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Decision-making techniques with similarity measures and OWA operators

José M. Merigó[1] and Anna M. Gil-Lafuente

Abstract

We analyse the use of the ordered weighted average (OWA) in decision-making giving special attention to business and economic decision-making problems. We present several aggregation techniques that are very useful for decision-making such as the Hamming distance, the adequacy coefficient and the index of maximum and minimum level. We suggest a new approach by using immediate weights, that is, by using the weighted average and the OWA operator in the same formulation. We further generalize them by using generalized and quasi-arithmetic means. We also analyse the applicability of the OWA operator in business and economics and we see that we can use it instead of the weighted average. We end the paper with an application in a business multi-person decision-making problem regarding production management.

MSC: 62C86

Keywords: Decision-making, OWA operator, similarity measure, Hamming distance, production management.

1. Introduction

Decision-making problems are very common in the literature (Canós and Liern 2008; Figueira et al 2005; A.M. Gil-Lafuente and Merigó 2010; Torra and Narukawa 2007; Wei 2009; Wei et al 2010; 2011). They are very useful in a lot of situations because people are almost all the time taking decisions. Sometimes, they take unconscious decisions or sometimes they simply take the usual decisions of their lives such as what to eat, what to see on TV and so on. In business and economics, people and organizations also take decisions almost all the time. Sometimes, they take decisions on how to do or

[1] Corresponding author: Tel: +34 93 402 19 62; Fax: +34 93 403 98 82. jmerigo@ub.edu
Department of Business Administration, University of Barcelona, Av. Diagonal 690, 08034, Barcelona, Spain.
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improve their work or sometimes the decisions are more global and affect a lot of decision-makers. Obviously, in these situations we also find a lot of unconscious decisions.

For the development of the decision-making process we can use a lot of tools for taking decisions such as individual decision-making, group decision-making, multi person decision-making, sequential decision-making and different statistical techniques. Among the different statistical techniques that we can use in decision-making, a very useful one is the aggregation operator because it permits to aggregate the information and obtain a single result that permits to continue with the decision process and make the decision. It is worth noting the ordered weighted averaging (OWA) operator (Yager 1988). It is a tool that provides a parameterized family of aggregation operators between the maximum and the minimum. Since its appearance, the OWA operator has been used in a wide range of studies and applications (Merigó et al 2010; Xu 2005; Xu and Da 2003; Yager 1993; 2004a; Yager and Kacprzyk 1997; Yager, Kacprzyk and Beliakov 2011; Zhao et al 2010; Zhou and Chen 2010).

In business and economics, it is very useful to use different similarity measures that also use aggregation operators such as the Hamming distance (Hamming, 1950). The Hamming distance is a very useful tool in decision-making because it permits to compare the available results with some ideal ones that are supposed to be the best ones. This is especially useful because, depending on the particular problem we are looking at, the best results are not always the best for the decision-maker because he may have different interests. An extreme example of this would be the concept of dumping, which means that the seller is selling the product with a price that is lower than its production cost. Thus, in the decision process of fixing this price, the seller obviously is looking for an ideal that it is not the best one. In the literature, we find a lot of studies that analyse the concept of the Hamming distance (Karayiannis 2000; Kaufmann 1975; Kaufmann and Gil-Aluja 1986; 1987; Xu 2010a; 2010b).

Recently, several authors (Karayiannis 2000; Merigó 2008; Merigó and Casanovas 2010a; 2011a; Merigó and A.M. Gil-Lafuente 2007; 2010; Xu and Chen 2008) have analysed the use of the OWA operator in the Hamming distance. We can refer to this new aggregation operator as the ordered weighted averaging distance (OWAD) operator. Its main advantage is that it provides a parameterized family of distance aggregation operators between the maximum and the minimum distance. The OWAD operator can be further extended by using other types of distances such as the Euclidean distance, the Minkowski distance and the quasi-arithmetic distance (Karayiannis 2000; Merigó 2008; Merigó and Casanovas 2011b; 2011c; Merigó and A.M. Gil-Lafuente 2011).

Other similarity measures that are very useful in business and economics are the adequacy coefficient (Kaufmann and Gil-Aluja 1986; 1987) and the index of maximum and minimum level (J. Gil-Lafuente 2001; 2002). The adequacy coefficient is an extension of the Hamming distance that analyses the results that are higher than the ideal by using a t-norm. This approach can also be extended by using the OWA operator, obtaining the ordered weighted averaging adequacy coefficient (OWAAC) operator

(Merigó and A.M. Gil-Lafuente 2010). Further developments on the OWAAC can be found in Merigó (2008) and Merigó et al. (2011a). The index of maximum and minimum level is a model that uses the Hamming distance and the adequacy coefficient in the same formulation using the one that is more appropriate for each variable considered. This tool can also be extended by using the OWA operator, forming the ordered weighted averaging index of maximum and minimum level (OWAIMAM) operator.

The aim of this paper is to introduce new decision-making techniques based on the use of the OWA operator and the weighted average in order to obtain a formulation that it is able to deal with the subjective beliefs of the decision-maker and with his attitudinal character. For doing so, we use the concept of immediate probabilities (Engemann et al 1996; Yager et al 1995) applied in situations where we use weighted averages instead of probabilities. Thus, we obtain the concept of immediate weights. We suggest the use of immediate weights with the OWAD operator, the OWAAC operator and the OWAIMAM operator. Therefore, we get the immediate weighted OWAD (IWOWAD), the immediate weighted OWAAC (IWOWAAC) and the immediate weighted OWAIMAM (IWOWAIMAM) operator. The main advantage of these similarity measures is that they are able to deal with the weighted Hamming distance and with the OWAD operator in the same formulation. Thus, we are able to represent the information in a more complete way because we can consider the degree of importance of the characteristics and the degree of "orness", that is, the tendency of the aggregation to the minimum or to the maximum. Thus, we can under or over estimate the results according to the interests we have in the aggregation.

We also extend this analysis by using generalized and quasi-arithmetic means, obtaining the generalized IWOWAD (GIWOWAD), the generalized IWOWAAC (GIWOWAAC) and the generalized IWOWAIMAM (GIWOWAIMAM) operator. The main advantage of these new generalizations is that they include a wide range of particular cases, including the usual arithmetic, geometric and quadratic aggregations. Thus, we obtain a more general formulation that permits to analyse the aggregation problem from different contexts.

We also analyse a wide range of applications that can be developed. Specially, we focus on a wide range of decision-making problems that can be implemented in business and economic scenarios. We study a business decision-making problem in production management by using different multi-person decision-making techniques based on the OWA operator such as the IWOWAD operator, the IWOWAAC operator and the IWOWAIMAM operator. Thus, we are able to construct new aggregation operators including the multi-person IWOWAD (MP-IWOWAD), the multi-person IWOWAAC (MP-IWOWAAC) and the multi-person IWOWAIMAM (MP-IWOWAIMAM). The main advantage of these aggregation methods is that they are able to deal with the opinion of several persons in the analysis providing results that represents the aggregated information of the group. We see that each method provides different results depending on the interests of the decision-maker. Therefore, we see that the results may lead to different decisions depending on the particular type of aggregation operator used. Thus,

we get a general overview of the different scenarios that may occur and select the one that is in more accordance with our interests.

The paper is organized as follows. In Section 2, we briefly review some basic decision-making techniques such as the Hamming distance, the adequacy coefficient, the index of maximum and minimum level and the OWA operator. Section 3 presents new decision-making techniques based on the use of immediate weights. Section 4 summarizes different applications that can be developed with the OWA operator in business and economics. In Section 5, we present a particular problem in a decision-making problem about the selection of production strategies in a company. In Section 6 we present a numerical example and in Section 7 we summarize the main conclusions of the paper.

2. Preliminaries

In this section, we briefly review some basic concepts to be used throughout the paper such as the Hamming distance, the adequacy coefficient, the index of maximum and minimum level and their extensions with the OWA operator.

2.1. The Hamming distance

The Hamming distance (Hamming 1950) is a useful technique for calculating the differences between two elements, two sets, etc. In fuzzy set theory, it can be useful, for example, for the calculation of distances between fuzzy sets, interval-valued fuzzy sets, intuitionistic fuzzy sets and interval-valued intuitionistic fuzzy sets. For two sets A and B, the weighted Hamming distance can be defined as follows.

Definition 1 *A weighted Hamming distance of dimension n is a mapping d_{WH}: $R^n \times R^n \to R$ that has an associated weighting vector W of dimension n with the sum of the weights being 1 and $w_j \in [0,1]$ such that:*

$$d_{WH}(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{i=1}^{n} w_i |x_i - y_i|, \tag{1}$$

where x_i and y_i are the ith arguments of the sets X and Y.

Note that the formulations shown above are the general expressions. For the formulation used in fuzzy set theory, see for example (Kaufmann 1975). Note also that if $w_i = 1/n$, for all i, then, the weighted Hamming distance becomes the normalized Hamming distance.

2.2. The adequacy coefficient

The adequacy coefficient (Kaufmann and Gil-Aluja 1986; 1987) is an index used for calculating the differences between two elements, two sets, etc. It is very similar to the Hamming distance with the difference that it neutralizes the result when the comparison shows that the real element is higher than the ideal one. For two sets A and B, the weighted adequacy coefficient can be defined as follows.

Definition 2 *A weighted adequacy coefficient of dimension n is a mapping* $K : [0,1]^n \times [0,1]^n \to [0,1]$ *that has an associated weighting vector W of dimension n with the sum of the weights 1 and* $w_j \in [0,1]$ *such that:*

$$K(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{i=1}^{n} w_i[1 \wedge (1 - x_i + y_i)], \tag{2}$$

where x_i *and* y_i *are the ith arguments of the sets X and Y.*

Note that if $w_i = 1/n$, for all i, then, the weighted adequacy coefficient becomes the normalized adequacy coefficient.

2.3. The index of maximum and minimum level

The index of maximum and minimum level is an index that unifies the Hamming distance and the adequacy coefficient in the same formulation (J. Gil-Lafuente 2001; 2002). For two sets A and B, the weighted index of maximum and minimum level can be defined as follows.

Definition 3 *A WIMAM of dimension n is a mapping* $K : [0,1]^n \times [0,1]^n \to [0,1]$ *that has an associated weighting vector W of dimension n with the sum of the weights 1 and* $w_j \in [0,1]$ *such that:*

$$\eta(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_u Z_i(u) \times |x_i(u) - y_i(u)| + \sum_v Z_i(v) \times [0 \vee (x_i(v) - y_i(v))], \tag{3}$$

where x_i *and* y_i *are the ith arguments of the sets X and Y.*

Note that if $w_i = 1/n$, for all i, then, the weighted index of maximum and minimum level becomes the normalized index of maximum and minimum level.

2.4. The OWA operator

The OWA operator (Yager 1988) provides a parameterized family of aggregation operators which have been used in many applications (Beliakov et al 2007; Merigó 2008; Xu 2005; Yager 1993; Yager and Kacprzyk 1997). It can be defined as follows.

Definition 4 *An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:*

$$\mathrm{OWA}(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{4}$$

where $b_j$is the jth largest of the a_i.

The OWAD operator (Merigó 2008; Merigó and A.M. Gil-Lafuente 2007; 2010) is an aggregation operator that uses OWA operators and distance measures in the same formulation. In this subsection, we focus on the Hamming distance. However, it is worth noting that it is also possible to use other types of distance measures with the OWA operator such as the Euclidean or the Minkowski distance (Merigó 2008). It can be defined as follows for two sets X and Y.

Definition 5 *An OWAD operator of dimension n is a mapping OWAD: $R^n \times R^n \to R$ that has an associated weighting vector W, with $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$ such that:*

$$\mathrm{OWAD}(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{j=1}^{n} w_j D_j, \tag{5}$$

where D_j represents the jth largest of the $|x_i - y_i|$.

The OWAAC operator (Merigó and A.M. Gil-Lafuente 2010) is an aggregation operator that uses the adequacy coefficient and the OWA operator in the same formulation. It can be defined as follows for two sets X and Y.

Definition 6 *An OWAAC operator of dimension n is a mapping OWAAC: $[0,1]^n \times [0,1]^n \to [0,1]$ that has an associated weighting vector W, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:*

$$\mathrm{OWAAC}(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{j=1}^{n} w_j K_j, \tag{6}$$

where K_j represents the jth largest of $[1 \wedge (1 - x_i + y_i)], x_i, y_i \in [0,1]$.

The OWAIMAM operator (Merigó 2008; Merigó et al. 2011b) is an aggregation operator that uses the Hamming distance, the adequacy coefficient and the OWA operator in the same formulation. It can be defined as follows.

Definition 7 *An OWAIMAM operator of dimension n, is a mapping OWAIMAM: $[0,1]^n \times [0,1]^n \to [0,1]$ that has an associated weighting vector W, with $w_j \in [0,1]$ and the sum of the weights is equal to 1, such that:*

$$\mathrm{OWAIMAM}\ (\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \sum_{j=1}^{n} w_j K_j, \tag{7}$$

where K_j represents the jth largest of all the $|x_i - y_i|$ and the $[0 \vee (x_i - y_i)]$.

3. Distance measures with immediate weights

In this section, we introduce a new approach for dealing with distance measures where we use the weighted average and the OWA operator in the same formulation. For doing so, we extend the concept of immediate probabilities (Engemann et al 1996; Merigó 2008; 2010; Yager et al 1995) for situations where we use the weighted average. Thus, instead of using immediate probabilities, we will use immediate weights in the analysis. Extending this to the use of distance measures implies the introduction of new distance and similarity measures such as the immediate weighted OWA distance (IWOWAD), the immediate weighted OWAAC (IWOWAAC) and the immediate weighted OWAIMAM (IWOWAIMAM) operator. The main advantage of these new models is that they can consider the information used in the weighted average (degree of importance) and in the OWA operator (degree of orness or optimism) in the same formulation. Thus, we get a more general formulation that is able to represent the information in a more complete way because in real world problems, it is very common that we have to combine in the same problem situations with weighted averages and with OWA operators. Before defining these three new distance aggregation operators let us recall the concept of immediate probabilities applied to the weighted average, that is, the immediate weights (IW). It can be defined as follows.

Definition 8 *An IW operator of dimension n is a mapping IW: $R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:*

$$\text{IW}\ (a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{8}$$

where b_j is the jth largest of the a_i, each a_i has associated a WA v_i, v_j is the associated WA of b_j, and $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$.

As we can see, if $w_j = 1/n$ for all j, we get the weighted average and if $v_j = 1/n$ for all j, the OWA operator. Now, we extend the measures commented in Section 2.4., by using immediate weights. Thus, for the OWAD operator, we get the IWOWAD operator and it is defined as follows.

Definition 9 *An IWOWAD operator of dimension n is a mapping IWOWAD: $R^n \times R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:*

$$\text{IWOWAD}(\langle x_1, y_1 \rangle, \ldots, \langle x_n, y_n \rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{9}$$

where b_j is the jth largest of the $|x_i - y_i|$, each $|x_i - y_i|$ has associated a WA v_i, v_j is the associated WA of b_j, and $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$.

In this case, if $w_j = 1/n$ for all j, we get the weighted Hamming distance and if $v_j = 1/n$ for all j, the OWAD operator. Note that the IWOWAD operator accomplishes similar properties that the OWAD operator with the exception of commutativity because the use of the weighted average does not allow the commutativity property. Note that if the weighting vector is not normalized, i.e., $\hat{V} = \sum_{j=1}^{n} \hat{v}_j \neq 1$, then, the IWOWAD operator can be expressed as: $IWOWAD/\hat{V}$.

If we use immediate weights in the OWAAC operator, we get the IWOWAAC operator. In this case, we have the same expression than in Eq. (9) with the difference that now b_j is the jth largest of the $[1 \wedge (1 - x_i + y_i)]$, x_i, $y_i \in [0,1]$ and each $[1 \wedge (1 - x_i + y_i)]$ has associated a WA v_i.

As we can see, if $w_j = 1/n$ for all j, we get the weighted adequacy coefficient and if $v_j = 1/n$ for all j, the OWAAC operator. Moreover, if $x_i \geq y_i$, for all i, then, the OWAAC operator becomes the OWAD operator.

Finally, if we use the OWAIMAM operator with immediate weights, we get the IWOWAIMAM operator. Note that we get the same formulation than Eq. (9) with the difference that now b_j is the jth largest of all the $|x_i - y_i|$ and the $[0 \vee (x_i - y_i)]$; x_i, $y_i \in [0,1]$, and each $|x_i - y_i|$ and $[0 \vee (x_i - y_i)]$ has associated a WA v_i.

Furthermore, we can a present a further generalization of the previous measures by using generalized and quasi-arithmetic means (Merigó and Casanovas 2010b; 2010c; 2011d; Merigó and Gil-Lafuente 2009). Note that in this paper we will use generalized means although it is straightforward to extend it by replacing the parameter λ of the generalized mean by the strictly continuous monotonic function g of the quasi-arithmetic mean (Merigó and Gil-Lafuente 2009). By generalizing the IWOWAD operator, we get the generalized IWOWAD (GIWOWAD) operator. It can be defined as follows.

Definition 10 *An GIWOWAD operator of dimension n is a mapping GIWOWAD:* $R^n \times R^n \to R$ *that has an associated weighting vector W of dimension n with* $w_j \in [0,1]$ *and* $\sum_{j=1}^{n} w_j = 1$, *such that:*

$$\text{GIWOWAD}(\langle |x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle) = \left(\sum_{j=1}^{n} \hat{v}_j b_j^{\lambda}\right)^{1/\lambda}, \tag{10}$$

where b_j *is the jth largest of the* $|x_i - y_i|$, *each* $|x_i - y_i|$ *has associated a WA* v_i, v_j *is the associated WA of* b_j, $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$, *and* λ *is a parameter such that* $\lambda \in (-\infty, \infty) - \{0\}$.

Note that if we extend the IWOWAD operator with quasi-arithmetic means we get the quasi-arithmetic IWOWAD (Quasi-IWOWAD) operator as follows:

$$\text{Quasi-IWOWAD}\left(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle\right) = g^{-1}\left(\sum_{j=1}^{n} \hat{v}_j g\left(b_j\right)\right), \tag{11}$$

where $g(b)$ is a strictly continuous monotonic function. Further generalizations in this direction can be developed by using norm aggregations following Yager (2010). If we generalize the IWOWAAC operator, we obtain the generalized IWOWAAC (GIWOWAAC) operator. It can be defined as follows.

Definition 11 *A GIWOWAAC operator of dimension n is a mapping GIWOWAAC:* $[0,1]^n \times [0,1]^n \to [0,1]$ *that has an associated weighting vector W of dimension n with* $w_j \in [0,1]$ *and* $\sum_{j=1}^n w_j = 1$*, such that:*

$$\text{GIWOWAAC}\left(\langle x_1, y_1\rangle, \ldots, \langle x_n, y_n\rangle\right) = \left(\sum_{j=1}^{n} \hat{v}_j b_j^{\lambda}\right)^{1/\lambda}, \tag{12}$$

where b_j *is the jth largest of the* $[1 \wedge (1 - x_i + y_i)]$*,* x_i*,* $y_i \in [0,1]$*, each* $[1 \wedge (1 - x_i + y_i)]$ *has associated a WA* v_i*,* v_j *is the associated WA of* b_j*,* $\hat{v}_j = (w_j v_j / \sum_{j=1}^n w_j v_j)$*, and* λ *is a parameter such that* $\lambda \in (-\infty, \infty) - \{0\}$*.*

And if we extend the IWOWAIMAM operator by using generalized means, we get the generalized IWOWAIMAM (GIWOWAIMAM) operator. It is defined as follows.

Definition 12 *A GIWOWAIMAM operator of dimension n is a mapping GIWOWAIMAM:* $[0,1]^n \times [0,1]^n \to [0,1]$ *that has an associated weighting vector W of dimension n with* $w_j \in [0,1]$ *and* $\sum_{j=1}^n w_j = 1$*, such that:*

$$\text{GIWOWAIMAM}\left(\langle x_1, y_{11}\rangle, \ldots, \langle x_n, y_n\rangle\right) = \left(\sum_{j=1}^{n} \hat{v}_j b_j^{\lambda}\right)^{1/\lambda}, \tag{13}$$

where b_j *is the jth largest of all the* $|x_i - y_i|$ *and the* $[0 \vee (x_i - y_i)]$*;* x_i*,* $y_i \in [0,1]$*, each* $|x_i - y_i|$ *and* $[0 \vee (x_i - y_i)]$ *has associated a WA* v_i*,* v_j *is the associated WA of* b_j*,* $\hat{v}_j = (w_j v_j / \sum_{j=1}^n w_j v_j)$*, and* λ *is a parameter such that* $\lambda \in (-\infty, \infty) - \{0\}$*.*

Note that in these two cases we can also consider the dual. Additionally, if we use quasi-arithmetic means we get the quasi-arithmetic IWOWAAC (Quasi-IWOWAC) and the quasi-arithmetic IWOWAIMAM (Quasi-IWOWAIMAM) operator.

These generalizations include a wide range of particular cases by using different types of weighting vectors and values in the parameter λ. In Table 1, we present some of the main particular cases.

Note that a lot of other families could be studied following the OWA literature for obtaining OWA weights. The main advantage of using these generalizations is that they provide a more robust formulation that includes a wide range of particular cases. Thus, we get a deeper picture of the different results that may occur in the specific problem considered.

Table 1: *Families of GIWOWAD, GIWOWAAC and GIWOWAIMAM operators.*

Particular type	GIWOWAD	GIWOWAAC	GIWOWAIMAM
$w_i = 1/n, \forall\, i$	OWAD	OWAAC	OWAIMAM
$v_i = 1/n, \forall\, i$	WHD	WAC	WIMAM
$g(a) = a^{\lambda}$	Quasi-IWOWAD	Quasi-IWOWAAC	Quasi-IWOWAIMAM
$\lambda = 1$	IWOWAD	IWOWAAC	IWOWAIMAM
$\lambda = 2$	Quadratic IWOWAD	Quadratic IWOWAAC	Quadratic IWOWAIMAM
$\lambda \to 0$	Geometric IWOWAD	Geometric IWOWAAC	Geometric IWOWAIMAM
$\lambda = -1$	Harmonic IWOWAD	Harmonic IWOWAAC	Harmonic IWOWAIMAM
$\lambda = 3$	Cubic IWOWAD	Cubic IWOWAAC	Cubic IWOWAIMAM
$\lambda \to \infty$	Maximum distance	Maximum adequacy coefficient	Maximum IMAM
$\lambda \to -\infty$	Minimum distance	Minimum adequacy coefficient	Minimum IMAM
Etc.			

4. Applicability of the OWA operator in business and economics

The OWA operator is a very useful tool for business and economics because it permits to reflect the attitudinal character (the degree of orness or optimism) of the decision-maker in the aggregation of the information.

First of all, it is clear that the OWA operator plays a key role in decision-making problems by unifying the classical decision criteria under uncertainty, that is, the optimistic criteria, the pessimistic criteria, the Laplace and the Hurwicz criteria. Thus, we can use them in a lot of situations such as individual decision-making, group decision-making, multi-attribute decision-making, multi-criteria decision-making, multi-person decision-making, sequential decision-making and dynamic decision-making.

Moreover, we can apply it in a lot of other decision-making contexts such as probabilistic decision-making (Engemann et al 1996; Yager et al 1995), minimization of regret (Yager 2004b), Dempster-Shafer theory of evidence (Yager 1992; Merigó and Casanovas 2009), analytic hierarchy process (Yager and Kelman 1999), neural networks (Yager 1994) and game theory (Yager 1999).

Focussing on business and economic decision-making, we see that the OWA operators, combined with one or more of the previous methods can be applied in a lot of situations. For example, we could use the OWA operator in business decision-making problems such as financial management, strategic management, human resource management and product management. Inside these business areas, we could use the OWA operator in different ways depending on the particular problem we are analyzing as mentioned in the previous paragraph. For example, in human resource management, we could be looking for a selection process between directors, mid-range jobs, low-range jobs, in public administration, in sports and so on.

When using the OWA operator in economics, we could relate it with political decision-making because they are very much connected. For example, when looking for general economic decisions, these ones have a strong impact in political decision-

making. For example, the economic decisions about the selection of monetary policies, fiscal policies and commercial policies involve both the economic and the political sector. Other economic decisions that could be considered are those that affect the public sector such as decisions from the ministries, decisions from the autonomic authorities and decisions from the local authorities.

Obviously, both business and economic decision-making are very much related and the situations mentioned above could be seen as general framework inside business and economics.

The OWA operator is also useful in a lot of other situations that are not directly related with decision-making. Basically, the OWA operator is very useful in those situations where it can be seen as a statistical technique representing a new type of weighted average. Thus, a lot of business and decisions problems that use some kind of weighted average can be reformulated using the OWA operator. For example, the OWA operator is very useful in statistics and econometrics. Thus, a lot of problems that use the weighted average could be revised including linear regression, multiple regressions and a lot of its extensions and applications. Thus, we see that the OWA operator can be used in a lot of business and economic environments that uses statistical techniques such as business economics, marketing, finance, management science, actuarial science, insurance, behavioural economics, macroeconomics, microeconomics, economic policy, applied economics, accounting, public economics, entrepreneurship, social choice and welfare, economic development, industrial organization, tourism management and sport management.

5. Multi-person decision-making in production management

In the following, we are going to consider a business multi-person decision-making problem in production management. The motivation for using the OWA operator in the selection of production strategies in all different kinds of areas, appears because the decision-maker wants to take the decision with a certain degree of optimism or pessimism rather than with a neutral position. Due to the fact that the traditional methods are neutral against the attitude of the decision-maker, the introduction of the OWA operator in these models can change the neutrality and reflect decisions with different degrees of optimism and pessimism. These techniques can be used in a lot of situations but the general ideas about it is the possibility of under estimate or over estimate the problems in order to get results that reflects this change in the evaluation phase.

The process to follow in the selection of production strategies with the OWA operator, is similar to the process developed in Gil-Aluja (1998) and Kaufmann and Gil-Aluja (1986; 1987) for the selection of human resources with the difference that the instruments used will include the OWA operator in the selection process. Note that similar models that use the OWA operator have been developed for other selection processes (Merigó and A.M. Gil-Lafuente 2010). The five steps to follow are:

Step 1: Analysis and determination of the significant characteristics of the available production strategies. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of finite alternatives, and $C = \{C_1, C_2, \ldots, C_n\}$, a set of finite characteristics (or attributes), forming the matrix $(x_{hi})_{m\times n}$. Let $E = \{E_1, E_2, \ldots, E_p\}$ be a finite set of decision-makers. Let $V = (v_1, v_2, \ldots, v_p)$ be the weighting vector of the weighted average such that $\sum_{k=1}^{p} v_k = 1$ and $v_k \in [0,1]$ and $U = (u_1, u_2, \ldots, u_p)$ be the weighting vector of the decision-makers that $\sum_{k=1}^{p} u_k = 1$ and $u_k \in [0,1]$. Each decision-maker provides their own payoff matrix $(x_{hi}^{(k)})_{m\times n}$.

Step 2: Fixation of the ideal levels of each significant characteristic in order to form the ideal production strategy. That is:

Table 2: *Ideal production strategy.*

	C_1	C_2	$\cdots$	C_i	$\cdot$	C_n
$P =$	x_1	x_2	$\cdots$	x_i	$\cdots$	x_n

where P is the ideal production strategy represented by a fuzzy subset, C_i is the ith characteristic to consider and $x_i \in [0,1]$; $i = 1, 2, \ldots, n$, is the valuation between 0 and 1 for the ith characteristic. Note that we assume that the ideal investment is given as a consensus between the opinions of the experts.

Step 3: Use the weighted average (WA) to aggregate the information of the decision-makers E by using the weighting vector U. The result is the collective payoff matrix $(x_{hi})_{m\times n}$. Thus, $x_{hi} = \sum_{k=1}^{p} u_k \left(x_{hi}^{(k)}\right)$.

Step 4: Comparison between the ideal production strategy and the different production strategies considered, and determination of the level of removal using the OWA operator. That is, changing the neutrality of the results to over estimate or under estimate them. In this step, the objective is to express numerically the removal between the ideal production strategy and the different production strategies considered. For this, it can be used the different available selection indexes such as those explained in the previous sections including the Hamming distance, the adequacy coefficient and the index of maximum and minimum level.

Step 5: Adoption of decisions according to the results found in the previous steps. Finally, we should take the decision about which production strategy select. Obviously, our decision will consist in choosing the production strategy with the best results according to the index used.

Note that when developing this decision process, we can summarize all the calculations in an aggregation process. We can do this with the IWOWAD obtaining the multi-person IWOWAD (MP-IWOWAD), with the IWOWAAC forming the multi-person

IWOWAAC (MP-IWOWAAC) and with the IWOWAIMAM obtaining the multi-person IWOWAIMAM (MP-IWOWAIMAM). They can be defined as follows.

Definition 13 *A MP-IWOWAD operator is an aggregation operator that has a weighting vector U of dimension p with* $\sum_{k=1}^{p} u_k = 1$ *and* $u_k \in [0,1]$*, and a weighting vector W of dimension n with* $\sum_{j=1}^{n} w_j = 1$ *and* $w_j \in [0,1]$*, such that:*

$$\text{MP-IWOWAD}(\langle x_1^1,\ldots,x_1^p),y_1\rangle,\ldots,\langle(x_n^1,\ldots,x_n^p),y_n\rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{14}$$

where b_j *is the* $|x_i - y_i|$ *largest individual distance, each* $|x_i - y_i|$ *has associated a weight* v_i*,* v_j *is the associated weighted average (WA) of* b_j*,* $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$*,* $x_i = \sum_{k=1}^{p} u_k x_i^k$ *and* x_i^k *is the argument variable provided by each person.*

Definition 14 *A MP-IWOWAAC operator is an aggregation operator that has a weighting vector U of dimension p with* $\sum_{k=1}^{p} u_k = 1$ *and* $u_k \in [0,1]$*, and a weighting vector W of dimension n with* $\sum_{j=1}^{n} w_j = 1$ *and* $w_j \in [0,1]$*, such that:*

$$\text{MP-IWOWAAC}(\langle x_1^1,\ldots,x_1^p),y_1\rangle,\ldots,\langle(x_n^1,\ldots,x_n^p),y_n\rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{15}$$

where b_j*is the jth largest of the* $[1 \wedge (1 - x_i + y_i)]$*,* x_i*,* $y_i \in [0,1]$*, each* $[1 \wedge (1 - x_i + y_i)]$ *has associated a weight* v_i*,* v_j *is the associated weighted average (WA) of* b_j*,* $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$*,* $x_i = \sum_{k=1}^{p} u_k x_i^k$ *and* x_i^k *is the argument variable provided by each person.*

Definition 15 *A MP-IWOWAIMAM operator is an aggregation operator that has a weighting vector U of dimension p with* $\sum_{k=1}^{p} u_k = 1$ *and* $u_k \in [0,1]$*, and a weighting vector W of dimension n with* $\sum_{j=1}^{n} w_j = 1$ *and* $w_j \in [0,1]$*, such that:*

$$\text{MP-IWOWAIMAM}(\langle(x_1^1,\ldots,x_1^p),y_1\rangle,\ldots,\langle(x_n^1,\ldots,x_n^p),y_n\rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \tag{16}$$

where b_j *is the jth largest of all the* $|x_i - y_i|$ *and the* $[0 \vee (x_i - y_i)]$*;* x_i*,* $y_i \in [0,1]$*, each* $|x_i - y_i|$ *and* $[0 \vee (x_i - y_i)]$ *has associated a weight* v_i*,* v_j *is the associated weighted average (WA) of* b_j*,* $\hat{v}_j = (w_j v_j / \sum_{j=1}^{n} w_j v_j)$*,* $x_i = \sum_{k=1}^{p} u_k x_i^k$ *and* x_i^k *is the argument variable provided by each person.*

The MP-IWOWAD, MP-IWOWAAC and MP-IWOWAIMAM have similar properties than those commented in Section 3. Thus, we can consider a wide range of extensions such as those that use generalized and quasi-arithmetic means obtaining the MP-GIWOWAD, the MP-GIWOWAAC and the MP-GIWOWAIMAM operators.

Furthermore, it is possible to consider a wide range of particular cases. For example, with the MP-IWOWAD we can consider the multi-person OWAD (MP-OWAD), the multi-person weighted Hamming distance (MP-WHD), the multi-person normalized Hamming distance (MP-NHD) and so on.

6. Illustrative example

In this Section, we present a simple numerical example where it is possible to see the applicability of the OWA operator in a business decision-making problem about selection of production strategies. Note that this example can be seen as a real world example although in this paper we do not use information from the real world.

Step 1: Assume an enterprise that produces cars is looking for its general strategy the next year and they consider that it should be useful for them to create a new production plant in order to be bigger and more competitive in the market. After careful evaluation of the information, the group of experts of the company constituted by three persons considers the following countries where it could be interesting to create a new production plant.

- A_1 = Produce in Russia.
- A_2 = Produce in China.
- A_3 = Produce in India.
- A_4 = Produce in Brazil.
- A_5 = Produce in Nigeria.

The economic evaluation of producing in these countries can be described considering the following characteristics $C = (C_1 =$ Benefits in the short term, $C_2 =$ Benefits in the mid term, $C_3 =$ Benefits in the long term, $C_4 =$ Risk of the strategy, $C_5 =$ Subjective opinion of the group of experts, $C_6 =$ Other variables).

Step 2: With this information, the group of experts of the company establishes the ideal results that the ideal production strategy should have. These results are represented in Table 3.

Table 3: *Ideal production strategy.*

	C_1	C_2	C_3	C_4	C_5	C_6
$P =$	0.8	0.9	1	0.9	0.9	0.8

Step 3: Fixation of the real level of each characteristic for all the different production strategies considered. For each of these characteristics, the following information is

given by each expert shown in Tables 4, 5 and 6. Note that we assume that each expert has the same degree of importance. That is: $U = (1/3, 1/3, 1/3)$.

Table 4: *Available production strategies-Expert 1.*

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.5	0.4	0.5	0.3	0.7	0.4
A_2	0.1	0.7	0.7	0.3	0.7	0.9
A_3	0.6	0.8	0.5	0.1	0.6	0.3
A_4	0.5	0.4	0.6	0.6	0.7	0.6
A_5	0.2	0.1	0.4	0.8	0.7	0.1

Table 5: *Available production strategies-Expert 2.*

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.6	0.8	0.7	0.4	0.8	0.7
A_2	0.1	0.9	0.9	0.5	0.7	0.7
A_3	0.8	1	0.7	0.3	0.8	0.6
A_4	0.4	0.4	0.6	0.9	0.5	0.9
A_5	0.6	0.2	0.8	0.9	0.7	0.1

Table 6: *Available production strategies-Expert 3.*

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.7	0.9	0.6	0.5	0.9	0.7
A_2	0.4	0.8	0.8	0.1	0.7	0.8
A_3	0.7	0.9	0.6	0.2	0.7	0.6
A_4	0.3	0.4	0.6	0.9	0.6	0.6
A_5	0.7	0	0.6	0.7	0.7	0.1

With this information, we can aggregate the information of the three experts in order to obtain a collective result of the available production strategies. The results are presented in Table 7.

Table 7: *Available production strategies-Collective results.*

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.6	0.7	0.6	0.4	0.8	0.6
A_2	0.2	0.8	0.8	0.3	0.7	0.8
A_3	0.7	0.9	0.6	0.2	0.7	0.5
A_4	0.4	0.4	0.6	0.8	0.6	0.7
A_5	0.5	0.1	0.6	0.8	0.7	0.1

Step 4: Comparison between the ideal production strategy and the different production strategies considered, and determination of the level of removal using the OWA operator. By using the Hamming distance, we will consider the normalized Hamming distance, the weighted Hamming distance, the OWAD, the AOWAD the median-OWAD

and the IWOWAD operator. In this example, we assume that the company decides to use the following weighting vectors: $W = (0.1, 0.1, 0.1, 0.2, 0.2, 0.3)$ and $V = (0.3, 0.2, 0.2, 0.1, 0.1, 0.1)$. Note that in the literature we have a wide range of methods for determining the weights (Merigó, 2010; Merigó and Gil-Lafuente, 2009; 2010; Yager, 1993). Thus, when using immediate weights for the IWOWAD, IWOWAAC and IWOWAIMAM, we use the following weights obtained by using Eq. (9), (10) and (11), shown in Table 8.

Table 8: *Immediate weights.*

A_1	0.066	0.133	0.2	0.266	0.133	0.2
A_2	0.1875	0.0625	0.0625	0.25	0.25	0.1875
A_3	0.055	0.111	0.055	0.111	0.333	0.333
A_4	0.1428	0.2142	0.1428	0.1428	0.1428	0.2142
A_5	0.125	0.0625	0.125	0.375	0.125	0.1875

Note that we have to calculate the individual distances of each characteristic to the ideal value of the corresponding characteristic forming the fuzzy subset of individual distances for each strategy. Once, we have the individual distances, we aggregate them with the appropriate aggregation operator. The results are shown in Table 9.

Table 9: *Aggregated results with the OWAD operator.*

	NHD	*WHD*	*OWAD*	*AOWAD*	*Median*	*IWOWAD*
A_1	0.266	0.26	0.22	0.32	0.2	0.2266
A_2	0.283	0.32	0.2	0.37	0.2	0.2375
A_3	0.283	0.23	0.2	0.38	0.25	0.1555
A_4	0.3	0.35	0.24	0.36	0.35	0.2927
A_5	0.316	0.43	0.24	0.42	0.25	0.35

If we develop the selection process with the adequacy coefficient, we will get the following. First, we have to calculate how close the characteristics are to the ideal production strategy. Once we have calculated all the different individual values, we will construct the aggregation. In this case, the arguments will be ordered using Eq. (6) and Eq. (12). The results are shown in Table 10.

Table 10: *Aggregated results with the OWAAC operator.*

	NAC	*WAC*	*OWAAC*	*AOWAAC*	*Median*	*IWOWAAC*
A_1	0.733	0.74	0.68	0.78	0.8	0.7734
A_2	0.716	0.68	0.63	0.8	0.6	0.7625
A_3	0.716	0.77	0.62	0.8	0.75	0.8445
A_4	0.7	0.65	0.64	0.76	0.65	0.7072
A_5	0.683	0.57	0.58	0.76	0.75	0.65

Finally, if we use the index of maximum and minimum level in the selection process as a combination of the normalized Hamming distance and the normalized adequacy

coefficient, we get the following. In this example, we assume that the characteristics C_1 and C_2 have to be treated with the adequacy coefficient and the other four characteristics have to be treated with the Hamming distance. Its resolution consists in the following. First, we calculate the individual removal of each characteristic to the ideal, independently that the instrument used is the Hamming distance or the adequacy index. Once calculated all the values for the individual removal, we construct the aggregation using Eq. (7) and Eq. (13). Here, we note that in the reordering step, it will be only considered the individual value obtained for each characteristic, independently that the value has been obtained with the adequacy coefficient or with the Hamming distance. The results are shown in Table 11.

Table 11: *Aggregated results with the OWAIMAM operator.*

	NIMAM	*WIMAM*	*OWAIMAM*	*AOWAIMAM*	*Median*	*IWOWAIMAM*
A_1	0.266	0.26	0.22	0.42	0.2	0.2266
A_2	0.283	0.32	0.2	0.37	0.2	0.2375
A_3	0.283	0.23	0.2	0.38	0.25	0.1555
A_4	0.3	0.35	0.24	0.36	0.35	0.2927
A_5	0.316	0.43	0.24	0.42	0.25	0.35

In order to see the optimal production strategies depending on the particular types of OWA aggregations used, we establish the following table with the ordering of the production strategies. Note that this is very useful when the decision-maker wants to consider more than one alternative. The results are shown in Table 12.

Table 12: *Ordering of the production strategies.*

	Ordering		Ordering
NHD	$A_1\}A_2 = A_3\}A_4\}A_5$	AOWAAC	$A_2 = A_3\}A_1\}A_4 = A_5$
WHD	$A_3\}A_1\}A_2\}A_4\}A_5$	Median	$A_1\}A_3 = A_5\}A_4\}A_2$
OWAD	$A_2 = A_3\}A_1\}A_4 = A_5$	IWOWAAC	$A_3\}A_1\}A_2\}A_4\}A_5$
AOWAD	$A_1\}A_4\}A_2\}A_3\}A_5$	NIMAM	$A_1\}A_2 = A_3\}A_4\}A_5$
Median	$A_1 = A_2\}A_3 = A_5\}A_4$	WIMAM	$A_3\}A_1\}A_2\}A_4\}A_5$
IWOWAD	$A_3\}A_1\}A_2\}A_4\}A_5$	OWAIMAM	$A_2 = A_3\}A_1\}A_4 = A_5$
NAC	$A_1\}A_2 = A_3\}A_4\}A_5$	AOWAIMAM	$A_1\}A_4\}A_2\}A_3\}A_5$
WAC	$A_3\}A_1\}A_2\}A_4\}A_5$	Median	$A_1 = A_2\}A_3 = A_5\}A_4$
OWAAC	$A_1\}A_4\}A_2\}A_3\}A_5$	IWOWAIMAM	$A_3\}A_1\}A_2\}A_4\}A_5$

As we can see, we get different orderings depending on the aggregation operator used. The main advantage of this analysis is that the company gets a more complete view of the different scenarios that could happen in the future depending on the method used. Although it will select the alternative that it is in accordance with its interests, it will be concerned on other potential results that could happen in the uncertain environment. Note that in this specific problem, we see that A_1 or A_3 seems to be the optimal choices.

7. Conclusions

We have studied the usefulness of the OWA operator in business and economics. For doing so, we have given special attention to business and economic decision-making problems. We have used some practical decision-making techniques that use similarity measures in the decision-making process such as the Hamming distance, the adequacy coefficient and the index of maximum and minimum level. We have reviewed the use of the OWA operator in these techniques, obtaining the OWAD operator, the OWAAC operator and the OWAIMAM operator. We have seen that these aggregation operators are very useful for decision-making because they permit to under or over estimate the results according to the attitudinal character of the decision-maker in the particular problem considered.

We have suggested new techniques by using immediate weights. That is, by using a framework that is able to deal with the weighted average and the OWA operator in the same formulation. We have presented the IWOWAD, the IWOWAAC and the IWOWAIMAM operator. Furthermore, we have generalized them by using generalized aggregation operators obtaining the GIWOWAD, the GIWOWAAC and the GIWOWAIMAM operator. The main advantage of these measures is that they include a wide range of particular cases that can be used in the aggregation process depending on the particular interests in analysis.

We have also seen that the OWA operator can be also used in a lot of other problems in business and economics, especially when we see it as a statistical (or aggregation) technique similar to the weighted average. We have mentioned different potential areas where we could use it and we have seen that the applicability is very broad because we can implement it in a lot of business problems such as in finance, marketing, production and tourism. We have also presented different applications in economics and we have seen that it has a strong connection with politics because national economic decisions are usually related with political ones.

In this paper, we have focussed on a business multi-person decision-making application concerning production management by using the IWOWAD, the IWOWAAC and the IWOWAIMAM operator. Thus, we have obtained the MP-IWOWAD, the MP-IWOWAAC and the MP-IWOWAIMAM operators. We have seen that they are very practical because we can assess the information of several persons (experts) in an efficient way. We have analysed a company that it is planning its production strategy for the next year. We have seen that depending on the particular type of aggregation operator used, the results may lead to different decisions.

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