) at  $q \rightarrow 0$  yields  $F_n(q)/F_p(q) \rightarrow 1 - (r_n + r_p)(r_n - r_p)q^2/6$ , which is driven by  $r_n - r_p$  ( $r_n + r_p \approx 11.1$  fm changes by less than 3% in the models). Though Coulomb distortions correct  $A_{PV}$  by more than 30%–40%, the correlation prevails in the DWBA result. One sees in Fig. 2 that any nuclear model accurately calibrated to masses and charge radii nearly falls on the best-fit line and that the confidence band of the regression is very narrow. Looking at Fig. 1, it can be realized that different models, similarly successful for the well-known observables, can give the same  $A_{PV}$  with different  $r_n$  (cf. MSkA, BCP, and SkM\*; Sk-Rs, Ska, and FSUGold; SkI5 and G2), but almost the same  $\Delta r_{np}$  are obtained with these forces. That the prediction band of the regression is wider horizontally in Fig. 1 than in Fig. 2 points to the same fact. Thus, one expects more accurate estimates of neutron observables using the correlation of Fig. 2. Having found  $\Delta r_{np}$ , one can get  $r_n$  by unfolding the finite size of the proton charge from the accurate  $^{208}\text{Pb}$  charge radius [12]. We note that our analysis allows one to deduce  $\Delta r_{np}$  and  $r_n$  from  $A_{PV}$  without assuming any

particular shape for the nucleon density profiles. Altogether, we believe our results firmly back the commissioning of an improved PREX run where  $A_{PV}$  can be measured more accurately. The present method will permit one to retain in  $\Delta r_{np}$  and  $r_n$  most of the experiment's accuracy. As recently proposed [26], if  $r_n$  is first precisely known, then a second measurement can be made at higher energy to constrain the surface thickness of the neutron density of  $^{208}\text{Pb}$ .

The correlation of  $A_{PV}$  with  $\Delta r_{np}$  is universal in the realm of mean field theory as it is based on widely different nuclear functionals. It is of interest to get further indications on it by looking at existing experiments. The  $^{208}\text{Pb}$  neutron densities found via proton elastic scattering at 0.8 GeV in [2] and 0.3 GeV in [3] were both deduced from the data in a way consistent with the experimental charge density of  $^{208}\text{Pb}$  (known by electron elastic scattering). We computed  $A_{PV}$  using the neutron and charge densities quoted in these works and plotted the results in Fig. 2 against the central  $\Delta r_{np}$  value of each experiment (0.14 fm in [2] and 0.21 fm in [3]). We did the same with the data deduced from the antiprotonic  $^{208}\text{Pb}$  atom [5] (now using the Fermi nucleon densities of Table VI of [5]). It is seen that the theoretical correlation of the models nicely agrees with these points. Our test value  $A_{PV} = 0.715$  ppm of 3% accuracy from PREX would give  $\Delta r_{np}$  as  $0.195 \pm 0.057$  fm (see Fig. 2). As reviewed in [11], we may recall that the recent constraints from strong probes, isospin diffusion, and pygmy dipole resonances favor a range 0.15–0.22 fm for the *central* value of  $\Delta r_{np}$  ( $^{208}\text{Pb}$ ). Recent information on the nuclear equation of state derived from observed masses and radii of neutron stars suggests a similar range 0.14–0.20 fm [24,28].

Finally, we analyze how PREX can constrain the density dependence of the nuclear symmetry energy  $E_{\text{sym}}(\rho)$  around normal density  $\rho_0$ , which is characterized by the slope coefficient  $L = 3\rho_0 \partial E_{\text{sym}}(\rho) / \partial \rho |_{\rho_0}$  in the literature [17–21]. A larger  $L$  value implies a higher pressure in neutron matter and a thicker neutron skin in  $^{208}\text{Pb}$ . Interest in  $L$  permeates many areas of active research, such as the structure and the reactions of neutron-rich nuclei [15–21], the physics of neutron stars [22–24], and events like giant flares [29] and gravitational radiation from neutron stars [30]. The available empirical estimates span a rather loose range  $30 \leq L \leq 110$  MeV, with the recent constraints seemingly agreeing on a value around  $L \sim 60$  MeV with  $\pm 25$  MeV spread [17–21]. A microscopic calculation with realistic nucleon-nucleon potentials and three-body forces predicts  $L = 66.5$  MeV [31]. Figure 3 displays the correlation between  $\Delta r_{np}$  ( $^{208}\text{Pb}$ ) and  $L$  [17–19] in the present analysis. Imposing the previous constraint  $\Delta r_{np} = 0.195 \pm 0.057$  fm yields  $L = 64 \pm 39$  MeV. While the central value depends on our test assumption  $A_{PV} = 0.715$  ppm, the spread following from a determination of  $A_{PV}$  to 3% accuracy essentially

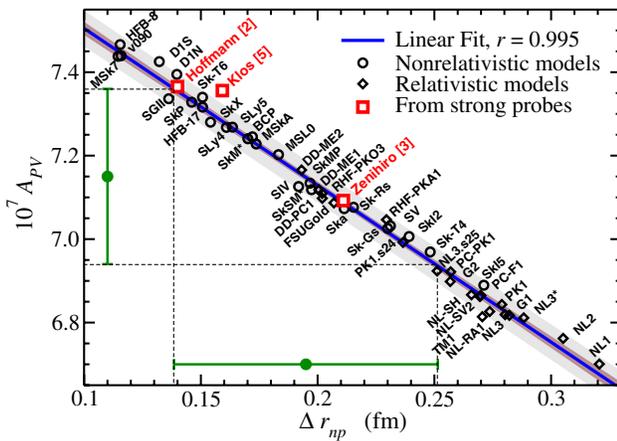


FIG. 2 (color online). Same as Fig. 1 against the neutron skin of  $^{208}\text{Pb}$ . The linear fit is  $10^7 A_{PV} = 7.88 - 3.75 \Delta r_{np}$ . The correlation is found to be quite stable; for example, if we remove the forces excluded by the depicted test constraint, then  $r = 0.990$ . The figure also shows the points calculated with the neutron densities deduced from experiment in Refs. [2,3,5].

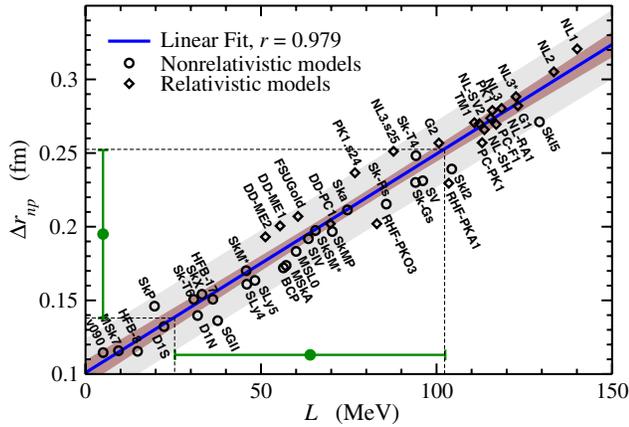


FIG. 3 (color online). Neutron skin of  $^{208}\text{Pb}$  against slope of the symmetry energy. The linear fit is  $\Delta r_{np} = 0.101 + 0.00147L$ . A sample test constraint from a 3% accuracy in  $A_{PV}$  is drawn.

does not. Then, we have to conclude that a 3% accuracy in  $A_{PV}$  sets modest constraints on  $L$ , implying that some of the expectations that this measurement will constrain  $L$  precisely may have to be revised to some extent. To narrow down  $L$ , though demanding more experimental effort, a  $\sim 1\%$  measurement of  $A_{PV}$  should be sought ultimately in PREX. Our approach can support it to yield a new accuracy near  $\delta\Delta r_{np} \sim 0.02$  fm and  $\delta L \sim 10$  MeV, well below any previous constraint. Moreover, PREX is unique in that the central value of  $\Delta r_{np}$  and  $L$  follows from a probe largely free of strong force uncertainties.

In summary, PREX ought to be instrumental to pave the way for electroweak studies of neutron densities in heavy nuclei [9,10,26]. To accurately extract the neutron radius and skin of  $^{208}\text{Pb}$  from the experiment requires a precise connection between the parity-violating asymmetry  $A_{PV}$  and these properties. We investigated parity-violating electron scattering in nuclear models constrained by available laboratory data to support this extraction without specific assumptions on the shape of the nucleon densities. We demonstrated a linear correlation, universal in the mean field framework, between  $A_{PV}$  and  $\Delta r_{np}$  that has very small scatter. Because of its high quality, it will not spoil the experimental accuracy even in improved measurements of  $A_{PV}$ . With a 1% measurement of  $A_{PV}$  it can allow one to constrain the slope  $L$  of the symmetry energy to near a novel 10 MeV level. A mostly model-independent determination of  $\Delta r_{np}$  of  $^{208}\text{Pb}$  and  $L$  should have enduring impact on a variety of fields, including atomic parity nonconservation and low-energy tests of the standard model [8,9,32].

We thank G. Colò, A. Polls, P. Schuck, and E. Vives for valuable discussions, H. Liang for the densities of the RHF-PK and PC-PK models, and K. Kumar for information on PREX kinematics. Work supported by the Consolider Ingenio Programme CPAN CSD2007 00042

and Grants No. FIS2008-01661 from MEC and FEDER, No. 2009SGR-1289 from Generalitat de Catalunya, and No. N N202 231137 from Polish MNiSW.

- [1] Special issue on The Fifth International Conference on Exotic Nuclei and Atomic Masses ENAM'08, edited by M. Pfützner [Eur. Phys. J. A **42**, 299 (2009)].
- [2] G. W. Hoffmann *et al.*, Phys. Rev. C **21**, 1488 (1980).
- [3] J. Zenihiro *et al.*, Phys. Rev. C **82**, 044611 (2010).
- [4] A. Krasznahorkay *et al.*, Nucl. Phys. **A731**, 224 (2004).
- [5] B. Klos *et al.*, Phys. Rev. C **76**, 014311 (2007).
- [6] E. Friedman, Hyperfine Interact. **193**, 33 (2009).
- [7] T. W. Donnelly, J. Dubach, and Ingo Sick, Nucl. Phys. **A503**, 589 (1989).
- [8] D. Vretenar *et al.*, Phys. Rev. C **61**, 064307 (2000).
- [9] C.J. Horowitz, S.J. Pollock, P.A. Souder, and R. Michaels, Phys. Rev. C **63**, 025501 (2001).
- [10] K. Kumar, P. A. Souder, R. Michaels, and G. M. Urciuoli, <http://hallaweb.jlab.org/parity/prex> (see section "Status and Plans" for latest updates).
- [11] M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. C **82**, 054314 (2010).
- [12] I. Angeli, At. Data Nucl. Data Tables **87**, 185 (2004).
- [13] B. A. Brown, Phys. Rev. Lett. **85**, 5296 (2000); S. Typel and B. A. Brown, Phys. Rev. C **64**, 027302 (2001).
- [14] R. J. Furnstahl, Nucl. Phys. **A706**, 85 (2002).
- [15] A. W. Steiner, M. Prakash, J. M. Lattimer, and P. J. Ellis, Phys. Rep. **411**, 325 (2005).
- [16] B. G. Todd-Rutel and J. Piekarewicz, Phys. Rev. Lett. **95**, 122501 (2005).
- [17] M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. **102**, 122502 (2009); M. Warda, X. Viñas, X. Roca-Maza, and M. Centelles, Phys. Rev. C **80**, 024316 (2009).
- [18] A. Carbone *et al.*, Phys. Rev. C **81**, 041301(R) (2010).
- [19] L. W. Chen *et al.*, Phys. Rev. C **82**, 024321 (2010).
- [20] B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008).
- [21] M. B. Tsang *et al.*, Phys. Rev. Lett. **102**, 122701 (2009).
- [22] C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. **86**, 5647 (2001).
- [23] J. Xu *et al.*, Astrophys. J. **697**, 1549 (2009).
- [24] A. W. Steiner, J. M. Lattimer, and E. F. Brown, Astrophys. J. **722**, 33 (2010).
- [25] O. Moreno, E. Moya de Guerra, P. Sarriguren, and J. M. Udías, J. Phys. G **37**, 064019 (2010).
- [26] S. Ban, C. J. Horowitz, and R. Michaels, arXiv:1010.3246.
- [27] N. R. Draper and H. Smith, *Applied Regression Analysis* (Wiley, New York, 1998), 3rd ed.
- [28] K. Hebeler, J. M. Lattimer, C. J. Pethick, and A. Schwenk, Phys. Rev. Lett. **105**, 161102 (2010).
- [29] A. W. Steiner and A. L. Watts, Phys. Rev. Lett. **103**, 181101 (2009).
- [30] D. H. Wen, B. A. Li, and P. G. Krastev, Phys. Rev. C **80**, 025801 (2009).
- [31] I. Vidaña, C. Providência, A. Polls, and A. Rios, Phys. Rev. C **80**, 045806 (2009).
- [32] T. Sil *et al.*, Phys. Rev. C **71**, 045502 (2005).