Unit II: The Euclidean Space

Exercise 1. Compute, whenever possible, the inner product of vectors $\vec{u}$ and $\vec{v}$ when

a) $\vec{u} = (2, 5, 6)$ and $\vec{v} = (3, 1, 1)$
b) $\vec{u} = (4, 2)$ and $\vec{v} = (5, 6)$
c) $\vec{u} = (1, 3, 1)$ and $\vec{v} = (5, 2)$
d) $\vec{u} = (2, 2, 0)$ and $\vec{v} = (3, -3, 1)$

Exercise 2. Let $\vec{u} = (-3, 1, 2)$, $\vec{v} = (5, 0, 3)$, $\lambda = 2$, and $\mu = 4$. Compute

a) $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v})$
b) $(\vec{u} \cdot \vec{v}) \cdot \vec{v}$
c) $(\lambda \vec{u}) \cdot (\mu \vec{v})$
d) $[\lambda(\vec{u} + \vec{v})] \cdot (\mu \vec{v})$

Exercise 3. Study whether the vectors $\vec{u}$ and $\vec{v}$ are orthogonal (perpendicular) or not in the following cases

a) $\vec{u} = (4, 1)$ and $\vec{v} = (-4, 1)$
b) $\vec{u} = (2, 1, 2)$ and $\vec{v} = (-2, -1, -2)$
c) $\vec{u} = (3, 1, -2)$ and $\vec{v} = (1, 5, 4)$
d) $\vec{u} = (3, 5, 1)$ and $\vec{v} = (0, 0, 0)$
e) $\vec{u} = (2, 3)$ and $\vec{v} = (3, -2)$.
f) $\vec{u} = (1, 2, 3)$ and $\vec{v} = (-4, -2, 3)$.
g) $\vec{u} = (2, -1, 1)$ and $\vec{v} = (3, 2, -2)$.
h) $\vec{u} = (2, -1, 1, 3)$ and $\vec{v} = (0, 0, 0, 0)$.

Exercise 4. Find the value of the parameter k that makes the inner product of vectors $(3, k, k)$ and $(1, -3, k)$ equal to 1.

Exercise 5. Find the values of the parameter k so that the inner product of the vectors $\vec{u} = (-1, k, 2)$ and $\vec{v} = (k, 1, k)$ equals 2.

Exercise 6. For what values of the parameter k are the vectors $\vec{u} = (k, 4, 5)$ and $\vec{v} = (k, -k, -1)$ orthogonal?

Exercise 7. Find the values of the parameter k for which the vectors $\vec{u} = (-1, k, 1)$ and $\vec{v} = (k, k, -6)$ are orthogonal?

Exercise 8. Let (a, b) and (c, d) be two orthogonal vectors of $\mathbb{R}^2$. Show that the vectors are linearly independent.

Exercise 9. Given $\vec{u} = (1, -2, 3)$ and $\vec{v} = (4, 0, 1)$, compute

a) $\|\vec{u}\|$ and $\|\vec{v}\|$
b) $\|2\vec{u}\|$
c) $\|\vec{u} - \vec{v}\|$
d) $\|2\vec{u} + \vec{v}\|$

Exercise 10. Let $\vec{u} = (2,3,4)$ and $\vec{v} = (3,3,2)$. Compute

a) $\|\vec{u}\|$ and $\|\vec{v}\|$

b) $\|3\vec{u}\|$

c) $\|-3\vec{u}\|$

d) $\|\vec{u}+\vec{v}\|$

Exercise 11. Check that the Schwartz and Triangle inequalities hold for the particular case of $\vec{u} = (1,-2,3)$ and $\vec{v} = (4,0,1)$.

Exercise 12. Check that the Schwartz and Triangle inequalities hold for the particular case of $\vec{u} = (1,3,4)$ and $\vec{v} = (0,-4,-3)$.

Exercise 13. Normalize the following vectors $\vec{u} = (2,5)$ and $\vec{v} = (1,2,2)$.

Exercise 14. Normalize the vectors $\vec{u} = (2,1,-2)$ and $\vec{v} = (4,-4,-4,4)$.

Exercise 15. Check if the following set of vectors is an orthogonal basis of $\mathbb{R}^3$:

$$\{(1,1,0),(1,-1,2),(-2,2,2)\}.$$

Exercise 16. Check if the following set is an orthonormal basis of $\mathbb{R}^3$:

$$\{(1,0,0),(0,6,8),(0,-8,6)\}.$$

Exercise 17. Check if the following set of vectors is an orthonormal basis of $\mathbb{R}^3$:

$$\left\{(0,0,-1)\,,\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0\right),\left(\frac{\sqrt{2}}{2},\frac{-\sqrt{2}}{2},0\right)\right\}.$$

Exercise 18. Find the value(s) of the parameter k for which the norm of the vector $(-4,k,0)$ is equal to 5?

Exercise 19. Find the angle between the vectors $\vec{u} = (1,0,1)$ and $\vec{v} = (4,3,0)$.

Exercise 20. Find the values of k that make the norm of the vector $\vec{u} = (3,-2,k)$ equal to 20.

Exercise 21. Compute the angle between the vectors $\vec{u} = (4,2,-6)$ and $\vec{v} = (0,3,-1)$.

Exercise 22. Compute the angle between the vectors $\vec{u} = (0,1,1)$ and $\vec{v} = (1,0,1)$.

Exercise 23. Find the angle between the vectors $\vec{u} = (2,0,2)$ and $\vec{v} = (3,0,3)$.

Exercise 24. Find the distance between the vectors $\vec{u} = (3,1,-2)$ and $\vec{v} = (0,1,2)$.

Exercise 25. Find the value of k for which the angle between the following vectors is 60^{o}:

$$\vec{u} = (1,0,1) \quad \text{and} \quad \vec{v} = (k,1,0).$$

Exercise 26. Compute the distance between the vectors $\vec{u} = (3, 0, 5)$ and $\vec{v} = (7, 2, 1)$.

Exercise 27. Find the value of k for which the distance between the following vectors is 5:

$$\vec{u} = (5, k) \quad \text{and} \quad \vec{v} = (8, 1).$$

Exercise 28. Find the value of k for which the distance between the following two vectors is 4:

$$\vec{u} = (0, -2, k) \quad \text{and} \quad \vec{v} = (k, 1, 0).$$

Exercise 39. Find the value (or values) of k for which the distance between the following two vectors is 13:

$$\vec{u} = (k, 2, k) \quad \text{and} \quad \vec{v} = (1, 2, -k).$$