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Incumbency (dis)advantage when citizens can propose

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Abstract: This paper analyses the problem that an incumbent faces during the legislature when deciding how to react to citizen proposals such as the outcome of referenda or popular initiatives. We argue that these proposals constitute a potential source of electoral disadvantage when citizens factor in their evaluation of the incumbent his reaction to these proposals. This is because an incumbent politician may jeopardize his re-election by implementing policies close to his preferred ones but unpopular among the electorate. We characterize conditions under which this potential disadvantage becomes in fact an electoral advantage for the incumbent. We fi nd that the choices of the
incumbent during the legislature will be closest to citizens policy proposals when the intensity of electoral competition is neither too soft nor too tough. Finally, we use our results to discuss some implications of the use of mechanisms such as referenda and popular assemblies on electoral competition and on the incumbency advantage phenomenon.

JEL Codes: D7, H1..

Keywords: Incumbency advantage, Referenda, Popular initiatives, Elections.

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1 Introduction

The incumbency advantage is a well documented phenomenon, according to which an incumbent politician is more likely to be reelected than a challenger candidate. Empirical studies, such as Gelman and King (1990) and Lee (2008), provide strong evidence in favor of the existence of such advantage in the US House.[1] Ansolabehere and Snyder (2002) find empirical support for the incumbency advantage hypothesis also in US state executives. These authors argue that this advantage does not originate in the strategic choices made by incumbents but in their innate characteristics.[2]

The present paper provides an explanation to the phenomenon of incumbency advantage that focuses on a strategic mechanism that can potentially constrain elected politicians. During his term in office, the incumbent must often implement some policies on new or emerging issues. These policies may be costly in terms of chances of reelection if they are unpopular among voters. The incumbent is thus facing implicit restrictions on the policies that he can implement if he wants to remain in office. We analyse the extent to which incumbents will make policy sacrifices during their time in office in order to enhance their electoral prospects. We show under which conditions these implicit restrictions can be transformed into an advantage and help the incumbent to be reelected.

Although our model is more general, we have in mind a specific type of issues and policy choices as the origin of this potential incumbency disadvantage: the outcomes of different forms of citizen direct political participation. The outcomes of processes like referenda, citizens' initiatives or popular assemblies may constrain incumbents because their reaction to these proposals factors into voters' evaluation of the incumbent's performance. This can create a disadvantage compared to the case of an incumbent who does not face such proposals. But also compared to the challenger, whose position does not require him to make any pre-election choice and who, as a result, may have

[1] Other studies assume that incumbents have better ways to influence voters' decisions than challengers through different mechanisms such as redistricting (Levitt and Wolfram 1997, Cox and Katz 2002), seniority (McKelvey and Reizman 1992), informational advantages (Krehbiel and Wright 1983), better access to campaign resources (Goodliffe 2001, Jacobson 2001), legislative irresponsibility (Fiorina 1989) or pork barrel politics (Cain, Ferejohn, and Fiorina 1987, Ansolabehere, Snyder, and Stewart 2000).

[2] In this line, Bevia and Llavador (2009) show that only good quality incumbents may enjoy an advantage. Ashworth and de Mesquita (2008) show that incumbents' quality and ability are higher on average than the challengers'. Gowrisankaran, Mitchell, and Moro (2008) find that incumbents face weaker challengers than candidates that face open seats and Stone, Maisel, and Maestas (2004) find that incumbents' personal qualities deter strong challengers from running for office.

a greater chance of winning unless the incumbent is ready to compromise.

We build a formal model of electoral competition with two candidates, two issues and three stages. In the first stage, the incumbent faces an exogenously given policy proposal on a certain issue, the *popular issue*. This proposal is non-binding so the incumbent can still choose which policy to implement in this issue. The implementation of his choice on the popular issue takes place during the legislature and before the next electoral campaign. In the second stage, both candidates announce simultaneously their policy platforms on a different issue, the *electoral issue*. The electoral issue is defined in the same way as in most models of electoral competition; the candidates' choices in this issue represent their campaign promises. Finally, in the third stage, voters vote for their most preferred candidate.

The model presents two types of asymmetries. First, voters evaluate the two candidates differently. We assume that voters use all the information they have available at the time of the election in order to decide who to vote for. They evaluate each candidate according to their campaign promises. When evaluating the incumbent, in addition to his campaign promises, they take into account his choice on the popular issue during the legislature.

The second asymmetry refers to the two issues. Citizens' evaluate the performance of the incumbent on the popular issue by comparing his policy choice with the proposal they had made to him. On the other hand, they evaluate candidates on the electoral issue by comparing their own preferred policy with each candidate's political platform. In addition, they assign different weights to the incumbent's choices in each one of the issues.

The optimal policy choices of the incumbent in both issues reflect the trade-off he faces between his own policy preferences and his chances of reelection. We find that for all parameter values the incumbent has a strategy that allows him to be reelected. The question is whether this winning strategy is always optimal for the incumbent. And the answer is no.

There are some instances where the incumbent prefers to forgo reelection because obtaining it is too costly in terms of policy. In those cases, the incumbent implements his ideal policy in the popular issue. For this to happen three conditions must hold: (1) the incumbent must care enough about policy, that is, the value of holding office must be low enough; (2) there must exist an intense conflict of interest between voters and the incumbent over the popular issue; and (3) competition on the electoral issue must be strong, that is, voters must assign a high weight to the electoral issue when evaluating the incumbent. The intuition for this result is the following: the incumbent has a disadvantage whenever he does not satisfy voters' demands on the popular issue. He will suffer the smallest disadvantage at the electoral competition stage when he fully satisfies citizens' demands on the popular issue. However, this is a costly strategy for a policy motivated incumbent. If the incumbent is policy motivated he will choose the platform that forces him to compromise as little as possible whilst guaranteeing reelection. But when the conflict of interests with voters in the popular issue is too intense or competition with the challenger is very strong, this strategy becomes too costly and the incumbent prefers to implement his ideal policy on the popular issue and lose reelection.

Otherwise, in equilibrium the incumbent chooses a winning strategy that consists of a combination of policies that depend on the weight that voters assign to his performance on each issue. The larger the weight that voters assign to the electoral issue, the more the incumbent will satisfy voters on that issue. Perhaps more surprisingly, this is not the case for the popular issue. The incumbent fully satisfies the voters' demands only when the weight citizens attach to the popular issue is neither too high nor too low. This is because the incumbent does not compete with the challenger in the popular issue. Hence, when citizens care a lot about it, the incumbent can implement a policy closer to his ideal one in this issue without jeopardizing his reelection. This cannot happen in the electoral issue because there the incumbent has to compete against the challenger.

The present work is related to the analysis of the effect of popular initiatives on policy outcomes by Besley and Coate (2008). These authors study a citizen-candidate model with two policy issues, one of which can be subject to popular initiatives of the type that exist in some US states. Contrary to our analysis, these initiatives bind politicians if passed . These authors show that these type of policy proposals can sometimes improve the congruence between citizens' preferences and policy outcomes. As in our case, the final effect depends on the relative salience of the issues. However, Besley and Coate (2008) do not analyse how initiatives affect the reelection prospects of incumbent candidates.

Our model relates also to the literature on spatial competition with valence initiated by Stokes (1963) and later developed by Ansolabehere and Snyder (2000), Groseclose (2001) and Aragones and Palfrey (2002). In those models, one of the candidates holds an advantage due to exogenous non policy factors, called valence factors, such as charisma, better campaign funds or higher intelligence. The difference between our model and these is that in ours the origin of the advantage (if any) is endogenous. In our case, a good performance of the incumbent in the popular issue provides him with an advantage that has an effect on electoral competition similar to the one that valence factors have in the models just mentioned. This choice becomes a source of disadvantage when the incumbent deviates too much from citizens' policy proposal on the popular issue. This trade-off that incumbents face in our model is similar to the one that emerges in dynamic settings with asymmetric information as in Reed (1994). In this context, Duggan (2000) and Banks and Duggan (2008) endogenize the trade-off between policy choices and re-election probabilities when elections are repeated, voters are fully rational, the challenger's preferences are privately known and policy spaces may be multidimensional.

The remainder of paper is organized as follows. In the next section we discuss two real political mechanisms our analysis can apply to, namely referenda and participatory democracy. Section 3 describes the formal model. Section 4 presents the results. The last section offers some concluding remarks. All technical proofs can be found in the Appendix.

2 Two sources of incumbency (dis)advantage

During their term in office, incumbents must make choices on new or common value issues. These choices might have a large negative effect on their chances of reelection if they are unpopular because citizens will factor these choices into their evaluation of the incumbent's performance. Jeopardizing reelection may not be optimal even for purely policy motivated incumbents. The policy implemented in case they lose reelection may be worse for them than the policy which could have granted them victory. Therefore, incumbents will have to compromise on some dimension if they want to be reelected

Two mechanisms that can generate this trade-off between policies and reelection chances are referenda and participatory democracy. The characteristics that both have in common are: (1) there is an issue that a significant part of the population considers to be very important; (2) the incumbent receives from citizens a policy proposal on this issue; (3) the incumbent must make a decision regarding that issue; (4) there is a significant proportion of voters that may base their voting decision on that issue. Next, we elaborate on how these two mechanisms fit in our main argument.

2.1 Referenda and popular initiatives

Facultative (non-mandatory) referenda may be initiated by a public authority or by some organized group of citizens. The latter case is known as popular initiative. Referenda may be either binding or non-binding. A non-binding referendum is merely or advisory. It is left to the government or legislature to interpret its results and react to its outcome (even by ignoring it altogether). If the incumbent chooses not to implement the policy corresponding to the referendum outcome he may be punished by voters. Therefore, incumbents will tend to follow the proposal emerged from the referendum.

The empirical evidence on the effect of referenda on congruence between policy outcomes and citizens' preferences is quite strong. Cross-section studies for Switzerland reveal that policy choices regarding provision of public goods correspond better with the preferences of voters in those cantons where referenda are more extensively used (Frey and Bohnet, 1993; Frey, 1994). Lutz and Hug (2006) run a cross-country study and find that the policy effects of referenda carry over to the national level. We argue in this paper that, apart from transmitting information about voters' preferences to candidates, referenda offer incumbents incentives to satisfy citizens' preferences. Therefore, incumbents can obtain an advantage through them. However, we also show in our model that incumbents will not be reelected if there exists a substantial disalignment between them and voters' preferences. Empirical evidence suggests that this disalignment between citizens and incumbents is frequent, especially in the case of local public services, as shown by Agreen, Dahlberg and Mork (2006) for a sample of Swedish municipalities. For the case of Switzerland, Frey and Bohnet (1993) report that 39% of the referenda held in that country between 1948 and 1990 yielded results that opposed the views of the Parliament.[3]

Still, a referendum initiated by the incumbent might have a weaker effect on voters' reaction than a referendum that originates with a popular initiative.[4] Popular initiatives sometimes take the form of legislative proposals that citizens can place in the ballot. This is the case in 24 US states, where petitions by citizens are voted after obtaining a number of signatures (between 2% and 15% of the voting population). Around 70% of the US population lives in either a state or a city in which initiatives are permitted. Popular initiatives are often regarded as an instrument that ensures a better congruence between citizens' preferences and policy outcomes. Gerber (1996) and Matsusaka (2005, 2010) find indeed that laws passed in US states that allow citizen initiatives reflect more closely the preferences of their electorate. Although these popular initiatives are binding, the reelection chances of an incumbent candidate may still depend on whether the candidate endorses or not such proposals.[5] The available evidence suggests that popular initiatives do provide incumbents with an electoral advantage. Bali and Davis (2007)

[3]More prominent examples are the two referenda called to decide whether the country should join the UN and the EU in 1986 and 1992 respectively, which yielded a majoritarian rejection (76% and 50%) despite the strong backing of all major political parties.

[4]Referenda called by the incumbent require them to perform strategy considerations as to when is optimal to called them. Xefteris (2011) analyzes this issue.

[5]Actually, some popular initiatives may express discontent with the incumbent legislator or even aim to weaken him (Bali and Davis, 2007).

show that in those US states which permit popular initiatives, incumbent legislators enjoy a 1% to 2% higher chance of being reelected. These effects are small but significant and suggest that although citizens' proposals may constraint incumbents' discretion, they can be used by them to obtain extra electoral support.

2.2 Participatory democracy

Participatory democracy is an extended version of the system of representative democracy in which citizens make policy proposals through popular assemblies. Real cases of participatory democracy can be found in the town meetings of New England; in the village governance system of the Indian states of Kerala and West Bengal; and in the participatory budgeting system of nearly two hundred Brazilian municipalities, where popular assemblies coexist with formal political parties and local elections. The most famous experience of this kind started in 1989 in the city of Porto Alegre.[6] Participatory budgeting has also been applied to school, university, and public housing budgets.

In all these cases, popular assemblies and deliberation emerge as governance mechanisms because citizens are interested on a certain issue, normally a local one, and they would like certain policies to be implemented. Because they care enough about these issues, their expected benefits from participating in the process overcome the costs of coordinating in order to elaborate a policy proposal. Typically, a policy proposal emerges from these meetings and is submitted to the incumbent. The incumbent has formally complete discretion regarding the policies he can implement. However, because the support to these policy proposals is significant within the population, the incumbent's chances of being re-elected critically depend on his policy choices on that issue. Hence, participatory processes can give incumbents an opportunity to obtain electoral advantage

A few years after the participatory budgeting system was first implemented in Porto Alegre, critics of the system claimed that it was being used as a partisan instrument by the ruling party, the Workers' Party. As a matter of fact, the party had won all municipal elections since 1989 by wide margins. Most studies indicate that the incumbent party did enjoy an advantage, as suggested by the higher levels of income redistribution and the patterns of citizen participation in the process (Aragones and Sanchez-Pages, 2009). However, the Workers' Party candidate was not reelected in the 2002 state

[6] The implications of participatory democracy on the behavior of citizens and politicians and on policy outcomes are analyzed in Aragones and Sánchez-Pagés (2009).

election. What was the difference between these two scenarios?

As argued by Goldfrank and Schneider (2006), one key issue was the degree of political competition. At the city level, the Workers' Party held strong support in Porto Alegre, and it is likely that the popular issue was dominant in voters' minds when casting their vote in subsequent elections. However, at the state level, the Workers' Party faced a much stronger opposition. In our model, we explicitly incorporate this factor. Its results suggest that a strong electoral competition erodes the electoral advantage that the incumbent could enjoy due to participatory democracy. Goldfrank and Schneider (2006) computed the difference between promised investments and actual investments completed for each municipality under the Workers' Party rule and then estimated a strong negative effect of these dashed expectations on the share of municipal votes of the party in the 2002 election. They show that departure between actual policy choices and proposals severely undermined the advantage that the incumbent party might have enjoyed in these municipalities.

3 The model

We assume that electoral competition takes place across two dimensions, denoted by x and y. Each dimension is represented by the unit interval of the real line $[0, 1]$. Dimension x represents the electoral issue and dimension y represents the popular issue. There are two candidates: the incumbent and the challenger. The model proceeds in three stages. The first stage takes place during the legislature: the incumbent receives a policy proposal on the popular issue and has to implement a policy on that issue. Both the policy proposed to the incumbent and the policy implemented by him on the popular issue are common knowledge to all candidates and all voters. The second stage is the electoral campaign: both candidates make policy announcements simultaneously on the electoral issue. Again all policy announcements are common knowledge to all candidates and all voters. It is assumed that the winner implements the announced policy on that issue. In the third stage of the game the election takes place: voters decide whether to reelect the incumbent or vote for the challenger. The winner is selected by majority rule and implements the policy announced on the electoral issue.

3.1 Candidates

The two candidates are denoted by L and R. Candidate L is assumed to be the incumbent. Candidates have single peaked preferences over the electoral issue. Without any loss of generality we assume that the ideal point of candidate L on the electoral issue is represented by $x_L = 0$ and the ideal point of candidate R is represented by $x_R = 1$. We assume that the incumbent has single-peaked preferences over the popular issue that are independent of his preferences on the electoral issue. The incumbent's ideal point on the popular issue is represented by $y_L = 0$. As we will argue below, it is not necessary to specify the preferences of the challenger over the popular issue.

Let us denote by $y(L)$ the policy chosen by the incumbent on the popular issue during the legislature. We assume the incumbent to be a unique decision maker. Thus, the present model applies to scenarios where the incumbent holds executive office, or to legislatures where a party holds a parliamentary majority and whose parliamentary representatives vote as a unified bloc.[7] Elections take place at the end of the legislature. When the electoral campaign starts, this choice $y(L)$ has already been made and it is taken as given. We model elections by means of a standard model of electoral competition on the issue x: the incumbent and the challenger simultaneously announce policy platforms denoted by $x(L)$ and $x(R)$ respectively. We assume full commitment, that is, the winner of the election will implement on the electoral issue the policy he announced during the campaign.

We assume that candidates have preferences over policies but that they are also office-motivated. Candidates' payoffs depend on the policy chosen by the incumbent on the popular issue and the policy announcements of both candidates on the electoral issue according to these utility functions:

$$U_L = -\left|y_L - y\left(L\right)\right| + \pi_L\left(K - \left|x_L - x\left(L\right)\right|\right) - \left(1-\pi_L\right)\left|x_L - x\left(R\right)\right|,$$

$$U_R = \left(1-\pi_L\right)\left(K - \left|x_R - x\left(R\right)\right|\right) - \pi_L\left(\left|x_R - x\left(L\right)\right|\right),$$

where $\pi_L = \pi_L(y(L), x(L), x(R))$ represents the probability that candidate L wins the election, and $1 - \pi_L$ denotes the probability that candidate R wins the election. The probability with which the incumbent is reelected depends on how the game unravels, that is, it depends on the policy choices made during the legislature (stage 1) and the policy announcements made during the campaign (stage 2).

K is a non-negative number that represents the utility or ego rent of holding office. $K = 0$ implies that candidates do not obtain any extra utility from holding office, they only derive utility from the policy implemented. In this case we would have two candidates that are purely policy motivated. The larger the value of K the more candidates value being in office. Thus

[7] Otherwise, it can be thought of as reduced form model of a more complex (and realistic) governmental system.

for larger values of K candidates care more about winning. When the value of K is high enough candidates become purely office motivated.

Note that the incumbent obtains a negative payoff whenever he implements a policy on the popular issue that does not coincide with his ideal point on that issue. Observe also that because we assume that the challenger has no power over policy implementation on the popular issue before or after the election, the policy choice of the incumbent on that issue $y(L)$ has an non reversible impact on his payoffs. We elaborate more on this below.

For simplicity, we have assumed that the incumbent cares equally about the two issues. Introducing a parameter in the incumbent's payoff function that represents the relative weight that each issue has on the incumbent overall payoffs would not change the main qualitative results obtained.

3.2 Voters

Voters have single-peaked preferences over the electoral issue x. We assume that their ideal points are uniformly distributed over x, thus the ideal point of the median voter on the electoral issue is $x_m = \frac{1}{2}$. Let the ideal point of society in issue y be denoted by $y_m > 0$. The parameter y_m is considered exogenous in our model. It can be interpreted as the outcome of a referendum or of a process of participatory democracy that took place before the beginning of the game. Notice that since the ideal point of the incumbent on the popular issue is assumed to be $y_L = 0$, the value of y_m measures the magnitude of the conflict of interests between the incumbent and the citizens with respect to the popular issue. Here we assume that this proposal is exogenous. In the final section of the paper, we discuss the consequences of endogenizing the policy proposal y_m.

When facing the election, voters observe the policies announced by both candidates on the electoral issue, $x(L)$ and $x(R)$, the policy implemented by the incumbent on the popular issue, $y(L)$, and then cast their vote. Voters use all the information available in order to evaluate the two candidates. Since they have different kinds of information about the performance of each candidate, their decision rule must exhibit some sort of asymmetry.

We assume that voter i evaluates the incumbent according to the function

$$V_i(L) = -(1-\mu)\left|y_m - y(L)\right| - \mu\left|x_i - x(L)\right|,$$

where μ is a parameter that measures the relative weight that voters assign to the electoral issue with respect to the popular issue, and $0 \leq \mu \leq 1$. Values of μ close to one mean that voters consider the popular issue to be very important. In this case, voters' evaluation of the incumbent would not be much affected by his policy choice on that issue. Values of μ close to zero mean that the popular issue is regarded as very important by voters and that their evaluation of the incumbent will be strongly affected by his policy choice on the popular issue.

Note that voters evaluate the incumbent on the electoral issue by comparing his electoral platform, $x(L)$, to their own ideal point x_i. However, they evaluate the incumbent on the popular issue by comparing the policy he implemented, $y(L)$, to the policy proposed initially by citizens y_m. Hence, citizens measure the performance of the incumbent on the popular issue in an homogeneous way. This assumption is justified whenever the policy proposal y_m, represents the outcome of referenda, citizens assemblies or polls, that is, when it represents the ideal policy on the popular issue of a substantial subset of the electorate. Our assumption of common evaluation of the incumbent's performance in the popular issue can alternatively be interpreted as an implicit commitment of citizens to punish politicians who do not follow the proposals submitted to them.

On the other hand, voter i evaluates the challenger according to the following function:

$$V_i(R) = -|x_i - x(R)|.$$

Voters evaluate the challenger according only to his promises on the electoral issue. The challenger could also make statements regarding the popular issue that might be incorporated by the voters in their evaluation. However, such statements are not actual facts as in the case of the incumbent, who had to implement a policy such as an annual budget, a reform of the abortion legislation, the participation or not in a war or the signature of an international treaty. The challenger could without cost state that her preference is fully aligned with y_m. We are then assuming that information about the popular issue, is only considered if it is hard information. We discuss the consequences of relaxing this assumption in the final section of the paper.[8]

Given voters' evaluations of both candidates, voter i will vote for candidate L if and only if

$$V_i(L) \geq V_i(R) \quad \Leftrightarrow \quad (1-\mu)|y_m - y(L)| + \mu |x_i - x(L)| \leq |x_i - x(R)|. \quad (1)$$

Notice that the lower the value of μ the more weight past choices have on the evaluation of the incumbent, that is, the more retrospective their

[8]The reader may argue that our evluation functions imply that voters evaluate $x(R)$ and $x(L)$ differently. This could be solved by assuming $V_i(R) = -\mu|x_i - x(R)|$. Note however that in that case we would be imposing an incumbency disadvantage to start with.

decisions become.[9] High values of μ mean that voters barely base their decision on past information, i.e. the performance of the incumbent during the legislature. In addition, the performance of the incumbent on the popular issue, i.e. the distance $|y_m - y(L)|$, has an effect on voters evaluations that is very similar to the effect of valence factors.[10]

[Insert Figure 1 here]

Figure 1 shows how citizens' evaluation of candidates changes with the platforms announced by them at the electoral stage. If the two candidates were to propose the same platform the incumbent would suffer a disadvantage proportional to the distance between his choice $y(L)$ and the citizens' proposal y_m. On the other hand, citizens' evaluation of the incumbent is less sensitive to changes in his electoral platform $x(L)$. This feature implies that voters with ideal points at both extremes of the distribution may decide to vote for the same candidate. In fact, when the distance between the policy implemented $y(L)$ and the policy proposal y_m is large enough, the set of voters who decide to vote for the incumbent becomes non-connected and the standard median voter analysis no longer applies. Expression (1) implies that a citizen with ideal point $x_i = x(L)$ will vote for candidate R whenever

$$\mu \leq 1 - \frac{|x_i - x(R)|}{|y_m - y(L)|}.$$

The set of voters who prefer to vote for the challenger but whose ideal policy x_i is closer to $x(L)$ enlarges as citizens care more about the popular issue and as the incumbent's choice departs more from the policy proposal y_m. This highlights that the existence of a policy proposal during the legislature can be a source of electoral disadvantage for the incumbent.

The present specification encompasses as particular cases some standard models of two-party competition. If $\mu = 1$, that is, if voters care only about the electoral issue, we have a standard model of electoral competition. In this case, for very large values of K candidates are purely opportunistic and the model describes a standard downsian framework. For relatively small values of K, candidates become policy motivated, and our model reproduces Wittman's (1983) model of electoral competition. On the other hand, the case of $\mu = 0$, that is, if voters only care about the popular issue, boils down

[9] There exists a distinguished literature in which voters base their decisions on past performance of parties. Examples include Barro (1973), Ferejohn (1986), Austen-Smith and Banks (1989) and Reed (1994).

[10] Ansolabehere and Snyder (2000), Groseclose (2001) and Aragones and Palfrey (2002).

to a more general version of our previous work on participatory democracy (Aragonès and Sánchez-Pagés, 2009).

The incumbent is reelected if an only if the set of voters who prefer the incumbent to the challenger contains a majority of the population.[11] Since the decisions on the two dimensions of the model are made sequentially, one at each stage, we do not have to deal with the complexities of electoral equilibrium in a multidimensional space. In fact, we can solve it as a one dimensional model within each stage. In the next section we study the equilibrium of this game for all values of the parameters K, μ and y_m.

4 Equilibrium results

4.1 Electoral stage

In order to solve the model described above we look for its subgame perfect equilibrium by using backward induction. We start by analyzing the electoral stage, taking as given the choice of the incumbent on the popular issue.

Citizens partially base their evaluation of the incumbent on his performance in the popular issue. Hence, he does not enter the election on the same grounds as the challenger. His choice on the popular issue will have an impact on electoral competition, as the following lemma illustrates.

Lemma 1 *If $x(L) = x(R)$, then L obtains at least $1 - 2|y(L) - y_m|$ of the votes and R obtains at most $2|y(L) - y_m|$.*

When both candidates choose the same position on the electoral issue, that is when $x(L) = x(R)$, only citizens at a distance of at least $|y(L) - y_m|$ from the policy proposed by both candidates vote for the incumbent. Thus, it is possible for the incumbent to capture the vote of extremists if he performs well enough in the popular issue, that is, when $|y(L) - y_m|$ is small enough. The incumbent's chances of being reelected will be higher the less his policy choice $y(L)$ in the popular issue departs from the society's most preferred policy y_m. As a matter of fact, there exists a threshold on this distance that is critical in determining whether the incumbent has an electoral advantage or not, as the next proposition shows.

Lemma 2 *If $|y(L) - y_m| < 1/4$, then L wins in the equilibrium of the electoral stage. Otherwise, R wins in equilibrium.*

[11] That is, we assume that if there is a tie the incumbent is reelected.

The incumbent obtains a decisive advantage when he compromises enough on the popular issue. If, on the contrary, his policy choice on the popular issue departs considerably from the proposal y_m, then he has to compromise so much on the electoral issue in order to win that he rather prefers to lose.

Let us now fully describe the equilibrium at the electoral competition stage given a choice of $y(L)$. The following two propositions characterize the strategies used by the winner of the election in equilibrium. These strategies define the equilibrium policy outcome of the electoral stage as well. First, we describe the equilibrium outcomes of the electoral stage for the case in which the incumbent is reelected in equilibrium.

Lemma 3 *If $|y(L) - y_m| \leq \frac{1}{4}$, then L's equilibrium strategies at the electoral stage are:*

$$x^*(L) = \begin{cases} 0 & \text{if } |y(L) - y_m| \leq \frac{1-3\mu}{4(1-\mu)} \\ \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m| & \text{otherwise} \end{cases}$$

This proposition illustrates the trade-off that the incumbent faces. The more he pleases the electorate on the popular issue, i.e. the smaller $|y(L) - y_m|$, the closer his winning electoral platform will be to his ideal policy. The incumbent can even guarantee his reelection by implementing his ideal point on the electoral issue if he satisfies voters enough on the popular issue. In order to achieve this, he will need to compromise more on the popular issue the larger the value of the weight voters put on the electoral issue μ (note that $\frac{1-3\mu}{4(1-\mu)}$ decreases with μ).

Otherwise, if the incumbent implements a policy on the popular issue that departs significantly from the policy proposal y_m, then the incumbent still wins the election in equilibrium but his electoral platform $x^*(L)$ includes a certain degree of compromise. His electoral platform will lie somewhere between his ideal point and the median voter's ideal point. And it will be closer to the median voter's ideal point the larger the distance between the policy he implemented in the popular issue $y(L)$ and the policy proposal y_m. This equilibrium policy choice will also be closer to y_m the more weight voters put on the electoral issue or, in other words, the tougher the competition at the electoral stage, which is represented by higher values of μ.[12] In the limit, when competition at the electoral stage attains a maximum, i.e. μ goes to 1, the policy announced by the incumbent on the electoral issue coincides with the median voter's ideal point. By the same token, as the popular issue becomes more important, i.e. μ decreases, the policy announced by the incumbent on the electoral issue approaches the incumbent's ideal point.

[12] Straightfoward calculations show that $\frac{\partial x^*(L)}{\partial \mu} = \frac{1}{\mu^2}\left(\frac{1}{4} - |y(L) - y_m|\right) \geq 0$.

The following proposition describes the equilibrium outcome of the electoral stage when the incumbent decides to forgo reelection. In that case, the equilibrium policy outcome in the electoral issue coincides with the strategies used by the challenger in the equilibrium of the electoral stage.

Lemma 4 *If $|y(L) - y_m| > \frac{1}{4}$, then R's equilibrium strategy at the electoral stage is $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.*

When the incumbent has departed significantly from the citizens' ideal point in the popular issue, the challenger wins with a moderate policy in the resulting equilibrium of the electoral stage. Observe that $x^*(R)$ is decreasing in μ so, as before, the tougher the competition at the electoral stage the closer the policy outcome will be to the median voter's ideal point. And the larger the distance between the policy proposal and the policy implemented on the popular issue, the closer the policy outcome on the electoral issue will be to the challenger's ideal point.

4.2 The popular issue

After solving for the equilibrium strategies of the electoral stage, we move backward in order to find the incumbent's optimal policy at the first stage.

Recall that when the incumbent is choosing which policy to implement on the popular issue he is facing a trade-off. If he implements a policy $y(L)$ that is relatively close to the citizens' proposal, y_m, he will be able to get reelected with an electoral platform relatively close to his ideal policy. The closer his choice $y(L)$ is to his own ideal policy on the popular issue, that is, the more his choice departs from y_m, the more he will have to compromise on the electoral issue if he wants to remain in office. This strategy may be too costly if the incumbent is sufficiently policy motivated. Instead, he can implement his most preferred policy on the popular issue and forgo reelection. The next few results characterize this trade-off. First we find the best winning strategies and best losing strategies for the incumbent. Then we show under which conditions the incumbent will prefer to be reelected.

The following Lemma describes the best winning policy choice of the incumbent given the degree of political competition with the challenger.

Lemma 5 *The incumbent's best wining strategies are:*

$$\{y^*(L), x^*(L)\} = \begin{cases} \{\max\left\{y_m - \frac{1-3\mu}{4(1-\mu)}, 0\right\}, 0\} & \text{if } \mu \leq \frac{1}{3} \\ \{y_m, \frac{3\mu-1}{4\mu}\} & \text{if } \frac{1}{3} \leq \mu \leq \frac{1}{2} \\ \{\max\left\{y_m - \frac{1}{4}, 0\right\}, \frac{1}{2}\} & \text{if } \mu \geq \frac{1}{2} \end{cases}$$

The relationship between the best winning electoral platform and μ is very intuitive: As competition on the electoral issue becomes tougher, i.e. μ goes up, the incumbent needs to select a platform closer to the median voter's ideal policy in order to win.

Perhaps more surprising is the non-monotonic effect that the weight that citizens put on the popular issue has on the best winning policy that the incumbent can implement in that issue. When the popular issue is important (low μ) the incumbent is virtually facing no opposition. Citizens care almost only about an issue in which the incumbent can act like a monopolist. Actually, when competition on the electoral issue is rather soft ($\mu \leq \frac{1}{3}$) the incumbent can win by implementing her most preferred policy on the electoral issue.

But as μ becomes larger the incumbent cannot longer win the election by implementing his ideal policy on the electoral issue. He can either please citizens on the popular issue by implementing their proposed policy y_m and in return choose a policy close to her ideal one on the electoral one, or alternatively he can pick the median voter's ideal policy on the electoral dimension and select a policy as close as possible to his own ideal one on the popular issue. For intermediate levels of μ the first option is better because electoral competition is relatively soft and he can implement a policy relatively close to his ideal policy on the electoral issue and still win. That winning electoral promise will be less favorable for the incumbent the tougher electoral competition becomes, that is, the larger the value of μ. However, when electoral competition is tough, i.e. $\mu > \frac{1}{2}$, the incumbent prefers the second option and he compromises substantially on the electoral issue, and implements the median voter's ideal point. Hence, the incumbent will implement the citizens' policy proposal on the popular issue only when electoral competition is neither too soft nor to tough.

Figure 2 depicts the incumbent´s winning strategy in both stages of the game as a function of μ.

[Insert Figure 2 here]

Next we find the incumbent's best losing strategy and the corresponding best response of the challenger.

Lemma 6 *The incumbent best losing strategies are* $y^*(L) = 0$ *and* $x^*(L) = 1/2$ *which in turn implies that* $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu} y_m$.

If the incumbent decides to forgo reelection, the best strategy that he can follow is to implement his preferred policy choice on the popular issue and force the challenger to become as moderate as possible in the electoral one. In equilibrium, he announces the median voter's ideal policy and the challenger wins the election by announcing a platform that will be closer to the median voter the higher the value of μ.

The last step of the analysis amounts to characterize when the incumbent prefers to win the election given the best winning strategies and the best losing strategies described above. His incentives to remain in office will depend on the level of disalignment with the population (simply measured by y_m), the relative weight that voters assign to the electoral issue, i.e. measured by μ, and the value that the incumbent attaches to office K.

Proposition 1 *In equilibrium the incumbent wins*

(i) When $y_m \leq \frac{1}{4}$ for any $K \geq 0$ and any $0 \leq \mu \leq 1$.

(ii) When $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ if and only if $K > \max\left\{\frac{2\mu}{1+\mu}y_m - \frac{1}{4}, 0\right\}$.

(iii) When $y_m \geq \frac{3}{8}$ if and only if

$$K > \begin{cases} \max\left\{\frac{2\mu}{1+\mu}y_m + \frac{5\mu-3}{4(1-\mu)}, 0\right\} & \text{if } \mu \leq \frac{1}{2} \\ \frac{2\mu}{1+\mu}y_m - \frac{1}{4} & \text{if } \mu \geq \frac{1}{2} \end{cases}$$

When the preferences of the incumbent on the popular issue are aligned with those of society, i.e. $y_m \leq \frac{1}{4}$, the incumbent prefers to win for all values of K and all values of μ. Otherwise, if the preferences of the incumbent on the popular issue are not aligned with the policy proposal y_m, the incumbent may decide to forgo the reelection. In this case, he will do so only when he is sufficiently policy motivated (for low enough values of K). The tougher electoral competition is, i.e. the higher μ, the larger is the range of values of K that induces the incumbent to forgo the election. Intuitively, the more intense electoral competition is and the more costly is to please voters in the popular issue, the more likely is that the incumbent will prefer to lose. The area under the curves in Figure 3 corresponds to the region of the parameter space where the incumbent is not reelected in equilibrium.

[Insert Figure 3 about here]

Incumbents that are highly policy motivated, i.e. have low values of K, are more likely to suffer a disadvantage from being in office. They may find too costly to make a policy choice that will guarantee their reelection when their preferences are not aligned with society's preferences. The cost of being reelected may also be too high when the degree of competition on the electoral issue is high, i.e μ close to 1. In that case, the incumbent will have to propose a very moderate policy on the electoral issue if he wants to beat the challenger. Otherwise, citizens' proposals can be used by the incumbent to obtain a decisive advantage in political competition and become reelected.

5 Concluding remarks

The main contribution of this paper is to study how electoral competition unravels when the policy choice of the incumbent in a pre-election issue factors into the citizens' evaluation of his performance. We assumed that the performance of the incumbent on that issue is assessed by the distance between the policy proposed by citizens and the policy that the incumbent finally implemented.

We have characterized conditions under which the incumbents can use this pre-electoral issue to their advantage. In all these cases, the incumbent has to adjust his policy choices in order to accommodate the policy proposals he receives, and the final policy outcome is relatively close to the policy outcome most preferred by society. But this is not always the case. When the policy proposals made by society are too far from the incumbent's preferred policy, the incumbent may decide to forgo reelection. In this case the final policy outcome is bad from voters' point of view.

We have assumed that voters use an asymmetric rule in order to evaluate the candidates. We identified two different kinds of asymmetries that had to be taken into account: (1) only the incumbent is responsible for the policy implemented on the popular issue, and (2) there is a policy proposal made only on the popular issue. Thus, we have assumed that voters evaluate the incumbent according to his performance on the two issues and the challenger only according to the platform he announces in the electoral issue.

We could relax the assumption by which both candidates are evaluated according to the two issues. The evaluation of the challenger with respect to the popular issue would just become an exogenous parameter given that the challenger cannot implement any policy during the legislature. This parameter would represent the performance of the challenger in the popular issues in the past. Our results would remain the same as long as the weight that voters assign to the electoral issue when they evaluate the incumbent is smaller than the one they use to evaluate the challenger. Otherwise the incumbent would suffer from a greater disadvantage but qualitatively our results would still go through.

Our modelling of citizens' evaluation rule constitutes a novel feature of our approach because it combines elements of both retrospective voting and prospective voting. Voters use retrospective voting to evaluate the performance of the incumbent with respect to the popular issues. And voters use prospective voting to evaluate the campaign promises that candidates announce during the electoral campaign. In order to use all the information available to them at the time of voting, voters combine these two different kinds of evaluations. Votes make use of past information regardless of whether past performance provides or not voters with information about future choices. In our model, citizens are fundamentally unhappy with an incumbent that deviated from the policy proposal they made to him during his time in office. This behavior in addition to the existence of such policy proposals generate a potential electoral advantage to the incumbent. From this point of view, and given that any electoral advantage comes from the incumbent's choices, the present model might be seen as a behavioral model of endogenous valence.[13]

We have characterized conditions under which the disadvantages generated are compensated by the advantages. In all these cases, the incumbent has to adjust his policy choices in order to accommodate the policy proposals he receives, and the final policy outcome is relatively close to the policy outcome most preferred by society. But this is not always the case. When the policy proposals made by society are too costly from the incumbent's point of view, the incumbent may decide to forgo reelection. In this case the final policy outcome is bad from voters' point of view.

The model assumes that these policy proposals are exogenous. It could be extended by making them endogenous. This issue was partially addressed in Aragones and Sanchez-Pages (2009) for the case when the proposal comes from popular assemblies and only the popular issue is relevant for voters, i.e. μ close to zero. Results above show that policy proposals that are not aligned with the policy preferences of the incumbent will tend to be neglected. This will be more likely to be the case as electoral competition becomes stronger. Therefore, if policy proposals were endogenous, demands on the popular issue that were aligned with the preferences of the incumbent would be satisfied more likely when the intensity of electoral competition were intermediate. Given this, it might be optimal for voters to submit policy demands that do not put too much pressure on the incumbent.

[13] We thank Maggie Penn for this observation.

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6 Appendix

Proof of Lemma 1. If $x(L) = x(R)$ then $-(1-\mu)|y_m - y(L)| - \mu|x_i - x(L)| \geq -|x_i - x(R)|$ becomes $|y_m - y(L)| \leq |x_i - x(R)|$. Thus L obtains votes from all i such that are at a distance from $x(R) = x(L)$ of at least $|y(L) - y_m|$. This means that R obtains at most $2|y(L) - y_m|$ votes, therefore L obtains at least $1 - 2|y(L) - y_m|$ votes. Notice that in this case R obtains exactly $2|y(L) - y_m|$ if $|y(L) - y_m| \leq x(L) = x(R) \leq 1 - |y(L) - y_m|$. ■

Proof of Lemma 2. First suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) = x(L)$. Then R cannot win because by the previous lemma R at most can obtain $2|y(L) - y_m| < 1/2$ votes.

Next suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) \neq x(L)$ and R wins. Then we must have

$$U_L(y(L), x(L), x(R)) = -y(L) - x(R).$$

Consider that L chooses instead $x'(L) = x(R)$. Then by the previous lemma L obtains at least $1 - 2|y(L) - y_m| > 1/2$ votes. Thus L wins and his utility is

$$U_L(y(L), x(R), x(R)) = -y(L) + K - x(R) > -y(L) - x(R),$$

In this case L prefers to win and has a winning strategy. Thus R cannot win in equilibrium.

Now suppose that $|y(L) - y_m| > 1/2$. If L is winning in equilibrium with $x(L)$ and $x(R)$, then consider $x'(R)$ such that $x'(R) = x(L)$ and notice that in this case R obtains more than half of the total vote. Thus R can win the election in this using this strategy, and it is also optimal for R to do so since he obtains an extra payoff of K and his deviation does not involve any change in the policy implemented. The reason is that if $x(L) \leq 1/2$ then R obtains a vote share equal to $x(L) + \min\{1 - x(L), |y(L) - y_m|\}$ which is a majority. Similarly if $x(L) \geq 1/2$ then R obtains $1 - x(L) + \min\{x(L), |y(L) - y_m|\}$ which is also a majority. Thus L cannot win in equilibrium with $|y(L) - y_m| > 1/2$.

Next suppose that $|y(L) - y_m| \in (\frac{1}{4}, \frac{1}{2})$

If $x(L) \in \left[\frac{1}{2} - |y(L) - y_m|, \frac{1}{2} + |y(L) - y_m|\right]$ then R can defeat it with $x(R) = x(L)$. In this case R obtains a vote share of $2|y(L) - y_m| > 1/2$ which allows R to win. R prefers to do so since by mimicking L he obtains an extra payoff of K and his deviation does not involve any change in the policy implemented..

If $x(L) \leq \frac{1}{2} - |y(L) - y_m|$ then R can defeat L with $x(R) \in (\frac{3-2\mu}{4}, \frac{3}{4})$. To show this, note that the set of supporters of R is the interval $[\frac{x(R)+\mu x(L)}{1+\mu} -$

$\frac{1-\mu}{1+\mu}\left|y\left(L\right)-y_{m}\right|,1]$ whenever $x\left(R\right)>(1-\mu)(1-\left|y\left(L\right)-y_{m}\right|)+\mu x\left(L\right).$ In addition, this number of voters constitutes a majority if and only if $x\left(R\right)<\frac{1+\mu}{2}+(1-\mu)\left|y\left(L\right)-y_{m}\right|-\mu x\left(L\right).$ This defines an interval of platforms that R can use to defeat L. Given the restrictions on $\left|y\left(L\right)-y_{m}\right|$ and the assumption on $x(L)$, this interval is at least as large as the interval $(\frac{3-2\mu}{4},\frac{3}{4})$. Hence, any platform in this interval guarantees R a victory against $x(L)$. Note again that R prefers to win rather than to let L win because, in addition to obtaining K, the policy outcome is closer to his ideal point.

If $x(L)\geq\frac{1}{2}+\left|y\left(L\right)-y_{m}\right|$ then the best winning policy for R is $x(R)=\mu x(L)+(1-\mu)(\frac{1}{2}+\left|y\left(L\right)-y_{m}\right|)$. We show this by following the same procedure as above to define the set of R's supporters and then check when it constitutes a majority. Next we need to see whether R actually uses this winning strategy. For this to be the case it need to hold that

$$\begin{aligned}K-1+\mu x(L)+(1-\mu)(\frac{1}{2}+\left|y\left(L\right)-y_{m}\right|) &> -1+x(L)\\ &\Leftrightarrow \quad x(L)<\frac{K}{1-\mu}+\frac{1}{2}+\left|y\left(L\right)-y_{m}\right|.\end{aligned}$$

Hence, L will not able to win with a $x(L)$ in $(\frac{1}{2}+\left|y\left(L\right)-y_{m}\right|,1]$ if $K>(1-\mu)(\frac{1}{2}-\left|y\left(L\right)-y_{m}\right|)$. If $K<(1-\mu)(\frac{1}{2}-\left|y\left(L\right)-y_{m}\right|)$ we need to check whether L prefers to win the election with such rightist policy. The best case scenario for L if he wants to win is when $x(L)=\frac{K}{1-\mu}+\frac{1}{2}+\left|y\left(L\right)-y_{m}\right|.$ In that case, his payoff is just $-y(L)-\frac{\mu}{1-\mu}K-\frac{1}{2}-\left|y\left(L\right)-y_{m}\right|.$ The best case scenario for L if in the contrary he decides to lose is to set $x(L)=\frac{1}{2}+\left|y\left(L\right)-y_{m}\right|$ given that that forces R to choose the same policy. His payoff is just $-y(L)-\frac{1}{2}-\left|y\left(L\right)-y_{m}\right|$, so he actually prefers to lose. ∎

Proof of Lemma 3. From the previous proposition we know that in this case L wins in equilibrium. Suppose that $x(L)$ and $x(R)$ is an equilibrium outcome such that L wins and $x(R)<x(L)$. Then we must have $U_{L}\left(y(L),x(L),x(R)\right)=-y(L)+K-x(L)$. Consider that L chooses instead $x'(L)=x(R)$. Then by lemma 1 L obtains at least $1-2\left|y\left(L\right)-y\left(A\right)\right|>1/2$ votes and his utility is $U_{L}\left(y(L),x(R),x(R)\right)=-y(L)+K-x(R)$.

Notice that $U_{L}\left(y(L),x(R),x(R)\right)=-y(L)+K-x(R)>-y(L)+K-x(L)=U_{L}\left(y(L),x(L),x(R)\right)$ since we assumed that $x(R)<x(L)$. Thus, $x(L)$ and $x(R)$ such that $x(R)<x(L)$ cannot be part of an equilibrium strategy and we must have $x(L)\leq x(R)$.

Let us first characterize the sets of voters that vote for candidate L given $y\left(L\right)$, $x(L)$ and $x(R)$.

The set of voters with $x_{i}<x\left(L\right)$ that vote for L is given by all x_{i} such

that

$$x_i < \frac{x\left(R\right) - \mu x\left(L\right)}{1-\mu} - \left|y\left(L\right) - y_m\right| \equiv \underline{x_i}.$$

Similarly, the set of voters with $x_i > x\left(R\right)$ that vote for L is given by all x_i such that

$$x_i > \frac{x\left(R\right) - \mu x\left(L\right)}{1-\mu} + \left|y\left(L\right) - y_m\right| \equiv \overline{x_i}$$

Since by proposition 2 $\frac{x(R)-\mu x(L)}{1-\mu} > x\left(R\right)$ then we have that $\overline{x_i} > x\left(R\right)$. Notice that if $\underline{x_i} < 0$ then $\overline{x_i} < 1$ for all $\left|y\left(L\right) - y_m\right| < \frac{1}{2}$.

Finally, the set of voters with $x\left(L\right) < x_i < x\left(R\right)$ that vote for L is given by all x_i such that

$$x_i < \frac{x\left(R\right) + \mu x\left(L\right)}{1+\mu} - \frac{1-\mu}{1+\mu}\left|y\left(L\right) - y_m\right| \equiv \widetilde{x_i}$$

Since by proposition 2 $\frac{x(R)+\mu x(L)}{1+\mu} < x\left(R\right)$ then we have that $\widetilde{x_i} < x\left(R\right) < \overline{x_i}$. However, the comparison between $\underline{x_i}$ and $\widetilde{x_i}$ is not clear-cut. We have that $\underline{x_i} < \widetilde{x_i} < x(L)$ if and only if

$$x(R) - x(L) < \left(1-\mu\right)\left|y\left(L\right) - y_m\right|.$$

Thus, two cases can emerge:

Case 1: If $x(R) - x(L) \geq \left(1-\mu\right)\left|y\left(L\right) - y_m\right|$ then we have that the votes that L obtains are given by $\widetilde{x_i} + \max\left\{0, 1-\overline{x_i}\right\}$.

Case 2: If $x(R) - x(L) < \left(1-\mu\right)\left|y\left(L\right) - y_m\right|$ then we have that the votes that L obtains are given by $\max\left\{0, \underline{x_i}\right\} + \max\left\{0, 1-\overline{x_i}\right\}$.

Suppose in the first place that $x(L) = 0$. Then the number of votes that L receives are

$$\#L = \left\{ \begin{array}{cl} 1 - \left|y\left(L\right) - y_m\right| - \frac{x(R)}{1-\mu} & \text{if } x\left(R\right) < \left(1-\mu\right)\left|y\left(L\right) - y_m\right| \\ 1 - \frac{2\mu}{1-\mu^2}x(R) - \frac{2}{1+\mu}\left|y\left(L\right) - y_m\right| & \text{if } x\left(R\right) \in \left[\left(1-\mu\right)\left|y\left(L\right) - y_m\right|, (1-\mu)(1-\left|y\left(L\right) - y\right.\right. \\ \frac{x(R)}{1+\mu} - \frac{1-\mu}{1+\mu}\left|y\left(L\right) - y_m\right| & \text{if } x\left(R\right) > (1-\mu)(1-\left|y\left(L\right) - y_m\right|) \end{array} \right.$$

that attains a minimum when $x\left(R\right) = (1-\mu)(1-\left|y\left(L\right) - y_m\right|$. The number of votes in that case is greater than $\frac{1}{2}$ if and only if

$$\left|y\left(L\right) - y_m\right| \leq \frac{1-3\mu}{4(1-\mu)}.$$

Note that if this holds, $x(L) = 0$ is a winning strategy for L. Otherwise, there exists a platform $x(R)$ that can defeat $x(L) = 0$.

Second, suppose that $\frac{1-3\mu}{4(1-\mu)} < |y(L) - y_m| > \frac{1}{4}$. Let us first show that any platform $x(L) \in (0, \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|)$ can be defeated by $x(R) = \frac{3-\mu}{4}$. First, note that we are in Case 1 since

$$x(R) - x(L) > (1-\mu)|y(L) - y_m| \Leftrightarrow x(L) < \frac{3-\mu}{4} - (1-\mu)|y(L) - y_m|$$

and in addition we have by assumption that

$$x(L) < \frac{3\mu - 1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y(A)| < \frac{3-\mu}{4} - (1-\mu)|y(L) - y(A)|$$

where the last inequality follows from simple algebra. One can also show that our assumption on $x(L)$ also implies that $\overline{x_i} > 1$ which means that the number of votes obtained by L is just $\widetilde{x}_i$ which in turn is smaller than $\frac{1}{2}$ if and only if

$$x(L) < \frac{3\mu - 1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|,$$

which holds by assumption. Hence, L is defeated if he chooses a platform in that interval. From the remainder, let us now show that $x(L) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|$ is a dominant strategy.

Again case we have to consider two cases:

1. Suppose that $x(R) > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}|y(L) - y_m|$. In that case, the number of citizens who vote for the incumbent are given by $\min\{1, \overline{x_i}\} - \widetilde{x}_i$. We need to consider two subcases depending on the value of the extremes of this interval.

   i. If $\overline{x_i} > 1$ then R gets $1 - \widetilde{x}_i$ votes and wins the election if and only if

   $$\widetilde{x}_i < \frac{1}{2} \rightarrow x(R) < \frac{3-\mu}{4}$$

   Since $\overline{x_i} > 1$ if and only if $x(R) > \frac{3-\mu}{4}$ then this case cannot arise.

   ii. If $\overline{x_i} < 1$ then R gets $\overline{x_i} - \widetilde{x}_i$ votes. This number of votes is greater than $\frac{1}{2}$ if and only if $x(R) > \frac{3-\mu}{4}$. Since $\overline{x_i} < 1$ if and only if $x(R) < \frac{3-\mu}{4}$ again this case is not possible.

2. Suppose instead that $x(R) < \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}|y(L) - y_m|$. This means necessarily that $\overline{x_i} < 1$ and that the challenger collects votes in $(\max\{0, \underline{x_i}\}, \overline{x_i})$. We need to consider then two different subcases:

i. If $\underline{x_i} < 0$ the challenger gets $\overline{x_i}$ votes and wins if and only if $x(R) \geq \frac{1+\mu}{4}$. But this leads to a contradiction because

$$\frac{1+\mu}{4} > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}|y(L) - y_m| \;\Leftrightarrow\; \frac{1-\mu}{4(1+\mu)} > -|y(L) - y_m|.$$

ii. If $\underline{x_i} > 0$ then R gets $\overline{x_i} - \underline{x_i} = 2|y(L) - y_m|$ votes. So here R cannot win either.

Thus R cannot win the election for any $x(R)$ he may choose. Still, observe that $x(R) = \frac{3-\mu}{4}$ is a dominant strategy for her.

Since we have shown that L wins in equilibrium when $|y(L) - y_m| \leq \frac{1}{4}$, we have that L's most preferred best response is an equilibrium strategy. ■

Proof of Lemma 4. First, suppose that $|y(L) - y_m| > 1/2$. If $x(L) > x(R)$ in equilibrium, consider $x'(R)$ such that $x'(R) = x(L)$ and notice that: 1) in this case R obtains more than $|y(L) - y_m|$ votes, that is, more than half of the total; and 2) the equilibrium policy outcome is larger, therefore better off for R'. Thus this is a profitable deviation for R and it implies that $x(L) > x(R)$ cannot hold in equilibrium.

Since we know that in equilibrium $x(L) \leq x(R)$ R's best winning strategy is defined by $\overline{x}_i > 1$ and $\widetilde{x}_i < \frac{1}{2}$.This implies that

$$\overline{x_i} = \frac{x(R) - \mu x(L)}{1-\mu} + |y(L) - y_m| > 1$$

and

$$\widetilde{x}_i = \frac{x(R) + \mu x(L)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by

$$(1-\mu)(1 - |y(L) - y_m|) + \mu x(L) < x(R) < \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$$

and among them R prefers the largest one $x(R) = \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$.

The best response for L in this case is the largest possible value of $x(L)$. So that R's best response to it corresponds to its smallest possible value. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$ implies $x(L) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. Thus in equilibrium $x(L) = x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.

Now suppose that $1/4 < |y(L) - y_m| < 1/2$. If $x(L) \in \left[0, \frac{1}{2} + |y(L) - y_m|\right]$ then R's best response, as in the previous proposition, is defined by $\overline{x_i} > 1$ and $\widetilde{x}_i < \frac{1}{2}$.

This implies that

$$\overline{x_i} = \frac{x(R) - \mu x(L)}{1 - \mu} + |y(L) - y_m| > 1$$

and

$$\widetilde{x_i} = \frac{x(R) + \mu x(L)}{1 + \mu} - \frac{1 - \mu}{1 + \mu} |y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by $(1 - \mu)(1 - |y(L) - y_m|) + \mu x(L) < x(R) < \frac{1+\mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$

and among them R prefers $x(R) = \frac{1+\mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$

And the best response for L in this case is the largest possible value of $x(L)$. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} + (1 - \mu)|y(L) - y_m| - \mu x(L)$ implies $x(L) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. Thus for $x(L) \in \left[0, \frac{1}{2} + |y(L) - y_m|\right]$ R's best response is $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.

Given that if $x(L) \in \left[\frac{1}{2} + |y(L) - y_m|, 1\right]$ we have that R's best response is $x(R) \in \left[\frac{1}{2} + |y(L) - y_m|, 1\right]$, and for $x(L) \in \left[0, \frac{1}{2} - |y(L) - y_m|\right)$ we have that R's best response is $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m| < \frac{1}{2} + |y(L) - y_m|$, this implies that L's optimal strategy will not be in $\left[\frac{1}{2} + |y(L) - y_m|, 1\right]$.

Therefore the equilibrium if $\frac{1}{4} < |y(L) - y_m| < \frac{1}{2}$ is given by $x(L) = x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. ■

Proof of Lemma 5. Let us start with the case when $|y(L) - y_m| \leq \frac{1-3\mu}{4(1-\mu)}$.Notice that it can emerge if and only if $\mu \leq \frac{1}{3}$. In that case the incumbent's payoff is increasing with $|y(L) - y_m|$ so his most preferred value of $y(L)$ in this range corresponds to $y(L) = y_m - \frac{1-3\mu}{4(1-\mu)}$. We already know from previous results that in this case that he will then set $x^*(L) = 0$

When $\frac{1-3\mu}{4(1-\mu)} \leq |y(L) - y_m| \leq \frac{1}{4}$, after plugging the incumbent's equilibrium platforms in the electoral issue, it is possible to rewrite his payoff as

$$U_L = -y_m + K - \frac{3\mu - 1}{4\mu} - \frac{1 - 2\mu}{\mu}|y(L) - y_m|, \tag{2}$$

which is decreasing with $|y(L) - y_m|$ as long as $\mu \leq \frac{1}{2}$ and increasing otherwise. In the former case, L's most preferred value of $y(L)$ corresponds to the minimal value of $|y(L) - y_m|$ in this range, that is, $y(L) = \max\left\{y_m - \frac{1-3\mu}{4(1-\mu)}, y_m\right\}$. Hence, if $\mu \leq \frac{1}{3}$ he will set again $y(L) = y_m - \frac{1-3\mu}{4(1-\mu)}$ (and then $x^*(L) = 0$)

whereas if $\frac{1}{3} \leq \mu \leq \frac{1}{2}$ he must set $y(L) = y_m$ which in turn implies that $x^*(L) = \frac{3\mu-1}{4\mu}$.

The third case occurs when $\mu \geq \frac{1}{2}$. Then (2) is increasing with $|y(L) - y_m|$. Thus while staying in this range his most preferred value of $y(L)$ corresponds to the one that maximizes $|y(L) - y_m|$, that is, $y(L) = y_m - \frac{1}{4}$, that from previous results it implies $x^*(L) = \frac{1}{2}$. ∎

Proof of Lemma 6. We know from previous results that if the incumbent decides to lose by setting $|y(L) - y_m| > \frac{1}{4}$, the challenger will win the election and set $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$. In that case, the incumbent receives the payoff

$$U_L = -y_m - \frac{1}{2} - \frac{2\mu}{1+\mu}|y(L) - y_m|,$$

which is increasing in $|y(L) - y_m|$. Thus while staying in this range, his most preferred value of $y(L)$ corresponds to the one that maximizes $|y(L) - y_m|$, that is, $y^*(L) = 0$, which implies that the challenger's best response in this case is $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}y_m$. ∎

Proof of Proposition 1. Previous results show that since $y_m < \frac{1}{4}$ implies that $|y(L) - y_m| < \frac{1}{4}$ then L prefers to win in this case.

If $y_m \geq \frac{1}{4}$, if the incumbent decides to lose then he receives a payoff equal to

$$U_L = -\frac{1}{2} - \frac{1-\mu}{1+\mu}y_m.$$

If the incumbent decides to use his best winnings strategy then he receives a payoff equal to

when $\mu \leq \frac{1}{3}$ his payoff boils down to

$$U_L = -y_m + K - \frac{3\mu - 1}{4(1-\mu)} \text{ if } \mu \leq \frac{1}{2} \tag{3}$$

and

$$U_L = -y_m + K - \frac{1}{4} \text{ if } \mu \geq \frac{1}{2} \tag{4}$$

Thus, when $\mu \geq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{1}{4} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}y_m$, that is, for

$$K > \frac{2\mu}{1+\mu}y_m - \frac{1}{4}$$

Notice that this value is strictly positive for all values of $\mu \in [0, 1]$ as long as $y_m > \frac{3}{8}$. For $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ we will have that the incumbent will decide to use a winning strategy for all values of K whenever $\frac{2\mu}{1+\mu}y_m - \frac{1}{4} > 0$, that is, $\mu > \frac{1}{8y_m-1}$. Notice that the incumbent decides to win for all K whenever $y_m = \frac{1}{4}$. Furthermore, the incumbent always decides to forgo reelection for some positive values of K whenever $y_m > \frac{3}{8}$.

Similarly, when $\mu \leq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{3\mu-1}{4(1-\mu)} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu}y_m$, that is, for

$$K > \frac{5\mu - 3}{4(1-\mu)} + \frac{2\mu}{1+\mu}y_m,$$

Notice that this value is strictly negative for small values of μ (in particular for all $\mu \leq \frac{1}{3}$). For those values the incumbent decides to win the election for all K. The set of values of K for which the incumbent decides to use a winning strategy is smaller for larger values of μ in this area. ■

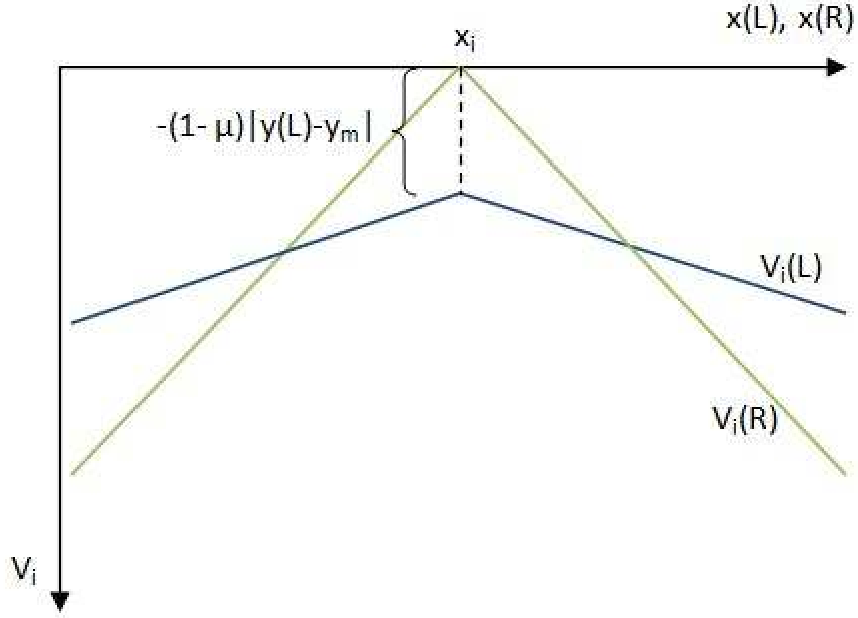

Figure 1: Voter's *i* evaluation of candidates at the electoral stage.

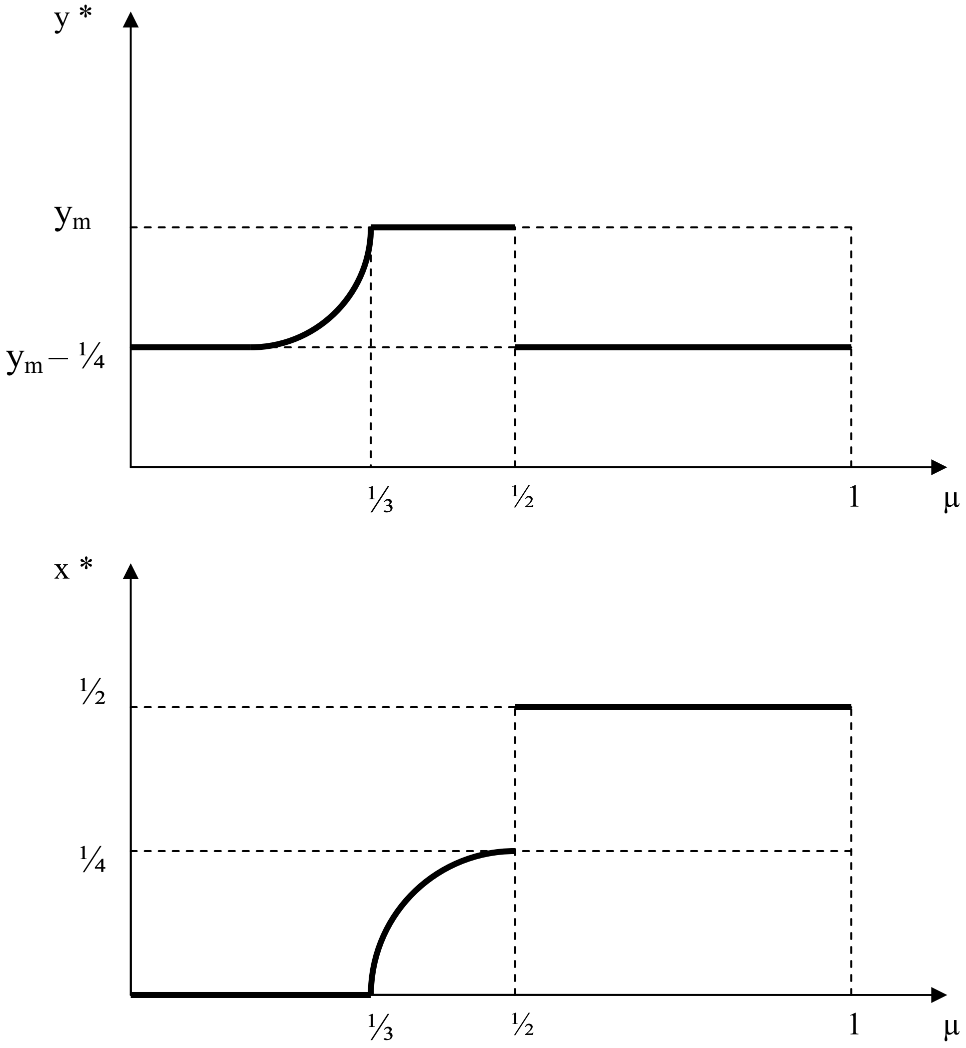

Figure 2: Incumbent’s best winning strategies.

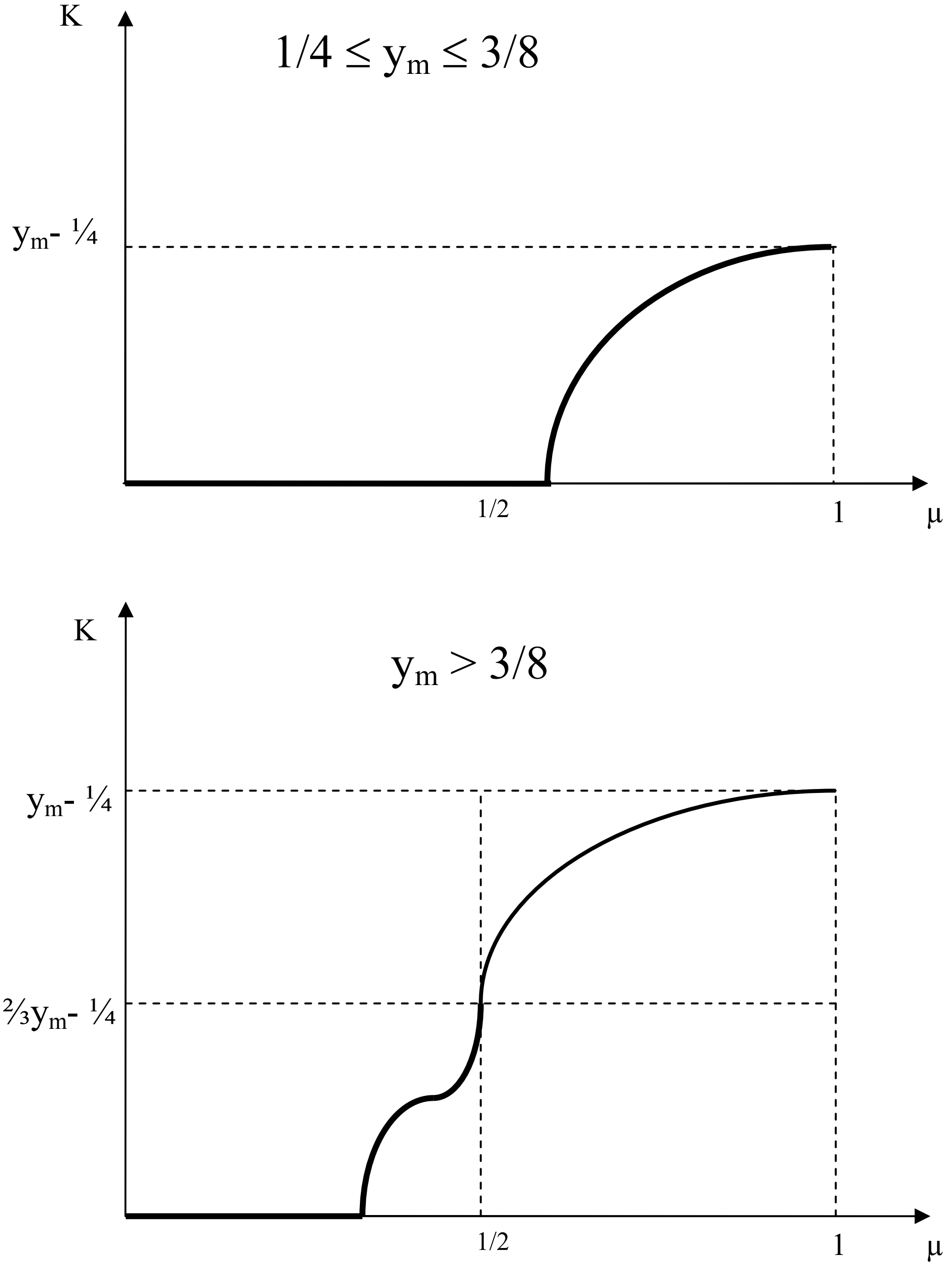

Figure 3 : Minimal values of K for which the incumbent prefers to use a winning strategy in equilibrium.