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Cosmological networks

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Abstract

Networks often represent systems that do not have a long history of study in traditional fields of physics; albeit, there are some notable exceptions, such as energy landscapes and quantum gravity. Here, we consider networks that naturally arise in cosmology. Nodes in these networks are stationary observers uniformly distributed in an expanding open Friedmann–Lemaître–Robertson–Walker universe with any scale factor and two observers are connected if one can causally influence the other. We show that these networks are growing Lorentz-invariant graphs with power-law distributions of node degrees. These networks encode maximum information about the observable universe available to a given observer.

Keywords: complex networks, cosmology, scale-free networks

1. Introduction

Network science is intrinsically multidisciplinary because the systems it studies come from different scientific domains. Complex networks are everywhere—in communication technologies, social and political sciences, biology, medicine, economics, and even linguistics [1–3]. This is why many scientific fields—computer science, social sciences, biology,

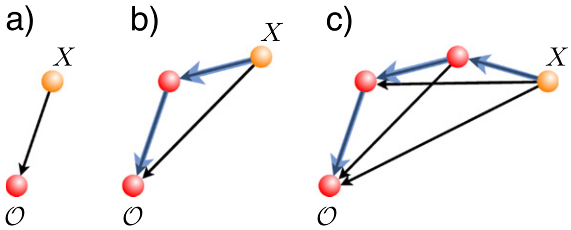

Figure 1. Direct versus indirect causal relations. Black edges show the direct causal relations between observers. In (b) and (c) the blue paths are indirect causal relations between observers X and O.

statistics, mathematics, and certainly physics—have contributed tremendously to network research over the last decade. Surprisingly, even though statistical physics has been applied with great success to understanding complex networks, the systems these networks represent can rarely display a long history of broad interest and focused research in traditional fields of physics. In fact, none of the preceding network examples provide an exception to this general rule. Exceptions, such as energy landscape networks [4] and networks in background-independent approaches to quantum gravity [5–8], are rare indeed.

Here, we add a class of networks that naturally arise in cosmology to this relatively short list of complex physical networks. Specifically, we consider evolving networks of causal connections among stationary (co-moving) observers, homogeneously distributed in any open Friedmann–Lemaître–Robertson–Walker (FLRW) spacetime [9]. These networks are purely classical. Nodes can represent classical particles, galaxies, or imaginary observers, scattered randomly throughout the space. The horizons of all the observers expand, and for any particular observer O at any given proper time τ, the network consists of all other observers within O's horizon, up to a certain cut-off time $\tau_\nu > 0$ in the past, which can be interpreted as the time of last scattering or the red shift beyond which the observer cannot observe [9]. A directed link from observer B to observer A exists in this network if B is within A's retarded horizon. The retarded horizon of A is A's horizon at earlier time $\tau_r < \tau$, such that light emitted by A at time τ_r reaches O at time τ. This means that if there are physical processes running at each observer, then directed paths between observers X and O in this network represent all possible causal relations between X and O, including indirect relations over paths longer than one hop (figure 1). Here, we show that this evolving network of maximum information about the universe, which any observer can collect by the proper time τ, is a growing power-law graph in any open homogeneous and isotropic (FLRW) spacetime.

We emphasize a critical difference between these cosmological networks and causal sets in de Sitter network cosmology considered in [6]. The latter are discretizations of four dimensional spacetime—nodes are elementary events (points in space and time)—and two events are connected if they are causally related, i.e., if they lie within each other's light cones. The resulting networks are directed acyclic graphs, and all linking dynamics are the appearance of new links connecting new nodes to the existing nodes lying in their past light cones. No new links appear between already existing nodes because any two events are either timelike-separated, and thus connected, or spacelike-separated and thus disconnected. The cosmological networks considered here are discretizations of three dimensional space. Time remains continuous. Therefore the evolution of nodes in these networks represent world-lines of co-moving observers. These networks have directed cycles, and new links not only connect new nodes to existing ones, but also appear at a certain rate between existing nodes, as they do in many complex networks [1–3].

2. Overlapping horizons in the Milne universe

The metric in an open FLRW spacetime is given by

$$ds^2 = -d\tau^2 + R(\tau)^2\left[d\chi^2 + \sinh^2\chi d\Omega_{d-1}^2\right], \tag{1}$$

where $\tau > 0$ and $\chi > 0$ are the cosmic time and 'radial' coordinates, $d\Omega_{d-1}^2$ is the metric on the unit $(d-1)$-dimensional sphere, and $R(\tau)$ is the scale factor of the universe given by the Friedmann equations [9]. The scale factor $R(\tau)$ is just a conformal factor in the spacial part of the metric, where coordinates (χ, Ω_{d-1}) describe the hyperbolic d-dimensional space $\mathbb{H}^d$ of constant curvature $K = -1$. The spacetime is thus foliated by d-dimensional hyperbolic spaces: for any time τ, the space is the hyperbolic d-dimensional space of constant curvature $K = -1/R(\tau)$. To simplify the calculations, we assume that $R(\tau) = \tau$, meaning that we are considering the Milne universe—a completely empty universe without any matter or dark energy [10]. The results presented henceforth do not depend on a particular form of scale factor $R(\tau)$. We discuss this important point at the end.

In $(2+1)$ dimensions (the generalization to $(d+1)$ with $d > 2$ is straightforward), the change of coordinates (τ, χ, θ) to

$$\begin{aligned} x &= \tau \sinh\chi \cos\theta \\ y &= \tau \sinh\chi \sin\theta \\ t &= \tau \cosh\chi \end{aligned} \tag{2}$$

transforms the metric in equation (1) into the Minkowski metric

$$ds^2 = -dt^2 + dx^2 + dy^2. \tag{3}$$

However, this transformation does not map the original spacetime in equation (1) to the entire Minkowski spacetime, but only to the future light cone of the event $t = x = y = 0$. The radial Minkowski coordinate $r = \sqrt{x^2 + y^2}$ of an event at coordinates (τ, χ, θ) is $r = t\tanh\chi$. This means that a stationary observer—that is, an observer at rest in the co-moving coordinates (χ, θ) in $\mathbb{H}^2$—is receding from the origin $x = y = 0$ at constant speed $v = \tanh\chi \leqslant 1$. Consistent with homogeneity and isotropy of the universe, we assume stationary observers are also homogeneously and isotropically distributed throughout space with constant density δ. These observers are therefore points distributed in the hyperbolic space $\mathbb{H}^2$ according to a Poisson point process with point density δ. In the Milne cosmology, an infinite number of such observers are thus initially at the origin of coordinates (the big bang), and then they all begin moving in all directions within a bubble—in the considered case, this bubble is a disk in $\mathbb{R}^2$—that expands at the speed of light [see the (x, y) plane in figure 2]. Because the distribution of observers is uniform in $\mathbb{H}^2$, any stationary observer will 'see' all other observers receding away with the Lorentz-invariant density of speeds v

$$\delta(v) \propto \delta\frac{v}{\left(1 - v^2\right)^{3/2}}. \tag{4}$$

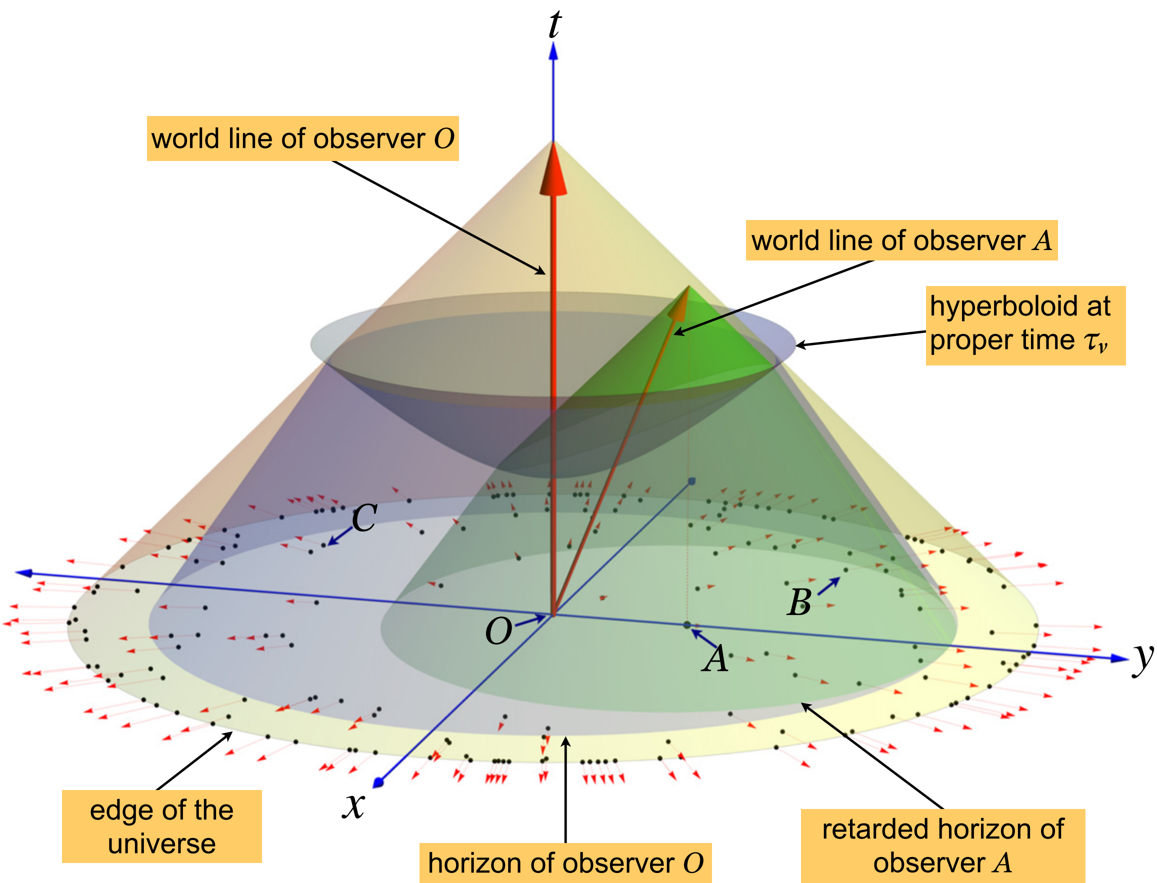

Figure 2. Milne universe with overlapping horizons as seen by observer O at proper time $\tau > \tau_\nu$. The horizontal plane is the (x,y) plane in a $(2+1)$-dimensional Minkowski spacetime. The vertical axis t is the proper time of observer O, who is at rest in the cosmic fluid, $\chi = 0$ and $x = y = 0$. At cosmic time $\tau = 0$, all particles are at the origin of this Minkowski spacetime and start moving away from O at velocities v, according to equation (4). Points and arrows in the (x, y) plane represent the position and velocity of such particles at proper time τ, as measured by O. The 'edge of the universe' corresponds to particles receding from O at the speed of light. Thus, this edge is a circle of radius $R_{edge} = \tau$ centered at O. Observer O does not observe all particles within this edge because particles are 'lit,' not at $\tau = 0$, but at $\tau_\nu > 0$. These events lie on the invariant hyperboloid $t^2 = \tau_\nu^2 + x^2 + y^2$, which is shown in blue. The horizon of any given observer is then induced by the intersection of the past light cone with this hyperboloid, and defines the maximum speed of a particle within the horizon. In particular, the radius of O's horizon in the (x, y) plane is $R_{horizon} = \tau[1 - (\tau_\nu/\tau)]/[1 + (\tau_\nu/\tau)]$ ($\tau_\nu = 1.5$ and $\tau = 5$ in the figure). The thick red arrows show the world lines of stationary observers O and A. Observer A is at rest at radial coordinate $\chi =$ const. The retarded horizon of observer A at proper time τ_χ is induced by the intersection of A's past light cone with the blue hyperboloid. Projected into the (x, y) plane, this retarded horizon encompasses all observers that can causally influence O indirectly via A (figure 1). Observer O has incoming connections from observers A, B, and C because they all lie within O's horizon. Observer A has incoming connections from O and B, but not from C, who is outside A's horizon.

Without loss of generality or breaking Lorentz invariance, in what follows, we focus on the stationary observer O at rest at coordinate $\chi = 0$, and therefore, also at rest at $x = y = 0$. According to equation (2), O's proper time τ is equal to the time coordinate t in the Minkowski spacetime. First, we determine the horizon of O at any given proper time τ. This horizon is the radius of the part of the universe that O can observe, up to the past cut-off time τ_ν, which can be any positive number, $0 < \tau_\nu < \tau$. This radius is determined by the intersection of O's past light cone with the hyperboloid at time τ_ν (figure 2). At time $\tau > \tau_\nu$, the farthest particle that O can observe is moving at a speed such that light emitted at proper time τ_ν reaches O at this time τ, yielding the following simple expression for the hyperbolic radius of O's horizon:

$$\chi_h = \ln\left(\frac{\tau}{\tau_\nu}\right). \tag{5}$$

The size of the network—i.e., the number of nodes in it—is, in this case, the number of other observers that O can observe, which is equal to the number of points within a hyperbolic disk of radius χ_h. This number grows asymptotically linearly with time τ:

$$N(\tau) = 2\pi\delta\left(\cosh\chi_h - 1\right) = \pi\delta\left[\frac{\tau}{\tau_\nu} + \frac{\tau_\nu}{\tau} - 2\right] \approx \pi\delta\frac{\tau}{\tau_\nu}. \tag{6}$$

Any two observers A and B in O's horizon are connected by a directed link from B to A if B lies within the retarded horizon of A. If A's radial coordinate is χ, then the retarded horizon of A is defined as its horizon at time $\tau_\chi = \tau \mathrm{e}^{-\chi}$. According to equation (5), τ_χ is such that if A emits light at the proper time τ_χ, this light reaches O at time τ. This means that if A has some physical state (possibly causally influenced by B) at time τ_χ, this state can causally influence O by time τ.

Figure 2 shows observer A lying within the horizon of observer O. Observer B is connected to A because B lies within A's retarded horizon at time τ_χ, the latest time in A's history that can influence O at time τ. Observer C is outside of this horizon and therefore is not connected to A. The link between O and A is bi-directed because they lie within each other's horizons.

Mapped to the hyperbolic plane, the horizon of observer O is a disk of radius χ_h, whereas the horizon of observer A is a disk of radius $\chi_h - \chi$ centered at A who is located at radial coordinate χ. This disk is tangent to O's horizon as illustrated in figure 3. The expected number of direct incoming connections to observer A, i.e., A's in-degree $\bar{k}_{in}(\chi)$, is thus given by the number of points within a disk of radius $\chi_h - \chi$:

$$\bar{k}_{in}(\chi) = 2\pi\delta\left(\cosh\left(\chi_h - \chi\right) - 1\right) \approx \pi\delta \mathrm{e}^{-(\chi - \chi_h)}. \tag{7}$$

Conversely, because observers are distributed uniformly according to the hyperbolic metric, their density located at radial coordinate χ is given by distribution

$$\rho(\chi) = \frac{\sinh\chi}{\cosh\chi_h - 1} \approx \mathrm{e}^{\chi - \chi_h}. \tag{8}$$

Thus, we have a combination of two exponential dependencies: $\bar{k}_{in}(\chi) \sim \mathrm{e}^{-\chi}$ and $\rho(\chi) \sim \mathrm{e}^{\chi}$. As one can verify [11], if, in general, the expected value $\bar{k}(x)$ of a variable k decays exponentially, $\bar{k}(x) \sim \mathrm{e}^{-\alpha x}$, $\alpha > 0$, as a function of random variable x whose distribution is also exponential, $\rho(x) \sim \mathrm{e}^{\beta x}$, $\beta > 0$, then the distribution of k is a power law, $P(k) \sim k^{-\gamma}$, with exponent $\gamma = \beta/\alpha + 1$. In our case, $\alpha = \beta = 1$, so that $\gamma = 2$:

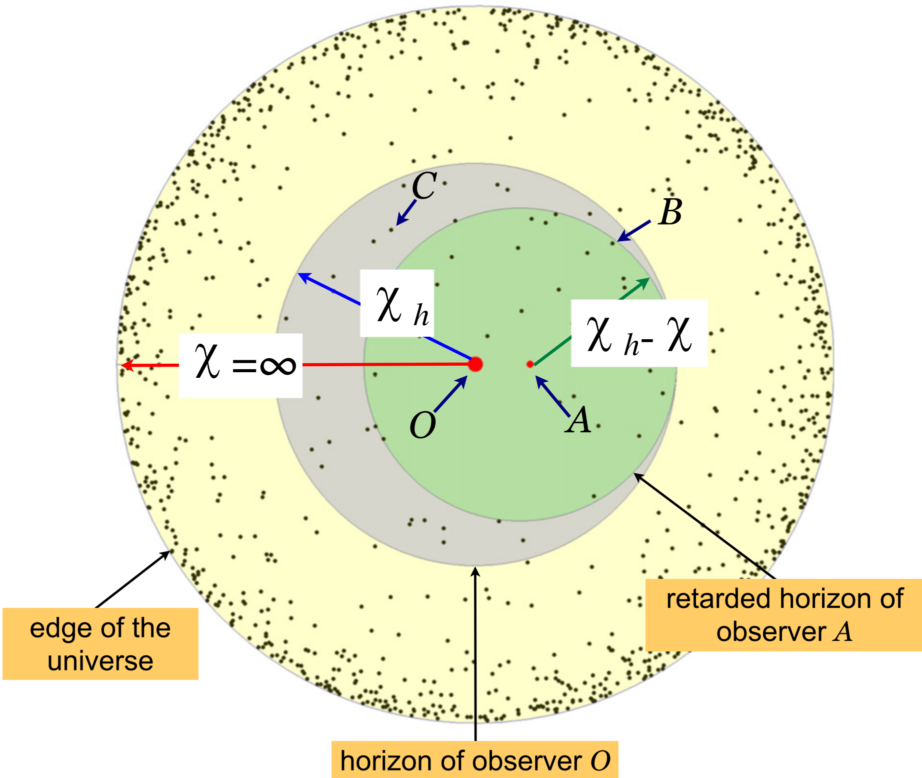

Figure 3. Milne universe projected onto the hyperbolic plane. All moving observers in figure 2 and their horizons can be mapped to the hyperbolic plane $\mathbb{H}^2$ via the change of coordinates in equation (2). After mapping, observers become static points on $\mathbb{H}^2$, whereas their horizons expand with cosmic time. The blue area is the horizon of observer O of hyperbolic radius $\chi_{\rm h}$. The green area is A's retarded horizon of radius $\chi_h - \chi$, centered at A and tangent to O's horizon. Nodes B and C are the same as in figure 2. The picture does not depend on scale factor $R(\tau)$, which determines only how horizon $\chi_{\rm h}$ grows with cosmic time τ.

$$P\left(k_{in}\right) \sim \frac{1}{k_{in}^2}, \quad \text{if } 1 \ll \mathrm{k}_{in} < \pi\delta \mathrm{e}^{\chi_h}. \tag{9}$$

In large networks with $\chi_h \gg 1$, the average in-degree scales as $\langle k_{in} \rangle \sim \pi\delta\chi_h \approx \pi\delta \ln(N/\pi\delta)$. The degree distributions in many large real networks are also close to power laws with exponents close to 2 [1–3].

Next we focus on the expected number of out-going connections, i.e., out-degree, of a node located at $(\chi, 0)$. It is equal to the number of points within a domain in $\mathbb{H}^2$ defined as the locus of points (χ', θ), such that their hyperbolic distances to the point $(\chi, 0)$, x, are smaller than the radius of their retarded horizons $\chi_h - \chi'$, that is,

$$\bar{k}_{out}(\chi) = 2\delta \int_0^\pi d\theta \int_0^{\chi_h} d\chi' \sinh\chi' \,\Theta\left(\chi_h - \chi' - x\right), \tag{10}$$

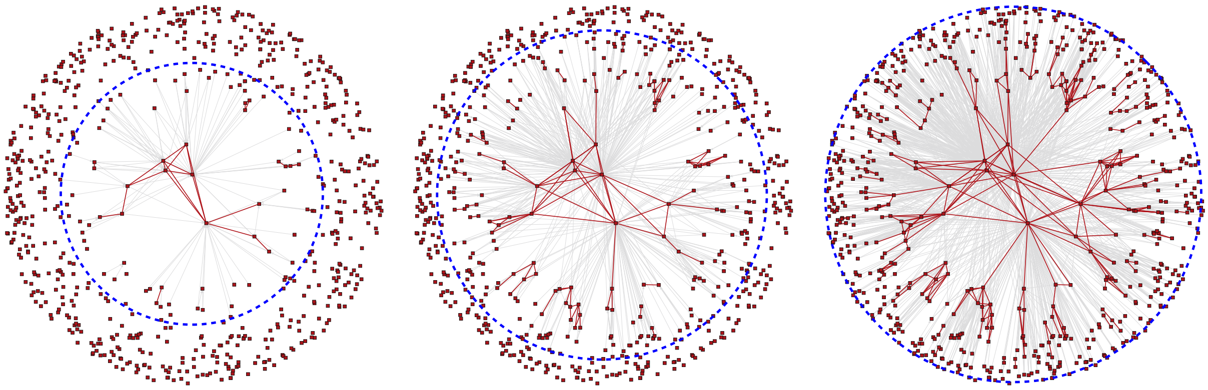

Figure 4. Evolution of a Milne network at three different proper times. The dashed blue circles represent the expanding horizons of the central observer. The grey and red links show directed and bi-directed (reciprocal) connections. The central observer and all its connections are suppressed.

where $\Theta(\cdot)$ is the Heaviside step function. In the limit $\chi_h \gg 1$, the integration yields

$$\bar{k}_{out}(\chi) \approx \begin{cases} 2\delta\sqrt{\dfrac{e^{\chi_h}}{\cosh\chi}}\, K(\tanh\chi) & \text{if } 0 \leqslant \chi < \chi_h, \\ 0 & \text{if } \chi = \chi_h, \end{cases} \tag{11}$$

where $K(\cdot)$ is the complete elliptic integral of the first kind. If $1 < \chi < \chi_h$, the average out-degree is well approximated by

$$\bar{k}_{out}(\chi) \approx 2\sqrt{2}\,\delta\chi \mathrm{e}^{(\chi_h-\chi)/2}. \tag{12}$$

For the same combination-of-exponentials reasons as in the in-degree case, this exponential scaling, combined with the one in equation (8), implies that the out-degree distribution scales as

$$P\left(k_{out}\right) \sim k_{out}^{-3}, \quad \text{for } \mathrm{k}_{out} \gg 1, \tag{13}$$

with logarithmic corrections. However, we notice that observers near (but not precisely at) the edge of the horizon have out-degrees approximately equal to χ_h. Therefore, the out-degree distribution is asymptotically a power law with a lower cut-off that grows as χ_h with time.

We note that new connections appear not only between new and existing nodes, but also between pairs of already existing nodes that were not previously connected. This type of linking creates directed cycles in the network. The appearance of new links between existing nodes is a simple consequence of the continuous expansion of the horizons of all observers. The resulting network dynamics are illustrated in figure 4, in which three snapshots of a growing network are taken. The horizon of the central observer O (the blue dashed circle) grows over time, discovering an exponentially increasing number of new observers. Gray connections indicate purely directed causal relations between observers, meaning one observer is aware of the other. As time goes on, directed connections are reciprocated (connections in red), meaning that an increasing number of pairs of observers are becoming mutually aware of each other.

Finally, we emphasize that our analysis is by no means limited to the Milne universe. Nearly the same results hold for any open FLRW universe with any scale factor $R(\tau)$. The same image as in figure 3 would apply. The only minor difference is the rate at which new nodes join the network, defined by the radius of the observer's horizon as a function of time. Specifically, given $R(\tau)$, this radius is

$$\chi_h = \int^t \frac{d\tau}{R(\tau)}, \tag{14}$$

generalizing equation (5).

3. Imperfect communication

Up to this point, we assumed all observers entering the horizon of another observer are detected with probability 1. If we assume that the probability of connection between observers decays exponentially with the hyperbolic distance x between them,

$$p(x) = pe^{-\beta x}, \tag{15}$$

then the average in-degree of an observer at coordinate χ is

$$\begin{aligned} \bar{k}_{in}(\chi) &= 2\pi\delta p \int_0^{\chi_h - \chi} \sinh \chi' \, e^{-\beta\chi'} d\chi' \\ &= 2\pi\delta p \frac{1 - \mathrm{e}^{\beta(\chi-\chi_h)}\left[\beta \sinh(\chi_h - \chi) + \cosh(\chi_h - \chi)\right]}{\beta^2 - 1}. \end{aligned} \tag{16}$$

If $\beta \geqslant 1$ and $\chi_h \gg 1$, the average in-degree of nodes is constant and the network becomes similar to a random geometric graph. In random geometric graphs, nodes lie in a geometric space and two nodes are connected if the distance between them in the space is below a given threshold. These graphs have Poisson distributions of node degrees [12]. We can show that the in-degree distribution in our imperfect networks with $\beta \geqslant 1$ is also Poisson. This is intuitively expected because, in this case, observers are connected only to other observers in their small neighborhoods. The case with $\beta < 1$ is more interesting. In this case, the average in-degree of an observer located at χ is $\bar{k}_{in}(\chi) \sim e^{(1-\beta)(\chi_h - \chi)}$. Consequently, for the same combination-of-exponentials reasons as given previously, the in-degree distribution scales asymptotically as a power law $P(k_{in}) \sim k_{in}^{-\gamma}$ with exponent

$$\gamma = 2 + \frac{\beta}{1 - \beta}, \tag{17}$$

which can take any value between 2 and ∞, as shown in figure 5.

This result may have interesting cosmological implications concerning what part of the universe our observers can observe. Indeed, in the case of imperfect communication with $\beta \in (0, 1)$, observer O directly detects only $\sim \mathrm{e}^{(1-\beta)\chi_h}$ other observers. Therefore, by the time the number of observers within O's horizon is $\sim N$, O detects only $\sim N^{1-\beta}$ of those observers, so the fraction of the universe that O sees directly ($\sim N^{1-\beta}/N$) approaches zero as time goes on. However, there are also indirect causal paths, which are shown in blue in figure 1. Any observer connected to O via either direct or indirect causal paths can still be detected by O. The question of what fraction of the universe can be observed by O becomes a variation of the bond percolation problem, which is well studied in network science. In the classical bond percolation problem, we are given a large network in which we retain or delete each link (also called a 'bond' for historical reasons) with probability p and $1 - p$. There often exists a critical value p_c of this probability corresponding to the phase transition in the system: if $p > p_c$, the network is

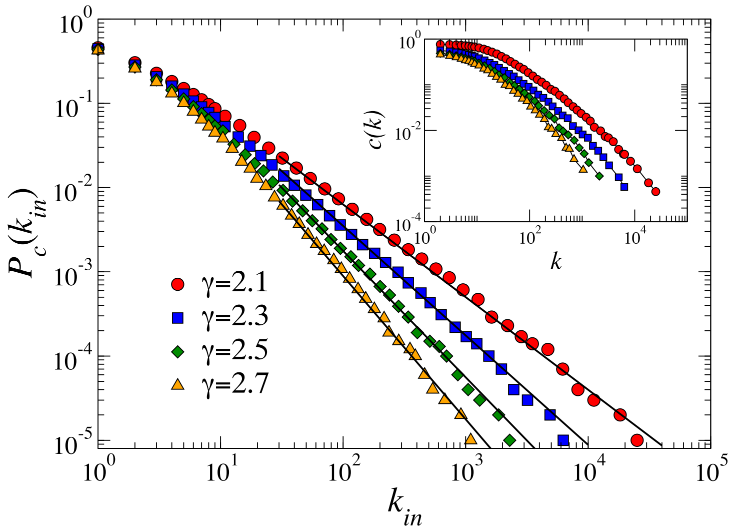

Figure 5. Complementary cumulative in-degree distribution $P_c(k_{in}) = \sum_{k_{in}' \geqslant k_{in}} P(k_{in}')$ in simulated Milne networks with exponents $\gamma = 2.1, 2.3, 2.5, 2.7$ grown up to $N = 10^5$ nodes. The solid lines are power laws with the same exponents. Inset: degree-dependent clustering coefficient for the undirected versions of the same networks. The average clustering coefficients excluding nodes of degree 1 are $\bar{C} = 0.67, 0.47, 0.41, 0.38$ for $\gamma = 2.1, 2.3, 2.5, 2.7$, respectively. The networks are disassortative, meaning that the correlations of degrees of connected nodes (not shown) are negative, due to structural constraints imposed by the scale-free degree distribution [13].

in the percolated phase, meaning that a macroscopical fraction of nodes belong to the largest connected component, whereas for $p < p_c$, the network decomposes into many small connected components. There is no such phase transition in random scale-free networks with power-law exponent $\gamma < 3$. They are always in the percolated phase, $p_c = 0$ [14]. In our imperfect cosmological networks with $\beta \in (0, 1)$, the given network is the perfect network, with $\beta = 0$ and $p = 1$, in which we retain links with probability in equation (15). The question is now what fraction of the network is connected to O via at least one causal path, direct or indirect. This problem is more involved than the standard bond percolation problem, but one may suspect that because the network is scale-free, there should exist a regime, perhaps with $\beta < 1/2$, in which the network is always percolated. This would imply that a macroscopic fraction of the universe can be observed by any observer.

Next, we support these expectations in simulations. Let g_O be the fraction of nodes within O's horizon that are connected to O via at least one causal path in an imperfect-communication network with the link existence probability (15). From the preceding exposition, including figure 1, the causal path is defined as a directed path $P = \{n_1, n_2, \ldots, O\}$ such that the retarded horizon H_{n_i} of any node n_i in the path, $i = 1, 2, \ldots$ (or equivalently the set of n_i's neighbors in the perfect network), contains all subsequent nodes in the path: $n_j \in H_{n_i}$ for any $j > i$. The problem of finding if such a path exists between a given node n_1 and O is likely to be an NP-hard combinatorial problem, because checking all directed paths between $n1$ and O seems unavoidable. We did not attempt to either prove the NP hardness of the problem or to find its computationally admissible solution because it is easier to provide upper and lower bounds for g_O. An upper bound g_O^u is simply the number of nodes that are connected to O via any directed

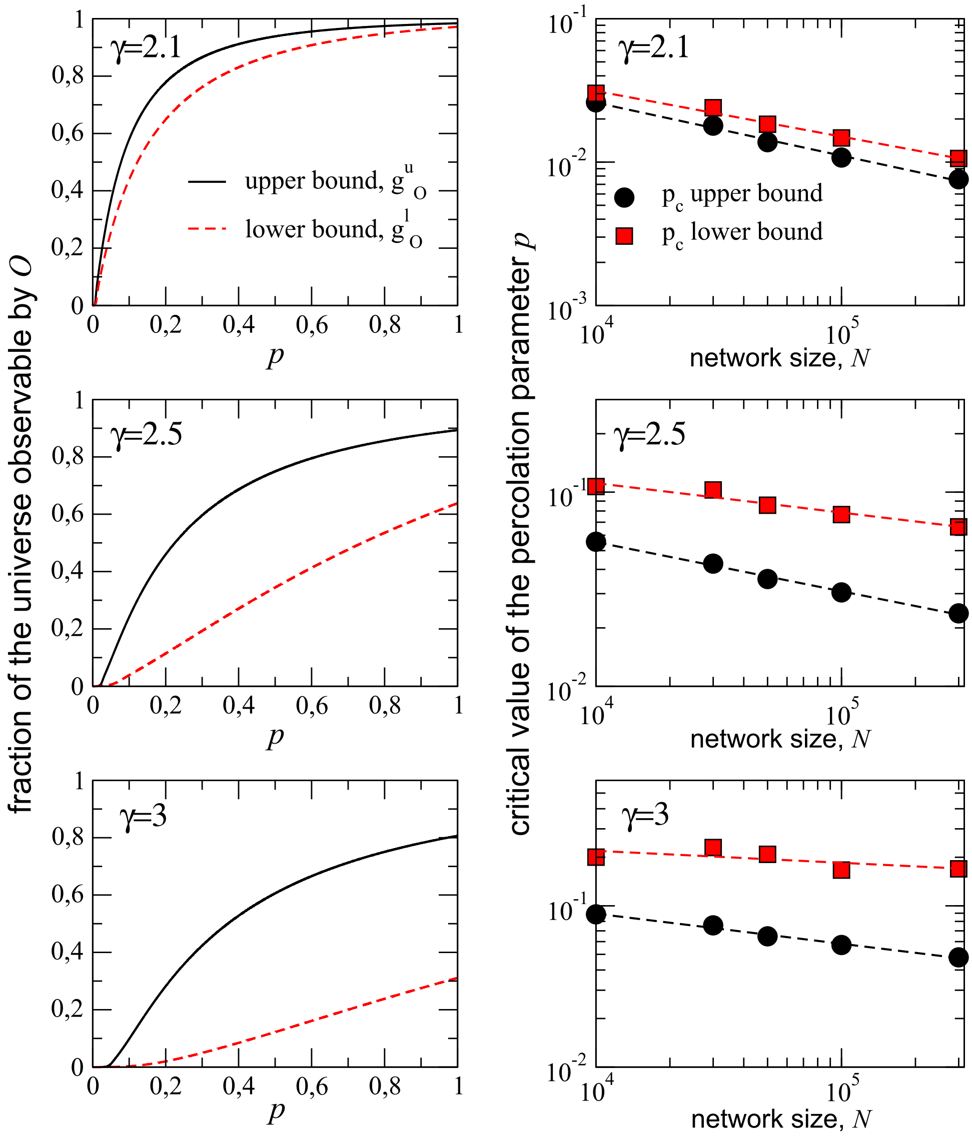

Figure 6. Bond percolation simulations of imperfect-communication networks. The left column shows the upper and lower bounds $g_O^{u,l}(p)$ for the fraction of nodes causally connected to O for different values of γ and $N = 3 \times 10^5$. The right column shows the critical values $p_c^{u,l}(N)$ for the same bounds, measured as the value of p that maximizes the susceptibilities $\xi^{u,l}$ in equation (18). The dashed lines are power law fits $p_c^{u,l}(N) \sim N^{-1/\nu}$ with exponents $1/\nu = 0.3(7), 0.3(2)$ for $\gamma = 2.1$, $1/\nu = 0.2(5), 0.1(5)$ for $\gamma = 2.5$, and $1/\nu = 0.1(8), 0.0(7)$ for $\gamma = 3$.

path, that is not necessarily causal. As a lower bound g_O^l, we use the number of nodes that are connected to O by at least one causal path and are located up to three hops away from O, which comprise a significant fraction of all nodes within O's horizon.

Figure 6 shows the results for these bounds in numerical simulations of networks with up to $N = 3 \times 10^5$ nodes, $\gamma = 2.1$, 2.5, 3 and $p \in [0, 1]$. The upper and lower bounds g_O^u and g_O^l increase monotonically as functions of p, suggesting that, as expected, the percolation threshold is zero. To verify if it is indeed zero, we measure the susceptibilities ξ^{u} and ξ^{l}, defined as

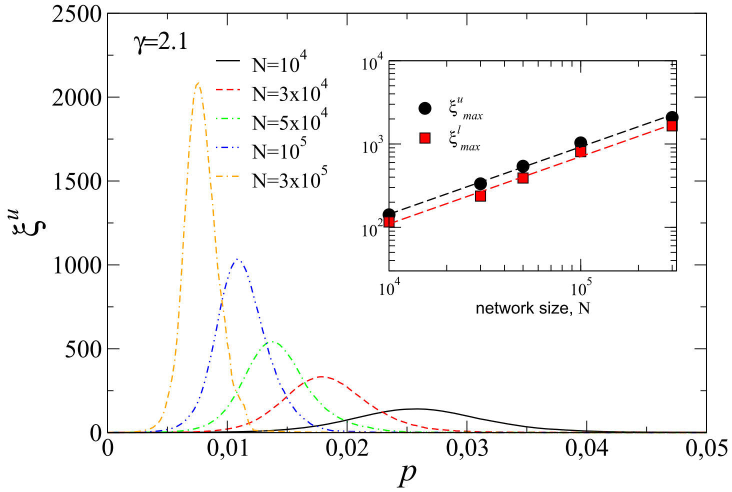

Figure 7. Percolation susceptibility of the upper bound ξ^{u} equation (18) as a function of p for different network sizes and $\gamma = 2.1$ (results for other values of γ are qualitatively similar). For all values of $\gamma < 3$, ξ^{u} and ξ^{l} show a peak that moves to the left as the system size increases. At the same time, the maximum value of ξ^{u} and ξ^{l} diverges as a function of N as $\xi^{u,l}_{max} \sim N^{\gamma'/\nu}$. Dashed lines in the inset are power law fits with exponents $\gamma'/\nu = 0.8(0)$.

$$\xi^{u,l} = N \frac{\left\langle \left[g_O^{u,l} \right]^2 \right\rangle - \left\langle g_O^{u,l} \right\rangle^2}{\left\langle g_O^{u,l} \right\rangle}, \tag{18}$$

where averages $\langle \cdot \rangle$ are taken over a large number (10,000 in our case) of different bond percolation realizations for each combination of values of N, γ, and p. In continuous phase transitions, the fluctuations of a property of interest (ξ in our case) diverge at a critical parameter value in the thermodynamic limit $N \to \infty$. In finite-size systems, this divergence manifests itself as a maximum of function $\xi(p)$ that becomes sharper for larger N (see figure 7). The value of $p = p_c$ corresponding to this maximum can be used as an estimate of the critical parameter value p_c [15]. The right column in figure 6 shows the values of thus estimated p_cs as functions of N for bounds $\xi^{u,l}(p)$ in our networks. For $\gamma < 3$, the critical points of both upper and lower bounds move to zero as power laws $p_c \sim N^{-1/\nu}$. This means that the percolation threshold is indeed zero in the thermodynamic limit ($p_c \to 0$ as $N \to \infty$), and that observer O can observe a finite fraction of the universe for any value of p. However, if $\gamma = 3$, then while the upper bound critical value goes to zero as N goes to infinity, the critical value corresponding to the lower bound becomes nearly size independent. This implies, that for $\gamma > 3$, there exists a critical point p_c below which our observer O can observe only the local neighborhood.

4. Conclusions

In summary, the physical network of (indirect) causal relations between observers uniformly distributed in any open FLRW universe is a Lorentz-invariant scale-free graph with strong clustering (see figure 5). This network represents the maximum information about the universe that any particular observer can collect by a certain time. More precisely, paths in this network are all possible communication channels between observers. Perhaps coincidentally, in the perfect case without information loss ($\beta = 0$), this network has the same statistical properties ($\gamma = 2$ and strong clustering) as the maximally navigable networks [16], i.e., networks that are most conductive with respect to targeted information signaling. The crucial requirement for this coincidence is that the universe must be open [see equation (1)]. Bubble universes are open in most inflationary cosmologies [17], and the current measurements of our universe do not preclude that it is open either; however, it is definitely close to being flat [18].

These results are of interest in both network science and cosmology. From the network science perspective, they may help to develop a 'general relativity' of networks, an analogy of the Einstein equations that would describe network dynamics within a unified framework, in which network nodes might be analogous to our observers or galaxies. Here, we have considered an idealized case where nodes are massless points distributed uniformly in the space. It remains unclear how the picture would change if points have masses, perhaps distributed according to some heterogeneous distributions similar to the distribution of the masses of galaxies in the universe [19], and if the spatial distribution of points deviates from uniform, as it does for galaxies [20] and real networks embedded in hyperbolic spaces [21].

From the cosmology perspective, it has been suggested that measures of photons from the cosmic microwave background scattered by high-energy electrons in clusters of galaxies could be used to probe the last scattering surface (LSS) at many different length scales, and thus overcome the limitations of the cosmic variance [22]. In this context, the cosmological networks we have considered here may be interesting because they contain not only direct connections within causal horizons, but also all possible indirect causal connections. The galaxy-scattered photons represent the latter indirect connections between the LSS and us, illustrated in figure 1(b), albeit made of only two hops. Yet the knowledge of the density of clusters of galaxies throughout the universe, coupled with our network representation, can be used to estimate the maximum information we could ever obtain from the LSS by counting the total number of causal paths connecting such a surface to us. In that respect, the discussed percolation problem on these networks may be of particular interest.

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