Hyperfine structure in hydrogen: the 21 cm line

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Abstract: Quantum Mechanics are used to describe how hydrogen ground state is split in two when the interaction between electron and proton's spins are considered. Using perturbation theory, the energetic difference between those levels is calculated. The corresponding wavelength is found to be close to $\lambda = 21$ cm. Lifetime of the decay is also calculated considering Fermi's Golden Rule. Some applications are discussed.

I. INTRODUCTION

Hydrogen ground state is split in two levels when the interaction between electron and proton magnetic moments is considered. We expect that the difference in energy between this two levels being of the order of $\mu_p\mu_e/a_0^3 \approx 10^{-6}$ eV, where a_0 is the Bohr radius. As we see, it is very small compared with hydrogen ground state energy ≈ 10 eV.

The energy between this two levels was calculated by Fermi in [1]. In 1944 Hendrik Christoffel van de Hulst suggested the possibility of measuring this radiation in the spectrum of galactic radiation in [2]. Few years later, in 1951, Harold Ewen and Edward M. Purcel from Harvard University measured for the first time this line. Their results can be found in [3].

What is the interest of this radiation? The answer is simple: hydrogen is the most abundant element of common matter present in the Universe and some of it will be in atomic ground state. As we will see, this will have important consequences in cosmological and astrophysical observations among others.

First of all some calculations are made in order to establish what the wavelength of this radiation is and determine the lifetime of the exited level. Then main applications are described.

II. HYPERFINE SPLITTING OF HYDROGEN GROUND STATE

For now on, we will work in the Coulomb gauge, so $\vec{\nabla} \cdot \vec{A} = 0$. To begin with, we consider the hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}' + \mathcal{W}$:

$$\mathcal{H}_0 = \frac{P^2}{2m} - \frac{e^2}{r} + \sum_{\vec{k}} \sum_{\lambda=1}^{2} a^\dagger_{\vec{k},\lambda} a_{\vec{k},\lambda}\ \hbar\omega_k$$
$$\mathcal{H}' = \frac{e}{mc}\vec{A} \cdot \vec{P} + \frac{g_s e}{2mc}\vec{S} \cdot \left(\vec{\nabla} \times \vec{A}\right) + \frac{e^2}{2mc^2}A^2 \tag{1}$$

where $\mathcal{H}_0$ is the sum of the hydrogen electrostatic Hamiltonian and the Hamiltonian of the electromagnetic field in the vacuum with no interaction between them (considering the quantized electromagnetic field inside a box of size L with periodic boundary conditions, taking the quantized vector potential using linear polarization: $\vec{A}(\vec{r}) = \sum_k \sum_{\lambda=1}^{2} \sqrt{\frac{2\pi\hbar c^2}{ckL^3}} (a_{\vec{k},\lambda}\vec{\epsilon}_{\vec{k},\lambda} e^{i\vec{k}\cdot\vec{r}} + a^\dagger_{\vec{k},\lambda}\vec{\epsilon}_{\vec{k},\lambda} e^{-i\vec{k}\cdot\vec{r}})$ and knowing that L will go to infinity at the end of the calculations as it is usually made), $\mathcal{H}'$ is the interaction between the electron and the electromagnetic field and $\mathcal{W}$ is the interaction between the spin of the proton and the spin of the electron that will be given in next section.

Here we are considering for simplicity a static proton ($m_p \gg m_e$), so m will be the mass of the electron. g_s is the gyromagnetic factor of the electron and we take it $g_s = 2$ as QED corrections are not considered.

A. Interaction between electron and proton spins

To study how the ground state is split, we take into account that both electron and proton behave as magnetic dipoles. So suppose that the proton with magnetic momentum $\vec{\mu}_p = \gamma_p \vec{S}_p$ ($\gamma_p = g_p \frac{\mu_N}{\hbar}$ where $\mu_N = \frac{e\hbar}{2m_p c}$ is the nuclear magneton and $g_p = 5.586$ is the gyromagnetic factor of the proton) is at $\vec{r} = \vec{0}$ and generates a magnetic field around it given by $\vec{B}(\vec{r}) = \vec{\nabla} \times \vec{A}(\vec{r})$, where $\vec{A}(\vec{r}) = -(\vec{\mu}_p \times \vec{\nabla})\frac{1}{r}$ [4]. Then,

$$\vec{B}(\vec{r}) = \vec{\nabla} \times \left(-\vec{\mu}_p \times \vec{\nabla}\right)\frac{1}{r} = -\left(\vec{\mu}_p \nabla^2 - \vec{\nabla}\left(\vec{\mu}_p \cdot \vec{\nabla}\right)\right)\frac{1}{r}$$

Inside that field we have an electron with magnetic momentum $\vec{\mu}_e = \gamma_e \vec{S}$ ($\gamma_e = -g_s \frac{\mu_B}{\hbar}$ where $\mu_B = \frac{e\hbar}{2m_e c}$ is the Bohr magneton). The interaction between the two dipoles is

$$\mathcal{W} = -\vec{\mu}_e \cdot \vec{B} = -\left(\vec{\mu}_e \cdot \vec{\mu}_p\ \nabla^2 - \left(\vec{\mu}_e \cdot \vec{\nabla}\right)\left(\vec{\mu}_p \cdot \vec{\nabla}\right)\right)\frac{1}{r}$$

In our case, the hydrogen atom will be in the ground state, and electron and proton will have their own spin, so we consider the uncoupled base $|nlm\rangle \otimes |m_e m_p\rangle = |100\rangle \otimes |m_e m_p\rangle$. In this case we have $\psi_{100}(\vec{r}) = \langle \vec{r}|\,100\rangle = \frac{e^{-\frac{r}{a_0}}}{\sqrt{\pi a_0^3}}$ and now we want to find the eigenvalues of $\mathcal{W}$.

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$$
\begin{aligned}
&\langle 100\ m_e m_p |\, \mathcal{W} \,| 100\ m_e m_p \rangle = \\
&= \langle m_e m_p | \int_{\mathbb{R}^3} \psi_{100}(\vec{r}) \mathcal{W} \psi^*_{100}(\vec{r}) \mathrm{d}^3\vec{r}\, | m_e m_p \rangle = \\
&= - \langle m_e m_p |\, \vec{\mu}_e \cdot \vec{\mu}_p \,| m_e m_p \rangle \int_{\mathbb{R}^3} |\psi_{100}(\vec{r})|^2 \nabla^2 \frac{1}{r} \mathrm{d}^3\vec{r} + \\
&\quad + \left\langle m_e m_p \left| \mu_e^i \mu_p^j \int_{\mathbb{R}^3} |\psi_{100}(\vec{r})|^2 \partial_i \partial_j \frac{1}{r} \mathrm{d}^3\vec{r} \right| m_e m_p \right\rangle = \\
&= - \langle m_e m_p |\, \vec{\mu}_e \cdot \vec{\mu}_p \,| m_e m_p \rangle\ \frac{2}{3} \int_{\mathbb{R}^3} |\psi_{100}(\vec{r})|^2 \nabla^2 \frac{1}{r} \mathrm{d}^3\vec{r} = \\
&= -\frac{8\pi}{3} \gamma_e \gamma_p \frac{1}{\pi a_0^3} \langle m_e m_p |\, \vec{S}_e \cdot \vec{S}_p \,| m_e m_p \rangle = \\
&= \mathcal{A} \frac{1}{\hbar^2} \langle m_e m_p |\, \vec{S}_e \cdot \vec{S}_p \,| m_e m_p \rangle
\end{aligned}
$$

Taking $\mathcal{A} = -\frac{8}{3}\frac{1}{a_0^3}\gamma_e\gamma_p\hbar^2$, and $\mathcal{A}$ have units of energy.

As we are looking for the eigenvalues of $\mathcal{W}$ we have to diagonalize it. Fortunately, it will not be too difficult if we define $\vec{S} = \vec{S}_e + \vec{S}_p$ and then

$\vec{S}_e \cdot \vec{S}_p = \frac{1}{2}\left(\vec{S}^2 - \vec{S}_e^2 - \vec{S}_p^2\right) = \frac{\hbar^2}{2}\left(S(S+1) - \frac{3}{2}\right)$

As S can take two different values, $S = 0, 1$ the ground state is split in two levels: the triplet and the singlet, the last having the lowest energy:

$$
\begin{aligned}
\langle \mathcal{W} \rangle &= \frac{\mathcal{A}}{\hbar^2}\frac{\hbar^2}{2}\left(S(S+1) - \frac{3}{2}\right) = \\
&= \begin{cases} E_+ = E_0 + \frac{\mathcal{A}}{4} & |S M_S\rangle = |1 M_S\rangle \\ E_- = E_0 - \frac{3\mathcal{A}}{4} & |S M_S\rangle = |00\rangle \end{cases}
\end{aligned}
$$

The difference between these two energies is the cause of the radiation we were talking about, which energy is:

$\varepsilon = E_+ - E_- = \mathcal{A} = -\frac{8}{3a_0^3}\gamma_e\gamma_p\hbar^2 = \frac{4}{3} g_s g_p \alpha^2 \frac{m_e}{m_p} |E_0|$

where $E_0 = -13,6$ eV is the hydrogen ground state energy, $g_s = 2$ and $g_p = 5.586$ as said before and $m_p = 1836 m_e$. Then we obtain: $\varepsilon = \mathcal{A} = 5.87 \cdot 10^{-6}$ eV

$$
\Rightarrow \qquad \lambda = 21.1 \text{ cm} \qquad \nu = 1420 \text{ MHz} \tag{2}
$$

III. EXCITED STATE LIFETIME

In the previous section we found the two levels in which hydrogen ground state is split when hyperfine structure is considered. As the atom is coupled to the electromagnetic field, if it is in the E_+ level ($S = 1$) it will decay to the E_- level ($S = 0$) emitting a photon. In this section we will answer what the lifetime of the $S = 1$ to $S = 0$ decay is.

To do this, we are going to take $\mathcal{H}'$ from (1) and consider it as a perturbation of $\mathcal{H}_0$ in lowest order perturbation theory. Recovering $\mathcal{H}'$ expression

$$
\begin{aligned}
\mathcal{H}' &= \frac{e}{mc}\vec{A}\cdot\vec{P} + \frac{g_s e}{2mc}\vec{S}\cdot\left(\vec{\nabla}\times\vec{A}\right) + \frac{e^2}{2mc^2}A^2 \\
&= \mathcal{H}'_1 + \mathcal{H}'_2 + \mathcal{O}(e^2) \\
\mathcal{H}'_1 &= \frac{e}{mc}\vec{A}\cdot\vec{P} \\
\mathcal{H}'_2 &= \frac{g_s e}{2mc}\vec{S}\cdot\left(\vec{\nabla}\times\vec{A}\right) = \frac{e\hbar}{2mc}\vec{\sigma}\cdot\left(\vec{\nabla}\times\vec{A}\right)
\end{aligned} \tag{3}
$$

The first we had observe is that the third term contains e^2 so it is a second order correction. As we are interested in first order correction we will ignore this term. First term will also disappear due to the fact that it does not acts on the spin, as we will see in the next section. Notice that for $\vec{A}(\vec{r})$ we consider the quantized vector potential given before.

A. Probability of the decay to a given final state

Consider we have an initial state $|i\rangle$ and a final state $|f\rangle$ and we want to compute what is the probability of $|i\rangle$ going to $|f\rangle$. By the one hand, $|i\rangle$ is the state of a hydrogen atom in the level with energy E_+ (so with spin $S = 1$). Due to the fact that the energy of the exited state is the same for $|11\rangle = |{\uparrow_e}{\uparrow_p}\rangle$, $|1-1\rangle = |{\downarrow_e}{\downarrow_p}\rangle$ and $|10\rangle = \frac{1}{\sqrt{2}}(|{\uparrow_e}{\downarrow_p}\rangle + |{\downarrow_e}{\uparrow_p}\rangle)$ we can arbitrarily choose one of them to make the calculation. This is consequence of the rotational symmetry of the problem and the fact that we will integrate over all directions as can be seen during calculations (we will comment this later). Then with an appropriate transformation we can choose $|{\uparrow_e}{\uparrow_p}\rangle$.

$|i\rangle = |100\rangle \otimes |{\uparrow_e}{\uparrow_p}\rangle \otimes |\emptyset\rangle$

By the other hand, $|f\rangle$ will be a hydrogen atom in the energy level with energy E_- (so with spin $S = 0$, $\frac{1}{\sqrt{2}}(|{\downarrow_e}{\uparrow_p}\rangle - |{\uparrow_e}{\downarrow_p}\rangle)$) and the electromagnetic field with a single photon with energy $\varepsilon = \hbar c k_0$ with momentum $\vec{k}_0$ and polarization λ_0:

$|f\rangle = |100\rangle \otimes \left(\frac{1}{\sqrt{2}}(|{\downarrow_e}{\uparrow_p}\rangle - |{\uparrow_e}{\downarrow_p}\rangle)\right) \otimes \left|\gamma_{\vec{k}_0,\lambda_0}\right\rangle$

As we said before, $\mathcal{H}'_1$ from (3) does not act on the spin, so we will have

$\langle f|\mathcal{H}'_1|\, i\rangle = (\langle 100| \otimes \langle\gamma_{\vec{k}_0}|)\mathcal{H}'_1(|100\rangle \otimes |\emptyset\rangle) \cdot \langle S = 0\ M_S = 0|\, S = 1\ M_S = 1\rangle = 0$

and it will not contribute to the probability, as said before. So we will study $\mathcal{H}'_2$.

B. Fermi's Golden Rule

The probability per unit of time of the decay is given by Fermi Golden's rule:

$$
\Gamma = \frac{2\pi}{\hbar}\sum_{\vec{k}}\sum_{\lambda=1}^{2} |\langle f\,|\mathcal{H}'_2|\, i\rangle|^2\, \delta\left(E_+ - (\hbar\omega + E_-)\right) \tag{4}
$$

Considering the quantized $\vec{A}(\vec{r})$ and taking $\mathcal{K}(k) = \frac{e\hbar}{2mc}\sqrt{\frac{2\pi\hbar c^2}{ckL^3}}$, we have for $\mathcal{H}'_2$:

$\mathcal{H}'_2 = \frac{e\hbar}{2mc}\vec{\sigma}\cdot\left(\vec{\nabla}\times\vec{A}\right) =$

$= \sum_{\vec{k}}\sum_{\lambda=1}^{2}\mathcal{K}(k) i\vec{\sigma}\cdot(\vec{k}\times\vec{\epsilon}_{\vec{k},\lambda})\left(a_{\vec{k},\lambda}e^{i\vec{k}\cdot\vec{r}} - a^\dagger_{\vec{k},\lambda}e^{-i\vec{k}\cdot\vec{r}}\right)$

It is important to observe that, as $\mathcal{H}'_2$ acts over the vacuum, the annihilation operator will not contribute. If the emitted photon of the state $|f\rangle$ has momentum

$\vec{k}^*$ and polarization λ^*, none of the $a^\dagger_{\vec{k},\lambda}$ with $\vec{k} \neq \vec{k}^*$ or different polarization will contribute because in that case

$$\left\langle \gamma_{\vec{k}^*,\lambda^*} \middle| a_{\vec{k},\lambda} \middle| \emptyset \right\rangle = \left\langle \gamma_{\vec{k}^*,\lambda^*} \middle| \gamma_{\vec{k},\lambda} \right\rangle = 0$$

So for each term $|\langle f | \mathcal{H}'_2 | i \rangle|^2$ of the summation we only have to consider one mode:

$|\langle f | \mathcal{H}'_2 | i \rangle|^2 = \left| \left\langle f \middle| H_{2,\vec{k},\lambda} \middle| i \right\rangle \right|^2$

with

$H_{s,\vec{k},\lambda} = \frac{e\hbar}{2mc} \sqrt{\frac{2\pi\hbar c^2}{ckL^3}} i\vec{\sigma} \cdot \left(\vec{k} \times \vec{\epsilon}_{\vec{k},\lambda}\right) \left(a^\dagger_{\vec{k},\lambda} e^{-i\vec{k}\cdot\vec{r}}\right)$

Taking this into account and in order to calculate Γ from (4), we make the volume go to infinity and we obtain $\left[\sum_{\vec{k}} \rightarrow \left(\frac{L}{2\pi}\right)^3 \int_{\forall \vec{k}} \mathrm{d}^3\vec{k}\right]$:

$$\Gamma = \frac{2\pi}{c\hbar^2} \int_{\forall \vec{k}} \frac{\mathrm{d}^3\vec{k}}{(2\pi/L)^3} \sum_{\lambda=1}^{2} \left| \left\langle f \middle| H_{2,\vec{k},\lambda} \middle| i \right\rangle \right|^2 \delta(k_0 - k) =$$
$$= \frac{2\pi}{c\hbar^2} \frac{L^3}{(2\pi)^3} \int_{\mathbb{S}^2} k_0^2 \sin\theta \mathrm{d}\theta \mathrm{d}\varphi \sum_{\lambda=1}^{2} \left| \left\langle f \middle| H_{2,\vec{k},\lambda} \middle| i \right\rangle \right|^2 \tag{5}$$

The next step is the calculation of the matrix elements $\left| \left\langle f \middle| H_{2,\vec{k},\lambda} \middle| i \right\rangle \right|^2$ were $k = k_0$ is fixed due to the energy conservation but the direction of $\vec{k}$ is not.

$$\langle f | H_{2,\vec{k},\lambda} | i \rangle = \langle 100 | e^{-i\vec{k}\cdot\vec{r}} | 100 \rangle \cdot \left\langle \gamma_{\vec{k},\lambda} \middle| a^\dagger_{\vec{k},\lambda} \middle| \emptyset \right\rangle \cdot$$
$$\cdot \langle \uparrow_e \uparrow_p | \vec{\sigma} \cdot \left(\vec{k} \times \vec{\epsilon}_{\vec{k},\lambda}\right) \left(\frac{1}{\sqrt{2}} \left(|\downarrow_e \uparrow_p \rangle - |\uparrow_e \downarrow_p \rangle \right) \right)$$

As $\lambda = 21$ cm and $r \approx a_0$ (which means that the probability of $\int |\psi_{100}(\vec{r})|^2 \mathrm{d}^3\vec{r} \approx 0$ if we integrate for $r > a_0$), we have that $\vec{k} \cdot \vec{r} \approx 10^{-9} \ll 1$ and it is a good approximation $e^{-i\vec{k}\cdot\vec{r}} = 1$ so $\langle 100 | e^{-i\vec{k}\cdot\vec{r}} | 100 \rangle = 1$

In order to perform the integral (5) it is convenient to choose the axis in this way: $\vec{k} = k_0(\sin\theta, 0, \cos\theta)$. Then, $\vec{\epsilon}_{\vec{k},2}$ can be chosen as $\vec{\epsilon}_{\vec{k},2} = (0, 1, 0)$. Finally $\vec{\epsilon}_{\vec{k},1} = \frac{1}{k_0} \vec{\epsilon}_{\vec{k},2} \times \vec{k} = (\cos\theta, 0, \sin\theta)$. Hence,

$\vec{k} \times \vec{\epsilon}_{\vec{k},1} = k_0 \vec{\epsilon}_{\vec{k},2} = (0, k, 0)$

$\vec{k} \times \vec{\epsilon}_{\vec{k},2} = -k_0 \vec{\epsilon}_{\vec{k},1} = k_0(-\cos\theta, 0, \sin\theta)$

Knowing how do $\vec{\sigma}$ act on $|\uparrow_e\rangle$ and $|\downarrow_e\rangle$ we compute $\vec{\sigma} \cdot \left(\vec{k} \times \vec{\epsilon}_{\vec{k},i}\right) |00\rangle$ (as $|\langle f | \hat{O} | i \rangle|^2 = |\langle i | \hat{O} | f \rangle|^2$ for an hermitian operator $\hat{O}$):

$$\vec{\sigma} \cdot (\vec{k} \times \vec{\epsilon}_{\vec{k},1}) 1/\sqrt{2} \left(|\downarrow_e \uparrow_p\rangle - |\uparrow_e \downarrow_p\rangle \right) =$$
$$= -1/\sqrt{2} i k_0 (|11\rangle + |1-1\rangle)$$
$$\vec{\sigma} \cdot (\vec{k} \times \vec{\epsilon}_{\vec{k},2}) 1/\sqrt{2} (|\downarrow_e \uparrow_p\rangle - |\uparrow_e \downarrow_p\rangle) =$$
$$= 1/\sqrt{2} k_0 \cos\theta (|11\rangle - |1-1\rangle) - \sin\theta |10\rangle$$

Here we see that for $|11\rangle$ and $|1,-1\rangle$ we have the same result:

$$\sum_{\lambda=1}^{2} \left| \left\langle f \middle| H_{2,\vec{k},\lambda} \middle| i \right\rangle \right|^2 = \mathcal{K}^2(k_0) \frac{1}{2} k_0^2 (1 + \cos^2\theta) \tag{6}$$

(For $|10\rangle$ we obtain $\mathcal{K}(k_0)^2 k_0^2 \sin^2\theta$ but the equality raises when the integral is made since $\frac{1}{2}\int_0^\pi (1 + \cos^2\theta) \sin\theta \mathrm{d}\theta = \int_0^\pi \sin^3\theta \mathrm{d}\theta$).

Now, going back to (5) we have

$$\Gamma = \frac{2\pi}{c\hbar^2} \frac{L^3}{(2\pi)^3} \int_{\mathbb{S}^2} k_0^2 \mathcal{K}^2(k_0) \frac{1}{2} k_0^2 (1 + \cos^2\theta) \sin\theta \mathrm{d}\theta \mathrm{d}\varphi$$
$$= \frac{2\pi}{c\hbar^2} \frac{L^3}{(2\pi)^3} k_0^4 \mathcal{K}^2(k_0) \frac{1}{2} \int_0^{2\pi} \int_0^{\pi} (1 + \cos^2\theta) \sin\theta \mathrm{d}\theta \mathrm{d}\varphi$$
$$= \frac{2\pi}{c\hbar^2} \frac{L^3}{(2\pi)^3} k_0^4 \mathcal{K}^2(k_0) \frac{1}{2} 2\pi \frac{8}{3}$$

And now every thing is known, as $\mathcal{K}^2(k_0) = \frac{e^2\hbar^3(2\pi)}{4m^2ck_0L^3}$

$$\Gamma = \frac{(2\pi)^2}{c\hbar^2} \frac{L^3}{(2\pi)^3} k_0^4 \frac{e^2\hbar^3(2\pi)}{4m^2ck_0L^3} \frac{1}{2} \frac{8}{3} = \frac{1}{3} \cdot \frac{e^2 k_0^3 \hbar}{m^2c^2} \tag{7}$$

As this is the probability per unit of time, it is directly related to the lifetime through the expression $\Gamma = 1/\tau$. So, $\tau = \frac{3m^2c^2}{e^2\hbar k_0^3} \approx 3.4 \times 10^{14}$ s $\approx 10^7$ years.

IV. HYPERFINE SPLITTING OF HYDROGEN GROUND STATE INSIDE HEAVY MAGNETIC FIELDS (ZEEMAN EFFECT)

If there exists an external magnetic field, $\vec{B}_0 = B_0 \hat{z}$ the situation changes a little, just because the triplet state is split in three different levels. This is because the new interaction $\mathcal{W}^*$ is

$$\mathcal{W}^* = \mathcal{W} - \vec{\mu}_e \cdot \vec{B}_0 - \vec{\mu}_p \cdot \vec{B}_0$$

and as $\gamma_p \ll \gamma_e$ we will not consider the last term. So now we have to diagonalize:

$$\mathcal{W}^* = \frac{\mathcal{A}}{\hbar^2} \vec{S}_e \cdot \vec{S}_p - \vec{\mu}_e \cdot \vec{B}_0 \tag{8}$$

where we can remember from (2) what $\mathcal{A}$ is. In the ordered coupled base of $|SM_S\rangle$, that is $\{|11\rangle, |1-1\rangle, |10\rangle, |00\rangle\}$, the matrix for (8) becomes:

$$\begin{pmatrix} \frac{1}{4}\mathcal{A} + \frac{1}{2}\hbar\gamma_e B_0 & 0 & 0 & 0 \\ 0 & \frac{1}{4}\mathcal{A} - \frac{1}{2}\hbar\gamma_e B_0 & 0 & 0 \\ 0 & 0 & \frac{1}{4}\mathcal{A} & \frac{1}{2}\hbar\gamma_e B_0 \\ 0 & 0 & \frac{1}{2}\hbar\gamma_e B_0 & -\frac{3}{4}\mathcal{A} \end{pmatrix}$$

The four eigenvalues and eigenstates are (taking $\tan 2\theta = \frac{e\hbar B_0}{m} / \mathcal{A}$):

$$\frac{\mathcal{A}}{4} + \frac{1}{2}\hbar\gamma_e B_0, \quad \varpi_1 = |11\rangle$$
$$\frac{\mathcal{A}}{4} - \frac{1}{2}\hbar\gamma_e B_0, \quad \varpi_2 = |1-1\rangle$$
$$-\frac{\mathcal{A}}{4} + \sqrt{\frac{\mathcal{A}^2}{4} + \frac{1}{4}(\hbar\gamma_e B_0)^2}, \quad \varpi_3 = \cos\theta |10\rangle + \sin\theta |00\rangle$$
$$-\frac{\mathcal{A}}{4} - \sqrt{\frac{\mathcal{A}^2}{4} + \frac{1}{4}(\hbar\gamma_e B_0)^2}, \quad \varpi_4 = -\sin\theta |10\rangle + \cos\theta |00\rangle$$

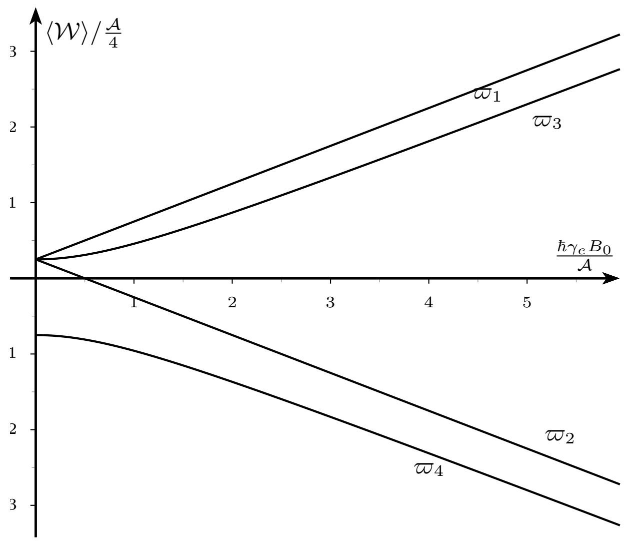

Figure 1: Splitting of hydrogen ground state inside an external magnetic field

V. RECENT APPLICATIONS

Now we are going to outline few relevant applications of 21 cm line in Physics nowadays. We have separated them in three: astrophysical and cosmological applications and precision measurement of constants while testing theories (like QED of General Relativity).

A. Astrophysical applications

Observation of 21 cm line allows us to obtain important astrophysical data. This application is based in calculating the density of atomic hydrogen in galaxies just measuring the intensity received of 21 cm line. We will explain three results:

First of all, this measure and the Doppler effect associated to the velocities of hydrogen clouds has given us an approximation of the structure of the Milky Way (radius, distance from the Sun to Galactic center,...); and also has helped to measure the interstellar medium density[5].

In second place, the 21 cm radiation is not only aplied to study the Milky Way, but also other galaxies and their kinematics. Thanks to this, we know that between 1 per cent and 10 per cent of the total mass galaxies is in atomic hydrogen form. The largest proportions are found in small galaxies $\sim 10^{10} M_{\odot}$[6]. With respect to the kinematics, not only circular motion around galactic centers are considered, but also some noncircular motions are distinguished and studied, such as motion associated to spiral arms, large-scale symmetric deviations, large-scale asymmetries (interaction with other close galaxies,...) and small-scale asymmetries (see [7]). This is certainly one of the most important applications, just because it has meant one of the evidences of the existence of dark matter, as the rotation curves of galaxies can not be explained with common matter alone. This is widely explained in [8].

Finally, we can add that the Zeeman splitting of the levels we have discussed is also used to detect intergalactic magnetic fields. Here Ly-α line is split in different lines and the difference between them is directly related with the magnetic field as we can see in Figure 1.

B. Cosmology: Epoch of Reionization

Recently it has been discussed the importance of the 21 line in Cosmology, specially in the study of the Epoch of Reionization (EoR). Using Statistical Mechanics, a spin temperature T_S is associated to the transition $S = 1$ to $S = 0$, being related to the proportion on hydrogen in $S = 0$ state and $S = 1$ state ($k_B\Theta_S \equiv \varepsilon$ of equation (2); if $T_S \gg \Theta_S$ almost all hydrogen will be in $S = 1$ level whereas $T_S \ll \Theta_S$ implies that it will be in $S = 0$ level).

When matter is decoupled from radiation and CMB is emitted, T_S evolves as kinetic temperature of matter T_K due to collisional coupling, which is below the temperature of the radiation T_γ. Then 21 cm line is observed as an absorption line in the CMB between $30 < z < 150$, being z the redshift.

This changes when density falls as the Universe expands and less collisions take place, so now T_S is coupled with CMB. Then between $10 < z < 30$ $T_S = T_\gamma$ and no information can be obtained with the 21 cm line. For $z < 10$ radiation from stars that begin to form makes T_S grater than T_γ, and from then on the line is detected as an emission line. This is an extremely recent field of research and it is very promising, although challenging measurement techniques and difficulties in the description of galaxy formation have to be faced and it is expected that it will give us constraints for EoR. A large explanation of the 21 cm line aplication to EoR is given in [11].

C. Precision Tests

The frequency corresponding to the 21 cm hyperfine transition is one of the best measured quantities in Physics, establishing that it is [12]:

$$\nu = 1\ 420\ 405.751\ 766\ 7(9)\ \text{kHz}$$

This has motivated using hyperfine transition between atoms to define the second in the International Sistem of Units and building atomic clocks:

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom. [...] This definition refers to a caesium atom at rest at a temperature of 0 K. [9]

Such accuracy made possible one of the experiments that gave support to Einstein's General Relativity in [10]. Placing one hydrogen maser in a rocket that goes up to 10.000 km and measuring the frequency received from the maser by an observer on the ground (and correcting the Doppler effect due to rocket velocity), gravitational redshift was tested with great precision. The experiment was in agreement with theory with an accuracy of $7{\cdot}10^{-5}$.

Even more, as we have great precision in the measurement of the ground state hydrogen hyperfine splitting, this has given a precision test of standard model and the extraction of the values of fundamental physical constants (fine structure constant, the electron and muon masses, proton charge radius,...); see for example [13].

Finally we mention that theoretical expression for the hyperfine splitting in the hydrogen can be corrected:

$$\Delta E^{HFS} = \varepsilon^{*}\left(1 + \delta^{QED} + \delta^{p,\ str} + \delta^{p,\ pol} + \delta^{HVP}\right)$$

where ϵ^{*} is ground state splitting taking into account finite mass of the proton, δ^{QED} represents the quantum electrodynamic contribution, $\delta^{p,\ str}$ and $\delta^{p,\ pol}$ are associated to the fact that proton is not an elementary particle and they correspond to proton structure and polarizability respectively and δ^{HVP} is the contribution of hadronic vacuum polarization.

In order to measure this quantities, hyperfine splitting in 2s and 1s levels are considered due to the fact that some corrections in both levels are equal and vanish when we make the difference, so the constants stated above can easier be determined. This higher order corrections are studied for example in [14], were theoretical predictions are made not only for hydrogen hyperfine structure but also for deuterium and helium-3 ion.

VI. CONCLUSIONS

The hyperfine splitting of hydrogen ground state is one of the most precisely measured quantities of Physics. Interacion between proton and electron magnetic momenta split the ground state in two levels; the wavelength of the transition between them is around 21 cm and the transition has a lifetime of about 10^{7} years that can be analytically calculated. Measurements of this radiation gives us a lot of information of Universe structure and evolution due to the high quantity of atomic hydrogen we can find.

In this sense, information is got from the Epoch of Reionization and from structure and motion of galaxies. Furthermore, from Zeeman effect on hyperfine levels we obtain information about intergalactic magnetic fields. In addition to this, the great precision we have measuring hyperfine splitting in hydrogen ground state gives us precision tests of General Relativity and Standard Model. Even more, 21 cm line has supported the existence of dark matter, a requirement of measurements that nowadays is still theoretically unexplained.

It is amazing how one of the less energetic transitions and with such a long lifetime opens such a wide window to the understanding of the Universe.

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