Labor mobility, structural change and economic growth

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Abstract: This paper develops a two-sector growth model in which the process of structural change in the sectoral composition of employment and GDP is jointly determined by non-homothetic preferences and labor mobility cost. This cost, paid by workers moving to another sector, limits structural change. Our model can explain the following patterns of development of the US economy throughout the period 1880-2000: (i) balanced growth of the aggregate variables in the second half of the last century; (ii) structural change in the sectoral composition of employment between agriculture and non-agriculture sectors; (iii) structural change process in the sectoral composition of GDP between these sectors; and (iv) wage convergence between the two sectors. We outline that the last two patterns can only be explained if labor mobility cost is introduced. Results reveal that mobility cost generates a misallocation of production factors. This implies a loss of GDP which amounts to over 30% of the GDP throughout initial periods according to the calibrated model. During the transition, the loss of GDP decreases and eventually vanishes. Thus, the elimination of the misallocation explains part of the increase in the GDP. Additionally, this study points out that misallocation introduces a mechanism through which cross-country differences in sectoral composition may account for cross-country income differences.

JEL Codes: O41, O47.

Keywords: Structural change, Non-homothetic preferences, Labor mobility.

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1. Introduction

Recent multisector growth literature has built models aimed at explaining the balanced growth of aggregate variables as well as the process of structural change observed in most developed economies (see Acemoglu and Guerrieri, 2008; Boppart, 2014; Dennis and Iscan, 2008; Melck, 2002; Foellmi and Zweimüller, 2008; Kongsamut et al., 2001; or Ngai and Pissaridis, 2007). On the one hand, the balanced growth of aggregate variables consists of an almost constant ratio of capital to GDP and an almost constant interest rate. On the other hand, the process of structural change consists in a large shift of both employment and aggregate production from agriculture to other sectors. This process, common to most economies, is illustrated in the first two columns of Table 1 for the US economy over the period 1880 to 2000.

[Insert Table 1]

The aforementioned literature explains both balanced growth of aggregate variables and the process of structural change in the sectoral composition of employment. This literature can be split into two different groups. One set of studies outlines that demand factors are the driving force of structural change (see, e.g., Kongsamut et al., 2001). These demand factors comprise income effects generated by non-homothetic preferences that drive structural change as the economy develops. The other set argues that supply factors are the driving force of structural change (see, e.g., Acemoglu and Guerrieri, 2008; or Ngai and Pissaridis, 2007). These factors encompass variations in relative prices that cause structural change through a substitution effect. More recently, the literature combines demand and supply factors to explain structural change (see, e.g., Boppart, 2014; or Dennis and Iscan, 2008). While these papers explain the process of structural change in the sectoral composition of employment, none of them explains the magnitudes of the two patterns of structural change: the shifts in employment and aggregate production from agriculture to other sectors. Buera and Kabosky (2009) argue that this literature does not explain these two features because it does not introduce sector specific factor distortions.

In this paper, we show that the two features of structural change can be explained when factor distortions cause sectoral wages differentials. In order to motivate this conclusion, we use the definition of the labor income share (LIS) at the sectoral level, and we decompose the ratio between the LIS in the agriculture sector and the LIS in the non-agriculture sector as the product of the following three other ratios: the ratio between wages in the agriculture and non-agriculture sectors; the ratio between the employment shares in agriculture and in the non-agriculture sector; and the ratio between the GDP shares in the non-agriculture and agriculture sectors. We can use the US data for the sectoral composition of employment and GDP shown in Table 1 to compute the value of the ratio between the two sectoral LIS that is compatible with the process of structural change in both employment and GDP. The fifth column of Table 1

\[ \text{LIS}_a / \text{LIS}_n = \left( \frac{w_a}{w_n} \right) \left( \frac{u_a}{u_n} \right) \left( \frac{\kappa_a}{\kappa_n} \right), \]

where \( w_i \) and \( \kappa_i \) are the employment and GDP shares in sector \( i \), respectively.

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1 The LIS in sector \( i \) is defined as \( \text{LIS}_i = w_i L_i / P_i Y_i \) where \( w_i \) is the wage in sector \( i \), \( L_i \) is the number of employed workers in this sector, \( P_i \) is the relative price and \( Y_i \) is the production in this sector. Using this definition, it is straightforward to obtain that the ratio between the LIS in sectors \( a \) and \( n \) is \( \text{LIS}_a / \text{LIS}_n = \left( \frac{w_a}{w_n} \right) \left( \frac{u_a}{u_n} \right) \left( \frac{\kappa_n}{\kappa_a} \right) \), where \( u_i \) and \( \kappa_i \) are the employment and GDP shares in sector \( i \), respectively.
shows that the value of this ratio should be equal to 2.15 in the year 1880 and it should
decrease to 1.05 in the year 2000 in the hypothetical case of equal wages across sectors.
These values are problematic for two reasons. First, they show a declining long-run
trend in the ratio of LIS, which can only be explained if we consider large departures
from the Cobb-Douglas production functions at the sectoral level. These departures are
not supported by empirical estimates of the long-run sectoral production functions. Second,
the value of the ratio between the two sectoral LIS consistent with the process
of structural change is completely different from actual estimates of this ratio, which
set its value at approximately equal to 0.68. This suggests that the two features of
structural change cannot be explained if we assume that wages are equal across sectors.
Furthermore, empirical evidence clearly demonstrates that wages are different across
sectors, especially in terms of the agriculture and non-agriculture sectors (see Helwege,
1992; Caselli and Colleman, 2001; and Herrendorf and Schoellmany, 2014). Table 1
shows the relative wage between agriculture and non-agriculture sectors. According to
the table, wages are lower in the agriculture sector and they have clearly converged
during the last century. However, wage differentials across sectors currently continue
to be large. Using this observed data on relative wages, we compute the ratio between
the sectoral LIS consistent with the two features of the process of structural change
when wages are unequal across sectors. The last column of Table 1 shows this ratio.
Note that after 1920 the value of this ratio was close to its empirical estimates and
does not exhibit a trend. This numerical analysis suggests that sectoral differences in
wages must be introduced to spell out the two features of structural change.

This paper purposes to show that a simple multisector growth model can illustrate
the two aforementioned features of structural change when wages do not equalize across
sectors. To this end, we develop an exogenous two-sector growth model with two main
features. First, preferences are non-homothetic owing to the introduction of minimum
consumption requirements, as in Kongsamut et al. (2001) or Alonso-Carrera and
Raurich (2014). Second, we introduce a labor mobility cost that generates differences in
wages across sectors. Literature explains sectoral differences in wages as the result of:
(i) differences in human capital across sectors (Caselli and Colleman, 2001; Herrendorf
and Schoellmany, 2014); (ii) barriers to mobility (Hayashi and Prescott, 2008); or (iii)
labor mobility cost (Lee and Wolpin, 2006; Raurich et al., 2014). Gollin et al. (2014)
show that labor productivity is lower in the agriculture sector even though we control
for human capital and for the number of hours employed. This signals that labor
mobility cost may explain part of the wage differences.

The labor mobility cost accounts for any cost that workers moving to another sector
must pay. This may include reallocation expenses (transport and housing costs), formal
training to acquire the skills used in another sector or an opportunity cost (the time

\footnote{Herrendorf et al. (2014) estimate the elasticity of substitution between capital and employment
and show that it is 1.58 for the agriculture sector, 0.8 for the manufacturing sector and 0.75 for the
service sector. They then conclude that Cobb-Douglas sectoral production functions capture the main
technological forces in the US postwar structural change.}

\footnote{This value is obtained from Valentinyi and Herrendorf (2008) that use data for the US in the period
1990-2000.}

\footnote{Before 1920 data on relative wages are controversial as has been explained by Caselli and Colleman
(2001). Therefore, measurement errors in the value of relative wages may explain the low values of the
ratio between sectoral LIS before 1920.}
spent looking for a job in a different sector). As moving out of the agriculture sector generally entails moving from a rural to an urban area, we consider that the relevant labor mobility cost is associated to reallocation expenses. As the expenses are not proportional to the wage, we assume that the unitary labor mobility cost is constant. Artuc et al. (2015) estimate labor mobility cost for both developed and developing economies. They show that this cost, as a fraction of annual wage, is larger in developing economies. This means that the labor mobility cost as a fraction of GDP declines along the development process. Note that this pattern is consistent with the assumption of a constant unitary labor mobility cost.

The introduction of the labor mobility cost segments the labor market into two sector specific labor markets. The existing number of workers in each sector determines the labor supply of the corresponding market. Thus this supply is determined by the sectoral employment share. The labor demand in each sectoral market rests on the demand for consumption goods in every sector that depends on economic development in a model with non-homothetic preferences. In every period, market clearing determines the wages paid in each sector. Therefore, sectoral wage differences exist because the labor mobility cost prevents workers from instantaneously moving to the higher wage sector. However, as the economy develops, the labor mobility cost, as a fraction of the GDP, declines. This triggers wage convergence across sectors. The process of structural change is driven by demand and supply factors. On the one hand, due to the non-homotheticity of preferences, the sectoral composition of consumption expenditures changes as the economy develops. Obviously, this is the classical demand factor explained in Kongsamut et al. (2001). Economic development reduces the effect of the minimum consumption requirement on the sectoral composition and this effect eventually vanishes. As a consequence, preferences are homothetic in the long run, so that the equilibrium converges to a balanced growth path (BGP). On the other hand, the supply factor is based on wage convergence, rather than on the standard mechanism in the literature which is based on changes in the relative prices of goods. Wage convergence implies faster-growing wages in the agriculture sector than in the non-agriculture sector. As a consequence, firms in the agriculture sector substitute labor for capital. This makes the technology more capital intensive and pushes workers out of the agriculture sector. This is the supply mechanism introduced by this paper.

We calibrate the proposed model to explain the process of structural change in the US for the period 1880-2000. From numerical simulations, we show that the model explains: (i) the balanced growth of aggregate magnitudes over time with structural change; (ii) the process of structural change in the sectoral composition of employment; (iii) the process of structural change in the sectoral composition of GDP; and (iv) the convergence of wages across sectors. We outline that in the absence of labor mobility cost, the model neither explains sectoral wage convergence nor the process of structural change in the sectoral composition of GDP.

The differences in sectoral wages introduce a misallocation of production factors: the sector with larger wages has a too large capital intensity. This misallocation causes a loss of GDP. This loss is not due to inefficiencies arising from barriers as, for instance,

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5We follow Acemoglu and Guerrierie (2008) and Alonso-Carrera and Raurich (2014), and we claim that the equilibrium follows a BGP with structural change when the growth rate of capital to GDP is almost null, whereas the growth rate of the employment share is clearly different from zero.
in Restuccia et al. (2008). Instead, the GDP loss in this paper must be interpreted as the reduction in GDP with respect to the level that would be attained in the absence of the labor mobility cost. Intuitively, moving a worker from a low to a high wage sector increases the GDP. Therefore, GDP loss will depend on the wage gap between the two sectors and on the size of the low wage sector (the agriculture sector). Both the wage gap and the size of the low wage sector were large in the US in the XIX century, which implies a large GDP loss. We use numerical simulation to quantify the GDP loss in our calibrated model. It turns out that this cost was about 30% of GDP in the last twenty years of the XIX century, it declined during the transition and eventually vanished. Consequently, part of the increase in the GDP during the transition, especially in the initial periods, is explained by the elimination of the misallocation.

GDP loss then introduces a mechanism through which cross-country differences in the sectoral composition of employment cause cross-country differences in income per capita. This mechanism indicates that those countries specialized in the sector with the lowest wage (the agriculture sector) will have a lower GDP. This conclusion is also obtained in Gollin et al. (2004, 2007). In these papers, the specialization in the low wage sector is explained by the presence of home production or minimum consumption requirements. By contrast, this paper explains this specialization as the result of a larger labor mobility cost, which can be justified by labor market regulations or larger reallocation expenses.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium. Section 4 solves the model numerically and obtains the main results. Finally, Section 5 includes some concluding remarks.

2. Model

We consider an exogenous two-sector growth model distinguishing between the agriculture and the non-agriculture sector. We assume that the latter is the numeraire of the economy and produces a single good that can either be consumed or invested. The agriculture sector produces a good that can only be devoted to consumption.

2.1. Household

The economy is populated by an infinitely lived representative household, formed by a continuum of members distributed on the interval [0, 1]. Every member inelastically supplies one unit of time so that the aggregate labor supply is inelastic and equal to unity. The household obtains income from renting capital and labor to firms. This income is devoted to consuming, investing or paying the cost of moving to another sector. Therefore, the budget constraint of the household is

\[ rk + w_a (1 - u) + uw_n = p c_a + c_n + \dot{k} + \pi \dot{u}, \]  

(2.1)

where \( r \) is the rental price of capital, \( k \) is the stock of capital, \( w_a \) is the wage obtained in the agriculture sector, \( w_n \) is the wage obtained in the non-agriculture sector, \( u \) is the fraction of workers employed in the non-agriculture sector, \( p \) is the relative price of agriculture goods in units of non-agriculture goods, \( c_a \) is the consumed units of the good produced in the agriculture sector, \( c_n \) is the consumed units of the good produced
in the non-agriculture sector, $\pi$ is the constant unitary labor mobility cost that every worker moving to another sector pays, and $\hat{u}$ is the fraction of workers that move every period.\footnote{We write the budget constraint assuming that workers move from the agriculture to the non-agriculture sectors. This pattern of structural change is obtained in equilibrium.}

The representative household’s utility function is

$$ U = \int_0^{\infty} e^{-\rho t} \left[ \theta \ln (c_a - \tilde{c}_a) + (1 - \theta) \ln c_n \right] dt, \quad (2.2) $$

where $\tilde{c}_a > 0$ is the minimum consumption requirement of the agriculture good; $\rho > 0$ is the subjective discount rate; and $\theta \in (0, 1)$ measures the weight of the agriculture good in the utility function. Note that this utility function is non-homothetic when $\tilde{c}_a \neq 0$.

The representative household chooses the amount of consumption expenditure, the sectoral composition of consumption expenditure and the number of members that move their labor supply to the non-agriculture sector every period in order to maximize the utility function (2.2) subject to the budget constraint (2.1). By using a standard procedure, we find the first order conditions in Appendix A and rearrange them to summarize the necessary conditions for optimality in the following three conditions:

$$ v = \theta + \frac{\tilde{E}}{E} (1 - \theta), \quad (2.3) $$

$$ \frac{\dot{E}}{E} = \left( \frac{E - \tilde{E}}{E} \right) (r - \rho) + \left( \frac{\tilde{E}}{E} \right) \frac{\dot{r}}{r}, \quad (2.4) $$

and

$$ w_n - w_a = r \pi, \quad (2.5) $$

where $E = pc_a + c_n$ is the value of consumption expenditure, $v = pc_a/E$ is the expenditure share in the agriculture good and $\tilde{E} = pc_a$ is the value at market prices of the minimum consumption requirement. Equation (2.3) determines the expenditure share in the good produced by the agriculture sector. Note that this share would be constant and equal to $\theta$ if $\tilde{c}_a = 0$. In contrast, if $\tilde{c}_a > 0$, preferences are non-homothetic and the fraction of expenditures devoted to the agriculture good decreases as the economy develops and consumption expenditure increases. This mechanism is the classical demand factor driving structural change. Equation (2.4) is the Euler condition governing the intertemporal decision between consumption expenditure and savings. Finally, equation (2.5) is a non-arbitrage condition between two investment decisions: investment in capital goods and investment in moving out of the agriculture sector. The left-hand side is the return from investing $\pi$ units of numeraire in moving a worker to another sector. The right-hand side is the return from investing these $\pi$ units in capital. This non-arbitrage condition implicitly determines the number of workers moving out of the agriculture sector in every period and thus determines the relative labor supplies in both sectors.
2.2. Firms

We assume that both sectors produce with the following constant returns to scale Cobb-Douglas technologies:

\[
Y_a = [(1 - s) k]^{\alpha_a} [A_a (1 - u)]^{1 - \alpha_a} = A_a (1 - u) z_a^{\alpha_a},
\]

and

\[
Y_n = (sk)^{\alpha_n} (A_n u)^{1 - \alpha_n} = A_n u z_n^{\alpha_n},
\]

where \( \alpha_a \in (0, 1) \) and \( \alpha_n \in (0, 1) \) are, respectively, the capital output elasticities in the agriculture and non-agriculture sector, \( A_a \) and \( A_n \) are efficiency units of labor, \( s \) is the fraction of capital devoted to the non-agriculture sector, and \( z_a = (1 - s) k/A_a (1 - u) \) and \( z_n = sk/A_n u \) measure capital intensities in the agriculture and non-agriculture sectors, respectively. We assume that efficiency units of labor grow for both sectors at the exogenous growth rate \( \gamma \). This implies that technological progress is unbiased and the long-run growth rate of GDP is \( \gamma \). Finally, perfect competition implies that each production factor is paid according to its marginal product, so that

\[
w_i = A_i p_i (1 - \alpha_i) z_i^{\alpha_i},
\]

and

\[
r = p_i \alpha_i z_i^{\alpha_i - 1} - \delta,
\]

where \( \delta \in [0, 1] \) is the depreciation rate, with \( i = a, n \). Capital can freely move across sectors, so the marginal product of capital is identical across sectors. By contrast, the introduction of the labor mobility cost implies that wages may be different across sectors. We define the relative wage between the two sectors by \( \lambda = w_a/w_n \). Using (2.8) and (2.9), we obtain that

\[
z_a = \left( \frac{\lambda \psi A_n}{A_a} \right) z_n,
\]

and

\[
p = \left( \frac{\alpha_n}{\alpha_a} \right) \left( \frac{\lambda \psi A_n}{A_a} \right)^{1 - \alpha_a} z_n^{\alpha_n - \alpha_a},
\]

where

\[
\psi = \left( \frac{\alpha_a}{\alpha_n} \right) \left( \frac{1 - \alpha_n}{1 - \alpha_a} \right).
\]

Equation (2.10) shows that the relationship between the sectoral capital intensities depends on the relative wage. As the economy develops, relative wage increases. This, in turn, causes an increase in the capital intensity of the agriculture sector relative to the capital intensity of the other sector. The intuition is as follows. An increase in the relative wage implies that wages in the agriculture sector increase relative to wages in the non-agriculture sector. As a consequence, firms in the agriculture sector choose a more capital-intensive technology by substituting labor for capital. This mechanism describes the supply factor driving structural change. This supply factor is different from the supply mechanism usually proposed by the literature, which is based on changes in relative price caused by either biased technological change (Ngai
and Pissariadis, 2007) or capital deepening jointly with sectoral differences in capital output elasticities (Acemoglu and Guerrieri, 2008).

The supply mechanism has two relevant implications. First, equation (2.11) shows that the relative price depends on: (i) the relative wage; (ii) the ratio between the efficiency units of labor in the non-agriculture sector and the efficiency units of labor in the agriculture sector; and (iii) capital deepening. The proposed supply mechanism, based on wage convergence across sectors, implies an increase in the relative price of agriculture. Yet, the biased technological mechanism and capital deepening both imply a reduction in this price. On the one hand, empirical evidence shows that TFP growth is larger in the agriculture sector. Thus, biased technological change reduces the relative price. On the other hand, capital deepening implies a reduction in the relative price because the estimates of the sectoral capital output elasticities suggest that this magnitude is larger in the agriculture sector (See Valentinyi and Herrendorf, 2008). As a consequence, this sector is the most benefited from capital deepening, which causes the reduction in the relative price. As in the model we combine two different supply mechanisms (capital deepening with wage convergence) the relative price can either increase or decrease along the development process. Interestingly, this is consistent with the observed differences in the patterns of relative prices along the development process.

Second, wage convergence implies that the agriculture sector becomes a more capital intensive sector as the economy develops. This helps to explain cross-country differences in sectoral capital intensities that clearly indicate the agriculture sector is more relatively capital intensive in developed economies (see Alvarez-Cuadrado, et al., 2013). It should be noted here that the aforementioned classical supply mechanisms of structural change would not explain this evidence. Using (2.10) and the definitions of \(z_n\) and \(z_a\), it follows to say that neither sectoral differences in capital-output elasticities nor biased technological change can explain cross-country differences in relative capital intensities when the production function is Cobb-Douglas. To the best of our knowledge, cross-country differences in sectoral capital intensity have only been explained by Alvarez-Cuadrado et al (2013). Using CES production functions, they claim these differences result from different sectoral elasticities of substitution between capital and employment. This paper therefore offers a complementary explanation based on wage convergence. Wage convergence contributes to explain differences in sectoral capital intensities even if the production function is Cobb-Douglas.

3. Equilibrium

The non-agriculture sector produces a commodity that can be devoted to consuming, investing and covering the cost of moving to a different sector. Therefore, the market-

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1 Dennis and Iscan (2009) show that TFP growth in the agriculture sector is larger than TFP growth in the non-agriculture sector for the US economy after 1930.

2 Dennis and Iscan (2009) provide evidence that relative prices of agriculture in the US increase during the XIX century and decrease after 1920. Alvarez-Cuadrado and Poschke (2011) find large disparities in the behavior of agriculture prices across both countries and time. Throughout the period 1920-1959 these prices grow for some countries (e.g., Canada, UK or Japan), whereas they decrease for others (e.g., Belgium, France or Netherlands). During the period 1960-2000 these prices decrease for the whole sample of the aforementioned study.
clearing condition in this sector is

\[ Y_n = c_n + k + \delta k + \dot{u} \pi. \]

By contrast, the agriculture sector only produces a consumption good so that the market clearing condition in this sector is \( c_a = Y_a \), which can be rewritten by using (2.6) as

\[ 1 - u = \frac{c_a}{A_a z_a}. \quad (3.1) \]

Let \( z = k/A_n \) be the stock of aggregate capital per efficiency units of labor in the economy. Thus, \( z \) measures the capital intensity of the economy. Using the definition of \( z \), we derive that

\[ z_n = \left( \frac{s}{u} \right) z, \quad (3.2) \]

and

\[ z_a = \left[ \frac{(1 - s) A_n}{(1 - u) A_a} \right] z. \]

From the last equation and (2.10), we get that

\[ \lambda \psi (1 - u) z_n = (1 - s) z. \quad (3.3) \]

From using the equilibrium condition in the capital market and equations (3.2) and (3.3), we obtain

\[ \frac{z}{z_n} = \lambda \psi (1 - u) + u \equiv \phi, \quad (3.4) \]

where \( \phi \) measures the capital intensity of the economy relative to the capital intensity of the non-agriculture sector.

Note that wage differentials between sectors would not emerge without mobility cost, so that \( \lambda = 1 \) and \( \phi = \psi (1 - u) + u \equiv \phi^* \) in this case. However, the labor mobility cost implies that during the transition \( \lambda < 1 \) and, therefore, \( \phi < \phi^* \). The introduction of the labor mobility cost, by increasing the wages of the non-agriculture sector, increases the capital intensity of this sector relative to the capital intensity of the whole economy. Thus, the labor mobility cost introduces a misallocation of inputs, measured by the gap between \( \phi \) and \( \phi^* \), which is given by

\[ \phi^* - \phi = (1 - u) \psi (1 - \lambda). \]

The misallocation will cause a GDP loss. In order to see this, we define GDP as \( Q = p Y_a + Y_n \). Using (2.10), (2.11) and (3.4), GDP can be rewritten as

\[ Q = A_n^{1 - \alpha_n} \Omega \Phi^{-\alpha_n} k^{\alpha_n}, \quad (3.5) \]

where

\[ \Omega = \left( \frac{\alpha_n}{\alpha_a} \right) \phi + u \left( \frac{\alpha_n - \alpha_n}{\alpha_a} \right), \quad (3.6) \]

and \( \Phi = \Omega \Phi^{-\alpha_n} \) measures the sectoral composition component of the total factor productivity (TFP), which is given by \( A_n^{1 - \alpha_n} \Omega \Phi^{-\alpha_n} \). By defining \( Q^* \) as the GDP level
that would be attained if \( \lambda = 1 \) (i.e., if \( \pi = 0 \)), we measure the GDP loss as a fraction of GDP by

\[
\frac{Q^* - Q}{Q} = \left( \frac{\Omega^*}{\Omega} \right) \left( \frac{\phi^*}{\phi} \right)^{-\alpha_n} - 1,
\]

where \( \Omega^* \) is the value of \( \Omega \) when \( \lambda = 1 \). Note that the loss of GDP depends on \( \lambda \) and on the employment share in agriculture \( 1 - u \). In the numerical simulations of Section 4, we show that the GDP loss has declined in the US during the last century as a result of wage convergence and the fall of the employment share in the agriculture sector.

An important remark that follows from the expression of the TFP is that differences in the sectoral composition of employment cause differences in the TFP when there are either differences in capital output elasticities or differences in wages across sectors.\(^9\) If we had assumed both \( \alpha_a = \alpha_n \) and \( \lambda = 1 \), then disparities in the sectoral composition would not imply differences in TFP levels since \( \Omega \phi^{-\alpha_n} = 1 \) in this case. In other words, TFP increases when economies specialize in sectors with larger capital output elasticities or in sectors with larger wages. In the numerical analysis performed in the next section, we will compare economies with different sectoral compositions and we will decompose the fraction of income differences explained by differences in sectoral wages and the fraction explained by differences in capital output elasticities. From this numerical analysis, we will show that the main mechanism explaining income differences through TFP is based on differences in sectoral wages.

### 3.1. Sectoral Composition

In this subsection, we obtain the sectoral composition of consumption expenditures, the sectoral employment shares and the relative wage, \( \lambda \), as a function of: the expenditure to GDP ratio, \( e = E/Q \); the capital intensity, \( z = k/A_n \); the intensity of the minimum consumption requirement, measured by the ratio \( \bar{e} = E/Q \); and the intensity of the labor mobility cost, measured by \( m = \pi/A_n \). Note that as the economy develops, the intensity of the minimum consumption requirement and of the labor mobility cost both decline and eventually converge to zero.

We first use (2.3) and the definitions of \( e \) and \( \bar{e} \) to directly obtain the sectoral share of expenditure as

\[
v = \theta + \frac{\bar{e}}{\bar{e} - e} (1 - \theta).
\]

However, the sectoral share of employment \( u \) and the relative wage \( \lambda \) are jointly determined by the market clearing conditions for the agriculture sector, which is given by (3.1), and for the labor market. Observe that the labor supply is determined by the non-arbitrage condition (2.5), whereas the labor demand is given by the condition (2.8). In Appendix B we manipulate these two market-clearing conditions to derive the following result characterizing the equilibrium value of the relative wage and the sectoral share of employment.

\(^9\)Observe that TFP is endogenously determined when either \( \alpha_a \neq \alpha_n \) or \( \lambda < 1 \). As a consequence, in our economy aggregate output cannot be represented by a Cobb-Douglas production function that uses capital and labor as inputs.
Proposition 3.1. The relative wage and the sectoral share of employment satisfy

\[ \lambda = \hat{\lambda}(e,z,m), \]  

and

\[ u = \frac{\lambda \psi \left( \frac{\alpha_a}{\alpha_n} \right)}{ve + \lambda \psi \left( \frac{\alpha_a}{\alpha_n} \right)} \left( 1 - ve \right). \]  

Furthermore, \( \partial \lambda / \partial m < 0 \) and \( \partial u / \partial \lambda > 0 \).

According to equation (3.9), sectoral structural change is driven by demand factors, measured by \( ve \), and supply factors, measured by \( \lambda \). In particular, as follows from Proposition 3.1, the relative wage \( \lambda \) is a decreasing function of the intensity of the labor mobility cost \( m \), and the employment share \( u \) is an increasing function of \( \lambda \). The latter relationship is obviously explained by the reduction in the demand for workers from the agriculture sector due to the increase in the relative wage. Therefore, a large mobility cost implies that the relative wage will be smaller and, thus, the employment share of agriculture will be larger. Both effects imply that the GDP loss increases with labor mobility cost.

From using (2.7), (3.4), and (3.5), we obtain

\[ \frac{Y_n}{Q} = u. \]  

(3.10)

The variable \( \Omega \) determines the relationship between the GDP share of the agriculture sector and the sectoral share of employment. Using (3.6), the variable \( \Omega \) can be rewritten as

\[ \Omega = 1 + (1 - u) \left( \frac{\alpha_a - \alpha_n}{1 - \alpha_a} \right) + (1 - u) \left( \frac{1 - \alpha_n}{1 - \alpha_a} \right) (\lambda - 1). \]  

(3.11)

Note that if there is no misallocation (i.e., \( \lambda = 1 \)) and there are no technological differences among sectors (i.e., \( \alpha_a = \alpha_n \)), then \( \Omega = 1 \). In this case, the relation between the sectoral shares of employment and GDP will be constant and these two shares will, in fact, be equal. However, as follows from Table 1, this is not consistent with actual data for the US economy. According to the data, the GDP share in agriculture is larger than the sectoral employment share. This implies that the value of \( \Omega \) should be smaller than one. As follows from the previous expression of \( \Omega \), the misallocation reduces the value of \( \Omega \) and, thus, makes the model consistent with actual data. On the contrary, technological differences increase the value of \( \Omega \), given that the agriculture sector seems to be more capital intensive than the non-agriculture sector. This analysis suggests that misallocation must be introduced to explain the two dimensions of structural change.

10 Following Valentinyi and Herrendorf (2008), \( \alpha_n = 0.33 \) and \( \alpha_a = 0.54 \). Then, \( \Omega = 1.2 \) in the US during the period 1880-1900 when we assume that \( \lambda = 1 \). In this period, the average value of \( u \) was 0.58 and that of \( Y_n/Q \) accounts to 0.75. According to these values, \( \Omega \) should be 0.75. Thus, in the absence of misallocation the sectoral composition of both GDP and employment cannot be jointly explained. In fact, a value of \( \Omega \) that is consistent with the sectoral composition of employment and GDP throughout the period 1880-1900 is attained when \( \lambda = 0.31 \).
3.2. Equilibrium Dynamics

In a supplementary appendix we obtain a full system of differential equations characterizing the time path of the transformed variables: $z$, $e$, $m$ and $\tilde{c}$. Given initial conditions $\tilde{c}_0$, $m_0$ and $z_0$, an equilibrium is a path of \{e, $\tilde{c}$, z, m, $\lambda$, $v$, u, $\phi$\} that solves this system of differential equations and satisfies equations (3.7), (3.8), (3.4), and (3.9), and the transversality condition $\lim_{t \to \infty} \frac{k}{\theta} e^{-\rho t} = 0$. Furthermore, we define a balanced growth path (BGP) as an equilibrium along which both the ratio of capital to GDP and the interest rate remain constant.

**Proposition 3.2.** There is a unique BGP along which the variables \{e, $\tilde{c}$, z, m, $\lambda$, $v$, u, $\phi$\} remain constant and their long-run values are $\hat{e}^* = 0$, $m^* = 0$, $\lambda^* = 1$, $v^* = \theta$,

\[
\begin{align*}
    e^* &= \frac{1 - \alpha_n \Delta}{1 + \Delta (\alpha_n - \alpha_n) \theta^*}, \\
    u^* &= \frac{\psi \alpha_n (1 - \theta e^*)}{\alpha_n \theta e^* + \psi \alpha_n (1 - \theta e^*)}, \\
    \phi^* &= \psi (1 - u^*) + u^*, \\
    z^* &= \left( \frac{\gamma + \delta + \rho}{\alpha_n} \right)^{\frac{1}{m-1}} \phi^*,
\end{align*}
\]

where $\Delta = (\delta + \gamma) / (\delta + \rho + \gamma)$. Furthermore, this BGP is saddle-path stable.\textsuperscript{11}

Note that the BGP is attained asymptotically, as $\hat{e}^*$ and $m^*$ converge to zero. Wages converge across sectors and, therefore, misallocation and GDP loss both disappear in the BGP. Moreover, there is no structural change along the BGP. Thus, the economy asymptotically converges to an equilibrium along which the interest rate and the ratio of capital to GDP remain constant and there is no structural change. As this only happens asymptotically, it is particularly significant to analyze the transitional dynamics. In the following section, we numerically analyze the transition and we demonstrate that aggregate variables exhibit a period of unbalanced growth followed by a long period in which they exhibit an almost constant time path of the interest rate and the ratio of capital to GDP. We also show that there is structural change over this period. We then conclude that (an almost) balanced growth of aggregate variables and structural change can simultaneously be observed in this economy.

The equilibrium is characterized by three state variables: capital intensity, $z$, intensity of minimum consumption requirements, $\tilde{c}$, and intensity of the labor mobility cost, $m$. Saddle-path stability implies that given initial conditions on these three state variables, a unique equilibrium path converges to the BGP. In the following section, the uniqueness of the equilibrium path is used to calibrate and simulate the economy.

\textsuperscript{11}Proof is in a supplementary appendix available from the authors upon request.
4. Transitional Dynamic Analysis: Structural Change

In this section we numerically simulate the economy in order to show how mobility cost affects the process of structural change. To this end, we first calibrate the parameters of the economy as follows. We define a period as a year to fit our model with data. A subset of parameter values are chosen based on targets that are independent of the equilibrium allocations. In particular, we set: (i) the initial values of the sectoral efficiency units of labor as $A_a(0) = A_n(0) = 1$ because these parameters only affect the unit of measurement of commodities $Y_a$ and $Y_n$; (ii) $\gamma = 0.02$ to obtain a long-run GDP growth rate equal to 2%, which is in the range used by the literature and corresponds with the growth rate of US GDP per capita between 1960 and 2000; (iii) $\alpha_a = 0.54$ and $\alpha_n = 0.33$ as estimated by Valentinyi and Herrendorf (2008); and (iv) $\theta = 0.01$ to fit the long-run expenditure share in agriculture obtained in Herrendorf et al. (2013). The remaining parameters $\rho$ and $\delta$ are respectively set to 0.32 and 0.056 by imposing the BGP to satisfy: (i) the interest rate equals to 5.2%; and (ii) the ratio of investment to capital is 7.6% (see, e.g., Cooley and Prescott, 1995). Table 2 reports the targets and the implied parameter values.

Secondly, we assume in all of the simulations $z_0 = 0.75z^*$. The initial value of this state variable mainly determines the length of the transition of aggregate variables. We choose an initial value that is consistent with an almost constant time path of both the interest rate and the ratio of capital to GDP over the last 50 years of the simulation. Finally, we simulate two benchmark models (labeled Model 1 and Model 2) that differ according to whether the labor mobility across sectors is a costless activity. In the simulation of Model 1 we assume that there is no mobility cost (i.e., $m_0 = 0$) and we set the initial condition on the other state variable, $\tilde{e}_0 = 0.59$, to match the employment share in the US in the initial year 1880. In the simulation of Model 2, we assume that there is labor mobility cost and we set the initial conditions on the two state variables, $\tilde{e}_0 = 0.28$ and $m_0 = 13.3$, to match the values of the shares of employment and GDP in the US agriculture sector in the year 1880. Table 3 summarizes the initial conditions of the two simulations.

[Insert Tables 2 and 3]

Figure 1 shows the first numerical simulation in which we assume that there is no mobility cost. In this case, wages equalize across sectors. This implies that the relative wage is equal to one and, thus, Model 1 does not explain wage convergence. This means there is no misallocation and thus there is no GDP loss. Panel (i) shows that this simulation reproduces practically the entire decline of the employment share in the agriculture sector. However, the model does not provide a reasonable explanation for the process of structural change in the sectoral composition of GDP. It overestimates the share of agriculture output in GDP along the whole transition. To provide intuition of this result, we can rewrite (3.10) as $\Omega = u(Q/Y_n)$. From Table 1, we observe that $u < Y_n/Q$ in actual data implying that $\Omega$ should be substantially lower than one in order to explain the two dimensions of structural change as described in Subsection 3.1. However, in our calibration $\alpha_a > \alpha_n$, so that $\Omega$ is larger than one when $\lambda = 1$ as follows

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12 In any case, the main results of our numerical analysis still hold under different initial values of $z$. 

13
from (3.11). Therefore, the model fails to explain the two dimensions of structural change in the absence of mobility cost. This means that the first numerical simulation overestimates the value of the production share. Thus, the simulation performs poorly in explaining the process of structural change in the sectoral composition of GDP.

[Insert Figure 1]

Figure 2 displays the second simulation, where a mobility cost is introduced. This cost, as a fraction of GDP, declines from 8% of GDP in the initial year to zero in the long-run. This ratio declines because GDP increases and the process of sectoral structural change declines in the long-run. The simulation of Model 2 replicates the declining path of the employment share in the agriculture sector, the declining path of the share of GDP produced in the agriculture sector and the process of wage convergence. Furthermore, this simulation explains practically all of the decline in employment and GDP shares of the agriculture sector. Regarding wage convergence, the simulation depicts the convergence of the relative wage, but it is not able to do so with the level of this relative wage. We interpret this as partial evidence of other relevant explanations concerning the sectoral wage differences apart from the mobility cost.\textsuperscript{13} Finally, the performance of this simulation in explaining the process of structural change in the sectoral composition of GDP is decidedly better than the previous simulation of Model 1.

[Insert Figure 2]

Table 4 provides three measures of performance for comparing the simulations of Models 1 and 2. Following these measures, both simulations are equally accurate in explaining the process of structural change in the sectoral composition of employment. For instance, the coefficient of determination is 75% in Model 1 and 78% in Model 2. Thus, the performance is very similar and only slightly better in Model 2. A comparable conclusion is attained if we compute the fraction of the reduction in the employment share of the agriculture sector over the period 1880-2000 explained by both simulations. Model 1 explains 96% of the reduction, whereas Model 2 explains 97%. The conclusion is completely different when we consider the performance of both simulations in explaining the process of structural change in the sectoral composition of GDP. Model 1, based on the absence of labor mobility cost, performs poorly. In the simulation of this Model, the coefficient of determination is negative and the reduction in the GDP share over the period 1880-2000 almost doubles the reduction in actual data. However, Model 2 based on the introduction of the labor mobility cost performs very well. The coefficient of determination is 96% and the fraction of the reduction explained by this simulation is 91%. We can then safely conclude that the model with mobility cost explains the process of structural change substantially better.

\textsuperscript{13}Candidates for explaining the slow convergence in wages are, among others, metapreferences associated to working in one sector or different skills across sectors. Other authors argue that wage differences can be explained by differences in the cost of living between urban and rural areas (see Esteban-Pretel and Sawada, 2014). If we assume that workers in urban areas are employed in the non-agriculture sector while workers in rural areas can be employed in the agriculture sector, permanent sectoral wage differences intend compensate for the differences in the cost of living.
As depicted in the previous section, the mobility cost introduces a misallocation of production factors that causes a loss of GDP. Panel (iv) in Figure 2 provides a measure of this loss as a percentage of GDP. This loss initially amounts to 35% of GDP, and it declines and converges to zero as the sectoral wage differences vanish and the labor share in the agriculture sector declines. Therefore, the elimination of the misallocation explains part of the increase in GDP during the transition.

The labor mobility cost also modifies the time path of the growth rate of GDP. Following Panel (vi) in Figure 2 we may see that the time path of the growth rate is hump-shaped. Interestingly, this finding is consistent with the observed development patterns.\textsuperscript{14} Christiano (1989) and, more recently, Steger (2000, 2001) explain this hump-shaped pattern in models with minimum consumption requirements. In these models, a sufficiently intensive minimum consumption requirement initially deters investment, which explains the initial low growth. As the economy develops, the intensity of the minimum consumption requirement declines and investment and growth both initially increase. Eventually, the interest rate goes down due to diminishing returns to capital and, therefore, capital accumulation and the growth rate decline until they converge to its long-run value. This paper contributes by showing that the interaction of both capital accumulation and labor mobility can explain the hump-shaped growth pattern. In this model, a large intensity of the labor mobility cost explains the initial low labor mobility in addition to a low initial capital accumulation. As this intensity declines, capital accumulation increases and the GDP loss declines because of the increase in the number of workers leaving the agriculture sector. These two changes point to an increase in the growth rate of GDP. Finally, diminishing returns to capital and labor imply that capital accumulation and labor mobility decline. This explains the final reduction in the growth rate of GDP. Note that both mechanisms (i.e., minimum consumption requirements and labor mobility cost) introduce complementary explanations for the hump-shaped time path of the GDP growth rate. Interestingly, the calibrated economy represented by Model 1, in which labor mobility cost is absent, cannot explain the hump-shaped time path of the GDP growth rate. This stresses the relevance of the complementarity between the two mechanisms in determining the time path of the GDP growth rate.

An important stylized fact of the patterns of development in the US economy since the second half of the last century is the balanced growth of the aggregate variables. Over this period, the interest rate and the ratio of capital to GDP remained almost constant, while the sectoral composition of employment and GDP changed. To illustrate that our simulations are consistent with this pattern, we follow Acemoglu and Guerrieri (2008) and we compute the average annual growth rate of: the capital to GDP ratio, the interest rate, the employment share in the agriculture sector, and the agriculture share in GDP over the last 50 years of the simulations. Results are displayed in Table 5. According to this table, the annual growth rates of the interest rate and the capital to GDP ratio are both almost null in both simulations. This is consistent with the balanced growth of the aggregate variables observed in the data. The annual growth

\textsuperscript{14}Papageorgiou and Perez-Sebastian (2005) illustrate that some fast growing economies exhibit a hump-shaped transition of the GDP growth rate.
rates of the employment share and the GDP share are greater than 1% and consistent with actual data. Thus, the calibrated model is consistent with balanced growth and structural change.

[Insert Table 5]

Figure 3 shows the simulated time path of the employment share in agriculture when demand and supply factors both drive structural change (dashed line) and when only demand factors drive structural change (continuous line). The former employment share is directly obtained from simulation of Model 2, whereas the latter is obtained from an alternative simulation of Model 2 in which the relative wage does not increase (i.e., in this simulation we maintain the initial value of relative wage corresponding to Model 2 constant along the equilibrium path). If we compare the two cases, it seems reasonable to say that demand factors explain most of the reduction in the employment share over the period 1880-2000. In fact, the supply factor based on wage convergence only explains 12% of the reduction in the employment share over the whole period, while wage convergence explains a much larger fraction of the reduction during the first part of the transition. As an example, wage convergence explains 40% of the fall in that share over the period 1880-1920.

[Insert Figure 3]

4.1. Sensitivity Analysis

This subsection intends to shed light on our understanding concerning the dynamic effects of the minimum consumption requirement and the labor mobility cost. To this end, we consider three comparative dynamic exercises in which we modify the value of the parameters in the calibrated economy labeled Model 2 (see Tables 2 and 3).

The first exercise is displayed in Figure 4. This figure shows the effects of changing the initial intensity of the minimum consumption requirement by comparing three economies which differ only in the initial value of \( \tilde{e}_0 \). The continuous line shows the calibrated economy named Model 2. In this benchmark economy, \( \tilde{e}_0 = 0.28 \). The dashed line is an economy with a lower value for the initial intensity of the minimum consumption requirement, \( \tilde{e}_0 = 0.15 \), and the dotted line is an economy with almost zero initial intensity, \( \tilde{e}_0 = 0.01 \). As follows from Panels (i) and (iii) of Figure 4, a larger minimum consumption requirement implies that the employment share and the GDP share of agriculture sector are both larger. The larger demand of labor in the agriculture sector causes an initially larger relative wage, as shown in Panel (ii). However, wage convergence is slower in this economy. This happens because a larger initial intensity of the minimum consumption requirement reduces the willingness of agents to substitute consumption intertemporally. Thus agents in these economies are less willing to reduce current consumption and invest either in capital or in moving to a different sector. As a consequence, the reduction in the employment share of the agriculture sector is at a lower rate. Obviously, this explains slower wage convergence.

Figure 5 shows the effects of changing the labor mobility cost by comparing three economies with different unitary mobility costs \( \pi \). The continuous line displays the benchmark economy labeled Model 2. The dashed line displays an economy with a
labor mobility cost that is 25% smaller than that of the benchmark economy, whereas the dotted line displays an economy with a mobility cost that is 75% smaller than that of the benchmark economy. From the comparison between these economies, it then follows to argue that a lower mobility cost causes: a lower amount of workers in the agriculture sector; a larger relative wage; a smaller GDP loss due to the misallocation; and a lower mobility cost as a percentage of GDP. These differences in the GDP loss affect the GDP growth rate, as shown in Panel (vi). Note that the time path of growth rate in the low mobility cost economy is not hump-shaped. This happens because the reduction of the GDP loss is almost null in this economy and, as previously mentioned, this reduction is necessary to explain the hump-shaped time path of the GDP growth rate. Let us also note that in the economy with a low mobility cost, the GDP growth rate converges from below given that the growth rate is below its long-run value for practically every period. In the following section, we will see that this low growth rate is explained by the negative impact of the sectoral composition on the growth rate in economies with a small reduction of the GDP loss.

[Insert Figures 4, 5 and 6]

Figure 6 compares three economies that are distinct in terms of the initial intensity in both labor mobility cost and minimum consumption requirement, whereas they initially exhibit identical sectoral composition of employment. The continuous line displays the benchmark economy labeled Model 2. The dotted line displays an economy without labor mobility cost, whereas the dashed line displays an intermediate situation with a positive but small labor mobility cost. In these economies, the initial intensity of the minimum consumption requirement has been calibrated so that the three economies have the same initial employment share. In fact, they exhibit a similar time path of the employment share in agriculture, as may be seen in Panel (i). However, the transitional dynamics of the other variables differ significantly given that these economies have a different labor mobility cost. A larger mobility cost implies a smaller relative wage and, therefore, a larger GDP loss. Note that economies revealing a similar process of structural change in employment will exhibit different levels of GDP due to the differences in the GDP loss generated by the misallocation. The misallocation can be observed from the dynamic comparison between the employment share and the GDP share of agriculture sector. Those economies with a larger GDP loss are economies with a lower GDP share in agriculture. In these economies, workers employed in the agriculture sector are much more unproductive according to the comparison between the employment share and the GDP share. This explains the lower level of GDP. We conclude from this analysis that understanding the effects of sectoral composition on GDP requires a prior explanation of the sectoral composition of employment and GDP through multisector growth models. Clearly, multisector growth models explaining only the time path of the employment share do not suffice to analyze the effects of structural change on GDP, as they neglect the differences in productivity across sectors.

4.2. Implications for Development

The conclusions in the previous subsection indicate that structural change derived from the presence of labor mobility cost may be an important mechanism driving the
observed differences in GDP levels across countries. The purpose of this subsection is to analyze how differences in the level of technology generate differences in sectoral structure that result in differences in the level of GDP. Equation (3.5) illustrates that GDP decomposes in: (a) the direct contribution of technology factor, $A_a^{-\alpha_n}$; (b) the contribution of sectoral composition, $\Phi = \Omega \phi^{-\alpha_n}$; and (c) the contribution of productive factors, $k^{\alpha_n}$. Changes in the level of technology propagate to the level of GDP by means of these three channels because these changes alter the capital accumulation and the sectoral structure. We are especially interested in quantifying the relative importance of the structural change as a propagation mechanism. As mentioned in this paper, sectoral composition affects GDP through two different mechanisms: misallocation and sectoral differences in capital output elasticities. According to the misallocation, a larger employment share in the agriculture sector reduces GDP per capita, given that this sector has a lower wage. In contrast, according to the second mechanism, a larger employment share in agriculture increases GDP per capita, as capital output elasticity is larger in the agriculture sector. In Appendix C we obtain the relative contribution of each mechanism in explaining cross-country GDP level differences.

Figures 7 and 8 compare two economies, say Rich and Poor, that differ only in terms of their initial level of technologies $A_a(0)$ and $A_n(0)$. The poor economy is the benchmark economy labeled Model 2 (see Tables 2 and 3), whereas the rich economy is built considering the values of $A_a(0)$ and $A_n(0)$ as twice the size of the respective levels in the benchmark economy. Figure 7 compares these two economies by displaying the time path of several variables. In consonance with Panel (i), the poor economy devotes a larger fraction of employment to the agriculture sector. Due to this larger labor demand in the agriculture sector, the relative wage is initially larger in the poor economy. In the more advanced technological economy, labor mobility is larger because it is a richer economy. This implies that the labor mobility cost is initially larger in the rich economy and the reduction of the employment share in the agriculture sector is faster. As a consequence, the relative wage converges more rapidly in the rich economy, which implies that the relative wage will eventually become larger in the rich economy. As may be seen in Panel (iv), the GDP loss is initially much the same for both economies. This happens because the initially larger relative wage compensates the effect of a larger employment share on the GDP loss in the poor economy. Nonetheless, differences in the GDP loss increase during the transition because the rich economy experiments a faster reduction in the GDP loss. This is driven by the faster reduction in employment share and the faster wage convergence. Finally, the differences in the time path of the GDP loss explain the differences in GDP growth rates. This once again stresses the importance of misallocation in explaining GDP growth patterns.

[Insert Figures 7 and 8]

Figure 8 shows the differences in terms of GDP levels. Panel (i) displays the ratio of GDP between the rich and the poor economy. The initial GDP differences are explained only by technological differences. In effect, as is shown in Panel (ii), the direct contribution of technology in explaining GDP differences is initially 100%. During the

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15 As mentioned in the previous subsection, our model also explains different levels of development as the result of different minimum consumption requirements or different labor mobility costs.
transition, a period of divergence is followed by a period of convergence in the levels of GDP. The impact of technological differences on both capital accumulation and sectoral composition explains this transition. On the one hand, the larger technological level entails a faster capital accumulation in the rich economy, which in turn drives a permanent divergence in GDP levels. As shown in Panel (iii), the contribution of capital permanently increases. On the other hand, the larger technological level also involves a faster structural change in the rich economy. This faster structural change drives an initial period of divergence that is followed by a period of convergence. The hump-shaped time path of the contribution of sectoral composition in Panel (iv) explains this. This hump-shaped contribution is obviously explained by the fact that both economies eventually converge into the same sectoral composition. Thus, technological differences only have temporary effects on the sectoral composition (see Panel (i) in Figure 7). Note that the contribution of sectoral composition is sizeable. It explains up to 18% of the GDP differences between the two economies. Moreover, this contribution explains the period of convergence between the two economies. In fact, in the absence of the effect of sectoral composition, there would not be a period of convergence in the levels of GDP. As previously mentioned, the contribution of sectoral composition on GDP differences is governed by two different mechanisms: misallocation and sectoral differences in capital output elasticities. Panel (v) shows that the misallocation channel explains slightly more than 100% of the contribution of sectoral composition. This means that the other mechanism slightly reduces the contribution of sectoral composition. This is because capital output elasticity is larger in the agriculture sector and it reduces the GDP gap between the two economies, given that the poor economy specializes in the agriculture sector.

5. Concluding Remarks

We have developed a two-sector growth model in which structural change is driven by both demand and supply factors. The demand factor is an income effect generated by non-homothetic preferences. The supply factor is a substitution effect generated by the change in relative wage between the two sectors. In order to calibrate the economy, we have identified the two sectors as the agriculture and non-agriculture sectors. We have shown that this model can explain the following patterns of development: (i) balanced growth of the aggregate variables; (ii) structural change in the sectoral composition of employment; (iii) structural change in the sectoral composition of GDP; and (iv) wage convergence across sectors. We have also illustrated that in the absence of the labor mobility cost the two patterns of structural change remain unexplained. We have then concluded that any model of structural change should also include a theory of sectoral differentials in wages consistent with the observed patterns of relative wages.

As wages are not equal across sectors, production factors are misallocated: the agriculture sector has smaller wages and lower capital intensity, whereas the non-agriculture sector has larger wages and larger capital intensity. Obviously, this misallocation causes a loss of GDP. We measure this loss and obtain that it initially amounts to 30% of the GDP. During the transition, the loss declines and finally vanishes. Therefore, the elimination of the misallocation not only explains part of the GDP growth, especially throughout the initial years of the transition, but it also affects the
shape of the path followed by the growth rate of GDP.

GDP loss introduces a relevant insight on cross-country income differences: these differences can be explained by differences in the sectoral composition of employment when wages are different across sectors. In this paper, wage differences are explained by an exogenous labor mobility cost. Future research should try to contribute to a better understanding of determinants of labor mobility cost. Among others, this could include the study of labor market regulations, fiscal policy, or geographical characteristics.
References


Appendix

A. Solution of the Representative Household Problem

The representative consumer maximizes the utility function (2.2) subject to the budget constraint (2.1). The Hamiltonian function is

\[ H = \theta \ln (c_a - \tilde{c}_a) + (1 - \theta) \ln c_n + \eta \{ r k + [w_a (1 - u) + uw_n] - p c_a - c_n - \varphi \} + \mu \varphi \]

where \( \varphi = \dot{u} \). The first order conditions with respect to \( c_a, c_n, \varphi, k \) and \( u \) are, respectively,

\[ \frac{\theta}{c_a - \tilde{c}_a} = \eta p, \quad (A.1) \]
\[ \frac{1 - \theta}{c_n} = \eta, \quad (A.2) \]
\[ \mu = \eta \pi, \quad (A.3) \]
\[ r - \rho = -\frac{\dot{\eta}}{\eta}, \quad (A.4) \]
\[ \frac{\eta}{\mu} (w_a - w_n) = \frac{\dot{\mu}}{\mu} - \rho. \quad (A.5) \]

From combining (A.1) and (A.2), we obtain (2.3) in the main text. We log-differentiate (A.1) to obtain

\[ \frac{\dot{c}_a}{c_a - \tilde{c}_a} + \frac{\dot{p}}{p} = -\frac{\dot{\eta}}{\eta} = \frac{\dot{c}_n}{c_n} \]

and from (A.3) we obtain

\[ \frac{\dot{\mu}}{\mu} = \frac{\dot{\eta}}{\eta}. \]

Using these two equations, we rewrite (A.4) as

\[ r - \rho = \frac{\dot{c}_a}{c_a - \tilde{c}_a} + \frac{\dot{p}}{p}, \quad (A.6) \]

and (A.5) as

\[ \frac{w_a - w_n}{\pi} = \frac{\dot{\mu}}{\mu} - \rho. \quad (A.7) \]

Using the definition of \( E \), (A.6) can be rewritten as (2.4) in the main text. Finally, (A.7) can be rewritten as (2.5) in the main text.

B. Proof of Proposition 3.1

The employment shares are obtained from combining (3.1), (2.10), (2.11) and (3.7) as follows

\[ 1 - u = v \left( \frac{\alpha_a}{\alpha_n} \right) \left( \frac{E}{\lambda \psi A_n \alpha_n} \right). \]
Using (3.4) and (3.5), the previous equation simplifies as follows

\[ 1 - u = ve \left( \frac{\alpha_a}{\alpha_n} \right) \left( \frac{\Omega}{\lambda^\psi} \right). \]  

(B.1)

By manipulating the definition of GDP, \( Q = pY_a + Y_n \), with the market clearing condition \( Y_a = c_a \), the definition of \( v \) and (3.5), we derive after some algebra

\[ \Omega = \frac{u}{1 - ve}. \]  

(B.2)

By introducing (B.2) into (B.1), we directly obtain (3.9).

We rewrite the labor supply, (2.5), as follows

\[ w_n - \lambda w_n = \pi r. \]

Rearranging terms and using the labor demand, (2.8), and equations (2.9) and (3.4), we obtain

\[ \lambda = 1 - m \left( \frac{\alpha_n}{\alpha_n - \delta} \right) \left( \frac{\xi}{\phi} \right)^{\alpha_n - 1}. \]  

(B.3)

We finally substitute (3.4) and (3.9) into (B.3) and we use the definition of \( m \) to obtain

\[ \left( \frac{1 - \lambda}{\lambda^\psi} \right) \left( 1 - \alpha_n \right) \left( \frac{\psi}{1 - ve} + \frac{\alpha_n}{\alpha_a} \lambda^\psi \right) = \alpha_n - \delta \left( \frac{z}{\lambda^\psi} \right)^{1 - \alpha_n} \left( \frac{\psi}{1 - ve} + \frac{\alpha_n}{\alpha_a} \lambda^\psi \right)^{1 - \alpha_n}. \]

This equation implicitly defines \( \lambda = \tilde{\lambda}(e, z, m) \). From using the implicit function theorem, we obtain

\[ \frac{\partial \lambda}{\partial m} = - \frac{\left( \frac{1 - \lambda}{m^2} \left( \frac{\psi}{1 - ve} + \frac{\alpha_n}{\alpha_a} \lambda^\psi \right) \right)}{\left( \frac{1}{m} \right) \left( \frac{\psi}{1 - ve} + \lambda \frac{\alpha_n}{\alpha_a} \lambda^\psi \right) + (1 - \alpha_n) \delta \left( \frac{z}{\lambda^\psi} \right)^{-\alpha_n} \left( \frac{\psi}{1 - ve} + \frac{\alpha_n}{\alpha_a} \lambda^\psi \right)^{-\alpha_n}} < 0. \]

C. Decomposition of GDP

The purpose of this appendix is to obtain the expression of the measures used in the comparative dynamic exercise of Figure 8. To this end, we use (3.5) to decompose GDP. Remember that \( A_n^{1 - \alpha_n} \Phi \) amounts for the TFP and \( \Phi = \Omega \phi^{-\alpha_n} \) measures the contribution of sectoral composition to the TFP. This contribution goes through two different mechanisms: (i) sectoral differences in technologies \( (\alpha_a \neq \alpha_n) \); and (ii) misallocation due to different sectoral wages \( (\lambda \neq 1) \).

In what follows we explain the differences in GDP per capita between two economies (Rich and Poor) as the result of differences in technology, capital and sectoral composition. In a second step, we measure the relevance of those two mechanisms.
driving the contribution of sectoral composition. In order to decompose the differences in GDP levels, we compute the logarithm of relative GDP as follows\textsuperscript{16}

$$\log \left( \frac{Q^R}{Q^P} \right) = (1 - \alpha_n) \log \left( \frac{A^R_n}{A^P_n} \right) + \log \left( \frac{\Phi^R}{\Phi^P} \right) + \alpha_n \log \left( \frac{k^R}{k^P} \right).$$

From this expression, we obtain the contribution to GDP of technology, capital and sectoral composition, that are, respectively,

$$C_A = \frac{(1 - \alpha_n) \log \left( \frac{A^R_n}{A^P_n} \right)}{\log \left( \frac{Q^R}{Q^P} \right)} \times 100,$$

$$C_\Phi = \frac{\log \left( \frac{\Phi^R}{\Phi^P} \right)}{\log \left( \frac{Q^R}{Q^P} \right)} \times 100,$$

and

$$C_k = \frac{\alpha_n \log \left( \frac{k^R}{k^P} \right)}{\log \left( \frac{Q^R}{Q^P} \right)} \times 100.$$

These magnitudes are displayed in Panels (ii), (iii), and (iv) of Figure 8.

We next measure the relevance of the two mechanisms determining the contribution of the sectoral composition. However, this decomposition cannot be done directly as these two mechanism generate complementaries. For our purpose, we follow the following steps:

1. First, note that if $\alpha_a = \alpha_n$ and $\lambda = 1$ then $\Phi = 1$. This implies that we can decompose $\Phi$ as $\Phi = 1 + \Phi_{\alpha} + \Phi_{\lambda}$ where $\Phi_{\alpha}$ measures the contribution of sectoral composition to GDP through sectoral different technologies and $\Phi_{\lambda}$ measures the contribution of sectoral composition to GDP through misallocation.

2. We obtain $\Phi_{\alpha}$ from measuring the value of $\Phi$ with $\lambda = 1$ but taking $u$ equal to the sectoral composition obtained when wages are different across sectors ($\lambda < 1$). We, therefore, obtain $\Phi_{\alpha}$ as follows:

$$\Phi_{\alpha} = (\Omega \phi^{-\alpha_n})|_{\lambda=1} - 1 = \left( \frac{\alpha_n}{\alpha_n} \right) \psi(1-u) + u \right] \psi(1-u) + u]^{-\alpha_n}.$$

3. We compute the contribution of sectoral composition to GDP through misallocation ($\Phi_{\lambda}$) by using $\Phi_{\alpha}$ and $\Phi$ as follows

$$\Phi_{\lambda} = \Phi - \Phi_{\alpha} - 1.$$

\textsuperscript{16}The superindex $R$ amounts for the rich economy and the superindex $P$ amounts for the poor economy.
4. We next compute the weight of the misallocation mechanism as the following ratio:

$$\varepsilon = \frac{1 + \Phi_R}{1 + \Phi} \cdot \frac{\Phi_R}{\Phi}.$$  

The numerator of this ratio measures the relative contribution of sectoral composition between the two countries due to the misallocation. Therefore, the ratio $\varepsilon$ measures the fraction of the differences between the two countries in the contribution of the sectoral composition explained by the misallocation. We label this measure as the weight of the misallocation mechanism, and we display it in Panel (v) of Figure 8.

5. Finally, we compute the contribution of the misallocation to GDP as $C_{\Phi, \lambda} = \varepsilon C_{\Phi}$. This magnitude is displayed in Panel (vi) of Figure 8.
D. Figures and Tables

Table 1. Structural change in the US economy

<table>
<thead>
<tr>
<th>Period</th>
<th>GDP share in Agriculture</th>
<th>Agriculture Employment share</th>
<th>Relative Wage</th>
<th>LIS$_a$/LIS$_n$ (1)</th>
<th>LIS$_a$/LIS$_n$ (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880-1900</td>
<td>0.251</td>
<td>0.412</td>
<td>0.203</td>
<td>2.151</td>
<td>0.438</td>
</tr>
<tr>
<td>1900-1920</td>
<td>0.174</td>
<td>0.304</td>
<td>0.257</td>
<td>2.082</td>
<td>0.535</td>
</tr>
<tr>
<td>1920-1940</td>
<td>0.117</td>
<td>0.222</td>
<td>0.333</td>
<td>2.169</td>
<td>0.723</td>
</tr>
<tr>
<td>1940-1960</td>
<td>0.071</td>
<td>0.135</td>
<td>0.413</td>
<td>2.021</td>
<td>0.834</td>
</tr>
<tr>
<td>1960-1980</td>
<td>0.041</td>
<td>0.049</td>
<td>0.602</td>
<td>1.202</td>
<td>0.723</td>
</tr>
<tr>
<td>1980-2000</td>
<td>0.021</td>
<td>0.022</td>
<td>0.697</td>
<td>1.054</td>
<td>0.735</td>
</tr>
</tbody>
</table>

Source: Historical statistics of the U.S; Caselli and Coleman (2001); Bureau of labor Statistic.

Notes: (1) This column shows the ratio of LIS obtained when wages are equal across sectors.
       (2) This column shows the ratio of LIS obtained when wages are not equal across sectors.

Table 2. Parameter values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.020</td>
<td>Long-run growth rate of GDP is 2%</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.032</td>
<td>Long-run interest rate is 5.2%</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.010</td>
<td>Long-run expenditure share in agriculture$^{(1)}$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.056</td>
<td>Long-run ratio of investment to capital to GDP is 7.6%</td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>0.540</td>
<td>Labor-income share in agriculture$^{(2)}$</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>0.330</td>
<td>Labor-income share in non-agriculture$^{(2)}$</td>
</tr>
</tbody>
</table>

Notes: (1) Herrendorf, et al. (2013).
       (2) Valentinyi and Herrendorf (2008).

Table 3. Initial conditions for simulations

<table>
<thead>
<tr>
<th>Values</th>
<th>Targets: Year 1880</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_0$</td>
<td>$A_n(0) = A_n(0)$</td>
</tr>
<tr>
<td>Model 1</td>
<td>0.75z$^*$</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.75z$^*$</td>
</tr>
</tbody>
</table>
Table 4. Performance of the simulations

<table>
<thead>
<tr>
<th></th>
<th>Employment share in Agriculture</th>
<th>GDP share in Agriculture</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SSR</td>
<td>U-Theil</td>
</tr>
<tr>
<td>Model 1</td>
<td>0.590</td>
<td>0.0636</td>
</tr>
<tr>
<td>Model 2</td>
<td>0.540</td>
<td>0.0607</td>
</tr>
</tbody>
</table>

Table 5. Average annual growth rate in the last 50 years

<table>
<thead>
<tr>
<th></th>
<th>r</th>
<th>k/Q</th>
<th>1 − u</th>
<th>pY₀/Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>−0.08%</td>
<td>−0.04%</td>
<td>−1.81%</td>
<td>−1.77%</td>
</tr>
<tr>
<td>Model 2</td>
<td>−0.009%</td>
<td>−0.02%</td>
<td>−1.86%</td>
<td>−1.63%</td>
</tr>
</tbody>
</table>
Figure 1. Numerical simulation without labor mobility cost
Figure 2. Numerical simulation with labor mobility cost
Figure 3. Demand and supply factors governing structural change
Figure 4. Economies with different initial minimum consumption intensity
Figure 5. Economies with different labor mobility cost
Figure 6. Economies with different labor mobility cost and minimum consumption requirements.
Figure 7  Economies with different initial technological levels.
Figure 8. Development accounting between two economies with different technology.