Optical encryption using photon-counting polarimetric imaging

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Abstract: We present a polarimetric-based optical encoder for image encryption and verification. A system for generating random polarized vector keys based on a Mach-Zehnder configuration combined with translucent liquid crystal displays in each path of the interferometer is developed. Polarization information of the encrypted signal is retrieved by taking advantage of the information provided by the Stokes parameters. Moreover, photon-counting model is used in the encryption process which provides data sparseness and nonlinear transformation to enhance security. An authorized user with access to the polarization keys and the optical design variables can retrieve and validate the photon-counting plain-text. Optical experimental results demonstrate the feasibility of the encryption method.

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References and links
1. Introduction

In recent years, optical security and encryption methods [1–16] have attracted the interest of many authors. In particular, double random phase-masks encryption (DRPE) is an effective method which is very suitable for encoding images that can be easily implemented in practice using a 4f system [3–8]. By means of this technique it is possible to encrypt a signal in such a way that becomes very difficult to be retrieved by an unauthorized user. Only those who have access to the phase mask in the Fourier plane can access the original signal. There are many variations of this technique which have been introduced by various authors some of which are

listed in [1–22]. Some authors have pointed out vulnerabilities of DRPE when facing specifically designed attacks [9–13], but security can be enhanced by increasing the number of degrees of freedom of this approach; for instance, changing the position of the phase masks in the optical setup [13,14]. Also, using randomized lens-phase functions can improve resistance against attack [15]. Another alternative is to use key masks that change the polarization direction and the phase difference between the electric field components [16–18]. Polarization has also been suggested as a method for extracting information from a scene [23]. Other methods to improve security have been proposed such as cryptographic blocks [19], a combination of real and virtual optics methods [20–22] or recording the encrypted distribution in low illumination conditions [24] using photon counting. Recording photon starved distributions is a highly nonlinear process in which only a few pixels of the encoded signal are recorded so that the corresponding decrypted image becomes very sparse and it does not reveal the original content. However, this distribution contains enough information to enable its validation when compared with the original image [24].

Interest in beams with non-uniform polarization patterns has increased significantly due to their special properties compared to homogeneously polarized beams, which can enhance the functionality of optical systems [25]. In particular, several methods based on interferometric configurations and liquid crystal displays have been suggested [26–31]. Recently, we reported a method for generating beams with arbitrary polarization based on a Mach-Zehnder setup combined with a translucent SLM in each path of the interferometer. The transverse components of the incident light beam are processed independently and modified by means of specifically designed holograms, recombined and imaged on a CCD camera. Our approach allows us to encode any polarization state at each point of the wave-front and the amplitude may also be modeled so as to obtain a particular shape in a given plane [32,33].

Random polarization distributions can be understood as encryption keys. In this paper, we describe an optical encryption processor based on a non-uniform arbitrary polarization setup. Moreover, security can be enhanced by recording the encrypted polarized signal under low-light illumination conditions using the photon-counting model. Our objective is to develop a Fresnel-like polarization encoder able to encrypt signals taking advantage of the information provided by the Stokes parameters. The encryption system is optically implemented whereas the decryption procedure is carried out numerically. The experiment demonstrates the feasibility of using this system as a polarimetric photon counting optical encoder for security applications.

Accordingly, the paper is organized as follows: in section 2, we describe the optical setup for generating wave-fronts with arbitrary polarization states and how this system can be used for optical encryption proposes. In section 3 we illustrate the proposed procedure by means of a set of numerical calculations whereas an experimental implementation of the system is presented in section 4. Finally, the main conclusions are summarized in section 5.

2. Polarization-based setup for optical encryption

The proposed system is based on a Mach-Zehnder setup as depicted in Fig. 1. A collimated, linearly polarized beam at 45° of the x-direction is split into two beams by means of polarizing beam splitter PBS1. The plain-text real-valued image \( t \) is set in one of the arms of the interferometer at a distant \( d_1 \) from spatial light modulator SLM1. In order to improve the encryption capabilities of the system, plain-text \( t \) is encoded by means of phase only mask \( M_0 \). Modulator SLM1 is located at the front focal plane of lens L1; two wave plates (\( \lambda/2 \) and \( \lambda/4 \), respectively labelled HWP and QWP in Fig. 1) are placed before SLM1 as to obtain the expected modulation curve. Amplitude-only random encryption key \( M_1 \) is displayed on this modulator. Note that this part of the optical system is equivalent to a lens-less Fresnel encryption processor [13].

A second modulator (SLM2) is placed in the other arm of the interferometer, displaying a phase-only distribution \( M_2 \). SLM2 is placed at the front focal plane of lens L1 as well. A
neutral density filter controls the amount of light reaching this modulator. The information propagated through the two arms of the interferometer is recombined by means of PBS2. Since the modulators used are pixelated displays, a spatial filter is used for removing high order diffracted terms. Then, the light passes through a 4f imaging system; note that f1 and f2 are selected in such a way that the area of the CCD matches the size of SLM1 and SLM2. Finally, the encrypted signal is recorded by the CCD placed at a distance $d_2$ from the back focal plane of lens L2. Accordingly, the vector cypher-text $T$ is described as

$$T = T_x e_x + T_y e_y = Pr[M_1 Pr[M_0, d_1], d_2] e_x + Pr[M_2, d_2] e_y,$$

where $\phi_0, \phi_2$ are uniformly distributed random variables in the interval $[-\pi, \pi]$ and C is a constant related with the amount of light reaching SLM2. Distribution $M_1$ is an amplitude encryption key in the range $[\varepsilon, 1]$ with $\varepsilon > 0$, $Pr[\cdot]$ stands for the propagation operator in Fresnel diffraction conditions; and vectors $e_x$ and $e_y$ are unit vectors in the x- and y-directions. $T_x = |T_x| \exp(i\tau_x)$ and $T_y = |T_y| \exp(i\tau_y)$ are the propagated fields in the first and second arms of the interferometer respectively. Note that since $T_x$ and $T_y$ are orthogonal complex valued distributions, the wave-front T is non-uniformly polarized. Moreover, no interference has to be detected at the CCD plane due to the orthogonality of $T_x$ and $T_y$.

Taking into account the vector character of $T$ it is possible to extract amplitude and phase information using polarimetric techniques. Polarization analysis is carried out by placing a linear polarizer (LP) and a quarter-wave plate (QWP) when required in front of the CCD camera. Stokes images provide a complete description of the state of polarization in each point of the wave-front. A set of six polarimetric images ($I(0^\circ, 0)$, $I(90^\circ, 0)$, $I(45^\circ, 0)$, $I(135^\circ, 0)$, $I(45^\circ, \pi/2)$ and $I(135^\circ, \pi/2)$) have to be recorded for calculating the four Stokes distributions ($S_0, S_1, S_2, S_3$) [34]:

$$S_0 = I(0^\circ, 0) + I(90^\circ, 0),$$

$$S_1 = I(0^\circ, 0) - I(90^\circ, 0),$$

$$S_2 = I(45^\circ, 0) - I(135^\circ, 0),$$

$$S_3 = I(45^\circ, \pi/2) - I(135^\circ, \pi/2).$$

$I(\alpha, 0)$ stands for the recorded intensity when the linear polarizer is set at an angle $\alpha$ with respect to the x direction whereas $I(\alpha, \pi/2)$ is the image recorded when a quarter wave-plate is used in addition to the polarizer. The Stokes parameters associated to vector cypher-text $T$ are used to recover amplitudes $|T_x|$ and $|T_y|$, and the phase difference $\tau_y - \tau_x$. The formulae that relate the polarization ellipse parameters with the Stokes vector ($S_0, S_1, S_2, S_3$) are:
\[ S_0 = |T_x|^2 + |T_y|^2 \]
\[ S_1 = |T_x|^2 - |T_y|^2 \]
\[ S_2 = 2 |T_x||T_y| \cos(\tau_y - \tau_x) \]
\[ S_3 = 2 |T_x||T_y| \sin(\tau_y - \tau_x) \]

or equivalently,
\[ |T_x| = \sqrt{\frac{S_0 + S_1}{2}} \quad |T_y| = \sqrt{\frac{S_0 - S_1}{2}} \quad \tan(\tau_y - \tau_x) = \frac{S_1}{S_2}. \]  

After encryption, encoded distribution \(|T_x| \exp(i(\tau_y - \tau_x))\) is sent through a secure channel. The original plain-text \(t\) can be retrieved if and only if keys \(M_1\) and \(M_2\), and distances \(d_1\) and \(d_2\) are known. Notice that \(M_i^{-1}\) is well posed since \(M_i \neq 0\) at any point thus inversion is possible:
\[ \tau_x = \tau_y - \arctan \frac{S_1}{S_2} = \arg \left[ \Pr [M_2, d_2] \right] - \arctan \frac{S_1}{S_2} \]
\[ r = \Pr [\Pr [|T_x| \exp(i\tau_x), -d_2] M_1^{-1}, -d_1]. \]

If a system works in low light illumination conditions, irradiance is recorded according to the photon-counting model. It is assumed that, in these conditions the image is statistically modeled by the Poisson distribution [35]. According to this law, the probability that no photon reaches pixel \((x_i, y_i)\) is
\[ P(0; (x_i, y_i)) = \exp(-n_p(x_i, y_i)), \]
where \(n_p(x_i, y_i)\), is the normalized irradiance at pixel \((x_i, y_i)\):
\[ n_p(x_i, y_i) = \frac{N_p \left| T_x(x_i, y_i) \right|^2}{\sum_{i,j} |T_x(x_i, y_i)|^2}. \]

\(N_p\) is the predetermined number of photon counts in the entire scene and \(N \times M\) is the total number of pixels. The photon-counting version \(|T_x|^{ph}\) of irradiance \(|T_x|\) is obtained according to
\[ |T_x|^{ph}(x_i, y_i) = \begin{cases} 0, & \text{if } \text{rand}(x_i, y_i) \leq P(0; (x_i, y_i)) \\ 1, & \text{otherwise} \end{cases} \]
where \(\text{rand}(x_i, y_i)\) is a uniformly distributed random number within the range [0,1]. In order to enhance security, photon-counting cypher-text \(|T_x|^{ph} \exp(i(\tau_y - \tau_x))\) is transmitted instead of using \(|T_x| \exp(i(\tau_y - \tau_x))\). Following Eq. (5), an authorized user with access to keys \(M_1\) and \(M_2\), and distances \(d_1\) and \(d_2\) can retrieve the photon-counting plain-text \(t^{ph}\):
\[ t^{ph} = \Pr \left[ \Pr [|T_x|^{ph} \exp \left( i \left( \arg \left[ \Pr [M_2, d_2] \right] - \arctan \frac{S_1}{S_2} \right) \right) - d_2] M_1^{-1}, -d_1 \right]. \]

Finally, to determine whether \(t^{ph}\) contains information related with plain text \(t\) or not, the non-linear cross-correlation \(t^{ph} \otimes t\) is calculated:
\[ t^{th} \otimes t = \text{FT}^{-1} \left[ \frac{\text{FT} \left[ t^{th} \right] \text{FT} \left[ t \right]}{\left| \text{FT} \left[ t^{th} \right] \right|^{-1} \left| \text{FT} \left[ t \right] \right|^{-1}} \right] \] (10)

Indexes \( k \) and \( l \) are selected in such ways that provide a good discrimination capability with a high peak-to-correlation energy [24, 36, 37].

3. Numerical test

To illustrate the encryption process a digital simulation is presented. A 1951 USAF glass slide resolution test is used as a true plain-text \( t \) (Fig. 2(a)). For convenience we used the same resolution test but rotated 90° as the false-class image \( f \) (Fig. 2(b)). Inputs images \( t \) and \( f \) are phase encoded with \( M_0 \). The simulation of the encryption and decryption processes is carried out using \( \lambda = 520 \) nm, \( d_1 = 250 \) mm and \( d_2 = 45 \) mm; and setting the size of the window to 10 mm. To prevent numerical problems in deriving the calculation of \( M_1^{-l} \), the minimum value of this key is set to \( \epsilon = 0.2 \).

![Fig. 2. (a) plain-text image \( t \). (b) false-class plain text image \( f \).](image)

![Fig. 3. Vector cipher-text \( \mathbf{T} \): (a) Stokes parameter \( S_0 = |\mathbf{T}|^2 \). Green and red ellipses indicate right-handed left-handed polarization respectively. (b) Histogram of |(\mathbf{T})|^2.](image)

Vector cipher-text \( \mathbf{T} = |T_x| \exp(i\tau_x) \mathbf{e}_x + |T_y| \exp(i\tau_y) \mathbf{e}_y \) is the combination of two uncorrelated random complex signals in the \( x \)- and \( y \)- directions (see Eq. (1)). The Stokes parameter \( S_0 \), \( |\mathbf{T}|^2 = |T_x|^2 + |T_y|^2 \), is shown in Fig. 3(a). The corresponding histogram exhibits a Gamma-like distribution profile as shown in Fig. 3(b). The state of polarization is also random since it is described by \( |T_x|, |T_y| \) and the phase difference \( \tau_y - \tau_x \). To provide more insight about the random nature of both irradiance and polarization, polarization ellipses have

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been calculated in some points of the recording plane. They appear superimposed in Fig. 3(a); green and red ellipses represent right-handed and left-handed polarization respectively.

The other three Stokes parameters are displayed in Fig. 4. These parameters were obtained from T and they are also used to retrieve the encoded full complex information $|T_{x}| \exp(i(\tau_{y} - \tau_{x}))$ (see Eq. (4)).

![Fig. 4. Stokes parameters](image)

To enhance security, the photon-counting version $|T_{x}|_{\text{ph}}$ of irradiance $|T_{x}|$ is obtained. $|T_{x}|_{\text{ph}}$ is calculated with $N_{p} \sim 1\%$ of the number of pixels of the image. An authorized user with access to the polarization encryption keys and the correct distances is able to access the photon-counting plaintext using Eq. (5) and (9) respectively. Nonlinear correlation indexes are estimated calculating the peak-to-correlation energy metric as a function of $k$ and $l$ from $t_{\text{ph}} \otimes t$. The results are shown in Fig. 5(a). Figure 5(b) shows a 3D plot of $t_{\text{ph}} \otimes t$; note that the presence of a sharp peak indicates that verification has been successful. A second numerical test is carried out using a false-class input image $f$ (Fig. 2(b)). In this case, the photon-counting plaintext $f_{\text{ph}}$ was obtained in the same conditions as in the previous case. Nevertheless, correlation $f_{\text{ph}} \otimes t$ does not show a peak so the image is labeled as fake (Fig. 5(c)).

4. Optical implementation

In this section we present an optical experiment to illustrate how the system works in real conditions. The encryption procedure is carried out by means of the polarized-based optical system sketched in Fig. 1. Design variables are summarized in Table 1. The SLMs used are two Holoeye HEO-0017 translucent displays whereas the camera is an AVT Stingray F-080B. The light source is a 15 mW green laser diode $\lambda = 520$ nm. Note that the size of the beams is limited by the frame of the collimating and imaging lenses.
Fig. 5. (a) Nonlinear indexes estimation: Peak to correlation energy as a function of \( k \) and \( l \). (b) True image \((k = 0.1\) and \( l = 0.7\)); (c) false image \((k = 0.1\) and \( l = 0.7\)).

Table 1. Values of the variables used in the optical implementation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength ( \lambda )</td>
<td>520 nm</td>
</tr>
<tr>
<td>Propagation distances ( d_1 ) and ( d_2 )</td>
<td>250 mm and 45 mm</td>
</tr>
<tr>
<td>Focal lengths ( f_1 ) and ( f_2 )</td>
<td>200 mm and 60 mm</td>
</tr>
<tr>
<td>( M ) minimum value ( \epsilon )</td>
<td>0.2</td>
</tr>
<tr>
<td>SLM</td>
<td>Pixel pitch 0.036 mm</td>
</tr>
<tr>
<td></td>
<td>Resolution (# of pixels) 1024 x 768 pixels</td>
</tr>
<tr>
<td></td>
<td>Diameter of the beam / encryption key 10 mm</td>
</tr>
<tr>
<td></td>
<td>CCD</td>
</tr>
<tr>
<td></td>
<td>Pixel pitch 0.00465 mm</td>
</tr>
<tr>
<td></td>
<td>Resolution (# of pixels) 1032 x 776 pixels</td>
</tr>
<tr>
<td></td>
<td>Diameter of the encrypted signal 3 mm</td>
</tr>
<tr>
<td>( N_p ) (number of photons)</td>
<td>Without phase mask ( M_0 ) ( N_p = 3000 ) (about 1%)</td>
</tr>
<tr>
<td></td>
<td>With phase mask ( M_0 ) ( N_p = 9000 ) (about 3%)</td>
</tr>
</tbody>
</table>

In section 2, masks \( M_1 \) and \( M_2 \) were designed as amplitude-only and phase only distributions. Nevertheless, in our experiment, the displays operate in phase mostly and high-contrast configurations (see Fig. 6); in both cases, a strong amplitude-phase coupling is obtained. For the case of display 1, the resulting \( M'_1 \) mask is no longer real because the effective modulation curve encodes the mask. The same happens with mask \( M_2 \) since key \( M'_2 \) is also complex valued. Despite the fact that modulation encoding is a continuous and smooth deterministic procedure, this step introduces a nonlinear change on the encryption keys. Consequently, modulation curves have to be known in advantage in order to allow fully reconstruction of the plain-text.
Precise alignment between the corresponding pixels of the two SLMs is required in order to achieve a correct encryption signal. The tuning procedure is carried out by displaying the same test pattern on both SLMs (Fig. 7(a)) and subsequently imaging them on the CCD (Fig. 7(b)). Note that both images are polarized in orthogonal directions and no interference is detected. Then, one of the images is electronically shifted until both distributions coincide (Fig. 7(c)). If required, final fine tuning is accomplished by displacing slightly one of the displays using a XY motorized stage.

Then, images $t$ and $f$ (Figs. 2(a) and 2(b)) are encrypted. In this first test, but without loss of generality, the phase mask $M_0$ is not used. For illustration purposes, the experimental Stokes parameters of the encrypted signal $T$ for the true-class image $t$ are shown in Figs. 8(a)-8(d).

Fig. 6. SLM modulation polar plot: (a) SLM$_1$: high-contrast configuration; (b) SLM$_2$: phase-mostly configuration.

Fig. 7. Alignment procedure: (a) test pattern; (b) Misaligned patterns; (c) aligned patterns.
Using the Stokes images, and by means of Eqs. (4) and (5), phase $\tau'_x$ and amplitude $|T_x|$ are determined. Then, a photon limited distribution $|T_x'|_{ph}$ with $N = 3200$ photons (about 1% of the total number of used pixels on the CCD) is generated. Both true-class image $t$ and false-class image $f$ photon-limited distributions are generated (see Figs. 9(a) and 9(b)).

Using Eq. (9) and the decryption information (keys, distances, and modulation curves), photon-counting plain-text texts $t_{ph}$ and $f_{ph}$ are calculated. The nonlinear cross-correlations $t_{ph} \otimes t$ and $f_{ph} \otimes t$ are obtained using Eq. (10) as shown in Fig. 10. The sharp peak present in $t_{ph} \otimes t$ validates $t$ whereas $f$ is rejected because of the low correlation signal in $f_{ph} \otimes t$. 

Fig. 8. Stokes parameters (a) $S_0 = \langle T \rangle^2$, (b) $S_1$, (c) $S_2$, (d) $S_3$.

Fig. 9. Photon counting encrypted signals $N = 3200$ (about 1% of the total encrypted signal): (a) true class $t_{ph}$; (b) false class $f_{ph}$. 
We repeated the experiment but now the plain text $t$ and the false class $f$ images are phase-encoded using a cello tape diffusor. The number of photons used is slightly higher, $N_p = 9500$ (around a 3% of the total number of used pixels on the CCD). Note that when the image is phase encoded, the propagated signal is a random pattern described by a Gamma probability distribution [38,39]. This fact produces a stronger encryption signal but more photons are required in $|T_x|_{ph}$ for obtaining good correlations outputs. Using the appropriate decryption information, photon-counting plaintexts are retrieved. The corresponding nonlinear cross-correlations $t_{ph} \otimes t$ and $f_{ph} \otimes t$ are shown in Figs. 11(a) and 11(b) respectively. Despite the fact that correlations are noisier, the true-class image is validated as shown in Fig. 11.

5. Concluding remarks

We have developed a polarimetric-based optical encoder setup for image encryption and verification purposes based on a Mach-Zehnder interferometer combined with translucent spatial light modulators. Stokes parameters information is used for retrieving full complex information of the encrypted vector beam. Moreover, security can be enhanced by recording the encrypted polarized signal under low-light illumination conditions using the photon-counting model. The addition of photon-counting model in the encryption process provides sparseness of recorded data and nonlinear transformation to enhance security. Provided that polarization keys and optical design variables are known, an authorized user can retrieve and validate the photon-counting plain-text. The experiments carried out demonstrate the feasibility of using such system as a polarimetric optical encoder for security applications.
Since we are using an interferometric system in the implementation of encryption process, our approach can be extended to encrypting 3D objects [40].

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