Micromagnetic study of magnetic droplet solitons in Spin Torque Oscillators

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Abstract: Spin transfer torque allows excitation of magnetization dynamics in ferromagnetic thin films. An electric current flowing through a nanocontact to a ferromagnetic thin film can create spin waves, which are governed by the Landau-Lifschitz-Gilbert-Slonczewski equation. In this project we study the behavior of localized spin-wave modes, which are called magnetic droplet solitons. The system is analyzed with micromagnetic numerical simulations of an array of spins. We study the stability of magnetic droplet solitons. We analyzed the conditions for creation and annihilation and the effect of external applied fields.

I. INTRODUCTION

A. Magnetic Droplet Solitons

An interesting phenomenon occurring in magnetic materials with uniaxial anisotropy is the formation of magnetic droplet solitons, i.e., a localized dynamical excitation consisting of reversed precessing magnetic moments. An example is shown in FIG. 1.

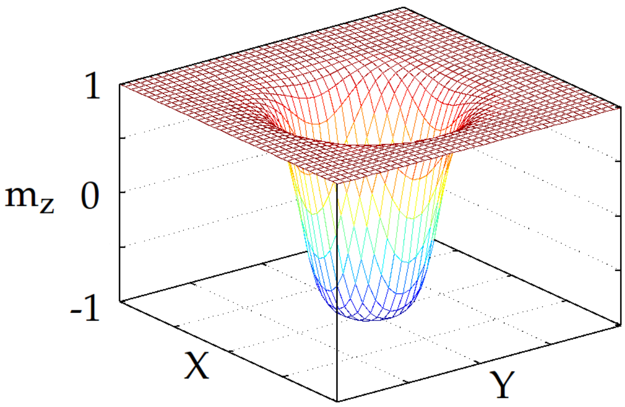

FIG. 1: Example of the result of one simulation. A magnetic droplet soliton has been created and it is stable. Notice that the magnetization is pointing upwards everywhere but below the contact, where it is reversed.

Experimental signatures of droplet solitons have been detected [3] and suggest that these excitations might only be stable under certain conditions. Further, recent experiments have shown a hysteretic behavior [2]. Here, we present numerical simulations that were performed on ferromagnetic films with perpendicular magnetic anisotropy to analyze the stability properties of the droplet solitons, such as creation and annihilation conditions, hysteresis, and the effect of longitudinal and transversal applied fields.

Magnetic droplets had been predicted as a solution for the magnetization dynamics with zero damping. When the damping term is introduced, droplets are no longer stable if no spin torque current is applied. Spin torque acting upon a ferromagnetic film with uniaxial magnetic anisotropy can create stable magnetic droplet solitons. In these droplets (see FIG. 1), the magnetic moment is almost reversed and precesses at a frequency smaller than the ferromagnetic frequency.

B. The Landau-Lifschitz-Gilbert Equation

The Landau-Lifschitz-Gilbert equation describes the magnetic moment dynamics in a magnetic material upon applied fields. The magnetic moment precesses around the effective magnetic field which can be controlled by the external applied field at GHz frequencies, and eventually aligns with it because of the damping. This equation describes the propagation of magnetic moment perturbations through the material, which are called spin waves. The precessional term of this equation (commonly known as Larmor's equation) can be easily derived from quantum mechanics. To do so, let us consider a small volume V inside the magnetic material. First of all, we notice that the classical magnetization $\mathbf{M}$ is the expected value of its corresponding quantum operator $\hat{\mathbf{M}}$,

$$\mathbf{M} \equiv \left\langle \hat{\mathbf{M}} \right\rangle . \tag{1}$$

Now, the Hamiltonian $\hat{H}$ of the volume element of the magnet under an effective magnetic field $\mathbf{H}_{\text{eff}}$ is:

$$\hat{H} = -\mu_0 V \hat{\mathbf{M}} \cdot \mathbf{H}_{\text{eff}}, \tag{2}$$

where μ_0 is the magnetic permeability of free space and $\mathbf{H}_{\text{eff}}$ is the effective magnetic field. According to the theory of magnetism, the magnetization operator can be written as follows:

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$$\hat{\mathbf{M}}V = -\gamma\hat{\mathbf{J}} \tag{3}$$

where $\hat{\mathbf{J}}$ is the angular momentum and γ is the gyromagnetic ratio. We can now use Ehrenfest's theorem, which relates the rate of change of the expected value of a quantum operator (e.g. the magnetization) with the Hamiltonian of the system:

$$\frac{d}{dt}\left\langle \hat{\mathbf{M}} \right\rangle = \frac{1}{i\hbar}\left\langle [\hat{\mathbf{M}}, \hat{H}] \right\rangle. \tag{4}$$

We may now compute the commutator in the last equation, by components. The summation over repeated indices is implicit.

$$[\hat{M}_k, \hat{H}] = -\frac{\gamma^2\mu_0}{V}\left(H_{\text{eff}}\right)_l [\hat{J}_k, \hat{J}_l]. \tag{5}$$

The last commutator is known to be $i\hbar\,\varepsilon_{kls}\,\hat{J}_s$, where ε_{kls} are the Levi-Civita symbols. Therefore,

$$[\hat{M}_k, \hat{H}] = i\hbar\gamma\mu_0\left(H_{\text{eff}}\right)_l \hat{M}_s\varepsilon_{kls}. \tag{6}$$

We can substitute the last expression into Ehrenfest's theorem:

$$\frac{dM_k}{dt} = \gamma\mu_0\left(H_{\text{eff}}\right)_l M_s\varepsilon_{kls}. \tag{7}$$

The last equation can be rewritten in vector form as:

$$\left.\frac{d\mathbf{M}}{dt}\right|_{\text{Larmor}} = -\gamma\mu_0\mathbf{M}\times\mathbf{H}_{\text{eff}}, \tag{8}$$

which is Larmor's equation. In addition to this term, there is a phenomenological damping term which accounts for the loss of energy.

$$\left.\frac{d\mathbf{M}}{dt}\right|_{\text{Damping}} = -\frac{2}{M_s^2T_2}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{\text{eff}}), \tag{9}$$

where M_s is the saturation magnetization and T_2 is the so called transverse relaxation time.

C. Ferromagnetic Thin Films and Spin Transfer Torque

A ferromagnetic thin film can therefore be seen as a huge array of nanometric damped oscillators. Because of the damping, any spin dynamics that occurs in the material will rapidly die out. However, Slonczewski predicted that this damping could be compensated using the spin torque caused by a polarized electric current flowing through a nanocontact [6]. The spin transfer torque is a purely quantum mechanical effect which is strongly related to giant magnetoresistance (GMR) [7], [8]. As an effect of GMR, when a flow of electrons encounters a ferromagnetic conducting metal, those electrons with the spin in the same direction as the magnetization are most likely transmitted while those electrons with the spin in the opposed direction will be mostly reflected.

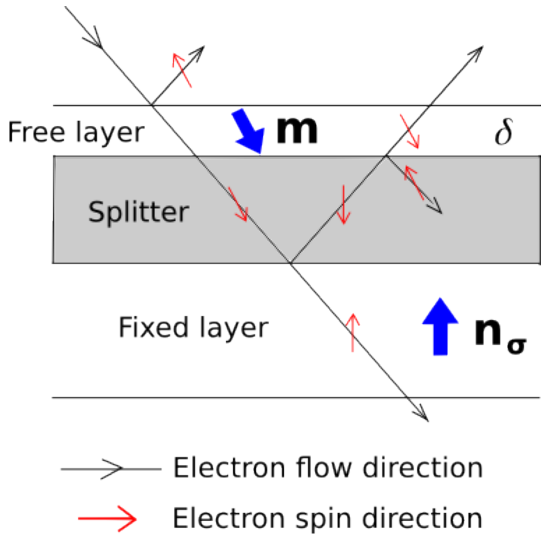

FIG. 2: Working principle of the spin transfer torque. Magnetization is represented by the blue big arrows, and the spin of the electron fluxes is shown as the red thin arrows. In this idealized example, only the electrons with their spin parallel to the magnetization are transmitted through the ferromagnetic layers, while those with antiparallel spin are reflected back.

Now consider the situation in FIG. 2. This situation is extremely analogous to the Stern-Gerlach experiment [9]. The electric current is flowing upwards, so the flow of electrons (which are negatively charged) is moving from top to bottom. The electrons coming from outside are not polarized at all. When an electron arrives to a magnetized layer its spin collapses to either parallel or antiparallel orientation with respect to $\mathbf{M}$. If it is parallel, the electron is transmitted; if it is antiparallel, it is reflected. As usual in quantum mechanics, the probability of collapse to a particular direction is the square of the projection of the spin wavefunction upon the corresponding eigenstate.

The thin ferromagnetic layer, which is called the "free" layer receives from below the electrons that have been reflected by the thick ferromagnetic layer called the "fixed" layer. As they have been reflected, these electrons are polarized in the direction of $\mathbf{n}_\sigma$, otherwise they would have crossed the fixed layer. When they arrive to the free layer, they collapse in the direction of the free magnetization, and therefore the component of the spin wavefunction is lost. This transverse component is proven to be [5]:

$$\mathbf{S}_T = \frac{\hbar}{2M_s^2}\,\mathbf{M}\times(\mathbf{M}\times\mathbf{n}_\sigma) \tag{10}$$

By conservation of angular momentum, this component must be transfered to the free layer. This is the origin of spin torque [6]. Considering that the free layer has thickness δ and the nanocontact has a radius of R_C, then the spin torque reads as:

$$\left.\frac{d\mathbf{M}}{dt}\right|_{\mathrm{ST}} = \frac{I\hbar\varepsilon\gamma}{2eM_s^2\delta\pi R_C^2}\theta(R_C-r)\,\mathbf{M}\times(\mathbf{M}\times\mathbf{n}_\sigma), \tag{11}$$

Here $\theta(x)$ is the Heaviside step function. This results in magnetization-dynamic excitations on the free film, such as spin waves. A way to excite dynamics in a ferromagnetic film is shown in FIG. 3. A nanocontact is built on top of the free ferromagnetic layer and an electric current is driven through it. The fixed layer is placed below, separated by a non magnetic splitter whose thickness is larger than the exchange length of the ferromagnets. The free layer, however, is thinner than the exchange length in order to have a two-dimensional problem.

Adding together the three terms one obtains the complete Landau-Lifschitz-Gilbert-Slonczewski, which is the differential equation governing our system:

$$\begin{aligned}\frac{d\mathbf{M}}{dt} = &-\gamma\mu_0\mathbf{M}\times\mathbf{H}_{\mathrm{eff}} - \frac{2}{M_s^2T_2}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{\mathrm{eff}})\\ &+\frac{I\hbar\varepsilon\gamma}{2eM_s^2\delta\pi R_C^2}\theta(R_C-r)\,\mathbf{M}\times(\mathbf{M}\times\mathbf{n}_\sigma),\end{aligned} \tag{12}$$

where e is the elementary charge, I is the electric current, and ε is an efficiency parameter.

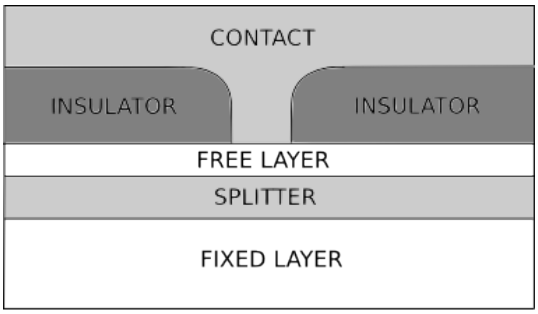

FIG. 3: Simplified version of the spin transfer torque setup. A nanometric hole is performed on an insulating layer (usually SiO_2), thus creating a nanocontact that is used to excite dynamics in the free ferromagnetic layer. The fixed layer is below, with a non magnetic splitter in order to affect the polarization of the moving electrons only.

II. COMPUTATIONAL TECHNIQUES

We performed the numerical simulations using the Landau-Lifschitz-Gilbert-Slonczewski equation, which describes the dynamics of the magnetization under the effect of a magnetic field. In order to make the numerical calculations easier, the equations have been nondimesionalyzed as described in [4].

$$\mathbf{m} = \frac{\mathbf{M}}{M_s}, \qquad \mathbf{h} = \frac{\mathbf{H}}{M_s}, \qquad \tau = \frac{\omega_M}{2\pi}t, \qquad \rho = \frac{r}{l_{\mathrm{ex}}}, \tag{13}$$

$$\omega_M = 2\pi\gamma\mu_0 M_s, \qquad l_{\mathrm{ex}} = \sqrt{\frac{D}{\gamma\mu_0 M_s\hbar}}, \tag{14}$$

where D is the so called exchange parameter. With this change of variables, the equation can be written as follows.

$$\frac{d\mathbf{m}}{d\tau} = -\,\mathbf{m}\times\mathbf{h}_{\mathrm{eff}} - \alpha\mathbf{m}\times(\mathbf{m}\times\mathbf{h}_{\mathrm{eff}}) + \sigma\mathbf{m}\times(\mathbf{m}\times\mathbf{n}_\sigma). \tag{15}$$

$$\mathbf{h}_{\mathrm{eff}} = \mathbf{h}_0 + (m_k-1)m_z\,\mathbf{e}_z + \nabla^2\mathbf{m}. \tag{16}$$

The σ and α parameters are the intensity of the polarized current and the damping strength, respectively. The effective magnetic field ($\mathbf{h}_{\mathrm{eff}}$) has three contributions. The first one is the external applied field ($\mathbf{h_0}$), which can be easily controlled by an electromagnet, for instance. The second one is the anisotropy term, that accounts for the tendency of the magnetization to preferably orient to some particular directions. First, since we are dealing with a thin film, the demagnetizing field plays an important role (shape anisotropy). The shape anisotropy favors the in-plane orientations. There can be other sources of anisotropy though, which can be used to achieve uniaxial (out of plane) anisotropy. These are included in the m_k parameter, which can be controlled by the crystal structure (crystal anisotropy) or by multilayers. If m_k is smaller than one, we have in-plane anisotropy, for $m_k = 1$ the system is isotropic, and if $m_k > 1$, which is the case studied here, uniaxial anisotropy shows up. The energy associated to anisotropy is $\varepsilon_a(\theta) = -(m_k-1)\cos^2\theta$, where θ is the polar angle of the spherical coordinates. A visual representation of this is shown in FIG. 4. The last term describes the exchange interaction between the spatially distributed magnetic moments, and it is the responsible for the collective phenomena. The origin of this exchange is the overlap of the wavefunctions of the neighbor atoms. This effect is modeled for a hydrogen molecule in [1]. The explicit form of the interaction between two atoms is $-J\mathbf{S}_1\cdot\mathbf{S}_2$, and in the continuous form it is taken to be the laplacian of $\mathbf{m}$.

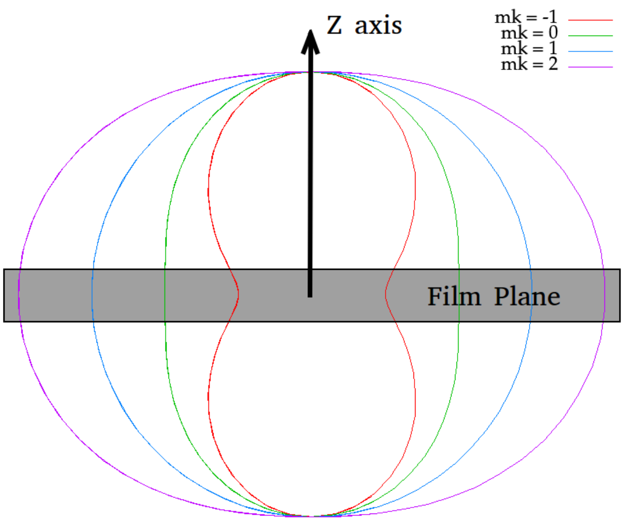

FIG. 4: Anisotropy energy as a function of the polar angle θ for different values of the anisotropy parameter m_k.

The simulations were done with the software *Octave*, using the *lsode* integration algorithm.

III. DROPLET STABILITY AND HYSTERESIS

We have done a study on the stability of the magnetic droplet solitons, and their creation and annihilation conditions of magnetic field and contact current.

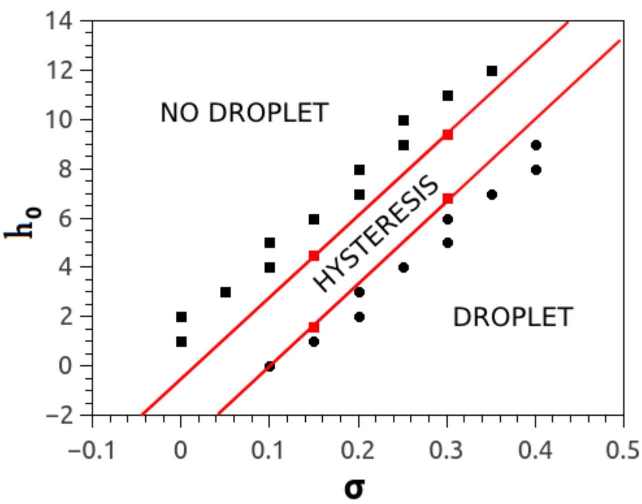

FIG. 5: Regions of stability of the droplets, for different values of the injected current σ and the external magnetic field $\mathbf{h_0}$. In the upper region the droplets cannot exist, while in the lower region droplets are always created. There is, however, a thin band between the two regions where the droplets are neither created nor destroyed (hysteretic behavior).

While the injected current is favoring the formation of the droplet, the external applied magnetic field ($\mathbf{h_0}$) is opposing it. As a consequence, for high enough magnetic fields, droplets will not be created or, if they were alredy there, they will be anihilated. On the other hand, with a high enough current, the droplet will be created and mantained undefinitely. Moreover, there exists a hysteretic behavior.

Hysteresis can be predicted using a simple macrospin model by the LLGS equation, neglecting the interaction between individual spins [2]. By doing it, one obtains the following linear equations for the frontiers of the stability region:

$$h_{0\uparrow} = \sigma/\alpha - (m_k - 1) \tag{17}$$
$$h_{0\downarrow} = \sigma/\alpha + (m_k - 1) \tag{18}$$

Notice that the size of the hysteresis band is $2(m_k - 1)$, which has been reproduced by the simulations. In this macrospin simple model, we are neglecting the exchange interaction between the spins, and therefore we are analyzing the stability properties of a single spin. However, in the simulation we introduce the interaction, so the dynamics becomes much more complex.

In FIG. 5 the three regions of stability are shown. The black dots are an initial bounding estimate of the frontiers that checks whether the minimum value of m_z is higher or lower than 0 at $\tau = 100$ to see if the droplet is present. In order to have more accurate data, another set of four simulations were run with a larger spin array until and the value for m_z at the center of the droplet was plotted as a function of time from $\tau = 0$ to $\tau = 200$. The graphs were visually examined in order to have a better estimation of the tendency of the system.

IV. TRANSVERSE FIELD

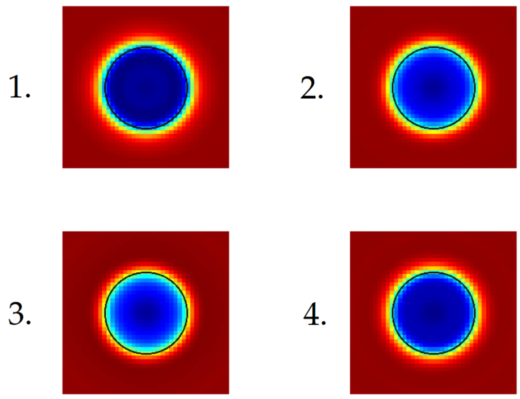

FIG. 6: Color map of the values of m_z under the nanometric contact. The contact position and shape is shown by the black circumferences. The pulsation, which affects the size and depth of the droplet can be seen.

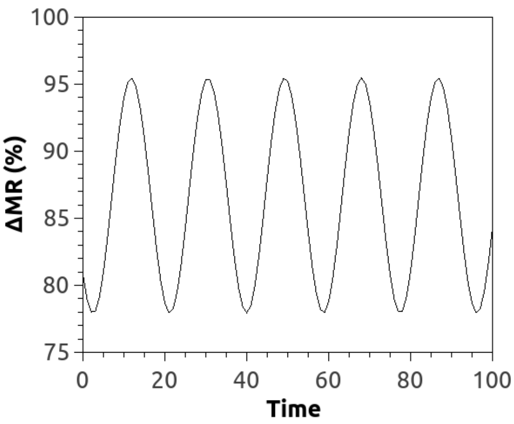

FIG. 7: Variation of the giant magnetoresistance of the contact region with time. Here, 0% represents the maximum conductivity ($m_z = 1$), when the magnetizations of the free and fixed layers are parallel. On the other hand, 100% corresponds to $m_z = -1$, and the magnetoresistance is maximal.

Also, we ran simulations with an external magnetic field and a current polarization (σ) that are not perpendicular to the ferromagnetic film, i.e., adding a component which is not perpendicular to the plane. First, we create a droplet, and then we introduce the transverse field. When the transverse field is added, and after a short transient regime, the droplet pulsates periodically. Namely, the z component of the magnetization inside the droplet, and the droplet diameter itself, increase and decrease periodically, as it can be seen in FIG. 6. Black circles correspond to the approximate size of the nanocontact. The variation of the droplet induces a magnetoresistance oscillation, which might be detected experimentally. Since magnetoresistance has a linear relation to m_z, it can be approximated (up to a constant factor) by averaging m_z over the whole surface of the nanocontact (see FIG. 7).

V. CONCLUSIONS

It is possible to create and control nanometric magnetic droplet solitons on a ferromagnetic layer by using an injected polarized current and with applied field. The behavior of the solitons can be modelled by the Landau-Lifschitz-Gilbert-Slonczewski equation, introducing an exchange term in numerical simulations. Droplets are created and annihilated depending on the current and magnetic field conditions, and they present hysteresis. A transverse field produces a pulsation of the droplet at the precession frequency, changing its magnetoresistance and therefore increasing the output power of the oscillation.

Acknowledgments

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