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Knowledge Licensing in a Model of R&D-driven Endogenous Growth

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Abstract: In this paper I present an endogenous growth model where the engine of growth is in-house R&D performed by high-tech firms. I model knowledge (patent) licensing among high-tech firms. I show that if there is knowledge licensing, high-tech firms innovate more and economic growth is higher than in cases when there are knowledge spillovers or there is no exchange of knowledge among high-tech firms. However, in case when there is knowledge licensing the number of high-tech firms is lower than in cases when there are knowledge spillovers or there is no exchange of knowledge.

JEL Codes: O30,O41, L16

Keywords: Knowledge Licensing, Intra-firm R&D, Competitive Pressure, Endogenous Growth.

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1 Introduction

A number of growth models treat private firms' intentional investments in R&D as the driver of long-run growth and welfare (e.g., Romer, 1990, Aghion and Howitt, 1992, Smulders and van de Klundert, 1995) . These models assume that there are knowledge spillovers in R&D process and R&D builds on a pool of knowledge. In this sense these growth models abstract from the role of knowledge (patent) licensing and from the details about the exchange of knowledge in the economy. Nevertheless, licensing and establishing consortia for exchanging patents is common in high-tech industries (e.g., Hagedoorn, 1993, 2002) .[1] Moreover, it has been argued extensively that exchanging patents plays a significant role for innovation in these industries (e.g., Grindley and Teece, 1997, Shapiro, 2001, Clark et al., 2000).[2] For instance, yet at the beginning of the previous century the major players in the Radio, Television and Communication Equipment industry in the United States experienced difficulties in innovating and advancing their products until the establishment of a patent consortium, RCA Corporation. Meanwhile, high-tech industries are the top private R&D performers and there is a large body of anecdotal and rigorous empirical evidence that they make a significant contribution to economic growth (e.g., Helpman, 1998, Jorgenson et al., 2005).

In this paper I present an endogenous growth model where high-tech firms engage in intra-firm (or in-house) R&D and that drives long-run growth. High-tech firms have exclusive rights to the type of their product. In a high-tech firm the innovation enhances firm/product-specific knowledge which reduces the firm's marginal costs or increases the quality of its product. High-tech firms finance their R&D expenditures from operating profits. They set prices and compete strategically in their output market. My point of departure is that I model knowledge (patent) licensing among high-tech firms. The knowledge generated in a high-tech firm cannot be used for free, but it can be licensed. Given that each high-tech firm produces a distinct type of good, for a high-tech firm the knowledge of other high-tech firms is complementary to its own. If a high-tech firm licenses the knowledge of another it can combine that knowledge with its own and improve its in-house R&D process since the latter builds on the knowledge that the firm possesses.

In such a setup I show how market concentration, intensity of competition as mea-

[1] In terms of 2-digit ISIC (Rev. 3), according to OECD STAN data high-tech industries as measured by R&D intensity are, for example, 24, 29, 30, 31, 32, 33, 34, 35, 64 and 72.

[2] Currently, there are virtually no comprehensive data for measuring the size of the market for patents and other types of intellectual property. According to some estimates (Robbins, 2009) in the US in 2002 corporate domestic income from licensing patents and trade secrets was $50 billion or approximately 25 percent of total private R&D expenditure. Moreover, it was expected to grown at more than 10 percent annual rate.

sured by the elasticity of substitution between high-tech goods, and type of competition (Cournot or Bertrand) can matter for innovation in the high-tech industry and aggregate performance. I contrast the inference from this setup to the inference from setups where there is no exchange of knowledge among high-tech firms and/or there are knowledge spillovers (i.e., firms obtain others' knowledge for free). Further, it is often conjectured that the use of high-tech goods such as phones and PCs entails positive externalities, which lower the transaction costs and increase the efficiency of users (e.g., Leff, 1984). I assess how innovation in the high-tech industry and aggregate performance depend on the magnitude of such externalities.

I show that when there is an exchange of knowledge among high-tech firms in the form of licensing or spillovers, innovation in the high-tech industry and economic growth increase with the number of high-tech firms. The driver behind this result are the relative price distortions which are due to price setting by high-tech firms. This distortion adversely affects the demand for high-tech goods. Given that high-tech firms interact strategically in the output market a higher number of firms implies lower mark-ups and lower distortion. This increases the demand for high-tech goods and implies higher output and investments in R&D in the high-tech industry.[3] However, if there is no exchange of knowledge among high-tech firms, then increasing the number of firms has two effects on innovation in the high-tech industry. One is the lower distortion, which is positive. Another is negative and is due to lower amount of R&D inputs available per high-tech firm. When the number of high-tech firms is relatively low the positive effect dominates, whereas for a relatively high number of firms the negative effect dominates. This negative effect when there is knowledge exchange among high-tech firms is offset by more complementary knowledge made available by high-tech firms.

I further show that in all the setups that I consider, innovation in the high-tech industry and economic growth increase with the intensity of competition. Tougher competition, which is defined as the type of competition with lower mark-ups (Bertrand vs. Cournot; Sutton, 1991), also implies more innovation in the high-tech industry and higher growth. These results are in line with the results of Smulders and van de Klundert (1995) and van de Klundert and Smulders (1997), and hold because both more intensive and tougher competition reduce mark-ups and the relative price distortions.[4]

The availability of complementary knowledge also motivates innovation in the high-tech industry. High-tech firms innovate more and the economy grows at a higher rate if

[3] O'Donoghue and Zweimüller (2004) have a similar result in a Schumpeterian growth model. Vives (2008) shows that such a result can also hold in partial equilibrium for various types of demand functions.

[4] The positive relation between innovation and different types of competitive pressure is consistent with the empirical findings of, for example, Blundell et al. (1999) and Nickell (1996).

there is an exchange of knowledge among high-tech firms than if there is no exchange. This is because R&D builds on a bigger pool of knowledge if there is an exchange of knowledge. Moreover, if there is no knowledge exchange high-tech firms might not innovate at all if there are many of them in the market. The driver behind this result is the scarcity of R&D inputs available per high-tech firm if there are many such firms. High-tech firms also innovate more and the economy grows at a higher rate when there is knowledge licensing compared to the case when there are knowledge spillovers among high-tech firms.

The higher magnitude of positive externalities from the use of high-tech goods implies lower innovation in the high-tech industry. Nevertheless, economic growth increases with the magnitude of these externalities. Innovation declines because the higher magnitude of positive extenralities brings no additional internalized benefit to high-tech firms and in equilibrium it implies a higher rate of interest. In turn, economic growth increases since the higher magnitude of externalities implies a higher contribution of innovation in the high-tech industry to growth.

Finally, I endogenize the number of high-tech firms assuming cost-free entry. Innovation in the high-tech industry and economic growth are the lowest in case when there is no exchange of knowledge among these firms. In turn, innovation in the high-tech industry and economic growth are the highest in case when there is knowledge licensing among these firms. This happens, however, at the expense of the number of high-tech firms (or the variety of high-tech goods.) In other words, the number of high-tech firms is the lowest in case when there is knowledge licensing and the highest in case there is no exchange of knowledge.

Increasing the intensity and/or toughness of competition reduces the number of firms. When there is an exchange of knowledge among high-tech firms this has no effect, however, on allocations, innovation in the high-tech industry, and economic growth. Meanwhile, allocations change and innovation and economic growth tend to increase with the intensity and toughness of competition if there is no exchange of knowledge among high-tech firms.

This paper is related to the endogenous growth literature (e.g., Romer, 1990, Aghion and Howitt, 1992, Smulders and van de Klundert, 1995) where the positive growth of the economy on a balanced growth path is a result of technological and preference factors. In particular, it is related to studies which in an endogenous growth framework suggest how the aggregate performance can be affected by imperfect competition in an industry where the firms engage in in-house R&D (e.g., van de Klundert and Smulders, 1997). It contributes to these streams of studies while showing how knowledge licensing in such an industry can affect innovation in that industry and the aggregate performance. It also contributes while showing how the positive externalities from the

use of the goods of such an industry can affect the decentralized equilibrium outcomes.

Further, there is a number of papers that model knowledge (patent) and technology licensing in standard Schumpeterian growth framework and show how patent policy and international technology licensing can affect innovation and growth (e.g., O'Donoghue and Zweimüller, 2004, Yang and Maskus, 2001, Tanaka et al., 2007). In these papers licensing happens between incumbents and entrants given that in standard Schumpeterian growth framework incumbents have no incentives to innovate. Licensing does not explicitly aid R&D process and licenses are essentially permits for production. In such a framework in order to maintain incentives for licensing, these papers assume that either licensors and licensees (incumbents and entrants) collude in the product market or licensees can access larger market (e.g., one of the countries bans FDI). The share in collective profits and licensing fees compensate incumbents' loss of the product market (and costs of technology transfer) and are either exogenous or exogenously determined by patent policy. In contrast, this paper has a non-tournament framework where incumbents innovate because that allows stealing market share and licensing happens among incumbents. Firms have incentive to license knowledge from other firms because that aids their R&D process. Further, license fees are determined by the structure of the market for knowledge, which can depend on patent policy, and supply and demand conditions. To that end, the framework and analysis of this paper can be thought to be complementary.

There is also a large body of firm- and industry-level studies that analyze the implications of patent licensing, patent consortia or pools, and knowledge exchange amongy firms on innovation and market conduct (e.g., Gallini, 1984, Gallini and Winter, 1985, Shapiro, 1985, Katz and Shapiro, 1985, Bessen and Maskin, 2009). This paper analyzes such issues at the aggregate level in a dynamic general equilibrium framework which assumes an undistorted market for knowledge/patents. This assumption allows to have tractable inference and can be justified to the extent that this paper aims to address long-run issues, for example. In turn, the dynamic general equilibrium framework allows to endogenize the growth rate of the economy and the effect of knowledge licensing on, for example, interest rate which affects the incentives to perform R&D. Licensing in this paper *ceteris paribus* motivates R&D. This, in turn, implies higher growth rate and higher rate of interest which reduces the incentives to perform R&D.

The next section offers the model. Section 3 analyzes the features of dynamic equilibrium. Section 4 concludes.

2 The Model

Households

The economy is populated by a continuum of identical and infinitely lived households of mass one. The representative household is endowed with a fixed amount of labor (L). It inelastically supplies its labor to firms which produce final goods and to high-tech firms. The household has a standard CIES utility function with an inter-temporal substitution parameter $\frac{1}{\theta}$ and discounts the future streams of utility with rate ρ ($\theta, \rho > 0$). The utility gains are from the consumption of amount C of final goods. The lifetime utility of the household is

$$U = \int_0^{+\infty} \frac{C_t^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt. \tag{1}$$

The household maximizes its lifetime utility subject to a budget constraint,

$$\dot{A} = rA + wL - C, \tag{2}$$

where A are the household's asset holdings [$A(0) > 0$], r and w are the market returns on its asset holdings and labor supply.

The optimal rule that follows from the household's optimal problem is the standard Euler equation,

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho). \tag{3}$$

This, together with budget constraint (2), describes the paths of the household's consumption and assets.

Final Goods

Final goods are homogeneous, Y. The household's demand for final goods is served by a representative producer. The production of final goods requires labor and X, which is a Dixit-Stiglitz composite of high-tech goods $\{x_i\}_{i=1}^N$ with an elasticity of substitution ε.

Ceteris paribus the increasing demand of X creates externalities in final goods production, which are measured by $\tilde{X}$. These externalities increase the productivity of the final goods producers. For example, these externalities stand for network effects that stem from using high-tech goods such as PCs and phones.

The production of the final goods has a Cobb-Douglas technology and is given by

$$Y = \tilde{X} X^{\sigma} L_Y^{1-\sigma}, \tag{4}$$

$$X = \left(\sum_{i=1}^{N} x_i^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{5}$$

$$1 > \sigma > 0, \varepsilon > 1,$$

where L_Y is the share of the labor force employed in final goods production.

For ease of exposition, the problem of the representative final goods producer is divided into two steps. In the first step the representative producer decides on the optimal combination of L_Y and X in Y and in the second step it decides on the optimal amounts of high-tech goods x in X.

Therefore, in the first step the representative producer solves the following problem.

$$\max_{L_Y, X} \{Y - wL_Y - P_X X\}$$

$$s.t.$$

$$(4),$$

where P_X is the "price" (private marginal value) of X and Y is the numeraire. The optimal rules that follow from this problem describe the final goods producer's demand for labor and the optimal amount of X for the production of Y,

$$[L_Y] : wL_Y = (1 - \sigma) Y, \tag{6}$$

$$[X] : P_X X = \sigma Y. \tag{7}$$

In the second step the producer solves

$$\max_{\{x_i\}_{i=1}^{N}} \left\{ P_X X - \sum_{i=1}^{N} p_{x_i} x_i \right\},$$

$$s.t.$$

$$(5),$$

where p_x is the price of x. This implies that the demand for a high-tech good is

$$[x_j] : x_j = X \left(\frac{P_X}{p_{x_j}} \right)^{\varepsilon}. \tag{8}$$

From this expression follow two equilibrium conditions,

$$P_X X = \sum_{i=1}^{N} p_{x_i} x_i, \tag{9}$$

$$P_X = \left(\sum_{i=1}^{N} p_{x_i}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}, \tag{10}$$

where the first means that there is no waste and the second implies that P_X is an index of p_x.

Further, I assume that the measure of externalities $\tilde{X}$ is given by

$$\tilde{X} = X^{\mu},$$

where μ measures the strength of these externalities $(1 - \sigma > \mu \geq 0)$.[5]

High-tech Goods

At any time t there are $N(t)$ producers in the high-tech industry.[6]

Production

Each high-tech firm owns a design (blueprint) of distinct high-tech good x, which it produces. The production of a high-tech good requires labor input L_x. The production function of a high-tech good x is

$$x = \lambda L_x, \tag{11}$$

where λ measures the producer's knowledge of production process or quality of the high-tech good. This knowledge is firm/product-specific since each high-tech firm produces a distinct good.

High-tech firms are price setters in their output market and discount their future profit streams π with the market interest rate r. I assume that high-tech firms cannot collude in the output market.

Knowledge Accumulation

High-tech firms can engage in R&D for accumulating knowledge and increasing λ. This can be interpreted as a process innovation that increases productivity (the firms are able to produce more of x), or as a quality upgrade (the firms are able to produce the

[5] It is necessary to have $1 - \sigma > \mu$ in order for the production function of final goods (4) to be concave in X in the Social Planner's problem.

[6] In order to avoid complications arising from integer constraints I allow N to be real number.

same amount of higher quality x). Knowledge is non-rival so that potentially it can be used at the same time in multiple places/firms.

In this section I offer three different settings of knowledge accumulation/R&D process. The differences stem from whether and how knowledge is exchanged among high-tech firms.[7,8]

Hereafter, when appropriate for ease of exposition I describe the properties of the high-tech industry while taking as an example high-tech firm j, $j \in (1, N]$. In order to improve its knowledge λ_j the firm needs to hire researchers/labor L_{r_j}. Researchers use the current knowledge of the firm in order to create better knowledge.

Knowledge Licensing: This is the benchmark setup (S.1). Knowledge in this setup can be licensed. In the market for knowledge the licensors (or the suppliers of knowledge) have the bargaining power in the sense that they can make a 'take it or leave it' offer.

The benefit from licensing knowledge is that it can be used in the in-house R&D process. If high-tech firm j decides to license knowledge from other high-tech firms, its researchers combine that knowledge with the knowledge available in the firm in order to produce new knowledge. The knowledge available in the firm is an essential input in the knowledge accumulation process of the firm. Moreover, it is the only essential input. This implies that the high-tech firm does not need to acquire knowledge from other firms in order to advance its own. However, it needs to have its knowledge for building on it. This is in line with that high-tech firms produce distinct goods.[9,10]

The knowledge accumulation/R&D process is given by

$$\dot{\lambda}_j = \xi \left[\sum_{i=1}^{N} (u_{i,j}\lambda_i)^{\alpha} \right] \lambda_j^{1-\alpha} L_{r_j}, \tag{12}$$
$$\xi > 0, 1 > \alpha > 0,$$

where ξ is an exogenous efficiency level, $u_{i,j}$ is the share of knowledge of firm i (λ_i) that firm j licenses, and $u_{j,j} \equiv 1$.[11]

It can be shown that in (12) the elasticity of substitution between the different types of knowledge that the high-tech firm licenses is equal to $\frac{1}{1-\alpha}$. It can also be shown that the elasticity of substitution between the high-tech firm's knowledge and

[7] The functional forms of the knowledge accumulation processes are selected so that they ensure a balanced growth path.

[8] In these setups each high-tech firm engages in in-house R&D and there is no R&D cooperation. Appendix E.1 analyzes the case when firms cooperate in R&D and compete in the product market.

[9] One way to think about this setup is that high-tech firms can use their knowledge (patents) and build around others' knowledge.

[10] I assume that license contracts do not allow sub-licensing.

[11] Appendix E.2 incorporates knowledge spillovers and depreciation in this R&D process.

any knowledge that it licenses is lower than $\frac{1}{1-\alpha}$ (see Appendix T.1). This restates the importance of the firm's knowledge for its knowledge accumulation process.

In this knowledge accumulation process because of summation the productivity of researchers increases linearly with knowledge licensed from an additional high-tech firm. Such a formulation can be justified if there are significant complementarities among the knowledge of high-tech firms.[12] Further, it might seem brave to assume that knowledge accumulation in a single firm can have non-decreasing returns.[13] This assumption allows to focus on the effect of market structure of the high-tech industry on innovation in that industry through competitive pressure. It can be relaxed setting $u_{j,j} \equiv 0$ in square brackets in (12). In such a case in this model knowledge licensing (or exchange of knowledge) is a necessary condition to ensure non-decreasing returns to knowledge accumulation and positive growth in the long-run (Appendix E.3 offers the main properties of the model if $u_{j,j} \equiv 0$). The knowledge accumulation process (12) can also be viewed as a simplification leading to tractable results. It ensures that there exists a balanced growth path, for example.

One way to think about this setup is that each high-tech firm can license the patented knowledge of other firms in order to generate its patented knowledge that helps to improve its production or output. The firm does not use the knowledge that it licensed directly in the production of its high-tech good because that knowledge needs to be combined with its own knowledge, and that requires investments in terms of hiring researchers (and time). The latter seems plausible for technologically sophisticated (e.g., high-tech) goods.

Knowledge Spillovers: In this case (S.2) there are knowledge spillovers among high-tech firms. In high-tech firm j the researchers combine the knowledge that spills over from other high-tech firms with the knowledge available in the firm while generating new knowledge. In order to maintain symmetry I assume also that the researchers do not fully internalize the use of the current knowledge available in the firm and have external benefits from it. Similar to the previous setup, this assumption allows to focus on the effect of market structure of the high-tech industry on innovation through competitive pressure.[14]

[12] Rivera-Batiz and Romer (1991) and Grossman and Helpman (1995) have a similar additive structure for knowledge in the R&D process in the context of knowledge spillovers between countries. Peretto (1998a) and Peretto (1998b) have a similar structure in the context of knowledge spillovers among firms in an industry.

[13] In this respect, a high-tech firm can be a firm that started with tabulating machines and reached the point of producing supercomputers and artificial intelligence systems (e.g., IBM).

[14] Appendix E.3 relaxes this assumption and offers the main properties of the model.

The knowledge accumulation process is

$$\dot{\lambda}_j = \xi \tilde{\Lambda} \lambda_j^{1-\alpha} L_{r_j}, \tag{13}$$

where I assume that in equilibrium $\tilde{\Lambda}$ is given by

$$\tilde{\Lambda} \equiv \sum_{i=1}^{N} \lambda_i^{\alpha}.^{15} \tag{14}$$

An interpretation of this case is that there are knowledge externalities/spillovers within high-tech firms and there is a market for knowledge where the potential licensees have a right to make a 'take it or leave it' offer. The licensees under this assumption receive the knowledge at no cost (i.e., there are knowledge spillovers) if the supply of knowledge is not elastic. The supply is necessarily inelastic if licensors do not have trade-offs and/or costs associated with licensing knowledge. It seems natural to assume that once knowledge is created its supply entails virtually no costs. Meanwhile, there would be no trade-offs if licensors do not take into account that the knowledge they license is used for business stealing: the licensees use it in order to reduce their prices and steal market share. I assume that licensors do not take into account this effect.

Such an assumption is not new to this line of literature. Many papers (e.g., van de Klundert and Smulders, 1997) assume that the originators of knowledge spillovers (here, high-tech firms) do not internalize the effect of spillovers (here, licensed knowledge) on other's knowledge accumulation and production processes. This assumption helps to avoid complications in differential games arising from the dependence of the current choice on the entire future (or history) of states.[16] Further, in the frames of this model this assumption is necessary in order to give such a market-based interpretation to knowledge spillovers, which links this setup (S.2) with the previous one (S.1).

In this model, similar to λ, the design of a high-tech good can be interpreted as knowledge/patent. In order to guarantee that high-tech firms have incentives to innovate it needs to be assumed that (at least for sometime) the knowledge on the design of high-tech goods does not spill over or cannot be used by other firms without appropriate compensation. Any high-tech firm, nevertheless, could sell the design of its high-tech good at market value: the discounted sum of profit streams earned selling the high-tech good.[17] Therefore, the market structure of knowledge on the production

[15] Peretto (1998a) and Peretto (1998b) model the knowledge accumulation process similar to (13) and (14), though these papers assume that $\alpha = 1$.

[16] In this model, under this assumption high-tech firms do not realize that the knowledge which they accumulate enters the knowledge accumulation process of other high-tech firms and from the next instance augments their rivals' productivity. If they realized that, then by integrating over the (future) changes of knowledge of their rivals they could track how their current investment in knowledge affects the productivity and market share of their rivals in the future.

[17] This simply implies that the name of the high-tech firm does not matter.

process or the quality of high-tech goods λ where the licensors have a right to make a 'take it or leave it' offer seems to be more appropriate in such a setup.

In this model λ can also be viewed as a patent on the production process or the quality of the product. Such market-based interpretations are then appropriate if, for example, there is strong enforcement of intellectual property rights and patent infringements are detectable. Given the recent history of the high number of patent infringement lawsuits in high-tech industries, both assumptions seem to be plausible. Meanwhile, the existence of lawsuits can indicate that the market structure of knowledge/patents where buyers have a right to make a 'take it or leave it' offer may not be realistic.

No Exchange of Knowledge: In this case (S.3) there is no exchange of knowledge among high-tech firms. Moreover, in the process of generation of new knowledge the researchers do not fully internalize the use of previous knowledge available in the firm and have external benefits from it.

The knowledge accumulation is given by

$$\dot{\lambda}_j = \xi \tilde{\lambda} \lambda_j^{1-\alpha} L_{r_j}, \tag{15}$$

where $\tilde{\lambda}$ stands for the external benefits and I assume that in equilibrium

$$\tilde{\lambda} \equiv \lambda_j^{\alpha}.^{18} \tag{16}$$

Clearly, in a symmetric equilibrium this case can also be interpreted as if there is an exchange of knowledge among high-tech firms in terms of spillovers and these spillovers are from average knowledge, i.e.,

$$\tilde{\lambda} \equiv \left(\frac{1}{N} \sum_{i=1}^{N} \lambda_i \right)^{\alpha}.$$

The spillovers from average knowledge, however, mask the existence of complementarity among the knowledge available in different firms. This is because the spillovers from an additional firm might bring no additional benefit. Therefore, I prefer avoiding this interpretation. In this respect, the knowledge accumulation process (15) and (16) can be interpreted as if the exchange of knowledge among high-tech firms is banned (or there is no appropriate institutional framework for it). It is clear that (12) and (13) reduce to (15) if there is no knowledge exchange among high-tech firms [i.e., (12) and (15) are equivalent if $u_{i,j} = 0$ for any $i \neq j$ and limiting case $\alpha = 0$; (13) and (15) are

[18] van de Klundert and Smulders (1997) have a similar formulation for the knowledge accumulation process.

equivalent if $\lambda_i = 0$ for any $i \neq j$].

Optimal Problem

The revenues of high-tech firm j are gathered from the supply of its high-tech good and in the case when there is knowledge licensing (S.1) from the supply of its knowledge $(u_{j,i}\lambda_j; \forall i \neq j)$. The costs are the labor compensations and license fees in case when there is knowledge licensing. The high-tech firm maximizes the present discounted value V of its profit streams subject to (8), (11), and either (12), or (13), or (15). Under Cournot competition, the high-tech firm chooses the supply of its product (i.e., L_{x_j}) given the (inverse) demand for it. In contrast, under Bertrand competition the firm chooses the price of its product (i.e., p_{x_j}) given the demand for it.

Formally, the problem of the high-tech firm is

$$\max_{\substack{Cournot:\ L_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1;(i\neq j)}^{N} \\ Bertrand:\ p_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1;(i\neq j)}^{N}}} V_j(\bar{t}) = \int_{\bar{t}}^{+\infty} \pi_j(t) \exp\left[-\int_{\bar{t}}^{t} r(s)\, ds\right] dt \tag{17}$$

$$s.t.$$

$$(8), (11) \text{ and either } (12), \text{ or } (13), \text{ or } (15),$$

where $\bar{t}$ is the entry date and

$$\begin{aligned} \pi_j = {} & p_{x_j} x_j - w\left(L_{x_j} + L_{r_j}\right) \\ & + \left[\sum_{i=1, i\neq j}^{N} p_{u_{j,i}\lambda_j}\left(u_{j,i}\lambda_j\right) - \sum_{i=1, i\neq j}^{N} p_{u_{i,j}\lambda_i}\left(u_{i,j}\lambda_i\right)\right]. \end{aligned} \tag{18}$$

In profit function π_j the term in square brackets stands for knowledge licensing, and $p_{u_{j,i}\lambda_j}$ and $p_{u_{i,j}\lambda_i}$ are the prices of $u_{j,i}\lambda_j$ and $u_{i,j}\lambda_i$.

The solution of the optimal problem implies that the supply of high-tech good x_j and the demand for labor for knowledge accumulation are

$$\left[L_{x_j}\right] : w = \lambda_j p_{x_j}\left(1 - \frac{1}{e_j}\right), \tag{19}$$

$$\left[L_{r_j}\right] : w = q_{\lambda_j} \frac{\dot{\lambda}_j}{L_{r_j}}, \tag{20}$$

where e_j is the elasticity of substitution between high-tech goods perceived by the high-tech firm and q_{λ_j} is the shadow value of knowledge accumulation.

The perceived elasticity of substitution (e_j) varies with competition type. It can

be shown that under Bertrand competition

$$e_j^{BR} \equiv e_j = \varepsilon - \left[\frac{(\varepsilon - 1)\, p_{x_j}^{1-\varepsilon}}{\sum\limits_{i=1}^{N} p_{x_i}^{1-\varepsilon}} \right], \tag{21}$$

and under Cournot competition

$$e_j^{CR} \equiv e_j = \varepsilon \left\{ 1 + \left[(\varepsilon - 1) \frac{x_j^{\frac{\varepsilon-1}{\varepsilon}}}{\sum\limits_{i=1}^{N} x_i^{\frac{\varepsilon-1}{\varepsilon}}} \right] \right\}^{-1}. \tag{22}$$

The terms in square brackets in (21) and (22) measure the impact of other high-tech firms on the demand of high-tech firm j. In other words, they measure the extent of strategic interactions among high-tech firms. Moreover, these terms indicate the difference between the perceived elasticity of substitution (e) and the actual elasticity of substitution (ε). Therefore, they indicate some of the distortions in the economy which stem from imperfect competition with a finite number of high-tech firms. In a symmetric equilibrium, when the number of firms increases, these distortions tend to zero since the terms in square brackets tend to zero.

In case when there is knowledge licensing (S.1) the returns on knowledge accumulation are

$$[\lambda_j] : \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left(\frac{e_j^k - 1}{e_j^k} \frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} + \sum_{i=1, i \neq j}^{N} \frac{p_{u_{j,i}\lambda_j} u_{j,i}}{q_{\lambda_j}} \right), \tag{23}$$
$$k = CR, BR,$$

where the first term in brackets is the benefit from accumulating knowledge in terms of increased output. The second term is the benefit in terms of higher amount of knowledge available for subsequent knowledge accumulation,

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi \left[1 + (1 - \alpha) \sum_{i=1, i \neq j}^{N} \left(\frac{u_{i,j} \lambda_i}{\lambda_j} \right)^{\alpha} \right] L_{r_j}. \tag{24}$$

In turn, the third term in brackets is the benefit in terms of increased amount of knowledge that can be licensed.

The demand for and the supply of knowledge in this case are

$$[u_{i,j}] : p_{u_{i,j}\lambda_i} = q_{\lambda_j}\xi\alpha\left(\frac{\lambda_j}{u_{i,j}\lambda_i}\right)^{1-\alpha} L_{r_j},\ \forall i \neq j, \tag{25}$$

$$[u_{j,i}] : u_{j,i} = 1,\ \forall i \neq j, \tag{26}$$

which means that the firm has a downward sloping demand for knowledge and licenses/supplies all its knowledge.

In case when there are knowledge spillovers among high-tech firms (S.2) the returns on knowledge accumulation are given by (23) but

$$p_{u_{j,i}\lambda_j} = 0,\ \forall i, \tag{27}$$

and

$$\frac{\partial\dot{\lambda}_j}{\partial\lambda_j} = \xi\left(1-\alpha\right)\left[\sum_{i=1}^{N}\left(\frac{\lambda_i}{\lambda_j}\right)^{\alpha}\right] L_{r_j}. \tag{28}$$

The first expression means that the licensees receive knowledge (patents) for free. In turn, there is a difference between (24) and (28) because in S.1 case there are no knowledge externalities within high-tech firms.

In turn, if there is no exchange of knowledge among high-tech firms (S.3) the returns on knowledge accumulation are given by (23) where the third term is absent and

$$\frac{\partial\dot{\lambda}_j}{\partial\lambda_j} = \xi\left(1-\alpha\right) L_{r_j}. \tag{29}$$

The expression for the price of knowledge (25) indicates that the licensees pay a fixed fee for it. The fee is equal to their marginal valuation of the knowledge that they acquire. This valuation includes all future benefits from using that knowledge for augmenting their current knowledge. Therefore, the licensors appropriate all the benefit from licensing knowledge (i.e., they make the 'take it or leave it' offer). With a continuous accumulation of knowledge, as given by (12), at each and every instant the licensees acquire new knowledge at a fixed fee.

It is clear from (23) that I have assumed that the firm does not take into account the effect of accumulating knowledge on the price of knowledge $p_{u_{j,i}\lambda_j}$. From (25) it follows that $p_{u_{j,i}\lambda_j}$ declines with λ_j. In this sense, I focus on a perfect market for knowledge (where the price of knowledge is equal to its marginal product and the licensors appropriate all benefit). An alternative assumption would be that the firm internalizes this effect. In such a circumstance there is an additional term in (23): the derivative of $p_{u_{j,i}\lambda_j}$ with respect λ_j.

Even though taking into account this effect changes the incentives of accumulating

knowledge, it does not affect the supply of knowledge (26). This is because supply entails no costs and/or trade-offs.[19]

In the frames of this model the assumption that the licensors of knowledge do not take into account that their knowledge is used for business stealing amounts to assuming that firm j takes q_{λ_i} for any i different than j as exogenous. This is in line with assuming that it takes $p_{u_{j,i}\lambda_j}$ as exogenous.

Finally, in equilibrium there is no difference if high-tech firms license their knowledge in return to wealth transfer or knowledge of other firms (plus-minus a fee). Therefore, knowledge licensing among high-tech firms can also be thought to resemble patent consortia or pools.

Firm Entry

I focus on two regimes of "entry" into the high-tech industry. In the first regime there are exogenous barriers to entry (i.e., there is no entry) and all firms in the market are assumed to have entered at time 0 ($t = 0$). In the second regime there are no barriers to entry into the high-tech industry. Moreover, entry entails no costs. To certain extent such a setup might be more appropriate for modelling exit rather than entry. This setup allows to have tractable results for the case when there is no exchange of knowledge among high-tech firms. Later in the text I offer and highlight the balanced growth path properties of a setup where entry entails endogenous costs for the cases when there is an exchange of knowledge among high-tech firms (S.1-2).

In order to support symmetric equilibrium, I assume that the entrants into the high-tech industry have the highest productivity available at that date. Further, I assume that high-tech firms do not coordinate on their entry and exit strategies.

3 Features of the Dynamic Equilibrium

I restrict the attention to a symmetric equilibrium in the high-tech industry.

I denote the growth rate of a variable Z by g_Z. Further, for subsequent analysis it is useful to define functions $I^N_{S.1-2}$ and $I^1_{S.2-3}$ as

$$I^N_{S.1-2} = \begin{cases} 1 \; for \; S.3, \\ N \; otherwise \end{cases}$$

[19] Appendix E.4 derives the model under this alternative assumption. It shows that high-tech firms innovate less if they take into account the effect of knowledge accumulation on the price of knowledge. This is because innovating decreases their returns on knowledge licensing.

and

$$I^1_{S.2-3} = \begin{cases} 0 \; for \; S.1, \\ 1 \; otherwise. \end{cases}$$

The growth rate of knowledge/productivity in cases when there is an exchange of knowledge among high-tech firms (S.1-2) is given by (12), (13), and (14). In case when there is no exchange of knowledge (S.3) it is given by (15). Using $I^N_{S.1-2}$ it can be written as

$$g_\lambda = \xi I^N_{S.1-2} L_r, \tag{30}$$

for all S.1-3 cases.

The (internal) rate of return on knowledge accumulation can be derived from the optimal rules of the high-tech firm (19), (20), and (23)-(29). It is given by

$$g_{q_\lambda} = r - g_\lambda \left(\frac{L_x}{L_r} + 1 - \alpha I^1_{S.2-3} \right). \tag{31}$$

This expression determines the allocation of labor to R&D in a high-tech firm relative to the allocation of labor to production. Here, this ratio does not (explicitly) depend on competitive pressure in the high-tech industry. This is because high-tech firms decide on the division of labor between production and R&D internally and L_x and L_r are paid the same wage.

From the high-tech firm's demand for labor for production (19), the representative final goods producer's optimal rules (6)-(7), and the relation between $P_X X$ and $p_x x$ (9) follows a relationship between NL_x and L_Y,

$$NL_x = \frac{\sigma}{1-\sigma} b^k L_Y, \tag{32}$$

where

$$b^k = \frac{e^k - 1}{e^k}. \tag{33}$$

This relationship shows the effect of price setting by high-tech firms. In symmetric equilibrium the perceived elasticities of substitution are

$$e^{BR} = \varepsilon - \frac{\varepsilon - 1}{N}, \tag{34}$$

$$e^{CR} = \frac{\varepsilon}{1 + \frac{\varepsilon - 1}{N}}. \tag{35}$$

Therefore, competition is tougher and mark-ups are lower if high-tech firms compete in prices, $e^{BR} > e^{CR}$. Moreover, mark-ups decline with the number of firms N and ε. This implies that the ratio $\frac{L_Y}{NL_x}$ declines with N, ε and toughness of competition. This is because as the competitive pressure in the high-tech industry increases the relative

price of x declines, which increases NL_x. Meanwhile, final goods producers substitute X for L_Y which reduces L_Y.

From (32) it is clear also that $\frac{L_Y}{NL_x}$ declines with σ and does not depend on μ. The first result holds because higher σ implies higher marginal product of X and lower marginal product of L_Y. The second result stems from the assumption that efficiency gains due to external effects are Hicks-neutral.

The relationship between NL_x and L_Y (32) together with labor market clearing condition,

$$L = L_Y + N\left(L_x + L_r\right), \tag{36}$$

implies a relationship between NL_x and NL_r,

$$NL_x = D^k\left(L - NL_r\right), \tag{37}$$

where

$$D^k = \frac{\sigma\left(e^k - 1\right)}{e^k - \sigma}.^{20} \tag{38}$$

Meanwhile, in the final goods market since either there is no entry or entry entails no costs and the assets in this economy are the high-tech firms it has to be the case that

$$Y = C, \tag{39}$$

which means that all final output is consumed.

Entry Regime 1: Exogenous Barriers to Entry

I take $N > 1$ and allow profits π in (18) to be negative. This is needed in order to characterize the behavior of labor force allocations and growth rate of knowledge for any $N > 1$, ε, and type of competition, and can be supported by subsidies, for example.

Decentralized Equilibrium

Since there are exogenous barriers to entry the number of firms is fixed,

$$\dot{N} = g_N = g_{\frac{e^k - 1}{e^k}} = 0.$$

[20] D^k measures the effect of competitive pressure in the high-tech industry on allocations of labor force. Appendix E.5 shows that in the limiting case when $\sigma = 1$ competitive pressure in the high-tech industry does not matter for these allocations. This is because in such a case there are no relative price distortions.

In such a case from (39) it follows that consumption and final output grow at the same rate,

$$g_C = g_Y. \tag{40}$$

Let the consumers be sufficiently patient so that $\theta \geq 1$, which is a standard stability condition in multi-sector endogenous growth models and seems to be the empirically relevant case.

Proposition 1. *Let the following parameter restriction hold for any sufficiently small N:*

$$\xi D^k \frac{I^N_{S.1-2}}{N} L > \rho. \tag{41}$$

In such a case, in decentralized equilibrium in all S.1-3 cases the economy makes a discrete "jump" to balanced growth path where labor force allocations and growth rates of knowledge/productivity and final output are given by

$$NL_r^{NE} = \frac{N}{\xi I^N_{S.1-2}} \frac{\xi D^k \frac{I^N_{S.1-2}}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + \alpha I^1_{S.2-3} + D^k}, \tag{42}$$

$$NL_x^{NE} = D^k \frac{\left[(\theta - 1)(\sigma + \mu) + \alpha I^1_{S.2-3}\right] L + \frac{N}{\xi I^N_{S.1-2}} \rho}{(\theta - 1)(\sigma + \mu) + \alpha I^1_{S.2-3} + D^k}, \tag{43}$$

$$L_Y^{NE} = \frac{1 - \sigma}{\sigma b^k} D^k \frac{\left[(\theta - 1)(\sigma + \mu) + \alpha I^1_{S.2-3}\right] L + \frac{N}{\xi I^N_{S.1-2}} \rho}{(\theta - 1)(\sigma + \mu) + \alpha I^1_{S.2-3} + D^k}, \tag{44}$$

and

$$g_Y^{NE} = (\sigma + \mu)\, g_\lambda^{NE}, \tag{45}$$

$$g_\lambda^{NE} = \frac{\xi D^k \frac{I^N_{S.1-2}}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + \alpha I^1_{S.2-3} + D^k}. \tag{46}$$

Proof. See Proofs Appendix. □

I use NE superscript for equilibrium labor force allocations and growth rates to denote the case when there is no entry. Parameter restriction (41) ensures that the inter-temporal benefit from allocating labor force to R&D outweighs its cost.

If parameter restriction (41) does not hold high-tech firms do not innovate. Therefore, the economy is static ($g_\lambda = g_Y = 0$) and the labor force allocations in all S.1-3

cases are given by

$$NL_r^{NE} = 0, \tag{47}$$

$$NL_x^{NE} = D^k L, \tag{48}$$

$$L_Y^{NE} = \frac{1-\sigma}{\sigma b^k} NL_x^{NE}. \tag{49}$$

This restriction may not hold for large N if there is no exchange of knowledge amonf high-tech firms (S.3) since when $I_{S.1-2}^N = 1$ the left-hand-side of the inequality tends to zero as N increases. In case when there is no exchange of knowledge, therefore, if N is sufficiently large then the economy is on balanced growth path where $g_\lambda = g_Y = 0$ and labor force allocations are given by (47)-(49). In this respect, if parameter restriction (41) holds for any sufficiently small $N > 1$ then it always holds in cases when there is an exchange of knowledge among high-tech firms (S.1-2). This is because when $I_{S.1-2}^N = N$ the left-hand-side of the inequality increases with N.

Without loss of generality, hereafter, I assume that (41) holds for any finite N and does not hold in case when there is no knowledge exchange among high-tech firms (i.e., $I_{S.1-2}^N = 1$) if N is arbitrarily large/infinite ($N = +\infty$).

Proposition 2. *Let parameter restriction (41) hold. If high-tech firms choose not to engage in R&D then labor force allocations are given by (47)-(49). Moreover, the value of high-tech firms is higher if none of the high-tech firms engages in R&D.*

Proof. See Proofs Appendix. □

I further assume that high-tech firms cannot collude and not innovate (for example, because of antitrust regulation or non-sustainability of collusion). In this respect, the reason why in decentralized equilibrium each high-tech firm prefers to engage in R&D is that R&D reduces its marginal cost. Therefore, *ceteris paribus* it allows the firm to lower its price and capture more market.

Social Optimum

The hypothetical Social Planner selects the paths of quantities so that to maximize the lifetime utility of the household (1). The Social Planner internalizes all externalities

and solves the following problem.

$$\max_{L_x, L_r} U = \int_0^{+\infty} \frac{C_t^{1-\theta} - 1}{1-\theta} \exp(-\rho t)\, dt \tag{50}$$

$s.t.$

$$C = \left(N^{\frac{\varepsilon}{\varepsilon-1}} \lambda L_x\right)^{\sigma+\mu} \left[L - N(L_x + L_r)\right]^{1-\sigma}, \tag{51}$$

$$\dot{\lambda} = \xi I_{S.1-2}^N \lambda L_r, \tag{52}$$

$$\lambda(0) > 0.$$

The Social Planner's optimal choices for L_x and L_r are given by

$$[L_x] : NL_x = D^{SP}(L - NL_r), \tag{53}$$

$$[L_r] : q_\lambda \xi I_{S.1-2}^N \lambda = \frac{(1-\sigma) N}{L - N(L_x + L_r)} C^{1-\theta}, \tag{54}$$

where

$$D^{SP} = \frac{\sigma + \mu}{1 + \mu}, \tag{55}$$

and I use SP superscript to make a distinction between the outcomes of Social Planner's choice and decentralized equilibrium outcomes. Meanwhile, its returns on knowledge accumulation are given by

$$[\lambda] : \dot{q}_\lambda = q_\lambda \rho - \left[q_\lambda \xi I_{S.1-2}^N L_r + (\sigma + \mu) \lambda^{-1} C^{1-\theta}\right]. \tag{56}$$

The optimal choice of L_x (53) together with labor market clearing condition (36) implies that

$$NL_x = \frac{1+\mu}{1-\sigma} D^{SP} L_Y. \tag{57}$$

This relation is the counterpart of (32) in decentralized equilibrium.

Proposition 3. *Let the following parameter restriction hold for any sufficiently small N:*

$$\xi D^{SP} \frac{I_{S.1-2}^N}{N} L > \rho. \tag{58}$$

In such a case, the Social Planner chooses labor force allocations such that the economy,

where there is "no entry", makes a discrete jump to balanced growth path, where

$$NL_r^{NE,SP} = \frac{N}{\xi I_{S.1-2}^N} \frac{\xi D^{SP} \frac{I_{S.1-2}^N}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}, \tag{59}$$

$$NL_x^{NE,SP} = D^{SP} \frac{(\theta - 1)(\sigma + \mu) L + \frac{N}{\xi I_{S.1-2}^N} \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}, \tag{60}$$

$$L_Y^{NE,SP} = \frac{1 - \sigma}{\sigma + \mu} D^{SP} \frac{(\theta - 1)(\sigma + \mu) L + \frac{N}{\xi I_{S.1-2}^N} \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}. \tag{61}$$

and

$$g_Y^{NE,SP} = (\sigma + \mu) g_\lambda^{NE,SP}, \tag{62}$$

$$g_\lambda^{NE,SP} = \frac{\xi D^{SP} \frac{I_{S.1-2}^N}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}. \tag{63}$$

Proof. See Proofs Appendix. □

Parameter restriction (58) necessarily holds as long as (41) holds since $D^{SP} > D^k$. As in decentralized equilibrium, this inequality states that the benefit from R&D outweighs its cost.

Given that C in (51) satisfies Inada conditions no corner solutions in terms of NL_x or L_Y satisfy the Social Planner's optimal problem.

Proposition 4. *Meanwhile, if (58) holds no corner solutions in terms of NL_r satisfy the Social Planner's optimal problem. In case, however, parameter restriction (58) does not hold the Social Planner sets*

$$NL_r = 0, \tag{64}$$

and the remaining labor force allocations according to

$$NL_x^{NE,SP} = D^{SP} L, \tag{65}$$

$$L_Y^{NE,SP} = \frac{1 - \sigma}{\sigma + \mu} D^{SP} L. \tag{66}$$

Proof. See Proofs Appendix. □

This parameter restriction does not hold if N is arbitrarily large/infinite and there is no knowledge exchange in the economy (S.3). It holds, however, for any N in cases when there is an exchange of knowledge (S.1-2) since I have assumed that so does (41).

I further assume that the Social Planner can choose between S.1-2 and S.3 cases. In terms of policies implemented by a government in decentralized equilibrium this cor-

responds to motivating or banning knowledge exchange in the economy.[21] Clearly, the Social Planner prefers S.1-2 over S.3 since it could set the same labor force allocations and have higher economic growth in S.1-2 cases. Therefore, in this sense it is socially desirable to have knowledge exchange in the economy.

Comparative Statics and Comparisons

Within the decentralized equilibrium outcomes, first, I discuss the case when the number of high-tech firms N is finite ($N < +\infty$). Next, I discuss the limiting case when the number of high-tech firms is infinite ($N = +\infty$) and, therefore, (41) does not hold if there is no exchange of knowledge among high-tech firms (S.3). In the end of the section I compare the decentralized equilibrium allocations and growth rates with the choice of the Social Planner.

Proposition 5. *In all S.1-3 cases the growth rate of knowledge/productivity (g_λ) and the growth rate of final output (g_Y) increase with the elasticity of substitution between high-tech goods (ε). Moreover, g_λ and g_Y are higher under Bertrand competition which is tougher than Cournot competition.*

Proof. These results follow from (33)-(35), (38), (45), and (46). □

The driver behind these results are the relative price distortions, which are due to price setting by high-tech firms. These distortions increase the demand for labor in final goods production. Increasing the elasticity of substitution or the toughness of competition reduces these distortions. The reduction of distortions motivates final goods producers to substitute (a basket of) high-tech goods for labor. Higher demand for high-tech goods and higher amount of available labor increase the incentives of high-tech firms to conduct R&D. This increases g_λ and g_Y.

Corollary 6. *In this respect, in all S.1-3 cases NL_r and NL_x grow and L_Y declines with the elasticity of substitution ε and toughness of competition.*

Proof. This result follows from (42)-(44). □

The comparative statics with respect to the number of high-tech firms in cases when there is an exchange of knowledge (S.1-2) are different from the case when there is no exchange of knowledge (S.3). The results are summarized in the following proposition.

Proposition 7. *In cases when there is an exchange of knowledge among high-tech firms (S.1-2), labor force allocations NL_r and NL_x and growth rates g_λ and g_Y increase with the number of firms N, whereas L_Y declines with it. If there is no exchange of knowledge (S.3), however, this result does not hold if the number of firms is relatively high.*

[21] An example for such policy/action is the establishment of the Radio Corporation of America (RCA Corporation) that fostered cross-licensing in the telecommunications industry in the US.

Proof. These results follow from (42)-(46). □

The driver behind the first result is the reduction in relative price distortions (or the intensification of competition) that the higher number of high-tech firms brings with it. Meanwhile, the second result holds because increasing the number of high-tech firms if there is no exchange of knowledge among high-tech firms (S.3) has two effects. It reduces the relative price distortions and the amount of labor force that can be devoted to R&D [see $\frac{I^N_{S.1-2}}{N}$ term in (46)]. The first effect motivates higher demand for NL_r and increases g_λ, whereas the second effect reduces NL_r and g_λ. The second effect is absent in cases when there is an exchange of knowledge among high-tech firms (S.1-2) because increasing the number of high-tech firms also increases the amount of complementary knowledge made available by these firms. Clearly, the result that these effects exactly offset each other hinges on the functional form assumptions for knowledge accumulation processes (12), (13) and (14).[22]

Proposition 8.

- *In cases when there is an exchange of knowledge among high-tech firms (S.1-2) g_λ and g_Y are concave functions of the number of firms N.*
- *In case when there is no exchange of knowledge (S.3) the derivative of g_λ, as well as g_Y, with respect to N is positive when N is close to 1 and it is negative for any N greater than 2.*

Proof. These results follow from (46) and that in cases when there is knowledge exchange among high-tech firms $I^N_{S.1-2} = N$, whereas $I^N_{S.1-2} = 1$ if there is no knowledge exchange. □

The first part of this proposition holds because competition intensifies more from adding a firm if there are few high-tech firms. Meanwhile, the second part holds because in case when there is no exchange of knowledge (S.3) at the higher levels of market concentration/lower levels of competition ($N \approx 1$) the positive effect of higher competition is dominant. Meanwhile, at the lower levels of market concentration/higher levels of competition ($N > 2$) the negative effect of the reduction in the amount of resource for R&D is dominant. The full characterization of the behavior of g_λ and g_Y for $N \in (1, 2)$ is not so straightforward, however. This is because of high non-linearity of g_λ in that interval. In the neighborhood of $N = 1$ the growth rate of knowledge/productivity g_λ

[22] One way to relax this assumption is to multiply (12) and (14) by a function $F(N)$, where $F' \leq 0$. As long as $F(N) D^k$ increases with N the results remain intact. If, however, $F(N) D^k$ decreases with N at least for sufficiently large N then the results would become similar to the case when there is no exchange of knowledge.

is increasing and concave in N and after a tipping point from $(1, 2)$ it becomes convex and decreasing.[23]

Proposition 9.

- *In all S.1-3 cases labor force allocations NL_r and NL_x and growth rates g_λ and g_Y increase with σ, whereas L_Y declines with it. In contrast, g_λ and NL_r decline with μ and g_Y, NL_x, and L_Y increase with it.*

- *In cases when there are knowledge spillovers/externalities (S.2-3) NL_r, g_λ and g_Y decline with α, whereas NL_x and L_Y increase with it.*

Proof. These results follow from (42)-(46). □

The first result holds because higher σ increases the marginal product of high-tech goods bundle X and reduces the marginal product of labor force employed in final goods production L_Y. Therefore, the demand for L_Y declines and labor force allocations NL_x and NL_r increase. According to (45) and (46) this implies that g_λ and the growth rate of final output g_Y increase with σ. In contrast, higher μ does not affect the balance between the demand for X and L_Y and in this sense does not alter the production and R&D incentives of high-tech firms. Meanwhile, *ceteris paribus* it increases the growth rate of final output g_Y and equilibrium interest rate r [see (3)], which discourages investments in R&D. Lower NL_r implies lower growth rate of knowledge/productivity g_λ. Finally, the second part of this proposition holds because in case there are knowledge spillovers/externalities as α increases the internalized returns on R&D decline and firms invest less in R&D. Therefore, more labor force is allocated to production activities, and g_λ and g_Y decline.

In order to preserve space, hereafter, unless stated otherwise, I exclusively discuss the results for the growth rate of knowledge/productivity g_λ while keeping in mind that the growth rate of final output g_Y is proportional to it.

Corollary 10. *If the number of high-tech firms is arbitrarily large/infinite*

- *when there is an exchange of knowledge among high-tech firms (S.1-2) labor force allocations and growth rate g_λ are given by (42)-(44), and (46) where*

$$D^k \equiv D = \frac{\sigma(\varepsilon - 1)}{\varepsilon - \sigma};$$

[23] This result implies that in case when there is no exchange of knowledge among high-tech firms (S.3) there is an "inverted-U" shape relationship between g_λ and the number of firms N. A similar result can be obtained also in cases when there is an exchange of knowledge among high-tech firms (S.1-2) assuming fixed management costs as in van de Klundert and Smulders (1997) or that (12) and (14) increase less than linearly with N.

- *when there is no exchange of knowledge among high-tech firms (S.3)* $g_\lambda = 0$ *and labor force allocations are given by (47)-(49) where* $D^k \equiv D$.

The first part of this corollary implies that when there is an exchange of knowledge among high-tech firms and $N = +\infty$ neither labor force allocations nor the growth rate of knowledge depend on the type of competition and the number of high-tech firms. It implies also that the remaining comparative statics stay intact in these cases. The second part of the corollary holds because when there is no exchange of knowledge among high-tech firms and $N = +\infty$ parameter restriction (41) does not hold. It can be shown that in this case, NL_x increases and L_Y declines with σ and ε and both NL_x and L_Y do not depend on the type of competition and parameters α and μ.

Corollary 11. *For both finite and infinite number of high-tech firms the comparison between S.1-3 cases yields the following relationships.*

$$\begin{aligned}
&NL_r^{NE,S.1} > NL_r^{NE,S.2} > NL_r^{NE,S.3},\\
&NL_x^{NE,S.1} < NL_x^{NE,S.2} < NL_x^{NE,S.3},\\
&L_Y^{NE,S.1} < L_Y^{NE,S.2} < L_Y^{NE,S.3},\\
&g_\lambda^{NE,S.1} > g_\lambda^{NE,S.2} > g_\lambda^{NE,S.3}.
\end{aligned} \tag{67}$$

This means that in decentralized equilibrium with no entry high-tech firms innovate the most in case when there is knowledge licensing (S.1). High-tech firms innovate the least if there is no exchange of knowledge among these firms (S.3). Therefore, for a given N the growth rate of final output is the highest in case when there is knowledge licensing and the lowest in case when there is no exchange of knowledge among high-tech firms.

In order to further highlight the contrast between all knowledge accumulation/R&D setups (S.1-3) Figure 1 plots g_λ for parameter values $\theta = 4$, $\rho = 0.01$, $\sigma = 0.3$, $\mu = 0.01$, $\varepsilon = 4$, $L = 1$, $\xi = 1$, and $\alpha = 0.1$ and Cournot and Bertrand types of competition.[24]

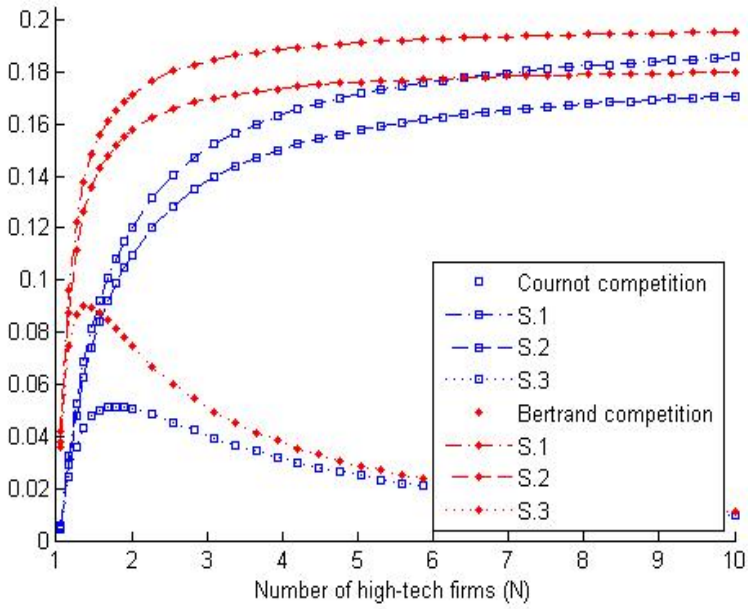
Comparisons Between Decentralized Equilibrium and Socially Optimal Results: Different types of competitive pressure matter for these decentralized equilibrium outcomes because of market interactions among high-tech firms. They do not matter, however, for the outcomes of the Social Planner's problem (59)-(63).

Corollary 12. *In contrast to the decentralized equilibrium results* NL_r^{SP}, NL_x^{SP}, g_λ, *and* g_Y *increase with* μ *and* L_Y^{SP} *declines with this parameter.*

Proof. This result follows from (59)-(63). □

[24]The parameter values were selected so that the growth rate of final output has a reasonable value.

Figure 1: *The Growth Rate of Productivity in S.1-3 Cases*



This result holds because the Social Planner internalizes μ and higher μ implies higher marginal product of X.

Corollary 13. *For both finite and infinite N the comparison between decentralized equilibrium growth rates and allocations and socially optimal growth rates and allocations yields the following relationships.*

$$
\begin{aligned}
&NL_r^{NE,SP,S.1-2} > NL_r^{NE,S.1},\\
&NL_x^{NE,SP,S.1-2} \lesseqgtr NL_x^{NE,S.3},\\
&NL_x^{NE,SP,S.1-2} \lesseqgtr NL_x^{NE,S.2},\\
&NL_x^{NE,SP,S.1-2} > NL_x^{NE,S.1},\\
&L_Y^{NE,SP,S.1-2} < L_Y^{NE,S.1},
\end{aligned}
$$

and

$$g_\lambda^{NE,SP,S.1-2} > g_\lambda^{NE,S.1},$$

where $\lesseqgtr$ indicates that the relation depends on model parameters.

This means that in decentralized equilibrium the economy innovates less than it is socially optimal and therefore grows at a lower rate. Moreover, in decentralized equilibrium it fails to have socially optimal labor force allocations. The driver behind these results are the relative price distortions and externalities. Due to these distortions final goods producers substitute labor for high-tech goods which lowers the output of high-

tech firms and the number of researchers that high-tech firms hire. The externalities in R&D have an effect of similar direction. If such externalities are present then high-tech firms do not fully internalize the returns on R&D. This reduces their incentives to invest in R&D and they hire lower number of researchers. Meanwhile, externalities in final goods production increase interest rate r. Since high-tech firms do not take into account these externalities they invest less than it is socially optimal. Final goods producers also do not take into account these externalities. Therefore, they demand less than socially optimal amount of high-tech goods.

The differences between socially optimal and decentralized equilibrium growth rates and labor force allocations in terms of the relative price distortions and externalities in final goods production are summarized by D^k and D^{SP}. It is easy to notice that for sufficiently high N

$$\lim_{\mu \to 0} D^{SP} = \lim_{\varepsilon \to +\infty} D^k.$$

This equality holds because for sufficiently high N the limiting case $\varepsilon = +\infty$ would imply perfect competition in the high-tech industry. In such a limiting case, however, in decentralized equilibrium high-tech firms make zero profits and have no market incentives to innovate.

In this respect, if there are no subsidies that keep the profits of high-tech firms non-negative, the positive relationship between innovation and ε holds as long as high-tech firms have sufficient profits to cover the costs of R&D. Profits of high-tech firms and ε are inversely related. Once profits net of R&D expenditures are equal to zero increasing ε reduces innovation to zero. Therefore, if there are no subsidies the relationship between intensity of product market competition (ε) and innovation has an "inverted-U" shape. Such a relation is consistent with Schumpeter's argument that firms need to be sufficiently big in order to innovate. Moreover, it is in line with the empirical findings of Aghion et al. (2005) and provides an alternative explanation for those findings.

Entry Regime 2: Cost-free Entry

In this section I endogenize the number of firms assuming that entry cost is zero.

Decentralized Equilibrium

From (18), (19) and (31) it follows that the profits of a high-tech firm are

$$\pi = wL_x \left[\frac{1}{e^k - 1} - \frac{g_\lambda}{r - g_{q_\lambda} - \left(1 - \alpha I^1_{S.2-3}\right) g_\lambda} \right].$$

Given that entry cost is zero the condition that endogenizes the number of high-tech firms is $\pi = 0$.

Denote

$$\bar{\pi} = \frac{1}{e^k - 1} - \frac{g_\lambda}{r - g_{q_\lambda} - \left(1 - \alpha I^1_{S.2-3}\right) g_\lambda}. \tag{68}$$

Therefore,

$$\pi = 0 \Leftrightarrow \bar{\pi} = 0. \tag{69}$$

Proposition 14. *At time 0 ($t = 0$) N makes a discrete jump to the balanced growth path equilibrium level.*

Proof. See Proofs Appendix. □

This implies that in decentralized equilibrium with cost-free entry the economy is on a balanced growth path (for any $t > 0$), where

$$\dot{N} = g_N = g_{\frac{e^k - 1}{e^k}} = 0.$$

Therefore, labor force allocations and growth rate of knowledge/productivity are given by (42)-(44) and (46), where the number of high-tech firms N is endogenous.

In turn, N can derived from the zero profit condition (69) and g_λ that solves the capital market equilibrium (46). The growth rate of productivity g_λ that solves the zero profit condition (69) is

$$g_\lambda = \frac{\rho}{e^k - 1 - \alpha I^1_{S.2-3} - (\theta - 1)(\sigma + \mu)}. \tag{70}$$

Let

$$\varepsilon - 1 - \alpha - (\theta - 1)(\sigma + \mu) > 0,$$

which implies that g_λ can be positive for sufficiently large N or, equivalently, there can exist decentralized equilibrium where high-tech firms innovate.

Hereafter, I call g_λ from (70) ZP – zero profit, and g_λ from (46) CME – capital market equilibrium. If $\alpha > 0$ and/or $\theta > 1$ the number of high-tech firms N that satisfies

$$e^k - 1 - \alpha - (\theta - 1)(\sigma + \mu) = 0$$

is strictly greater than 1. Denote it by N^*. For $N \in (1, N^*)$ it can be shown that g_λ in (70) or ZP is negative, decreasing, and convex function of N and

$$\lim_{N \to N^* -} g_\lambda = -\infty.$$

Meanwhile for $N > N^*$ it can be shown that ZP is positive, decreasing, and convex

function of N and

$$\lim_{N \to N^{*}+} g_{\lambda} = +\infty.$$

Proposition 15. *In decentralized equilibrium with endogenous entry it cannot happen so that $N \in (1, N^{*})$.*

Proof. This is because for $N \in (1, N^{*})$ high-tech firms do not innovate, which implies that the profit of each firm is

$$\pi = wL_x \frac{1}{e^k - 1} > 0.$$

Therefore, there will be entry that will increase the number of high-tech firms above N^{*}. □

Both CME and ZP are continuous functions of N for $N > N^{*}$, the values of CME are finite for any $N > 1$, and ZP is arbitrarily large around N^{*}. Therefore, at least for N sufficiently close to N^{*} it has to be the case that ZP is higher CME. This means that there exists decentralized equilibrium where high-tech firms innovate.

If ZP crosses CME from above then the decentralized equilibrium determined by the intersection is stable in the sense that the entry of firms reduces $\bar{\pi}$ in (68) and exit increases it. The number of firms and the growth rate of productivity can be solved from the intersection of CME and ZP in such a case. Moreover, if at time 0 ($t = 0$) the number of high-tech firms is higher than (and in S.3 case sufficiently close to) the number determined by the intersection of ZP and CME then high-tech firms will exit the market till ZP and CME are equal. Considering such a setup, or exit of high-tech firms instead of entry, can support the zero entry costs assumption.

In order to have meaningful equilibrium in each of S.1-3 cases [i.e., (69) holds] I further assume that the parameters are such that there exists N^{**} where ZP crosses CME under Cournot competition in case when there is no exchange of knowledge (S.3). Given that (46) shifts up and (70) shifts down with the elasticity of substitution ε this can be equivalent to assuming that the elasticity of substitution ε is sufficiently high. It implies that ZP crosses CME in all the remaining S.1-3 cases.[25]

The previous section showed that if there is an exchange of knowledge (S.1-2) the growth rate of knowledge g_{λ} from (46), or CME, is monotonically increasing function of N.

Corollary 16. *In cases when there is an exchange of knowledge among high-tech firms (S.1-2) ZP crosses CME from above and the number of high-tech firms under Bertrand*

[25] van de Klundert and Smulders (1997) offers a model which resembles the case when there is no exchange of knowledge among high-tech firms (S.3). The authors assume parameter values such that ZP crosses CME from above. Clearly, such a set of parameter values is restrictive for cases when there is an exchange of knowledge among high-tech firms (S.1-2).

and Cournot types of competition can be found from

$$e^k = \frac{\xi\sigma L\left[1 + \alpha I^1_{S.2-3} + (\theta - 1)(\sigma + \mu)\right]}{\xi\sigma L - \rho}, \tag{71}$$

where $k = CR, BR$ and e^b and e^c are given by (34) and (35).

If there is no exchange of knowledge (S.3), however, CME is not a monotonic function for all N. It is monotonically increasing function in the neighborhood of $N = 1$ and monotonically decreasing after some $N \in (1, 2)$. Moreover, it is continuous and finite for any N and negative for $N = 1$ and $N = +\infty$. Therefore, given that ZP is a monotonically decreasing function and it is positive for any N, ZP crosses CME at least twice.

Corollary 17. *If there is no exchange of knowledge among high-tech firms then the number of firms under Bertrand and Cournot types of competition can be found from*

$$e^k = \frac{\xi\sigma\frac{1}{N}L\left[1 + \alpha + (\theta - 1)(\sigma + \mu)\right]}{\xi\sigma\frac{1}{N}L - \rho}. \tag{72}$$

It is straightforward to show that (72) is a quadratic equation in N. This means in case there is no exchange of knowledge among high-tech firms (S.3) ZP crosses CME twice. It does so from above and from below. The smaller root of (72) corresponds to the stable equilibrium where ZP crosses CME from above. Meanwhile, the bigger root corresponds to the case when ZP crosses CME from below and the equilibrium is not stable. Denote it by N_2^{**}. If the economy starts with a number of firms greater or equal to N_2^{**} then $\bar{\pi}$ does not decline to zero as N increases. In order to rule this out I further assume that the economy starts with a number of high-tech firms that is lower than N_2^{**}. Therefore, depending on whether ZP is higher or lower than CME, firms exit or enter till (the point where) ZP crosses CME from above.[26]

Social Optimum

In this case the hypothetical Social Planner solves the optimal problem (50) and chooses N.

The Social Planner's optimal choice for N in case when there is an exchange of knowledge (S.1-2) is given by

$$[N] : \frac{\sigma + \mu}{\varepsilon - 1}\frac{C^{1-\theta}}{N} \geq 0$$

[26] The functional forms of knowledge accumulation process in cases when there is an exchange of knowledge among high-tech firms (S.1-2) help to avoid this assumption.

or simply

$$N = +\infty, \tag{73}$$

whereas if there is no exchange of knowledge (S.3) it is given by

$$[N] : \frac{\sigma + \mu}{\varepsilon - 1} C^{1-\theta} \geq q_\lambda \xi \lambda L_r, \tag{74}$$

The former result (73) holds because if there is an exchange of knowledge then $I^N_{S.1-2} = N$ and the Social Planner has no trade-offs while increasing N.[27] In contrast, if there is no exchange of knowledge then $I^N_{S.1-2} = 1$ and it has a trade-off. Higher N implies lower growth rate.

In order to be able to solve the optimal control problem in cases when there is an exchange of knowledge ($I^N_{S.1-2} = N$) with first order conditions C needs to be rescaled by N so that at time zero $C < +\infty$ (i.e., C needs to be divided to $N^{\frac{\sigma+\mu}{\varepsilon-1}}$).

Proposition 18. *The Social Planner selects labor force allocations and N such that the economy makes a discrete jump to balanced growth path.*

- *If there is an exchange of knowledge, on this path labor force allocations and growth rate of knowledge g_λ are given by (59)-(61) and (63) and $N = +\infty$.*
- *If there no exchange of knowledge and (74) is binding then*

$$N = \frac{\xi (\sigma + \mu)}{\rho} \frac{\varepsilon - 1 - (\theta - 1)(\sigma + \mu)}{\varepsilon (1 + \mu) - (1 - \sigma)} L, \tag{75}$$

$$g_\lambda^{CFE,SP,S.3} = \frac{\rho}{\varepsilon - 1 - (\theta - 1)(\sigma + \mu)}, \tag{76}$$

where CFE stands for cost-free entry.

Proof. See Proofs Appendix. □

If there is no exchange of knowledge and (74) is binding labor force allocations can be derived from (52), (53), (57), and (76), where the expression (76) is the counterpart of ZP (70) with $N = +\infty$ and $\alpha = 0$.

Comparing the lifetime utility of the household it can be shown, however, that the Social Planner prefers to set $N = +\infty$ also in case when there is no exchange of knowledge. Therefore, (74) does not bind. The following proposition summarizes this result.

[27] In cases when there is an exchange of knowledge (S.1-2) the Social Planner selects at time zero $N = +\infty$ because of the assumption that firm entry or creating high-tech goods entails no costs. If there were costs associated with entry (or costs associated with maintaining the goods/firms as in van de Klundert and Smulders, 1997) the Social Planner might not select at time zero (or at any time) $N = +\infty$.

Proposition 19. *In case when there is no exchange of knowledge the Social Planner sets*

$$N = +\infty,$$
$$g_{\lambda}^{CFE,SP,S.3} = NL_r^{CFE,SP,S.3} = 0, \tag{77}$$
$$NL_x^{CFE,SP,S.3} = D^{SP}L, \tag{78}$$
$$L_Y^{CFE,SP,S.3} = \frac{1-\sigma}{\sigma+\mu}D^{SP}L. \tag{79}$$

Proof. See Proofs Appendix. □

As it was shown in the Social optimum section of Entry Regime 1 this implies that the Social Planner prefers the case when there is an exchange of knowledge (S.1-2) over the case when there is no exchange of knowledge (S.3). This result is not stemming from the cost-free entry assumption. Even if there were fixed costs associated with entry the Social Planner could set the number of firms in case when there is an exchange of knowledge (S.1-2) equal to the number of firms it finds optimal in case when there is no exchange of knowledge (S.3). In such a circumstance according to (63) it would have higher growth rate and, therefore, welfare in case when there is an exchange of knowledge (S.1-2).

Comparative Statics and Comparisons

The following proposition establishes the comparative statics results for the number of high-tech firms.

Proposition 20.

- *In all S.1-3 cases, there are fewer high-tech firms in equilibrium under Bertrand competition than under Cournot competition. Further, the number of firms declines with ε and increases with μ.*
- *In cases when there are knowledge spillovers/externalities (S.2-3) the number increases with α. It does not depend on α in case when there is knowledge licensing (S.1).*

Proof. See Proofs Appendix. □

The number of firms declines with toughness of competition and ε since tougher competition and higher ε imply lower mark-ups, which reduces $\bar{\pi}$ for a given N. In turn, it increases with μ since higher μ implies lower R&D investments (fixed costs), which increases $\bar{\pi}$ for a given N. Higher α in cases when there are knowledge

spillovers/externalities (S.2-3) also implies lower R&D investments. The comparative statics with respect σ depend on model parameters.

Proposition 21. *In cases when there is an exchange of knowledge among high-tech firms*

- g_λ *and labor force allocations do not depend on the type of competition,* ε*, and* N*.*
- g_λ *and* NL_r *decrease with* α *and* μ*,* NL_x *increases with these parameters, and* g_Y *declines with* α *but increases with* μ*.*

Proof. The first part of the proposition holds because in cases when there is an exchange of knowledge e^k in (71) does not depend on the type of competition, ε, and N. See Proofs Appendix for the second part of the proposition. □

The analytical derivations for comparative statics with respect to σ and for labor force allocation to final goods production L_Y are not trivial. Numerical simulations where L is normalized to 1 and the remaining parameters are from the following intervals

$$\begin{aligned} &\theta \in [1, 10], \rho \in [0.01, 0.1], \sigma \in [0.01, 0.99], \\ &\mu \in [0.01, 0.99], \xi \in [0.1, 10], \alpha \in [0.01, 0.99], \end{aligned} \tag{80}$$

show that:where $+$ means positive relationship, $-$ negative, and $\pm$ that the relationship

Table 1: *Numerical Comparative Statics for S.1-2 Cases*

	g_λ	NL_r	NL_x	L_Y	g_Y
σ	$+$	$+$	$+$	$-$	$+$
μ	$-$	$-$	$+$	$\pm$	$+$
α	$-$	$-$	$+$	$\pm$	$-$

Note: This table offers numerical comparative statics for the cases when there is an exchange of knowledge among high-tech firms (S.1-2). The values of parameters are from intervals (80) and satisfy parameter restrictions. Grids are equally spaced and each has 5 points.

depends on model parameters.

If there is no exchange of knowledge among high-tech firms (S.3) it is not straightforward to derive the relationship between the toughness of competition and growth rate of productivity g_λ. This is because of high non-linearity of CME in this case. Nevertheless, it is possible to show that if ZP crosses CME from above in the region of N where CME is monotonically increasing then the growth rate of productivity g_λ is

higher under Bertrand competition. The comparison of the labor allocations using analytical techniques also is not trivial. Numerical simulations show that under Cournot competition g_λ, NL_r are lower and NL_x is higher than under Bertrand competition. Meanwhile, depending on model parameters L_Y can be both higher and lower.

The analytical derivation of comparative statics for labor force allocations and growth rates of final output and knowledge with respect to parameters ε, σ, μ, and α also are not trivial in case when there is no knowledge exchange among high-tech firms. Numerical simulations show that:

Table 2: *Numerical Comparative Statics for S.3 Case*

	g_λ	NL_r	NL_x	L_Y	g_Y
ε	$+$	$+$	$-$	$\pm$	$+$
σ	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$
μ	$\pm$	$\pm$	$+$	$\pm$	$\pm$
α	$-$	$-$	$+$	$\pm$	$-$

Note: This table offers numerical comparative statics for the cases when there is no exchange of knowledge among high-tech firms (S.3). The values of parameters are from intervals (80) and satisfy parameter restrictions. Grids are equally spaced and each has 5 points.

The following proposition summarizes the comparisons among different settings for R&D process.

Proposition 22. *The growth rate of knowledge/productivity g_λ is higher in case when there is knowledge licensing (S.1) compared to the case when there are knowledge spillovers among high-tech firms (S.2). Moreover, it is higher in case when there are knowledge spillovers among high-tech firms (S.2) compared to the case when there is no exchange of knowledge (S.3), i.e.,*

$$g_\lambda^{CFE,S.1} > g_\lambda^{CFE,S.2} > g_\lambda^{CFE,S.3}.$$

Proof. The first inequality follows from that g_λ declines with α. In turn, the second inequality follows from (67) given that ZP is monotonically decreasing function of N and CME in cases there is an exchange of knowledge (S.1-2) is monotonically increasing function of N. □

Given that R&D investments are fixed costs, this implies that there are more high-tech firms in case when there are knowledge spillovers among these firms (S.2) than in case when there is knowledge licensing (S.1). Moreover, there are more high-tech firms in case when there is no exchange of knowledge among these firms (S.3) compared to the case when there are knowledge spillovers (S.2), i.e.,

$$N^{CFE,S.3} > N^{CFE,S.2} > N^{CFE,S.1}.$$

The differences between labor force allocations in cases there is an exchange of knowledge (S.1-2) and there is no exchange of knowledge (S.3) depend on model parameters.

These results indicate that high-tech firms innovate more in cases when there is an exchange of knowledge compared to the case when there is none. Moreover, these firms innovate more in case when there is knowledge licensing compared to the case there are knowledge spillovers/externalities. Meanwhile, using (42)-(44), (46), (59)-(61), and (63) it can be shown that in all S.1-3 cases in decentralized equilibrium with cost-free (endogenous) entry into the high-tech industry the economy invests in R&D less than it is socially optimal. Therefore, it grows at a lower than socially optimal rate. Further, it fails to have socially optimal number of high-tech firms.

Policies leading to the first best outcome in decentralized equilibrium

In this section I offer policies that if implemented in decentralized equilibrium lead to the first best outcome. I assume that there is knowledge licensing in decentralized equilibrium. This can amount to assuming that the government has motivated knowledge exchange among high-tech firms that happens in a market where the licensors have the right to make a 'take it or leave it' offer (i.e., they have the bargaining power). In this respect, such an action is one of the necessary policy instruments for increasing welfare in decentralized equilibrium.[28]

I assume that the set of policy instruments includes marginal taxes on or subsidies to purchases of high-tech goods (τ_x) and high-tech firms' expenditures on buying knowledge (τ_λ). It also includes lump-sum transfers to high-tech firms (T_π) and households (T). The latter balances government expenditures.

Under such a policy from the final goods producer's problem it follows that (8) and (9) need to be rewritten as

$$x_j = X\left[\frac{P_X}{(1-\tau_x)\,p_{x_j}}\right]^{\varepsilon},$$

$$P_X X = (1-\tau_x)\sum_{i=1}^{N} p_{x_i} x_i.$$

[28]This is because in case when there is no exchange of knowledge among high-tech firms there is no set of (orthodox) policy instruments in terms of welfare transfers which in decentralized equilibrium equates labor force allocations and growth rate of knowledge to their socially optimal counterparts.

In turn, the profit function of high-tech firm j is

$$\pi_j = p_{x_j} x_j - w\left(L_{x_j} + L_{r_j}\right) + \left[\sum_{i=1, i\neq j}^{N} p_{u_{j,i}\lambda_j}\left(u_{j,i}\lambda_j\right) - (1-\tau_\lambda)\sum_{i=1, i\neq j}^{N} p_{u_{i,j}\lambda_i}\left(u_{i,j}\lambda_i\right)\right] + T_\pi.$$

Therefore, the demand for knowledge of the high-tech firm (25) needs to be rewritten as

$$[u_{i,j}] : (1-\tau_\lambda)\, p_{u_{i,j}\lambda_i} = q_{\lambda_j}\xi\alpha\left(\frac{\lambda_j}{u_{i,j}\lambda_i}\right)^{1-\alpha} L_{r_j},\ \forall i \neq j.$$

Considering symmetric equilibrium and combining these optimal rules with (6), (7), (24) and labor market clearing condition (36) gives the counterparts of the relation between NL_x and L_Y (32), returns on knowledge accumulation (31), and the relation between NL_x and NL_r (37):

$$NL_x = \frac{1}{1-\tau_x}\frac{\sigma}{1-\sigma} b^k L_Y, \tag{81}$$

$$g_{q_\lambda} = r - g_\lambda\left(\frac{L_x}{L_r} + 1 + \alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda}\right), \tag{82}$$

$$NL_x = D^{GO}\left(L - NL_r\right), \tag{83}$$

where D^{GO} is the counterpart of D^k,

$$D^{GO} = \left[(1-\tau_x)\frac{1-\sigma}{\sigma}\frac{1}{b^k} + 1\right]^{-1},$$

and I use GO to denote decentralized equilibrium with government.

Proposition 23. *Let the marginal tax rates be constant. In such a case, labor force allocations and the growth rate of knowledge g_λ are*

$$NL_r = \frac{1}{\xi}\frac{\xi D^{GO} L - \rho}{(\theta-1)(\sigma+\mu) + D^{GO} - \alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda}},$$

$$NL_x = D^{GO}\frac{\left[(\theta-1)(\sigma+\mu) - \alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda}\right] L + \frac{1}{\xi}\rho}{(\theta-1)(\sigma+\mu) + D^{GO} - \alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda}},$$

$$L_Y = (1-\tau_x)\frac{1-\sigma}{\sigma b^k} NL_x,$$

$$g_\lambda = \xi NL_r.$$

Proof. See Proofs Appendix. □

Therefore, in order to have socially optimal growth rate and allocations it is suffi-

cient to have

$$NL_r = NL_r^{SP}, NL_x = NL_x^{SP}.$$

To achieve such an outcome it is sufficient to subsidize the purchases of high-tech goods,

$$\tau_\lambda = 0, \tag{84}$$

$$\tau_x = \frac{e^k \mu + \sigma}{e^k (\sigma + \mu)}, \tag{85}$$

where τ_x equates D^{GO} to D^{SP}. It is enough to subsidize the demand for high-tech goods because the returns on knowledge accumulation are fully appropriated.[29]

Although under this policy labor force allocations and growth rate of knowledge in decentralized equilibrium are equal to their socially optimal counterparts, welfare is not. This is because in decentralized equilibrium there is lower number of high-tech firms/goods. The policy instrument that can correct for this is T_π. It is straightforward to show that it is sufficient to set

$$T_\pi = wL_x\tau_\pi, \tag{86}$$

where τ_π is such that for any finite N the profits of high-tech firms are greater than zero, but for $N = +\infty$ profits are zero.

Corollary 24. *The rate τ_π can be derived from a zero profit condition and is given by*

$$\begin{aligned}\tau_\pi = &\frac{\varepsilon - 1 + D^{SP}}{(\varepsilon - 1)\left[(\theta - 1)(\sigma + \mu)\xi D^{SP} L + D^{SP}\right]} \\ &\times \left[\frac{\varepsilon - 1 - (\theta - 1)(\sigma + \mu)}{\varepsilon - 1 + D^{SP}} \xi D^{SP} L - \rho\right].\end{aligned} \tag{87}$$

Proof. See Proofs Appendix. □

The expression in the second line of τ_π needs to be positive in order to have $N > 1$ in (75). Therefore, τ_π is greater than zero implying that entry into high-tech industry needs to be subsidized. Such subsidies are in the spirit of R&D subsidies in Romer (1990) model to the extent that entry can be thought to be a result of R&D that generates new types of high-tech goods.[30]

[29] Appendix E.6 offers a policy which instead of subsidizing the purchases of high-tech goods subsidizes the production and R&D expenditures of high-tech firms. It shows that the subsidy rates to these expenditures should be equal in order to have first best allocations and growth rates. This is because in a high-tech firm the allocations of labor to production and R&D are affected by relative price distortions equally.

[30] Appendix E.4 shows how τ_λ can be used together with τ_x in case when high-tech firms do not take the price of knowledge as exogenous. If $\tau_\lambda \neq 0$ then subsidy rate τ_π is not given by (87).

The result that τ_π is greater than zero is not stemming from the cost-free entry assumption. Next section shows that even if entry into the high-tech industry entailed positive costs then still it could be that at least in very long-run the Social Planner sets $N = +\infty$ whereas in decentralized equilibrium the market is saturated for $N < +\infty$. The Social Planner can prefer having $N = +\infty$ because as λ grows the marginal product of N increases.

Entry Regime 3: Costly Entry

In this section I assume that entry into the high-tech industry entails endogenous costs. I focus on the cases when there is an exchange of knowledge among high-tech firms. Further, I do not assume that parameters are such that CME necessarily crosses ZP. This restriction can be lifted since in case when entry entails endogenous costs positive profits can be allowed.

Firm Entry

In order to enter into the high-tech industry and to generate its distinct type of high-tech good, the potential producer has to invest. The investment is in terms of final goods. The entrant should borrow the resources for the investment from the household at the market interest rate r.

The creation of the distinct type of high-tech good is given by

$$\dot{N} = \eta S, \ \eta \geq 0, \tag{88}$$

where $\dot{N}$ is the new high-tech good created by the investment S and η is the efficiency of investments.

The entrants are assumed to break-even on a zero net-value constraint,

$$V\dot{N} = S. \tag{89}$$

From this expression, (2), (6), (7), (9), (18), (89), and Hamilton-Jacobi-Bellman equation $\dot{V} = rV - \pi$ it follows that for $\eta \in (0, +\infty)$

$$Y = C + S, \tag{90}$$

given that the assets in this economy are the high-tech firms ($A = VN$). Meanwhile, in terms of previously analyzed cases of entry, $\eta = 0$ in (88) corresponds to the case when there are exogenous barriers to entry. In such a case (89) does not bind. The limiting case $\eta = +\infty$ corresponds to cost-free endogenous entry. In such a case any

infinitesimally small investment leads to entry. Given that this investment is a cost, the entrants would select to invest 0 and enter. Therefore, in both limiting cases $\eta = 0$ and $\eta = +\infty$ (39) holds.

Hereafter, I assume that η is a small number ($\eta \approx 0$). Such a restriction allows to have no transition in the hypothetical Social Planner's solution.

Decentralized Equilibrium

It is instructive to derive the profit function of a high-tech firm first. As in case when entry entails no cost it can be written as

$$\pi = wL_x\bar{\pi},$$

where

$$\bar{\pi} = \frac{1}{e^k - 1} - \frac{g_\lambda}{r - \left(g_w - I^0_{\dot{N}=0} g_N\right)}, \; k = CR, BR, \tag{91}$$

and

$$I^0_{\dot{N}=0} = \begin{cases} 1 \; for \; \dot{N} \neq 0, \\ 0 \; otherwise. \end{cases}$$

Corollary 25. *$\bar{\pi}$ in (91) is monotonically decreasing function of N.*

Proof. See Proofs Appendix. □

The competition intensifies with the number of firms N. When strategic interactions in the product market are non-negligible, the intensity of competition and profits are related negatively. The negative relation between N and $\bar{\pi}$ reflects exactly this point.

Hereafter, I focus only on the balanced growth path analysis. Depending on the household's preferences, final goods production technology and the high-tech firm's knowledge accumulation process, there are two cases when the economy grows at constant rates. In the first case there are so many high-tech firms that the new entrant's impact on others' demand is negligible. Whereas in the second case, the next entrant will have negative profit streams (i.e., there are endogenous barriers to entry).[31]

In the first case, the counterpart of CME in (46) is always lower than the counterpart of ZP in (70). On the balanced growth path there are infinitely many high-tech firms and there is permanent entry ($N = +\infty$, $\dot{N} > 0$).

[31] This ordering is possible given that $\bar{\pi}$ in (91) is negatively related to the number of firms and the investments in knowledge accumulation are fixed costs.

Proposition 26. *The growth rates of final output and knowledge are*

$$g_Y^{CE} = Bg_\lambda^{CE},$$
$$g_\lambda^{CE} = \frac{\xi DL - \rho}{\left(\theta - 1 + I^0_{\dot{N}=0}\right) B + \alpha I^1_{S.2-3} + D}, \tag{92}$$

where I use superscript CE - costly entry - in order to distinguish the outcomes of this setup and

$$B = \frac{(\varepsilon - 1)(\sigma + \mu)}{\varepsilon - 1 - I^0_{\dot{N}=0}(\sigma + \mu)}.$$

Proof. See Proofs Appendix. □

Labor force allocations in this case can be derived from (30), (36), and (37). In the denominator of g_λ^{CE} (92) $I^0_{\dot{N}=0}$ captures the effect of continuous entry into the high-tech industry on innovation incentives of high-tech firms. Continuous entry erodes the returns on innovation. *Ceteris paribus* this leads to lower investments in R&D.

In the second case, let N^{**} ($< +\infty$) be the last high-tech firm that will have non-negative profit streams if it enters. There is no entry after N^{**} (i.e., $\dot{N} = 0$) because for any $N > N^{**}$ the value V would be negative.[32] When there is no entry, the economy is on a balanced growth path; therefore, N^{**} is determined from the intersection of CME and ZP curves. In such a case, labor force allocations, growth rates and the number of firms under different types of competition can be obtained from (42)-(46) and (71).[33]

Social Optimum

The hypothetical Social Planner's problem is given by (50)-(52) and (88). I assume that the Social Planner can make negative investments in the high-tech industry (i.e., in N) and η is close to zero. These assumptions allow to have no transition in the social optimum.

Proposition 27. *The socially optimal growth rates of final output and knowledge are*

[32] Strictly speaking, the firm that has zero profits invests zero; therefore, according to (88), it also does not enter. Therefore, N^{**} is an upper bound for the number of firms in the high-tech industry. However, since $\bar{\pi}$ in (91) is a continuous function of the number of firms, N^{**} is exactly the number of firms in high-tech industry.

[33] In case when there is no exchange of knowledge and the counterpart of ZP crosses the counterpart of CME from above at finite N then the balanced growth path properties of the model are summarized in the section Entry Regime 2. However, in case when ZP does not cross CME on balanced growth the economy needs to be static in case B is finite and positive.

given by

$$g_Y^{CE,SP} = B^{SP} g_\lambda^{CE,SP}, \tag{93}$$

$$g_\lambda^{CE,SP} = \frac{\xi D^{SP} L - \rho}{(\theta - 1) B^S + D^{SP}}, \tag{94}$$

where

$$B^{SP} = \frac{(\varepsilon - 1)(\sigma + \mu)}{\varepsilon - 1 - (\sigma + \mu)}.$$

Proof. See Proofs Appendix. □

The socially optimal labor force allocations can be found from

$$\begin{aligned} NL_r^{CE,SP} &= \frac{1}{\xi} g_\lambda^{CE,SP}, \\ NL_x^{CE,SP} &= D^S \left(L - NL_r^{CE,SP} \right), \\ L_Y^{CE,SP} &= L - NL_x^{CE,SP} - NL_r^{CE,SP}. \end{aligned}$$

Corollary 28. *There is permanent entry in the social optimum.*

The permanent entry result is due to the absence of market incentives in the social optimum. It stands in contrast to the decentralized equilibrium result where it may be the case that there are endogenous barriers to entry. It holds because the accumulation of knowledge (R&D) increases the marginal product of N.

Comparisons and Policy Inference

It is straightforward to show that in both cases when there are endogenous barriers to entry in decentralized equilibrium ($\dot{N} = 0$) and there are no barriers to entry ($\dot{N} > 0$) the following relationships hold:

$$g_\lambda^{CE,SP} > g_\lambda^{CE,S.1} > g_\lambda^{CE,S.2}.$$

Further, similar to the previous sections, it is straightforward to show that in both cases when $\dot{N} = 0$ and $\dot{N} > 0$ in decentralized equilibrium the economy fails to have socially optimal labor force allocations. From (93) and (94) it also follows that in the social optimum the growth rate of final output is higher if there is continuous entry compared to the case when there is no continuous entry.

Corollary 29. *If there is continuous entry into the high-tech industry ($\dot{N} > 0$) and knowledge licensing among high-tech firms, then the following policy delivers socially*

optimal allocations and growth rates as a decentralized equilibrium outcome.

$$\tau_x = \frac{e^k \mu + \sigma}{e^k (\sigma + \mu)},$$
$$\tau_\lambda = \frac{\frac{N}{N-1}\frac{1}{\alpha}B}{1 + \frac{N}{N-1}\frac{1}{\alpha}B}.$$

Proof. See Proofs Appendix. □

In this policy τ_x is the same as in (85) and subsidizes the purchases of final goods. In contrast, τ_λ in this policy is greater than zero which means that this policy subsidizes also knowledge licensing. It does so in order to motivate R&D in the high-tech industry and alleviate the negative effect of continuous entry on innovation incentives of high-tech firms.

Continuous entry, in turn, can be guaranteed with lump-sum transfers to high-tech firms (86) which make the profits of these firms marginally greater than zero for any N.

4 Conclusions

The model presented in this paper incorporates knowledge (patent) licensing into a stylized endogenous growth framework, where the engine of growth is high-tech firms' in-house R&D. The inference from this model suggests that if there is knowledge licensing high-tech firms innovate more and economic growth is higher than in cases when there are knowledge spillovers and/or there is no knowledge exchange among these firms. The results also suggest that innovation in the high-tech industry and economic growth increase with the intensity and toughness of competition in that industry. Such an inference holds also for the number of high-tech firms if there is an exchange of knowledge among these firms in the form of licensing or spillovers. Increasing the number of high-tech firms increases innovation in the high-tech industry and the growth rate of the economy. However, if there is no exchange of knowledge among high-tech firms, then increasing the number of firms can also discourage innovation in the high-tech industry and reduce economic growth.

Innovation in the high-tech industry declines with the magnitude of externalities which stem from the use of high-tech goods. However, the rate of economic growth increases with it. Further, the existence of such externalities creates a wedge between resource allocations in decentralized equilibrium and socially optimal allocations. In this model, this implies that the existence of externalities also creates a wedge between growth rates in decentralized equilibrium and the socially optimal growth rate.

If entry (or exit) is endogenous and entails no costs, innovation in the high-tech industry and economic growth are again higher in case when there is knowledge licensing. However, this happens at the expense of lower number of high-tech firms. More intensive and/or tougher competition reduce the number of high-tech firms. If there is an exchange of knowledge among these firms the intensity and toughness of competition do not affect, however, allocations, innovation in the high-tech industry, and economic growth. In contrast, allocations change and innovation and economic growth tend to increase with the intensity and toughness of competition if there is no exchange of knowledge among high-tech firms.

If entry entails no costs a policy consisting of four instruments can be sufficient for achieving the first best outcome in decentralized equilibrium. The policy gives the bargaining power in the market for knowledge to the licensors so that they appropriate all the benefit. Further, it subsidizes the purchases of high-tech goods so that it offsets the negative effect of price setting by high-tech firms and takes into account the externalities from the use of high-tech goods. Finally, it subsidizes entry into the high-tech industry and uses lump-sum taxes to cover all these subsidies.

Meanwhile, if entry entails endogenous costs then in the social optimum there is continuous entry into the high-tech industry. In decentralized equilibrium continuous entry erodes the returns on innovation and therefore reduces R&D effort of high-tech firms. In order to alleviate this effect and achieve first best outcomes in decentralized equilibrium the policy also subsidizes knowledge licensing.

Appendix

Proofs Appendix

Proof of Proposition 1: The growth rates of quantities and prices that characterize the essential dynamics of this model can be obtained from (3)-(9), (11), (19), and (20). These growth rates are

$$g_C = \frac{1}{\theta}(r-\rho), \tag{95}$$

$$g_Y = (\sigma+\mu)\, g_X + (1-\sigma)\, g_{L_Y}, \tag{96}$$

$$g_X = \frac{\varepsilon}{\varepsilon-1} g_N + g_x, \tag{97}$$

$$g_Y = g_w + g_{L_Y}, \tag{98}$$

$$g_x = g_\lambda + g_{L_x}, \tag{99}$$

$$g_w = g_{q_\lambda} + g_N + g_\lambda. \tag{100}$$

Combining (31) with (19), (20), (30), (32), (36), (37), (40), and (95)-(100) gives a differential equation in L_r,

$$\begin{aligned}\dot{L}_r = &\frac{L - NL_r}{N\left[(1+\mu)(\theta-1)+1\right]} \\ &\times \left\{\left[(\theta-1)(\sigma+\mu) + \alpha I^1_{S.2-3} + D^k\right] \xi I^N_{S.1-2} L_r - \left(\xi D^k \frac{I^N_{S.1-2}}{N} L - \rho\right)\right\},\end{aligned} \tag{101}$$

for all S.1-3 cases.

Let parameter restriction (41) hold. The first term of the differential equation (101) is non-negative. Without that term, the characteristic root of the differential equation is positive, $\frac{\partial \dot{L}_r}{\partial L_r} > 0$. This, together with neoclassical production function of final goods (4), implies that there is a unique L_r such that (101) is stable and $NL_r, NL_x, L_Y \in (0, L)$,

$$L_r^{NE} = \frac{1}{\xi I^N_{S.1-2}} \frac{\xi D^k \frac{I^N_{S.1-2}}{N} L - \rho}{(\theta-1)(\sigma+\mu) + \alpha I^1_{S.2-3} + D^k}. \tag{102}$$

Combining this expression with the relations between NL_x and L_Y (32) and NL_x and NL_r (37) gives the equilibrium allocations of labor force (42)-(44). Given that allocations of labor force are constant from (40), (96), and (99) it follows that

$$\begin{aligned} g_C^{NE} &= g_Y^{NE} = g_w^{NE} = (\sigma+\mu)\, g_X^{NE}, \\ g_X^{NE} &= g_x^{NE} = g_\lambda^{NE}, \end{aligned}$$

where g_λ is given by (30),

$$g_\lambda^{NE} = \frac{\xi D^k \frac{I_{S.1-2}^N}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + \alpha I_{S.2-3}^1 + D^k}.$$

Therefore, in decentralized equilibrium with no entry if (41) holds the economy makes a discrete "jump" to balanced growth path in all S.1-3 cases.

Proof of Proposition 2: The value of a high-tech firm if high-tech firms innovate [i.e., $NL_r \in (0, L)$] is

$$V^{NL_r \in (0,L)} = \frac{1}{(\theta - 1)(\sigma + \mu) g_\lambda + \rho} \pi(t) \exp\left[-(\sigma + \mu) g_\lambda t\right],$$

where I have dropped the superscript NE and $\pi(t)$ can be derived from (4), (5), (7), (9), (11), (18), (19), (42) and (43),

$$\pi(t) = \frac{1}{N}\sigma \left(N^{\frac{1}{\varepsilon-1}} \lambda(0) NL_x\right)^{\sigma+\mu} L_Y^{1-\sigma} \frac{1}{e^k} \exp\left[(\sigma + \mu) g_\lambda t\right]$$
$$\times \left\{1 - \frac{e^k - 1}{D^k} \frac{\xi D^k \frac{I_{S.1-2}^N}{N} L - \rho}{\left[(\theta - 1)(\sigma + \mu) + \alpha I_{S.2-3}^1\right] \xi \frac{I_{S.1-2}^N}{N} L + \rho}\right\}.$$

In turn, if none of the high-tech firms innovates then the economy is static ($g_Y = g_\lambda = 0$) and each high-tech firm's profits and value are given by

$$\pi = \frac{1}{N}\sigma \left(N^{\frac{\varepsilon}{\varepsilon-1}} \lambda(0) L_x\right)^{\sigma+\mu} L_Y^{1-\sigma} \frac{1}{e^k},$$
$$V^{NL_r=0} = \frac{1}{N}\sigma \left(N^{\frac{\varepsilon}{\varepsilon-1}} \lambda(0) L_x\right)^{\sigma+\mu} L_Y^{1-\sigma} \frac{1}{e^k} \frac{1}{\rho}.$$

It can be easily shown that

$$V^{NL_r \in (0,L)} < V^{NL_r=0},$$

which means that the value of any high-tech firm is higher if none of the high-tech firms engages in R&D.

Proof of Proposition 3: Using (54), the expression for the returns on knowledge accumulation (56) can be rewritten as

$$g_{q_\lambda} = \rho - \left(\frac{1 - \sigma}{1 + \mu} \xi I_{S.1-2}^N L_r + \xi D^{SP} \frac{I_{S.1-2}^N}{N} L\right). \tag{103}$$

Meanwhile, from (51)-(54) and (57) it follows that

$$g_{L_x} = g_{L_Y} = -\frac{N\dot{L}_r}{L - NL_r}, \tag{104}$$

$$g_C = (\sigma + \mu)(g_\lambda + g_{L_x}) + (1 - \sigma) g_{L_x}, \tag{105}$$

$$g_\lambda = \xi I^N_{S.1-2} L_r, \tag{106}$$

$$g_{q_\lambda} = -g_\lambda - g_{L_x} - (\theta - 1) g_C. \tag{107}$$

Combining (103)-(107) gives a differential equation in L_r,

$$\dot{L}_r = \frac{L - NL_r}{N[(\theta - 1)(1 + \mu) + 1]} \times \left\{ \left[(\theta - 1)(\sigma + \mu) + D^{SP}\right] \xi I^N_{S.1-2} L_r - \left(\xi D^{SP} \frac{I^N_{S.1-2}}{N} L - \rho \right) \right\}. \tag{108}$$

Without the first non-negative term this expression implies that $\frac{\partial \dot{L}_r}{\partial L_r} > 0$. Therefore, there is unique L_r such that (108) is stable and $NL_r \in (0, L)$,

$$L_r^{NE,SP} = \frac{1}{\xi I^N_{S.1-2}} \frac{\xi D^{SP} \frac{I^N_{S.1-2}}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}. \tag{109}$$

The numerator in (109) is positive if (58) is positive.

Combining (109) with (53) and (57) gives the socially optimal (interior) allocations of labor force (59)-(61).

Given that labor force allocations are constant from (39) and (105) it follows that

$$g_Y^{NE,SP} = (\sigma + \mu) g_\lambda^{NE,SP},$$

where $g_\lambda^{NE,SP}$ can be derived from (52) and (109),

$$g_\lambda^{NE,SP} = \frac{\xi D^{SP} \frac{I^N_{S.1-2}}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}.$$

Therefore, the Social Planner chooses allocations such that the economy, where there is "no entry", makes a discrete jump to balanced growth path.

Proof of Proposition 4: Lifetime utility of the representative household in case when the Social Planner innovates is

$$U^{NE,SP,NL_r\in(0,L)} \equiv U = \frac{1}{1-\theta}\frac{1}{(\sigma+\mu)(\theta-1)\,g_\lambda^{NE,SP}+\rho}\times \\ \left[\left(N^{\frac{1}{\varepsilon-1}}\lambda(0)\,NL_x^{NE,SP}\right)^{\sigma+\mu}\left(L_Y^{NE,SP}\right)^{1-\sigma}\right]^{1-\theta} - \frac{1}{\rho}\frac{1}{1-\theta}.$$

where $NL_x^{NE,SP}$, $L_Y^{NE,SP}$, and $g_\lambda^{NE,SP}$ are given by (60), (61), and (63). In turn, in case when the Social Planner does not innovate it is

$$U^{NE,SP,NL_r=0} \equiv U = \frac{1}{1-\theta}\frac{1}{\rho}\left[\left(N^{\frac{1}{\varepsilon-1}}\lambda(0)\,NL_x^{NE,SP}\right)^{\sigma+\mu}\left(L_Y^{NE,SP}\right)^{1-\sigma}\right]^{1-\theta} - \frac{1}{\rho}\frac{1}{1-\theta}.$$

where $NL_x^{NE,SP}$ and $L_Y^{NE,SP}$ are given by (65) and (66).

Using (60), (61), (63), (65) and (66) it can be shown that the inequality

$$U^{NE,SP,NL_r=0} \leq U^{NE,SP,NL_r\in(0,L)}$$

is equivalent to

$$\left(\xi D^{SP}\frac{I_{S.1-2}^N}{N}L\frac{1}{\rho}\right)^{(\theta-1)(1+\mu)} \leq \left[\frac{(\theta-1)(1+\mu)\,\xi D^{SP}\frac{I_{S.1-2}^N}{N}L\frac{1}{\rho}+1}{(\theta-1)(1+\mu)+1}\right]^{(\theta-1)(1+\mu)+1}.$$

Denote

$$z = \xi D^{SP}\frac{I_{S.1-2}^N}{N}L\frac{1}{\rho}$$

and take the natural logarithm of both sides of this inequality:

$$0 \leq [(\theta-1)(1+\mu)+1]\left[\ln((\theta-1)(1+\mu)z+1) - \ln((\theta-1)(1+\mu)+1)\right] \\ -(\theta-1)(1+\mu)\ln z.$$

The derivative of the left-hand side of this inequality with respect to z is greater than zero. Meanwhile, the left-hand side is equal to zero in case when $z=1$. Therefore, given that (58) holds $z>1$ and

$$U^{NE,SP,NL_r=0} \leq U^{NE,SP,NL_r\in(0,L)}.$$

Proof of Proposition 14: It is straightforward to show that if the number of firms N is fixed the economy is on a balanced growth path. Further, it is straightforward to show that $\bar{\pi}$ in (68) declines with N (see also proposition 29). This, together with

cost-free entry and that $\bar{\pi}$ in (68) is constant on balanced growth path, implies that at time zero ($t = 0$) N makes a discrete jump to the balanced growth path equilibrium level where $\bar{\pi} = 0$. Therefore, thereafter in decentralized equilibrium with cost-free entry the economy is always on a balanced growth path.

Proof of Proposition 18: If (74) and the remaining optimal rules/constraints are binding, then for the case when there is no exchange of knowledge ($I^N_{S.1-2} = 1$) it is straightforward to show that the optimal labor force allocations are

$$NL_r^{CFE,SP,S.3} = \frac{\sigma + \mu}{\varepsilon (1 + \mu) - (1 - \sigma)} L, \tag{110}$$

$$NL_x^{CFE,SP,S.3} = \frac{(\varepsilon - 1)(\sigma + \mu)}{\varepsilon (1 + \mu) - (1 - \sigma)} L, \tag{111}$$

$$L_Y^{CFE,SP,S.3} = \frac{(\varepsilon - 1)(1 - \sigma)}{\varepsilon (1 + \mu) - (1 - \sigma)} L. \tag{112}$$

It can be further shown that the returns on knowledge accumulation are given by

$$g_{q_\lambda} = \rho - \xi \frac{I^N_{S.1-2}}{N} \frac{\varepsilon (\sigma + \mu)}{\varepsilon (1 + \mu) - (1 - \sigma)} L. \tag{113}$$

In turn, from (51), (52), (74) and (110)-(112) it follows that

$$g_{L_Y} = g_{NL_x} = g_{NL_r} = 0, \tag{114}$$

$$g_C = (\sigma + \mu) \left(\frac{1}{\varepsilon - 1} g_N + g_\lambda \right), \tag{115}$$

$$g_\lambda = \xi L_r, \tag{116}$$

$$g_{q_\lambda} = -g_\lambda - (\theta - 1) g_C + g_N. \tag{117}$$

Combining these conditions with (113) gives a differential equation in N,

$$g_N = -\frac{\varepsilon - 1}{\varepsilon - 1 - (\theta - 1)(\sigma + \mu)} \times \left[\xi (\sigma + \mu) \frac{\varepsilon - 1 - (\theta - 1)(\sigma + \mu)}{\varepsilon (1 + \mu) - (1 - \sigma)} L \frac{1}{N} - \rho \right].$$

Since $\frac{\partial g_N}{\partial N} > 0$ the only stable solution is (75),

$$N = \frac{\xi (\sigma + \mu)}{\rho} \frac{\varepsilon - 1 - (\theta - 1)(\sigma + \mu)}{\varepsilon (1 + \mu) - (1 - \sigma)} L,$$

which implies that

$$g_N = 0.$$

Therefore, from (52) and (110) it follows that (76) holds,

$$g_{\lambda}^{CFE,SP,S.3} = \frac{\rho}{\varepsilon - 1 - (\theta - 1)(\sigma + \mu)}.$$

This implies that the economy needs to make a discrete jump to balanced growth path at time zero.

Proof of Proposition 19: In order to check whether (74) is binding denote

$$\bar{U}^{CFE,SP,S.3,N<+\infty} = U^{CFE,SP,S.3,N<+\infty} + \frac{1}{1-\theta}\frac{1}{\rho}.$$

From (51) it follows that

$$\bar{U}^{CFE,SP,S.3,N<+\infty} = \frac{1}{1-\theta}\frac{1}{(\theta-1)(\sigma+\mu)\, g_{\lambda}^{SP} + \rho} \times \left[\left(N^{\frac{1}{\varepsilon-1}} \lambda(0)\, NL_x\right)^{\sigma+\mu} L_Y^{1-\sigma}\right]^{1-\theta},$$

where N, g_{λ}^{SP}, NL_x, and L_Y are given by (75), (76), (111), and (112) correspondingly.

In case when $\theta > 1$

$$\bar{U}^{CFE,SP,S.3,N=+\infty} = 0,$$

whereas

$$\bar{U}^{CFE,SP,S.3,N<+\infty} \leq 0.$$

Meanwhile, in case when $\theta = 1$

$$\bar{U}^{CFE,SP,S.3,N=+\infty} = +\infty,$$

whereas

$$\bar{U}^{CFE,SP,S.3,N<+\infty} < +\infty.$$

Clearly, therefore,

$$\bar{U}^{CFE,SP,S.3,N=+\infty} > \bar{U}^{CFE,SP,S.3,N<+\infty},$$

implying that the solution with finite N is not optimal.

Therefore, in case when there is no exchange of knowledge ($I_{S.1-2}^{N} = 1$) the Social

Planner sets

$$
\begin{aligned}
&N = +\infty, \\
&g_{\lambda}^{CFE,SP,S.3} = NL_{r}^{CFE,SP,S.3} = 0, \\
&NL_{x}^{CFE,SP,S.3} = D^{SP}L, \\
&L_{Y}^{CFE,SP,S.3} = \frac{1-\sigma}{\sigma+\mu}D^{SP}L,
\end{aligned}
$$

and the economy is static.

Proof of Proposition 20: If there is an exchange of knowledge among high-tech firms, the expression for perceived elasticity of substitution e^k (71) indicates that e^k does not depend on the type of competition. Since for any given number of firms the perceived elasticity of substitution is higher under Bertrand competition ($e^{BR} > e^{CR}$), from (71) it follows that in equilibrium there are fewer high-tech firms under Bertrand competition than under Cournot competition. Further, given that perceived elasticities of substitution monotonically increase with the number of firms and actual elasticity of substitution, from (71) it follows that the number of firms under both types of competition declines with ε and increases with μ. It also increases with α if there are knowledge spillovers (S.2) and does not depend on α if there is knowledge licensing (S.1).

If there is no exchange of knowledge among high-tech firms the right-hand side of (72) and perceived elasticity of substitution e^k from (34) and (35) are increasing in N and $e^{BR} > e^{CR}$ for any N. Therefore, also in this case there are more firms under Cournot competition than under Bertrand competition.[34] Moreover, the number of firms N declines with ε and increases with μ and α.

Proof of Proposition 21: In cases when there is an exchange of knowledge among high-tech firms

$$
\begin{aligned}
\frac{\partial}{\partial\alpha}g_{\lambda} &= \frac{(\theta-1)(\sigma+\mu)\xi L\frac{\partial}{\partial\alpha}D + \alpha I_{S.2-3}^{1}\xi L\frac{\partial}{\partial\alpha}D - (\xi DL-\rho) + \rho\frac{\partial}{\partial\alpha}D}{\left[(\theta-1)(\sigma+\mu)+\alpha I_{S.2-3}^{1}+D\right]^{2}}, \\
\frac{\partial}{\partial\mu}g_{\lambda} &= \frac{\xi L\left[(\theta-1)(\sigma+\mu)+\alpha I_{S.2-3}^{1}\right]\frac{\partial}{\partial\mu}D - (\theta-1)(\xi DL-\rho) + \rho\frac{\partial}{\partial\mu}D}{\left[(\theta-1)(\sigma+\mu)+\alpha I_{S.2-3}^{1}+D\right]^{2}}.
\end{aligned}
$$

[34] It can be shown also that the quadratic polynomial in (72) opens upward and under Bertrand competition for any N it is lower than under Cournot competition. Since stable equilibrium corresponds to the smaller roots of the polynomials, the number of firms is lower under Bertrand competition.

Therefore the sign of $\frac{\partial}{\partial\alpha}g_\lambda$ is equivalent to the sign of

$$(\theta-1)(\sigma+\mu)\xi L\frac{\partial}{\partial\alpha}D+\alpha I^1_{S.2-3}\xi L\frac{\partial}{\partial\alpha}D-(\xi DL-\rho)+\rho\frac{\partial}{\partial\alpha}D, \tag{118}$$

where

$$D=\frac{\sigma\left\{\xi\sigma L\left[\alpha I^1_{S.2-3}+(\theta-1)(\sigma+\mu)\right]+\rho\right\}}{\xi\sigma L\left[1-\sigma+\alpha I^1_{S.2-3}+(\theta-1)(\sigma+\mu)\right]+\sigma\rho},$$
$$\frac{\partial}{\partial\alpha}D=\frac{\xi\sigma L(1-\sigma)\sigma(\xi\sigma L-\rho)}{\left\{\xi\sigma L\left[1-\sigma+\alpha I^1_{S.2-3}+(\theta-1)(\sigma+\mu)\right]+\sigma\rho\right\}^2}.$$

Using these expressions and manipulating (118) gives the following expression

$$\xi\sigma L(1-\sigma)-\{\xi\sigma L[1-\sigma+\alpha+(\theta-1)(\sigma+\mu)]+\sigma\rho\}$$

which has a negative value. This implies that g_λ and NL_r decline with α. Moreover, since $NL_x=D(L-NL_r)$ and $\frac{\partial}{\partial\alpha}D>0$ this implies that NL_x increases with α.

In turn, the sign of $\frac{\partial}{\partial\mu}g_\lambda$ is equivalent to the sign of

$$\xi L\left[(\theta-1)(\sigma+\mu)+\alpha I^1_{S.2-3}\right]\frac{\partial}{\partial\mu}D-(\theta-1)(\xi DL-\rho)+\rho\frac{\partial}{\partial\mu}D, \tag{119}$$

where

$$\frac{\partial}{\partial\mu}D=\frac{\xi\sigma L(1-\sigma)\sigma(\xi\sigma L-\rho)(\theta-1)}{\{\xi\sigma L[1-\sigma+\alpha+(\theta-1)(\sigma+\mu)]+\sigma\rho\}^2}.$$

Using this expression and manipulating (119) gives the following expression

$$\xi\sigma\sigma L(1-\sigma)-\{\xi\sigma L[1-\sigma+\alpha+(\theta-1)(\sigma+\mu)]+\sigma\rho\}$$

which has a negative value. This implies that g_λ and NL_r decline with μ. Moreover, since $NL_x=D(L-NL_r)$ and $\frac{\partial}{\partial\mu}D>0$ this implies that NL_x increases with μ. In contrast,

$$\frac{\partial}{\partial\mu}g_Y=\frac{(\theta-1)(\sigma+\mu)(\sigma+\mu)\xi L\frac{\partial}{\partial\mu}D+\alpha I^1_{S.2-3}\left[(\xi DL-\rho)+(\sigma+\mu)\xi L\frac{\partial}{\partial\mu}D\right]}{\left[(\theta-1)(\sigma+\mu)+\alpha I^1_{S.2-3}+D\right]^2}$$
$$+\frac{D(\xi DL-\rho)+(\sigma+\mu)\rho\frac{\partial}{\partial\mu}D}{\left[(\theta-1)(\sigma+\mu)+\alpha I^1_{S.2-3}+D\right]^2}$$

which is positive.

Proof of Proposition 23: Let the marginal tax rates be constant. This implies that (95)-(100) hold. Combining (82), (83), and (95)-(100) gives the counterpart of (101),

$$\dot{L}_r = \frac{L - NL_r}{N\left[(1+\mu)(\theta-1)+1\right]} \times \left\{\left[(\theta-1)(\sigma+\mu) + D^{GO} - \alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda}\right]\xi N L_r - \left(\xi D^{GO} L - \rho\right)\right\}.$$

The stationary solution of this differential equation is given by

$$L_r = \frac{1}{\xi N}\frac{\xi D^{GO} L - \rho}{(\theta-1)(\sigma+\mu) + D^{GO} - \alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda}}.$$

Remaining labor force allocations can be derived from (81) and (83).

Proof of Corollary 24: Subsidy/tax rate τ_π can be derived from zero profit condition

$$\pi = 0 \Leftrightarrow \tau_\pi = \frac{L_r}{L_x} - \frac{1}{\varepsilon - 1}.$$

where

$$\frac{L_r}{L_x} = \frac{L_r^{SP}}{L_x^{SP}} = \frac{\xi D^{SP} L - \rho}{(\theta-1)(\sigma+\mu)\xi D^{SP} L + D^{SP}\rho}.$$

Proof of Corollary 25: To prove that $\bar{\pi}$ is monotonically decreasing in N consider its first term. It can be shown that

$$\frac{\partial e^k}{\partial N} > 0 \; k = CR, BR.$$

This implies that the first term is monotonically decreasing function of N. For the second term,

$$\frac{\partial}{\partial N}\frac{g_\lambda}{r - (g_w - \delta g_N)} = \frac{\frac{NL_x}{L_Y}\left(\frac{\partial}{\partial N}\frac{NL_r}{L_Y}\right) - \frac{NL_r}{L_Y}\left(\frac{\partial}{\partial N}\frac{NL_x}{L_Y}\right)}{\left(\frac{NL_x}{L_Y}\right)^2},$$

where

$$\frac{\partial}{\partial N}\frac{NL_r}{L_Y} = \frac{1}{b}\left(\frac{NL_r + L_Y}{L_Y}\right)\frac{\partial b}{\partial N},$$
$$\frac{\partial}{\partial N}\frac{NL_x}{L_Y} = \frac{1}{b}\frac{NL_x}{L_Y}\frac{\partial b}{\partial N}.$$

Therefore,

$$-\frac{\partial}{\partial N}\frac{g_\lambda}{r - (g_w - \delta g_N)} = -\left(\frac{1}{b^k}\right)^2\frac{1-\sigma}{\sigma}\frac{\partial b^k}{\partial N},$$

where

$$\frac{\partial b^k}{\partial N} > 0.$$

Therefore, the second term is monotonically decreasing function of the number of firms as well. Hence, $\bar{\pi}$ is monotonically decreasing function of N.

An alternative proof for $\bar{\pi}' < 0$ uses the labor market clearing condition (36), final and telecom goods production functions (4), (11), and the relation between labor demand in final goods and high-tech goods production. A sufficient condition to observe the desired is $b\sigma L\frac{1-\sigma}{1+\mu} < NL_x$, which can be shown to hold from the labor market clearing condition.

Proof of Proposition 26: The growth rates and labor force allocations can be derived from (30)-(38) and (95)-(100). In case when there is continuous entry into the high-tech industry the growth rate of knowledge is

$$g_\lambda^{CE} = \frac{\xi DL - \rho}{\left(\theta - 1 + I^0_{\dot{N}=0}\right) B + \alpha I^1_{S.2-3} + D}, \tag{120}$$

The growth rate of consumption, final output, number of firms and savings are

$$\begin{aligned} g_C^{CE} &= g_Y^{CE} = g_N^{CE} = g_S^{CE} \\ &= Bg_\lambda^{CE}. \end{aligned}$$

Proof of Proposition 27: Given that in this case N is endogenous state variable it is convenient to rewrite labor force allocations to knowledge accumulation and production of high-tech goods as

$$\begin{aligned} \bar{L}_r &= NL_r, \\ \bar{L}_x &= NL_x. \end{aligned}$$

The hypothetical Social Planner's then solves:

$$\max_{S,\bar{L}_x,\bar{L}_r} U = \int_0^{+\infty} \frac{C_t^{1-\theta}-1}{1-\theta} \exp\left(-\rho t\right) dt.$$

$$s.t.$$

$$Y = C + S, \tag{121}$$

$$Y = \left(N^{\frac{1}{\varepsilon-1}} \lambda \bar{L}_x\right)^{\sigma+\mu} \left(L - \bar{L}_x - \bar{L}_r\right)^{1-\sigma}, \tag{122}$$

$$\dot{\lambda} = \xi \lambda \bar{L}_r, \tag{123}$$

$$\dot{N} = \eta S, \tag{124}$$

$$\lambda\left(0\right) > 0, N\left(0\right) > 1.$$

The Social Planner's optimal choice for accumulation of N is given by

$$[N] : \dot{q}_N = q_N \rho - \frac{\sigma+\mu}{\varepsilon-1} Y \frac{1}{N} C^{-\theta}. \tag{125}$$

The remaining optimal rules are as follows

$$\left[\bar{L}_x\right] : \bar{L}_x = \frac{\sigma+\mu}{1-\sigma} L_Y \tag{126}$$

$$\left[\bar{L}_r\right] : q_\lambda \xi \lambda = \left(1-\sigma\right) C^{-\theta} \frac{Y}{L_Y} \tag{127}$$

$$[\lambda] : \dot{q}_\lambda = q_\lambda \rho - \left[q_\lambda \xi \bar{L}_r + \left(\sigma+\mu\right) C^{-\theta} \frac{Y}{\lambda}\right] \tag{128}$$

Since C and S are in the same terms it has to be that

$$C^{-\theta} = \eta q_N. \tag{129}$$

Using expression (127) and labor market clearing condition (36) the returns on knowledge accumulation (128) can be rewritten as

$$g_{q_\lambda} = \rho - \left(\frac{1-\sigma}{1+\mu} \xi \bar{L}_r + \frac{\sigma+\mu}{1+\mu} \xi L\right). \tag{130}$$

In turn, from (36) and (122)-(127) it follows that

$$g_Y = (\sigma + \mu)\left(\frac{1}{\varepsilon - 1} g_N + g_\lambda + g_{\bar{L}_x}\right) + (1 - \sigma)\, g_{L_Y}, \tag{131}$$

$$g_\lambda = \xi \bar{L}_r, \tag{132}$$

$$g_N = \eta \frac{S}{N},$$

$$g_{\bar{L}_x} = g_{L_Y} = -\frac{\partial \bar{L}_r}{\partial t} \frac{1}{L - \bar{L}_r},$$

$$g_{q_\lambda} = -\theta g_C + g_Y - g_{L_Y} - g_\lambda. \tag{133}$$

From these expressions and (125) it is possible to derive a differential equation in L_Y,

$$g_{L_Y} = -\frac{\sigma + \mu}{\mu} \xi L + \xi \frac{\sigma + \mu}{1 - \sigma} L_Y + \frac{\sigma + \mu}{(\varepsilon - 1)\, \mu} \eta \frac{C}{N}. \tag{134}$$

Since growth rate of L_Y increases with L_Y the only stationary solution of this equation is $g_{L_Y} = 0$. This implies that labor force allocations are constant in the social optimum

$$g_{\bar{L}_r} = g_{\bar{L}_x} = g_{L_Y} = 0.$$

Moreover, (134) implies a relation between N and λ on balanced growth path and

$$g_C = g_N.$$

These results, together with (121)-(125) and (128), imply that

$$g_N = const$$

and

$$g_N = g_S = g_C = g_Y.$$

From (131), (132), (133) and labor market clearing condition (36) then it follows that

$$g_Y^{CE,SP} = B^{SP} g_\lambda^{CE,SP},$$

$$g_\lambda^{CE,SP} = \frac{\xi D^{SP} L - \rho}{(\theta - 1)\, B^{SP} + D^{SP}},$$

and

$$NL_r^{CE,SP} = \frac{1}{\xi}\frac{\xi D^{SP}L - \rho}{(\theta-1)B^{SP} + D^{SP}}, \tag{135}$$

$$NL_x^{CE,SP} = D^{SP}\frac{1}{\xi}\frac{\xi(\theta-1)B^{SP}L + \rho}{(\theta-1)B^{SP} + D^{SP}}, \tag{136}$$

$$L_Y^{CE,SP} = \frac{1-\sigma}{\sigma+\mu}D^{SP}\frac{1}{\xi}\frac{\xi(\theta-1)B^{SP}L + \rho}{(\theta-1)B^{SP} + D^{SP}}. \tag{137}$$

It can be shown that as long as there can be negative investments in N and η is sufficiently low in the social optimum the economy makes a discrete jump to balanced growth path at time zero ($t = 0$). This holds because when the economy is relatively abundant of N [(137) does not hold] then the Social Planner at time zero selects negative investments in N so that (137) holds from the following instance. Meanwhile, sufficiently low η guarantees that balanced growth path value of N is so low that when the economy is relatively abundant of λ there are sufficient resources for savings that (immediately) cover the gap between initial and balanced growth path value of N. The Social Planner in such a case also selects savings so that the economy makes a discrete jump to balanced growth path.

Proof of Corollary 29: Let $\dot{N} > 0$ and

$$\tau_x = \frac{e^k\mu + \sigma}{e^k(\sigma+\mu)}$$

so that D^{GO} and D^{SP} are equivalent. Combining equations (95)-(100) with (82) and (83) gives the counterpart of (101),

$$N\dot{L}_r = \frac{L - NL_r}{\theta B\frac{1+\mu}{\sigma+\mu} + 1}\left[\left(\theta B + D^{SP} - \alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda}\right)\xi NL_r - \left(\xi D^{SP}L - \rho\right)\right].$$

If

$$\alpha\frac{N-1}{N}\frac{\tau_\lambda}{1-\tau_\lambda} = B,$$

or equivalently

$$\tau_\lambda = \frac{\frac{N}{N-1}\frac{1}{\alpha}B}{1 + \frac{N}{N-1}\frac{1}{\alpha}B},$$

then labor force allocations and growth rates in decentralized equilibrium coincide with the choices of the Social Planner.

Appendix E.1

In this section I present a setup where high-tech firms cooperate in R&D and select optimal rules for R&D so that to maximize joint profits. High-tech firms later compete in the product market. I call this case CO - R&D cooperation.[35]

I offer below the setup of the high-tech industry and the optimization problem of high-tech firms in the stage of R&D cooperation.

R&D Cooperation: Each high-tech firm has its knowledge. At R&D cooperation stage high-tech firms establish a research joint venture where they pool their knowledge and jointly hire researchers. In a "laboratory" a group of researchers combines the knowledge of different firms in order to produce a better one for a firm. There are as many laboratories (or different knowledge production processes) as many there are high-tech firms. This research joint venture takes into account the effect of the accumulation of one type of knowledge on the accumulation of other types of knowledge.[36]

In such a case high-tech firms take (19) as given and jointly solve the following optimal problem.

$$\max_{NL_r} \int_{\bar{t}}^{+\infty} \left\{ \sum_{j=1}^{N(t)} \pi_j(t) \exp\left[-\int_{\bar{t}}^{t} r(s)\, ds \right] \right\} dt$$

$s.t.$

$$\sum_{j=1}^{N} \pi_j = \sum_{j=1}^{N} \left(p_{x_j} \lambda_j - w \right) L_{x_j} - wNL_r, \tag{138}$$

$$x_j = \lambda_j L_{x_j}, \tag{139}$$

$$\dot{\lambda}_j = \xi \left(\sum_{i=1}^{N} \lambda_i^{\alpha} \right) \lambda_j^{1-\alpha} L_{r_j}. \tag{140}$$

The optimal rules for R&D that follow from this problem are

$$\left[L_{r_j} \right] : w = q_{\lambda_j} \frac{\dot{\lambda}_j}{L_{r_j}}, \tag{141}$$

$$\left[\lambda_j \right] : \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left(\sum_{j=1}^{N} \frac{e_j^k - 1}{e_j^k} \frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} \right), \tag{142}$$

[35] It might be argued that firms' cooperation in R&D increases the odds that they will collude in the product market. I rule this out in order to focus on the differences between knowledge exchange mechanisms.

[36] An alternative cooperation mode is that high-tech firms in R&D stage jointly hire researchers and produce the same knowledge for all. In such a case the knowledge accumulation process is $\dot{\lambda} = \xi \lambda N L_r$. It can be easily shown that the decentralized equilibrium outcome of this cooperation mode is no different than the outcome of the cooperation mode offered in this section.

where

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi L_{r_j} \tag{143}$$
$$\times \left\{ 1 + (1-\alpha) \left(\sum_{i=1, i \neq j}^{N} \frac{\lambda_i}{\lambda_j} \right)^{\alpha} + \alpha \left[\sum_{i=1, i \neq j}^{N} \left(\frac{\lambda_i}{\lambda_j} \right)^{-(1-\alpha)} \frac{\partial \lambda_i}{\partial \lambda_j} \right] \right\},$$

and

$$\frac{\partial \lambda_i}{\partial \lambda_j} = \frac{\partial \lambda_i}{\partial t} \frac{\partial t}{\partial \lambda_j} = \left(\frac{\lambda_i}{\lambda_j} \right)^{1-\alpha} \frac{L_{r_i}}{L_{r_j}}. \tag{144}$$

The third term in the second line of (143) illustrates effect of the accumulation of the jth type of knowledge (the knowledge of high-tech firm j) on the accumulation of remaining types of knowledge.

In symmetric equilibrium, according to (140) the growth rate of knowledge is

$$g_\lambda = \xi N L_r. \tag{145}$$

The rate of return on knowledge accumulation can be derived from (19), (141)-(144). It is the same as (31) where $I^1_{S.2-3} = 0$,

$$g_{q_\lambda} = r - g_\lambda \left(\frac{L_x}{L_r} + 1 \right).$$

The growth rates of quantities and prices that characterize the essential dynamics of this model if there is R&D cooperation are given by (95)-(99) and

$$g_w = g_{q_\lambda} + g_\lambda. \tag{146}$$

This equation is the counterpart of (100).

From high-tech firm's demand for labor for production (19), final goods producer's optimal rules (6)-(7), and the relation between $P_X X$ and $p_x x$ (9) follows a relationship between NL_x and L_Y (32). This relationship together with labor market clearing condition (36), implies a relationship between NL_x and NL_r (37).

Meanwhile, in the final goods market (39) holds

$$Y = C.$$

Combining (31) with (19), (32), (36), (37), (95)-(99), (141), (145), and (146) gives

a differential equation in L_r,

$$\dot{L}_r = \frac{L - NL_r}{N\left[(1+\mu)(\theta-1)+1\right]} \times \left\{\left[(\theta-1)(\sigma+\mu)+D^k\right]\xi NL_r - \left(\xi D^k L - \rho\right)\right\}, \tag{147}$$

which is the counterpart of (101).

Let $\theta \geq 1$ and (41) hold. Therefore, given that the first term of this differential equation is non-negative there is unique L_r such that (147) is stable and $NL_r, NL_x, L_Y \in (0, L)$,

$$L_r = \frac{1}{\xi N}\frac{\xi D^k L - \rho}{(\theta-1)(\sigma+\mu)+D^k}.$$

Combining this expression with the relations between NL_x and L_Y (32) and NL_x and NL_r (37) and (145) gives the equilibrium allocations of labor force and growth rates of final output and knowledge

$$\begin{aligned}
NL_r^{NE} &= \frac{1}{\xi}\frac{\xi D^k L - \rho}{(\theta-1)(\sigma+\mu)+D^k},\\
NL_x^{NE} &= D^k\frac{(\theta-1)(\sigma+\mu)L + \frac{1}{\xi}\rho}{(\theta-1)(\sigma+\mu)+D^k},\\
L_Y^{NE} &= \frac{1-\sigma}{\sigma b^k}D^k\frac{\left[(\theta-1)(\sigma+\mu)\right]L + \frac{1}{\xi}\rho}{(\theta-1)(\sigma+\mu)+D^k},\\
g_Y^{NE} &= (\sigma+\mu)\, g_\lambda^{NE},\\
g_\lambda^{NE} &= \frac{\xi D^k L - \rho}{(\theta-1)(\sigma+\mu)+D^k}.
\end{aligned}$$

Therefore, in decentralized equilibrium with no entry and R&D collaboration if (41) holds the economy makes a discrete jump to balanced growth path. Further, the growth rates and labor force allocations are the same in cases when there is knowledge licensing (S.1) and R&D collaboration (CO). This means that if there is no (continuous) entry knowledge licensing and R&D cooperation deliver equivalent equilibrium outcomes. Therefore, the policy (84)-(85) also leads to the first best outcome in terms of allocations and growth rates in this case.

Further, in line with the results offered in the section where I discuss policies in order to have socially optimal number of high-tech firms there need to be lump-sum transfers to high-tech firms given by (86). These transfers make sure profits are greater than zero for any finite N and are zero for $N = +\infty$.

When there is continuous entry into the high-tech industry equations (100) and (146) identify the difference between R&D cooperation (CO) and knowledge licensing (S.1). The rate of return on knowledge accumulation in case when there is knowledge

licensing declines with continuous entry of firms ($\dot{N} > 0$). In contrast, in case when there is R&D cooperation it does not do so. This is because in R&D cooperation case firms choose R&D expenditures to maximize joint profits. Meanwhile, in case when there is knowledge licensing entry erodes the profits and returns on knowledge accumulation of high-tech firms.

It can be easily shown that when there is continuous entry and R&D cooperation the growth rate of knowledge/productivity is

$$g_{\lambda}^{CE} = \frac{\xi DL - \rho}{(\theta - 1)\frac{(\varepsilon-1)(\sigma+\mu)}{\varepsilon-1-(\sigma+\mu)} + D}. \tag{148}$$

This implies that, the policy (84)-(85) leads to the first best outcome in terms of allocations and growth rates in this case.

Comparing (120) and (148) it is straightforward to notice that

$$g_{\lambda}^{CE} > g_{\lambda}^{CE,S.1-2}.$$

This is because continuous entry ($\dot{N} > 0$) into high-tech industry decreases the returns on knowledge accumulation if high-tech firms engage in R&D disjointly.

Appendix E.2

In this section I show that adding knowledge depreciation and spillovers in case when there is knowledge licensing does not alter the main results. I consider exclusively S.1 and S.3 cases and the decentralized equilibrium of the model. I further assume that there are exogenous barriers to entry into the high-tech industry.

In order to support symmetric equilibrium I assume that the rate of depreciation of knowledge is the same across high-tech firms, δ (> 0). This implies that the knowledge accumulation processes in case when there are is no knowledge exchange among high-tech firms (S.3) can be written as

$$\dot{\lambda}_j = \xi \tilde{\lambda} \lambda_j^{1-\alpha} L_{r_j} - \delta \lambda_j. \tag{149}$$

Meanwhile, adding spillovers in the knowledge accumulation process in case there is knowledge licensing results in

$$\dot{\lambda}_j = \xi \left[\sum_{i=1}^{N} \hat{\lambda}_i \left(u_{i,j} \lambda_i \right)^{\alpha_1} \right] \lambda_j^{\alpha_2} L_{r_j} - \delta \lambda_j, \tag{150}$$

$$\alpha_1 + \alpha_2 > 1 - \alpha,$$

where I assume that in equilibrium

$$\hat{\lambda}_i \equiv (u_{i,j}\lambda_i)^{1-\alpha_1-\alpha_2}.$$

The optimal problem of high-tech firm j in such a case is

$$\max_{\substack{Cournot:\ L_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1;(i\neq j)}^N \\ Bertrand:\ p_{x_j}, L_{r_j}, \{u_{j,i}, u_{i,j}\}_{i=1;(i\neq j)}^N}} V_j(\bar{t}) = \int_{\bar{t}}^{+\infty} \pi_j(t) \exp\left[-\int_{\bar{t}}^{t} r(s)\, ds\right] dt$$

$$s.t.$$

$$\pi_j = p_{x_j} x_j - w\left(L_{x_j} + L_{r_j}\right)$$

$$+\left[\sum_{i=1,i\neq j}^{N} p_{u_{j,i}\lambda_j}(u_{j,i}\lambda_j) - \sum_{i=1,i\neq j}^{N} p_{u_{i,j}\lambda_i}(u_{i,j}\lambda_i)\right],$$

$$(8), (11), (149) \text{ or } (150).$$

The demand functions for labor force for production and R&D are then given by

$$\left[L_{x_j}\right] : w = \lambda_j p_{x_j}\left(1 - \frac{1}{e_j^k}\right), \tag{151}$$

$$\left[L_{r_j}\right] : w = q_{\lambda_j}\frac{\partial \dot{\lambda}_j}{\partial L_{r_j}}. \tag{152}$$

In case when there is knowledge licensing (and spillovers; S.1) the returns on knowledge accumulation are

$$[\lambda_j] : \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left(\frac{e_j^k - 1}{e_j^k}\frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} + \sum_{i=1,i\neq j}^{N} \frac{p_{u_{j,i}\lambda_j} u_{j,i}}{q_{\lambda_j}}\right),$$

where

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi \lambda_j^{\alpha_2 - 1} L_{r_j}\left[\alpha_2 \sum_{i=1}^{N} \hat{\lambda}_i (u_{i,j}\lambda_i)^{\alpha_1} + \alpha_1 \hat{\lambda}_j \lambda_j^{\alpha_1}\right] - \delta,$$

and the supply of and demand for knowledge are

$$[u_{j,i}] : u_{j,i} = 1,\ \forall i \neq j,$$

$$[u_{i,j}] : p_{u_{i,j}\lambda_i} = q_{\lambda_j}\xi\alpha_1 \hat{\lambda}_i (u_{i,j}\lambda_i)^{\alpha_1 - 1} \lambda_j^{\alpha_2} L_{r_j},\ \forall i \neq j.$$

Meanwhile, in case when there is no exchange of knowledge among high-tech firms

(S.3) the returns on knowledge accumulation are

$$[\lambda_j] : \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left[\frac{e_j^k - 1}{e_j^k} \frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + (1-\alpha)\, \xi \tilde{\lambda} \lambda_j^{-\alpha} L_{r_j} - \delta \right].$$

In a symmetric equilibrium, in cases when there is knowledge licensing (S.1) and no exchange of knowledge among high-tech firms (S.3) returns on knowledge accumulation can be rewritten as

$$g_{q_\lambda} = r + \delta - (g_\lambda + \delta) \left(\frac{L_x}{L_r} + I_{S.3}^{1-\alpha} \right), \tag{153}$$

where

$$I_{S.3}^{1-\alpha} = \begin{cases} \alpha_1 + \alpha_2 \ for \ S.1, \\ 1 - \alpha \ for \ S.3. \end{cases}$$

Using (37), (95)-(100), (151) and (152) this expression can be rewritten as a differential equation in L_r,

$$\begin{aligned} \dot{L}_r = & \frac{L - NL_r}{N\left[(\theta-1)(1+\mu)+1\right]} \times \\ & \left(\left[(\theta-1)(\sigma+\mu) + D^k + 1 - I_{S.3}^{1-\alpha} \right] \xi I_{S.1-2}^N L_r - \right. \\ & \left. \left\{ \xi D^k \frac{I_{S.1-2}^N}{N} L + \left[(\theta-1)(\sigma+\mu) + D^k \right] \delta - \rho \right\} \right), \end{aligned}$$

Let

$$\xi D^k \frac{I_{S.1-2}^N}{N} L - \left(1 - I^S\right) \delta - \rho > 0.$$

This differential equation is stable if

$$L_r = \frac{1}{\xi I_{S.1-2}^N} \frac{\xi D^k \frac{I_{S.1-2}^N}{N} L + \left[(\theta-1)(\sigma+\mu) + D^k \right] \delta - \rho}{(\theta-1)(\sigma+\mu) + D^k + 1 - I_{S.3}^{1-\alpha}}.$$

This implies that the economy immediately jumps to balanced growth path where labor force allocations and growth rates of final output and knowledge are

$$\begin{aligned} NL_r &= \frac{N}{\xi I_{S.1-2}^N} \frac{\xi D^k \frac{I_{S.1-2}^N}{N} L + \left[(\theta-1)(\sigma+\mu) + D^k \right] \delta - \rho}{(\theta-1)(\sigma+\mu) + D^k + 1 - I_{S.3}^{1-\alpha}}, \\ NL_x &= D^k \left(L - NL_r \right), \\ L_Y &= \frac{1-\sigma}{\sigma b^k} NL_x, \\ g_Y &= (\sigma+\mu)\, g_\lambda, \\ g_\lambda &= \frac{\xi D^k \frac{I_{S.1-2}^N}{N} L - \left(1 - I_{S.3}^{1-\alpha}\right) \delta - \rho}{(\theta-1)(\mu+\sigma) + D^k + 1 - I_{S.3}^{1-\alpha}}. \end{aligned}$$

Therefore,

$$\frac{\partial g_\lambda}{\partial \delta} < 0, \frac{\partial NL_r}{\partial \delta} > 0, \frac{\partial NL_x}{\partial \delta} < 0, \frac{\partial L_Y}{\partial \delta} < 0,$$

and

$$\frac{\partial g_\lambda}{\partial I_{S.3}^{1-\alpha}} > 0, \frac{\partial NL_r}{\partial I_{S.3}^{1-\alpha}} > 0, \frac{\partial NL_x}{\partial I_{S.3}^{1-\alpha}} < 0, \frac{\partial L_Y}{\partial I_{S.3}^{1-\alpha}} < 0. \tag{154}$$

Relationships (154) imply that the growth rate of productivity and labor force allocation to productivity/knowledge accumulation are decreasing with the degree of not appropriated returns on knowledge accumulation. Meanwhile, NL_x and L_Y are increasing with it. This is analogous to the results in section Entry Regime 1.

Appendix E.3

In this section I relax the assumption that there are externalities within high-tech firms in two ways and present the main properties of the model. First I assume that there are decreasing returns to knowledge accumulation at firm-level unless there is an exchange of knowledge among high-tech firms. Next, I assume instead that there no externalities in high-tech firms and, as in the main text, returns on knowledge accumulation are constant even if there is no exchange of knowledge.

I have assumed that N is a real number. If N also changes continuously then in the sums in (12) and (13) each firm has zero size. Since λ of each firm is finite dropping firm j or any finite number of firms from those sums makes no difference for the inference.

If N changes discretely (and each firm has unit size) I assume that $N - 1 > 1$ so that knowledge exchange can only increase the productivity of researchers. In such a circumstance I assume that if there is knowledge licensing the knowledge accumulation process of high-tech firm j is given by

$$\dot{\lambda}_j = \xi \left[\sum_{i=1, i \neq j}^{N} (u_{i,j} \lambda_i)^\alpha \right] \lambda_j^{1-\alpha} L_{r_j}. \tag{155}$$

This is the counterpart of (12) where $u_{j,j} \equiv 0$. In turn, if there are knowledge spillovers the knowledge accumulation process is given by (13) where

$$\tilde{\Lambda} \equiv \sum_{i=1, i \neq j}^{N} \lambda_i^\alpha. \tag{156}$$

If there is no knowledge exchange among high-tech firms I assume that knowledge accumulation process is given by (15) where

$$\tilde{\lambda} \equiv 1. \tag{157}$$

Therefore, the counterparts of (24), (28), and (29) are given by

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi (1-\alpha) \left[\sum_{i=1, i\neq j}^{N} \left(\frac{u_{i,j}\lambda_i}{\lambda_j} \right)^{\alpha} \right] L_{r_j}, \tag{158}$$

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi (1-\alpha) \left[\sum_{i=1, i\neq j}^{N} \left(\frac{u_{i,j}\lambda_i}{\lambda_j} \right)^{\alpha} \right] L_{r_j}, \tag{159}$$

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi (1-\alpha) \lambda_j^{-\alpha} L_{r_j}. \tag{160}$$

I further focus on symmetric equilibrium analysis in the high-tech industry. For the subsequent analysis it is useful to define function $I_{S.1-2}^{N-1}$ as

$$I_{S.1-2}^{N-1} = \begin{cases} \lambda^{-\alpha} \ for \ S.3, \\ N-1 \ otherwise. \end{cases}$$

In such a case the growth rate of knowledge in the high-tech industry in all setups (S.1-3) can be rewritten as

$$g_\lambda = \xi I_{S.1-2}^{N-1} L_r. \tag{161}$$

The (internal) rate of return on knowledge accumulation can be obtained from the optimal rules of the high-tech firm (19), (20), and (23), (27), (158)-(160). It is given by (31),

$$g_{q_\lambda} = r - g_\lambda \left(\frac{L_x}{L_r} + 1 - \alpha I_{S.2-3}^{1} \right),$$

where

$$g_\lambda = \xi I_{S.1-2}^{N-1} L_r.$$

Combining (31) with (19), (20), (32), (36), (37), (40), (95)-(100) and (161) gives the counterpart of (101),

$$\begin{aligned} \dot{L}_r = & \frac{L - NL_r}{N\left[(1+\mu)(\theta-1)+1\right]} \\ & \times \left\{ \left[(\theta-1)(\sigma+\mu) + \alpha I_{S.2-3}^{1} + D^k \right] \xi \frac{I_{S.1-2}^{N-1}}{N} NL_r - \left(\xi D^k \frac{I_{S.1-2}^{N-1}}{N} L - \rho \right) \right\}. \end{aligned} \tag{162}$$

Assuming that

$$\xi D^k \frac{N-1}{N} L - \rho > 0,$$

if there is exchange of knowledge the stable solution of this differential equation is

$$NL_r = \frac{1}{\xi} \frac{\xi D^k L - \frac{N}{N-1}\rho}{(\theta-1)(\sigma+\mu) + \alpha I_{S.2-3}^{1} + D^k}.$$

Therefore, g_λ is given by

$$g_\lambda = \frac{\xi D^k \frac{N-1}{N} L - \rho}{(\theta - 1)(\sigma + \mu) + \alpha I^1_{S.2-3} + D^k}.$$

This implies that the comparative statics with respect to σ, μ, α, ε, and type of competition presented in the section Entry Regime 1 hold. Moreover, g_λ increases with N and at least for sufficiently high N ($N > 2$) it is concave in N.

Meanwhile, in case there is no exchange of knowledge the expression (162) is second order differential equation in knowledge λ. It describes the path of λ. In the steady-state the growth rate of knowledge and labor force allocation to knowledge accumulation are zero. Therefore, labor force allocations to high-tech and final goods production are given by (48) and (49).

If there are no knowledge externalities within high-tech firms

In this section I assume that everything else the same in case when there are knowledge spillovers among high-tech firms the knowledge accumulation process is given by

$$\dot{\lambda}_j = \xi \left[\lambda_j + \tilde{\Lambda} \lambda_j^{1-\alpha} \right] L_{r_j}, \tag{163}$$

where I assume that in equilibrium $\tilde{\Lambda}$ is given by (156). Meanwhile, in case when there is no exchange of knowledge among high-tech firms I assume that the knowledge accumulation process is given by

$$\dot{\lambda}_j = \xi \lambda_j L_{r_j}. \tag{164}$$

From (163) it follows that (28) needs to be rewritten as

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi \left[1 + (1-\alpha) \sum_{i=1, i \neq j}^{N} \left(\frac{\lambda_i}{\lambda_j} \right)^\alpha \right] L_{r_j}. \tag{165}$$

In turn, from (164) it follows that (29) needs to be rewritten as

$$\frac{\partial \dot{\lambda}_j}{\partial \lambda_j} = \xi (1-\alpha) L_{r_j}. \tag{166}$$

The (internal) rate of return on knowledge accumulation can be derived from the optimal rules of the high-tech firm (19), (20), and (23), (27), (163), (164), (165), and

(166). In a symmetric equilibrium, in case there are knowledge spillovers it is given by

$$g_{q_\lambda} = r - g_\lambda \left[\frac{L_x}{L_r} + \frac{1 + (1-\alpha)(N-1)}{N} \right], \tag{167}$$

and in case when there is no exchange of knowledge among high-tech firms it is given by (31) where $I^1_{S.2-3} = 0$,

$$g_{q_\lambda} = r - g_\lambda \left(\frac{L_x}{L_r} + 1 \right). \tag{168}$$

From (168) it follows that if the knowledge accumulation process is given by (164) then the growth rate of knowledge and labor force allocations are given by (42)-(46) where $I^1_{S.2-3} = 0$ and $I^N_{S.1-2} = 1$. Therefore, the comparative statics with respect to σ, μ, ε, N and type of competition presented in the section Entry Regime 1 hold. Meanwhile, g_λ does not depend on α.

Further, combining (167) with (19), (20), (30), (32), (36), (37), (40), and (95)-(100) gives the counterpart of (101),

$$\begin{aligned} \dot{L}_r = & \frac{L - NL_r}{N\left[(\theta-1)(1+\mu)+1\right]} \times \\ & \left\{ \left[(\theta-1)(\sigma+\mu) + D^k + \alpha \frac{N-1}{N} \right] \xi N L_r - \left(\xi D^k L - \rho \right) \right\}. \end{aligned}$$

Therefore, the stable solution of this differential equation is

$$NL_r = \frac{1}{\xi} \frac{\xi D^k L - \rho}{(\theta-1)(\sigma+\mu) + D^k + \alpha \frac{N-1}{N}}.$$

This implies that the growth rate of knowledge is given by

$$g_\lambda^{S.2,NEx} = \frac{\xi D^k L - \rho}{(\theta-1)(\sigma+\mu) + D^k + \alpha \frac{N-1}{N}}.$$

Therefore, the comparative statics with respect to σ, μ, α, ε, and type of competition presented in the section Entry regime 1 hold.

If N changes continuously then $\frac{N-1}{N}$ can be replaced by 1 and $g_\lambda^{S.2,NEx}$ is increasing and concave in N. Meanwhile, in case when N changes discretely $g_\lambda^{S.2,NEx}$ is increasing and concave in N if parameters θ, ρ (and σ and μ) are sufficiently high and N is sufficiently small. However, if θ and ρ are low (e.g., $\theta = 1, \rho = 0$) or N is high then $g_\lambda^{S.2,NEx}$ is decreasing and convex in N. It can be further shown that

$$\begin{aligned} & g_\lambda^{S.1} > g_\lambda^{S.2,NEx} > g_\lambda^{S.2}, \\ & \lim_{N\to+\infty} g_\lambda^{S.2,NEx} = \lim_{N\to+\infty} g_\lambda^{S.2}. \end{aligned}$$

Appendix E.4

In this section I present the main properties of the model if high-tech firms take into account the effect of knowledge accumulation on the price of knowledge $p_{u_{j,i}\lambda_j}$. Further, I offer a policy that if implemented in decentralized equilibrium leads to socially optimal outcomes.[37]

The high-tech firms in this case internalize the demand (23). Therefore, the profit function of high-tech firm j "at the stage" when it designs its supply of knowledge and knowledge accumulation is

$$\pi_j = p_{x_j} x_j - w\left(L_{x_j} + L_{r_j}\right) + \left[\alpha\xi \sum_{i=1, i\neq j}^{N} q_{\lambda_i} \left(u_{j,i}\lambda_j\right)^{\alpha} \lambda_i^{1-\alpha} L_{r_i} - \sum_{i=1, i\neq j}^{N} p_{u_{i,j}\lambda_i} \left(u_{i,j}\lambda_i\right)\right].$$

This implies that everything else the same (25) needs to be rewritten as

$$[\lambda_j] : \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left[\frac{e_j^k - 1}{e_j^k} \frac{p_{x_j}}{q_{\lambda_j}} L_{x_j} + \frac{\partial \dot{\lambda}_j}{\partial \lambda_j} + \alpha^2 \xi \sum_{i=1, i\neq j}^{N} \frac{q_{\lambda_i} \left(u_{j,i}\lambda_j\right)^{\alpha} \lambda_i^{1-\alpha} L_{r_i}}{q_{\lambda_j} \lambda_j}\right].$$

Therefore, in symmetric equilibrium the rate of return on knowledge accumulation is

$$g_{q_\lambda} = r - g_\lambda \left[\frac{L_x}{L_r} + 1 - \alpha\left(1 - \alpha\right) \frac{N-1}{N}\right]. \tag{169}$$

In this expression the third term in square brackets captures the adverse effect of higher knowledge accumulation on the price of knowledge.

Combining (95)-(100), (37), and (169) gives the counterpart of (101),

$$\dot{L}_r = \frac{L - NL_r}{N\left[\left(1+\mu\right)\left(\theta - 1\right) + 1\right]} \times \left\{\left[\left(\theta - 1\right)\left(\sigma + \mu\right) + D^k + \alpha\left(1 - \alpha\right)\frac{N-1}{N}\right] \xi N L_r - \left(\xi D^k L - \rho\right)\right\}.$$

[37] I assume that price discrimination is not feasible. This is necessary in order to avoid the problem with determination of the price of durable goods (Coase, 1972). In this framework it can be supported, for example, by an assumption that the licensors have to license their entire knowledge (at a uniform price). Another assumption that could support this is that licensors rent (but not sell) their knowledge and cannot monitor its use.

Therefore, in equilibrium

$$
\begin{aligned}
NL_r^{NE,M} &= \frac{1}{\xi}\frac{\xi D^k L - \rho}{(\theta-1)(\sigma+\mu) + D^k + \alpha(1-\alpha)\frac{N-1}{N}}, \\
NL_x^{NE,M} &= D^k \frac{\left[(\theta-1)(\sigma+\mu) + \alpha(1-\alpha)\frac{N-1}{N}\right] L + \frac{1}{\xi}\rho}{(\theta-1)(\sigma+\mu) + D^k + \alpha(1-\alpha)\frac{N-1}{N}}, \\
L_Y^{NE,M} &= \frac{1-\sigma}{\sigma b^k} NL_x, \\
g_\lambda^{NE,M} &= \xi NL_r^{NE,M},
\end{aligned}
$$

where I use M in order to indicate that the firms are price setters in the market for knowledge in the sense that they internalize the effect of knowledge accumulation on the price of knowledge. If N changes continuously then $\frac{N-1}{N}$ can be replaced by 1 in all these expressions.

Comparing these results with (42)-(46) it is clear that for any given N

$$
\begin{aligned}
&NL_r^{NE,S.1} > NL_r^{NE,M} > NL_r^{NE,S.2}, \\
&NL_x^{NE,S.1} < NL_x^{NE,M} < NL_x^{NE,S.2}, \\
&L_Y^{NE,S.1} < L_Y^{NE,M} < L_Y^{NE,S.2}, \\
&g_\lambda^{NE,S.1} > g_\lambda^{NE,M} > g_\lambda^{NE,S.2}.
\end{aligned}
$$

Therefore, with cost-free entry assumption

$$g_\lambda^{CFE,S.1} > g_\lambda^{CFE,M} > g_\lambda^{CFE,S.2},$$

and

$$N^{CFE,S.1} < N^{CFE,M} < N^{CFE,S.2}.$$

This is because ZP is monotonically decreasing function of N.

If N changes continuously then $g_\lambda^{NE,M}$ is increasing and concave in N. It is increasing and concave in N also in case N changes discretely if parameters θ, ρ (and σ and μ) are sufficiently high and N is sufficiently small. However, if θ and ρ are low (e.g., $\theta = 1, \rho = 0$) or N is high then $g_\lambda^{NE,M}$ can be decreasing and convex in N.

These results imply that if high-tech firms take into account the effect of knowledge accumulation on the price of knowledge they innovate less. Therefore, the economy grows at a lower rate than the economy where high-tech firms do not take into account this effect. Moreover, since

$$g_\lambda^{SP} > g_\lambda^{S.1}$$

the economy (again) fails to grow at the socially optimal rate and fails to have socially

optimal labor allocations.

A policy that can equate decentralized equilibrium allocations and growth rates to their socially optimal counterparts subsidizes the demand for high-tech goods and high-tech firms' demand for knowledge. It can be shown that this policy is

$$\tau_x = \frac{e^k \mu + \sigma}{e^k (\sigma + \mu)},$$
$$\tau_\lambda = 1 - \alpha.$$

Further, in line with the results offered in the section where I discuss policies in order to have socially optimal number of high-tech firms there need to be lump-sum transfers to high-tech firms given by (86). These transfers make sure profits are greater than zero for any finite N and are zero for $N = +\infty$.

The profit function of high-tech firms can be rewritten as

$$\pi = wL_x \left[\frac{1}{e^k - 1} - \left(1 - \alpha \frac{N-1}{N} \frac{\tau_\lambda}{1 - \tau_\lambda} \right) \frac{L_r}{L_x} + \tau_\pi^M \right], \tag{170}$$

where

$$\frac{L_r}{L_x} = \frac{L_r^{SP}}{L_x^{SP}} = \frac{\xi D^{SP} L - \rho}{(\theta - 1)(\sigma + \mu) \xi D^{SP} L + D^{SP} \rho}.$$

Therefore,

$$\tau_\pi^M = \alpha \frac{L_r}{L_x} - \frac{1}{\varepsilon - 1}.$$

This implies that unlike τ_π from (87) the rate τ_π^M can be negative, for example, if $\alpha \approx 0$.[38]

Appendix E.5

In this section I present the main properties of the model in case when final goods producers do not hire labor ($\sigma = 1$) or L_Y is fixed.

If $\sigma = 1$ then (4) is given by

$$Y = \tilde{X} X. \tag{171}$$

and final goods producers' demand for high-tech goods bundle is given by

$$P_X = \tilde{X}. \tag{172}$$

[38] In order to have a meaningful policy a parameter restriction is required so that τ_π^M which solves zero profit condition for (170) is increasing in N.

Assuming symmetric equilibrium this implies that (96) needs to be rewritten as

$$g_Y = (1+\mu)\, g_X, \tag{173}$$

and (98) needs to be replaced by

$$g_{q_\lambda} = \mu\left(g_\lambda + g_{L_x}\right), \tag{174}$$

which follows from (5), (10), (11), (19), (20), and (172).

Since $L_Y = 0$ the labor market clearing condition is

$$L = NL_x + NL_r. \tag{175}$$

Combining (31) with (30), (40), (95), (97), (99), and (173)-(175) gives a differential equation in L_r,

$$\begin{aligned}\dot{L}_r = &\frac{L - NL_r}{N\left[(\theta-1)(1+\mu)+1\right]}\\ &\times\left\{\left[(\theta-1)(1+\mu)+\alpha I^1_{S.2-3}+1\right]\xi I^N_{S.1-2}L_r - \left(\xi\frac{I^N_{S.1-2}}{N}L-\rho\right)\right\}.\end{aligned}$$

Therefore, labor force allocations and growth rates of final output and knowledge/productivity are given by

$$\begin{aligned}NL_r &= \frac{N}{\xi I^N_{S.1-2}}\frac{\xi\frac{I^N_{S.1-2}}{N}L-\rho}{(\theta-1)(1+\mu)+\alpha I^1_{S.2-3}+1},\\ NL_x &= \frac{\left[(\theta-1)(1+\mu)+\alpha I^1_{S.2-3}\right]L+\frac{N}{\xi I^N_{S.1-2}}\rho}{(\theta-1)(1+\mu)+\alpha I^1_{S.2-3}+1},\\ g_Y &= (1+\mu)\, g_\lambda,\\ g_\lambda &= \frac{\xi\frac{I^N_{S.1-2}}{N}L-\rho}{(\theta-1)(1+\mu)+\alpha I^1_{S.2-3}+1}.\end{aligned}$$

Given that in this case $L_Y = 0$ these expressions coincide with (42)-(46) in the limit when $\sigma = 1$. They suggest that if $\sigma = 1$ labor force allocations and, therefore, growth rates do not depend on competitive pressure in the high-tech industry. This is because in this case there are no relative price distortions in the sense that all prices are affected in a similar manner.

In case, however, $L_Y \equiv \zeta_1 > 0$ then from (32) and (36) it follows that

$$NL_x = \frac{\sigma}{1-\sigma}\frac{e^k - 1}{e^k}\zeta_1,$$
$$NL_r = L - \frac{e^k - \sigma}{(1-\sigma)\, e^k}\zeta_1.$$

Increasing competitive pressure in the high-tech industry increases e in these expressions. Therefore, NL_x increases with e, whereas NL_r declines with it, which means that increasing the competitive pressure in this case increases the output of the high-tech industry but reduces the amount of resources devoted to innovation. This is because increasing the competitive pressure increases NL_x and since L_Y is fixed that reduces NL_r.

In case when the wage of researchers L_r is given $w_{L_r} \equiv \zeta_2 Z$ the demand for labor for R&D of high-tech firm j is given by

$$w_{L_r} = q_{\lambda_j}\frac{\dot{\lambda}_j}{L_{r_j}}.$$

Combining this expression with (19) gives the relative demand for labor for production. In symmetric equilibrium the relative demand is

$$\xi I^N_{S.1-2}\frac{w}{w_{L_r}} = \frac{e-1}{e}\frac{p_x}{q_\lambda}.$$

Combining these expressions with returns on knowledge accumulation (23) and (24)-(29) gives

$$g_{w_{L_r}} = r - g_\lambda\left(\frac{w}{w_{L_r}}\frac{NL_x}{NL_r} - \alpha I^1_{S.2-3}\right).$$

Assuming that $g_{w_{L_r}} = g_w$ from this expression, (40), and (95)-(99) it follows then

$$-g_{L_x} = \frac{1}{(\theta-1)(1+\mu)+1}$$
$$\times\left\{\left[(\theta-1)(\sigma+\mu) + \alpha I^1_{S.2-3}\right] g_\lambda + \rho - g_\lambda\frac{w}{w_{L_r}}\frac{NL_x}{NL_r}\right\}.$$

In turn, from (37) it follows that

$$\dot{L}_r = \frac{L - NL_r}{N\left[(1+\mu)(\theta-1)+1\right]} \times \left\{ \left[(\theta-1)(\sigma+\mu) + \alpha I^1_{S.2-3} + \frac{w}{w_{L_r}} D^k \right] \xi I^N_{S.1-2} L_r - \left(\xi \frac{I^N_{S.1-2}}{N} \frac{w}{w_{L_r}} D^k L - \rho \right) \right\}.$$

Therefore, labor force allocation to R&D in the high-tech industry and growth rate of knowledge are given by

$$NL_r = \frac{N}{\xi I^N_{S.1-2}} \frac{\xi \frac{I^N_{S.1-2}}{N} \frac{w}{w_{L_r}} D^k L - \rho}{(\theta-1)(\sigma+\mu) + \alpha I^1_{S.2-3} + \frac{w}{w_{L_r}} D^k},$$

$$g_\lambda = \frac{\xi \frac{I^N_{S.1-2}}{N} \frac{w}{w_{L_r}} D^k L - \rho}{(\theta-1)(\sigma+\mu) + \alpha I^1_{S.2-3} + \frac{w}{w_{L_r}} D^k}.$$

This implies that reducing the relative wage $\frac{w}{w_{L_r}}$ reduces innovation.

Appendix E.6

In this section I show that subsidies to production of high-tech goods (τ_{L_x}) and R&D expenditures (τ_{L_r}) can also lead to first best labor force allocations and growth rates. Under such a policy the profit function of high-tech firm j is

$$\pi_j = p_{x_j} x_j - (1-\tau_{L_x}) w L_{x_j} - (1-\tau_{L_r}) w L_{r_j} + \left[\sum_{i=1, i\neq j}^{N} p_{u_{j,i}\lambda_j} (u_{j,i}\lambda_j) - \sum_{i=1, i\neq j}^{N} p_{u_{i,j}\lambda_i} (u_{i,j}\lambda_i) \right] + T_\pi.$$

In turn, its demand for labor for the production of its high-tech good (19) and demand for labor for R&D (20) are given by

$$\left[L_{x_j}\right] : (1-\tau_{L_x}) w = \lambda_j p_{x_j} \left(1 - \frac{1}{e_j}\right),$$

$$\left[L_{r_j}\right] : (1-\tau_{L_r}) w = q_{\lambda_j} \frac{\dot{\lambda}_j}{L_{r_j}}.$$

Assuming symmetric equilibrium and combining these optimal rules with (6), (7), (24) and labor market clearing condition (36) gives the counterparts of the relation between NL_x and L_Y (32), returns on knowledge accumulation (31), and the relation

between NL_x and NL_r.

$$\begin{aligned}
NL_x &= \frac{1}{1-\tau_{L_x}}\frac{\sigma}{1-\sigma}b^k L_Y, \\
g_{q_\lambda} &= r - g_\lambda\left(\frac{1-\tau_{L_x}}{1-\tau_{L_r}}\frac{L_x}{L_r}+1\right), \\
NL_x &= D^{GO}\left(L - NL_r\right),
\end{aligned}$$

where

$$D^{GO} = \left[\left(1-\tau_{L_x}\right)\frac{1-\sigma}{\sigma}\frac{1}{b^k}+1\right]^{-1}.$$

Assuming that subsidy rates are constant and combining these conditions with (95)-(100) gives the counterpart of (101),

$$\begin{aligned}
\dot{L}_r = &\frac{L-NL_r}{N\left[\left(1+\mu\right)\left(\theta-1\right)+1\right]} \\
&\times\left\{\left[\left(\theta-1\right)\left(\sigma+\mu\right)+D^{GO}\frac{1-\tau_{L_x}}{1-\tau_{L_r}}\right]\xi NL_r \right. \\
&\left. -\left(\xi D^{GO}\frac{1-\tau_{L_x}}{1-\tau_{L_r}}L-\rho\right)\right\}.
\end{aligned}$$

Labor force allocations and the growth rate of knowledge g_λ then are

$$\begin{aligned}
NL_r &= \frac{1}{\xi}\frac{\xi D^{GO}\frac{1-\tau_{L_x}}{1-\tau_{L_r}}L-\rho}{\left(\theta-1\right)\left(\mu+\sigma\right)+D^{GO}\frac{1-\tau_{L_x}}{1-\tau_{L_r}}}, \\
NL_x &= D^{GO}\frac{\left(\theta-1\right)\left(\mu+\sigma\right)L+\frac{1}{\xi}\rho}{\left(\theta-1\right)\left(\mu+\sigma\right)+D^{GO}\frac{1-\tau_{L_x}}{1-\tau_{L_r}}}, \\
L_Y &= \left(1-\tau_x\right)\left(1-\tau_{L_x}\right)\frac{1-\sigma}{\sigma b^k}NL_x, \\
g_\lambda &= \xi NL_r.
\end{aligned}$$

Therefore, in order to have socially optimal growth rate and labor allocations it is sufficient to have

$$NL_r = NL_r^{SP}, NL_x = NL_x^{SP}.$$

In order to achieve such outcomes it is sufficient to subsidize the expenditures of high-tech firms

$$\tau_{L_x} = \tau_{L_r} = \frac{e^k\mu+\sigma}{e^k\left(\sigma+\mu\right)},$$

In this case, τ_{L_x} and τ_{L_r} are equal because in decentralized equilibrium the relative price distortions affect wages of L_x and L_r in the same way.

Appendix T.1

The elasticities of substitution between the knowledge that high-tech firm j licenses from other firms and between its knowledge and the knowledge of other firms can be derived from (12).

The elasticity of substitution between the knowledge licensed from firm m and firm k ($m \neq k$) is given by

$$\varepsilon^{\lambda}_{m,k} = \frac{1}{1-\alpha}.$$

In turn, the elasticity of substitution between the knowledge bought from firm k and firm j's own knowledge can be derived in the following way.

$$\varepsilon^{\lambda}_{j,k} = \frac{d\ln\left(\frac{u_{k,j}\lambda_k}{\lambda_j}\right)}{(1-\alpha)\,d\ln\left(\frac{u_{k,j}\lambda_k}{\lambda_j}\right) + d\ln\left[(1-\alpha)\sum\limits_{i=1,i\neq j}^{N}\left(\frac{u_{i,j}\lambda_i}{\lambda_j}\right)^{\alpha}+1\right]}.$$

Denote

$$\frac{u_{k,j}\lambda_k}{\lambda_j} = z,$$

and rewrite $\varepsilon^{\lambda}_{j,k}$ as

$$\varepsilon^{\lambda}_{j,k} = \frac{1}{1-\alpha+\alpha\frac{(1-\alpha)z^{\alpha}}{(1-\alpha)z^{\alpha}+(1-\alpha)\sum\limits_{i=1,i\neq j,k}^{N}\left(\frac{u_{i,j}\lambda_i}{\lambda_j}\right)^{\alpha}+1}}.$$

Since the third term in the denominator of $\varepsilon^{\lambda}_{j,k}$ is positive

$$\varepsilon^{\lambda}_{j,k} < \varepsilon^{\lambda}_{m,k}.$$

This means that the elasticity of substitution between the firm's knowledge with the knowledge that it licenses from other firms is lower than the elasticity of substitution between the different types of knowledge that it licenses from other firms.

References

Aghion, P., N. Bloom, R. Blundell, R. Griffith, and P. Howitt (2005). Competition and innovation: An inverted-U relationship. *The Quarterly Journal of Economics 120*(2), 701–728.

Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica 60*(2), 323–351.

Bessen, J. and E. S. Maskin (2009). Sequential innovation, patents, and imitation. *The RAND Journal of Economics 40*(4), 611–635.

Blundell, R., R. Griffith, and J. M. van Reenen (1999). Market share, market value and innovation in a panel of British manufacturing firms. *Review of Economic Studies 66*(3), 529–554.

Clark, J., J. Piccolo, B. Stanton, and K. Tyson (2000). Patent pools: A solution to the problem of access in biotechnology patents? US Patent and Trademark Office, white paper (last accessed 23.10.2012). `http://www.uspto.gov/web/offices/pac/dapp/opla/patentpool.pdf`.

Coase, R. H. (1972). Durability and monopoly. *Journal of Law and Economics 15*(1), 143–149.

Gallini, N. T. (1984). Deterrence by market sharing: A strategic incentive for licensing. *The American Economic Review 74*(5), 931–941.

Gallini, N. T. and R. A. Winter (1985). Licensing in the theory of innovation. *The RAND Journal of Economics 16*(2), 237–252.

Grindley, P. C. and D. J. Teece (1997). Managing intellectual capital: Licensing and cross-licensing in semiconductors and electronics. *California Management Review 39*(2), 8–41.

Grossman, G. M. and E. Helpman (1995). Technology and trade. In G. M. Grossman and K. Rogoff (Eds.), *Handbook of International Economics* (1 ed.), Volume 3. North-Holland: Elsevier B.V.

Hagedoorn, J. (1993). Understanding the rationale of strategic technology partnering: Interorganizational modes of cooperation and sectoral differences. *Strategic Management Journal 14*(5), 371–385.

Hagedoorn, J. (2002). Inter-firm R&D partnerships: An overview of major trends and patterns since 1960. *Research policy 31*(4), 477–492.

Helpman, E. (Ed.) (1998). *General purpose technologies and economic growth*. Cambridge, MA: MIT Press.

Jorgenson, D. W., M. S. Ho, and K. J. Stiroh (2005). *Information technology and the American growth resurgence*. Cambridge, MA: MIT Press.

Katz, M. L. and C. Shapiro (1985). On the licensing of innovations. *The RAND Journal of Economics 16*(4), 504–520.

Leff, N. H. (1984). Externalities, information costs, and social benefit-cost analysis for economic development: An example from telecommunications. *Economic Development and Cultural Change 32*(2), 255–276.

Nickell, S. J. (1996). Competition and corporate performance. *Journal of Political Economy 104*(4), 724–746.

O'Donoghue, T. and J. Zweimüller (2004). Patents in a model of endogenous growth. *Journal of Economic Growth 9*(1), 81–123.

Peretto, P. F. (1998a). Technological change and population growth. *Journal of Economic Growth 3*(4), 283–311.

Peretto, P. F. (1998b). Technological change, market rivalry, and the evolution of the capitalist engine of growth. *Journal of Economic Growth 3*(1), 53–80.

Rivera-Batiz, L. A. and P. M. Romer (1991). Economic integration and endogenous growth. *The Quarterly Journal of Economics 106*(2), 531–555.

Robbins, C. A. (2009). Measuring payments for the supply and use of intellectual property. In M. Reinsdorf and M. J. Slaughter (Eds.), *International trade in services and intangibles in the era of globalization*. Chicago, IL: University of Chicago Press.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy 98*(5), 71–102.

Shapiro, C. (1985). Patent licensing and R&D rivalry. *The American Economic Review 75*(2), 25–30.

Shapiro, C. (2001). Navigating the patent thicket: Cross licenses, patent pools, and standard setting. In *Innovation Policy and the Economy*, Volume 1, pp. 119–150. Cambridge, MA: MIT Press.

Smulders, S. and T. van de Klundert (1995). Imperfect competition, concentration and growth with firm-specific R&D. *European Economic Review 39*(1), 139–160.

Sutton, J. (1991). *Sunk costs and market structure: Price competition, advertising, and the evolution of concentration*. Cambridge, MA: MIT Press.

Tanaka, H., T. Iwaisako, and K. Futagami (2007). Dynamic analysis of innovation and international transfer of technology through licensing. *Journal of International Economics 73*(1), 189–212.

van de Klundert, T. and S. Smulders (1997). Growth, competition and welfare. *Scandinavian Journal of Economics 99*(1), 99–118.

Vives, X. (2008). Innovation and competitive pressure. *The Journal of Industrial Economics 56*(3), 419–469.

Yang, G. and K. E. Maskus (2001). Intellectual property rights, licensing, and innovation in an endogenous product-cycle model. *Journal of International Economics 1*(53), 169–187.