Sound transmission through simple and double infinite covers

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Abstract: The sound transmission of simple and double infinite walls or covers is studied in this paper. The proposing model is used to describe the acoustic behaviour of these structures. The complexity of the dynamical system requires the use of an electrical analogy that allows using Kirchhoff's laws and the impedance matrix of a quadrupole. The characteristic impedance and the propagation constant of each element are required as input parameters. This model was computed to calculate transmission loss for plane incident sound waves. Then, the results were both compared with the simplified London-Beranek model, which can only be applied to double walls with the same mass, and with experimental data. The simulations also showed the influence of the mass of each wall and the thickness of the air cavity on sound transmission.

I. INTRODUCTION

Prediction models for the acoustical behaviour of simple and double covers or walls are of practical interest in building industry. In general, sufficient sound insulation by air can only be created in buildings when multilayer structures are used. A specific type of these structures is the double wall that is used in practice for other reasons- i.e. thermal insulation.

The transmission of sound in simple or multi-layered walls is produced by the vibration of the structure, thus the mass and frequency are relevant variables. There are other variables that may affect the sound transmission, such as the angle of incidence, rigidity, density, sound damping for the materials and cavity dimensions. In fact, in a real case sound transmission does not only depend on the materials and dimensions of the wall structure. It also depends on the real construction details regarding the walls and their surrounding elements and how the sound wave reaches the walls.

To evaluate the acoustical efficiency of simple or multi-layered structures, transmission loss (TL) is defined as a log of radiated sound power ratio of the analysed structure against free transmission. If the TL value is positive —which means the ratio is less than one—, the wall structure will reduce sound radiation.

Prediction of sound transmission through simple and double-layer walls has been analytically and experimentally studied by many authors using a variety of approximations. All of these studies use some simplifications only treating a small set of parameters. Some of these models are described below since they allow us to understand the acoustic phenomena involved.

London used the conclusions of his previous study about simple walls [1] in order to solve the case of sound transmission through identical double walls [2]. Using this model he became the first person who obtained good results, but his model works only for frequencies below their critical frequency and does not take into account the effect of wall resonances. Subsequently, Josse [3] proposed a simplified method which considers walls with the same composition but different thicknesses.

Different numerical methods as 'Statistical Energy Analysis' (SEA), 'Finite Element Method (FEM) and boundary element method (BEM) have been used to calculate the sound transmission trough single and double walls given the finite dimensions of the structure [4, 5].

Fringuellino and Gulglielmone [6] analysed the sound transmission using a simplified approach based on modelling the sound transmission by the impedances found by the acoustic waves during its propagation through the walls system. This method called Progressive Impedance Method (PIM) represents an alternative to the use of the analytical theory which requires a solution for the wave equations and the study of boundary conditions. However, this simplification requires the knowledge of the characteristic impedance and the propagation constant of each layer; furthermore, it only works if the walls have no limits, and cavity losses are not consider.

More sophisticated new methods, which can incorporate different layer properties, can be found in the bibliography: the progressive-wave method [7] and the transfer matrix method [8].

This paper provides an analysis of simple and double walls. Section II presents two models which describe the acoustic behaviour of the system mentioned before. The H. Arau model [9, 10] is used to describe plane wave propagation through infinite simple and double walls using an electrical analogy. As a comparison, the simplified London-Beranek model [11] is built by extending the London theory to consider the effects of wall resonances.

Section III describes de air cavity and panel mass effects about a sound transmission. Finally, the two models are compare to experimental data [12].

II. ACOUSTIC MODELS

A. H. Arau model

The studied model is based on an elastic analysis of the system and its subsequent electrical treatment through the impedances of each system element in order to reduce the complexity of the model. The electrical treatment is similar to the PIM method [6] although the proposed model will consider the effect of the air cavity.

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1. Simple wall

The study is addressed from a dynamic point of view and considering that the wall has a constant stiffness. If the wall is subjected to the action of dynamic forces produced by a variation of the sound pressure, a modal vibration will appear within the wall. This modal vibration will consist of stationary waves, which are created by overlaying two free bending waves travelling in opposite directions within the wall. According to the elasticity theory, the wall satisfies the elastic bending of plate's equation

$$B\nabla^4 s + m\frac{\partial^2 s}{\partial t^2} = \widetilde{p_1} - \widetilde{p_2} \quad (1)$$

where B is the rigidity, s is the warp displacement produced in the plate, m is the mass per unit area and $\widetilde{p_{1,2}}$ are the incident and transmitted complex sound pressures respectively.

This acoustic analysis does not consider the modal effect, which requires the solution of the equation's wave, applies boundary conditions and also involves the use of Fourier Transforms. Therefore, the infinite wall's condition is applied. The effect of internal friction of the material is considered by using a complex rigidity,$B\ (1 - j\eta)$ where η is defined as a 'loss factor' [13].

Whereas the incident wave is a planar acoustic wave with a certain pulse ω and an angle of incidence ϑ, the next equation is obtained:

$$\widetilde{p_1} - \widetilde{p_2} = \left[jm\omega - j\frac{B\omega^3 \sin^4(\vartheta)}{c^4} + \frac{\eta B\omega^3 \sin^4(\vartheta)}{c^4}\right]\tilde{v} \quad (2)$$

where c is the fluid medium's velocity, and $\tilde{v}$ is the wall's particles velocity.

The first term in eq. (2) corresponds to walls with no elasticity (ideal walls). The second complex term shows the elastic behaviour of the wall and it also depends on the incidence angle. The third term appears as a real linked term to loss factor and only affects the transmission system for high frequencies.

Given that the velocity depends on time through a $e^{i\omega t}$ factor, once it has obtained the dynamic equation (2) it can treat the simple wall from an electrical analogy that allows using the Kirchhoff equation (3). Thus, voltage and current will be equal to the sound pressure and the wall's particle velocity respectively.

$$L\frac{di}{dt} + R \cdot i(t) + \frac{1}{C}\int i\, dt = V(t) \quad (3)$$

where inductance, resistance and capacity are defined as

$$L = m$$
$$R\,(\omega, \vartheta) = \frac{\eta B\omega^3}{c^4}\sin^4(\vartheta)$$
$$C(\omega, \vartheta) = \frac{c^4}{B\omega^4 \sin^4(\vartheta)} \quad (4)$$

At this point, the impedance of each system's element is defined. The impedance associated with the wall, called Cramer Impedance [13], is given by

$$\widetilde{Z_p} = \left[jm\omega - j\frac{B\omega^3 \sin^4(\vartheta)}{c^4} + \frac{\eta B\omega^3 \sin^4(\vartheta)}{c^4}\right] =$$
$$= jm\omega\left[1 - (\frac{f}{f_c})^2 \sin^4(\vartheta)\right] + \eta m\omega(\frac{f}{f_c})^2 \sin^4(\vartheta) \quad (5)$$

where the critical or coincidence frequency is defined as

$$f_c = \frac{c^2}{2\pi}\sqrt{\frac{m}{B}} \quad (6)$$

One should think of the critical frequency defined here as a particular case for normal incidence ($\vartheta = 0°$).

It must also consider the effect of exterior fluid medium that surrounds both sides of the wall. This effect is placed in the radiation impedances, defined as

$$Z_R = \frac{Z_O}{\cos(\vartheta)} = \frac{\rho^f c}{\cos(\vartheta)} \quad (7)$$

where Z_O is the characteristic impedance by the fluid medium and ρ^f is the fluid density.

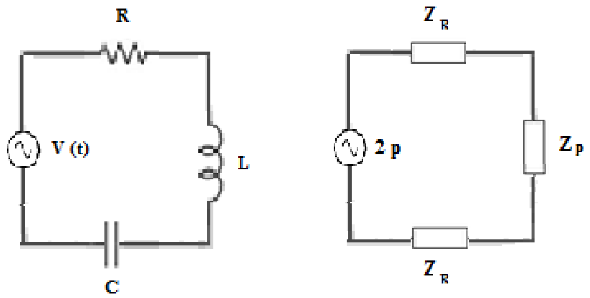

FIG 1. The first figure (left) shows the electrical circuit formed from eq. (3). The second figure (right) describes the simple wall electrical circuit. Extracted from Ref [9].

In order to predict the sound reduction of this structure, one has to take an average transmission coefficient ($\bar{\tau}$) over all incident angles. This transmission coefficient is defined as the ratio between the radiated power per unit area in a transmission system and the free transmission. Assuming a diffuse sound field state and knowing the radiated power of free transmission, the transmission coefficient is obtained as

$$\bar{\tau} = \frac{W}{W_o} = 8(\rho^f c)^2 \int_0^{\vartheta_{lim}} \left|\tilde{Z}\right|^{-2} \tan(\vartheta)\, d\vartheta \quad (8)$$

where $\left|\tilde{Z}\right|^2$is the global quadratic system impedance calculated as $\left|\tilde{Z}\right|^2 = |\tilde{v}/2\tilde{p}|^{-2}$ and $\tilde{v}$ is comparable to the input current of the circuit. This impedance is calculated from Kirchhoff's current and voltage laws and formulating the mesh equation from fig. 1 (right).

In the practical applications, the integration is often limited to a maximum angle of incidence that usually lies between 78˚ and 85˚.

Finally, the transmission loss (TL), a.k.a. sound reduction index (R) is determined as

$$TL = -10\log(\bar{\tau}(\omega)) \quad (9)$$

This parameter is measured in decibels (dB) and gives an idea of the sound that has passed through the wall.

2. Double wall

2.1. Influence of the air cavity

To analyse the sound insulation produced by a double wall system, a study of the air cavity's effect is needed. Following the methodology of the previous subsection, the treatment will begin by proposing a mechanical model and then it will provide an electrical analogy.

According to the elastic theory, vibration due to the sound pressure produces a longitudinal wave. Therefore, the behaviour of the air cavity can be compared to the case of a mass-spring-mass system with constant stiffness k in one dimension. Hooke's law describes the relationship between the deformations produced in a fluid layer containing dynamic forces. If the Newton second law is used to find the temporal dependence of the deformation, this system of equations will be obtained:

$$\begin{aligned}\frac{\partial \tilde{F}}{\partial x} &= -j\omega m_L \cdot \tilde{v} \\ \frac{\partial \tilde{v}}{\partial x} &= -\frac{j\omega}{kh} \cdot \tilde{F}\end{aligned} \tag{10}$$

where $\tilde{F}$ is a complex force associated with the mass-spring-mass system, h is the thickness of the air cavity, m_L is a lineal mass ($m_L = \rho h$) and k is the constant stiffness of the spring and is determinate as $k = \rho^f c^2/(h \cos^2(\vartheta))$ [14].

Then, the equations of the electrical system of a transmission line [13] are presented as:

$$\begin{aligned}\frac{\partial \tilde{V}}{\partial x} &= -(j\omega L + R) \cdot \tilde{\imath} \\ \frac{\partial \tilde{\imath}}{\partial x} &= -(j\omega C + G) \cdot \tilde{V}\end{aligned} \tag{11}$$

The parameters of these equations are related to the eq. (10) as shown in Tab. I. To make both equation systems equivalent, the values of resistance (R) and conductance (G) must be zero. To solve the equations system, the transfer matrix (A) is generated:

$$A = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \tag{12}$$

where the quadrupole parameters (α_{ij}) are defined as

$$\begin{aligned}\alpha_{11} &= \alpha_{22} = \cos(\omega\sqrt{\frac{m}{k}}) \\ \alpha_{12} &= j\sqrt{mk}\,\sin(\omega\sqrt{\frac{m}{k}}) \\ \alpha_{21} &= -\frac{1}{j\sqrt{mk}}\sin(\omega\sqrt{\frac{m}{k}})\end{aligned} \tag{13}$$

Knowing that the mass-spring-mass system is equivalent to a T-network type quadrupole shown in fig. 2 (a) [15], the impedances associated with air cavity structure can be obtained as

$$\begin{aligned}\tilde{Z_1} &= \frac{\alpha_{11}-1}{\alpha_{21}} = j\rho^f c \tan(\beta) \\ \tilde{Z_2} &= \frac{1}{\alpha_{21}} = -j\rho^f c \frac{1}{\sin(2\beta)}\end{aligned} \tag{14}$$

where $\beta = (\omega h/2c)\cos(\vartheta)$.

Mechanical parameters	Electrical parameters
Force, $\tilde{\boldsymbol{F}}$	Voltage, $\tilde{\boldsymbol{V}}$
Panel velocity, $\tilde{\boldsymbol{v}}$	Intensity, $\tilde{\boldsymbol{\imath}}$
Lineal mass, $\boldsymbol{m_L}$	Inductance, **L**
Constant stiffness of the spring, **k**	Inverse of capacity, $\boldsymbol{C^{-1}}$

TABLE I. The table shows the equivalence between the mechanical proposal parameters and the electrical analogy used to solve the equations system.

2.2. Sound transmission of double walls

Once known the air cavity impedances, the transmission loss can be calculated following the treatment described in the previous subsection. Using the impedances presented in eq. (5, 7 and 14), two mesh equations can be built to calculate the global quadratic impedance of the system (electrical circuit shown in fig. 2 (right)). Then, the eq. (8) is used to find the average transmission coefficient, from which the TL value is calculated.

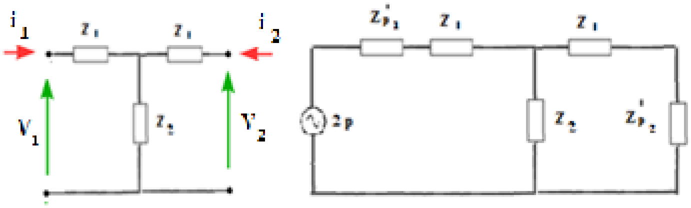

FIG 2. The first figure (left) shows the electrical equivalence to the air cavity. The second figure (right) describes the double wall electrical circuit. The Cramer impedances also include the radiation impedance. Extracted from Ref [9].

B. London-Beranek model

In this section, it is proposed a simplified model to easily calculate the sound reduction index. This model called London-Beranek was built from a method proposed by London and was later extended by Beranek in order to encompass a wider range of frequencies. This model defines sound reduction as

$$TL_\vartheta = 10\log\left(1 + \left(\frac{\omega M_S}{c\rho^f}\right)^2 cos^2(\vartheta)\left(cos(\varsigma) - \frac{\omega M_S}{2c\rho^f} cos(\vartheta)\, sin(\varsigma)\right)^2\right)$$

with $\quad \varsigma = \frac{2\omega c\rho^f}{\omega_0^2 M_S}\cos(\vartheta)$

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{2c^2\rho^f}{hM_S}} \tag{15}$$

where f_0 is the resonance frequency taken at normal incidence ($\vartheta = 0°$), h is the thickness of the air cavity and M_S is the mass per unit area of both panels.

This simplified method is only applicable to infinite double walls with identical mass, M_S, subjected to plane waves. In order to compare the two models presented in this section, the eq. (15) should be integrated for the range of incidence angles considered in eq. (8).

III. SIMULATION RESULTS

The numerical values for both models' parameters are given in Table II. In the two cases, the sound transmission coefficient has been calculated by implementing the trapezoidal method on a wide frequency range. To improve the effectiveness of this method a change of variable $x = \cos(\vartheta)$ has been applied. The critical frequency has been removed from the audible range (i.e. 100 to 4000 Hz) to avoid its TL effects in figures 3 and 4.

Parameter, Symbol	Values (Units)
air density, ρ^f	1.22 (kg·m^{-3})
sound velocity on air, c	343.6 (m·s^{-1})
critical frequency, f_{c1}	10000 (Hz)
loss factor, η	0.02

TABLE II. The input parameters in both simulations.

Figure 3 shows the influence of the mass per unit area on the TL calculated from Arau's model. At a low frequency, a TL dips are observed. These sound insulation dips correspond to the resonance frequency of the walls, and they coincide with the frequencies predicted by the theory [15]

$$f_o = \frac{1}{2\pi}\sqrt{\frac{\rho^f c^2}{L\cos^2(\vartheta)} \cdot \frac{(m1+m2)}{m1m2}} \qquad (14)$$

As the masses of walls are increased, the resonance frequency decreases.

The lineal range follows a behaviour defined by the law of masses [15], which is widely used in buildings. An improvement of several decibels can be observed if the global mass of the system is increased in this range.

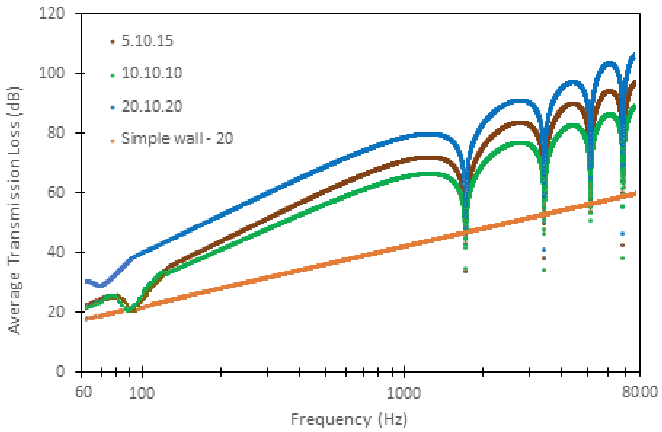

FIG 3. Influence of superficial mass on TL of a simple and double walls. In the legend, the notation shows 'superficial mass of the first wall (Kg/m^2). thickness of air cavity (cm). superficial mass of the second wall (Kg/m^2)' respectively.

The sound insulation dips in higher frequencies correspond to the resonance frequency of the air cavity and can be usually called limit frequency. It can be seen that the position of the dips do not depend on the masses. These local insulation effects occur when the frequency and the thickness of the air cavity are related as $f_k(Hz) = ck/2h$, where $k = 1,2, \ldots$

One can see that, for identical global masses, the TL value improves as the asymmetry increases.

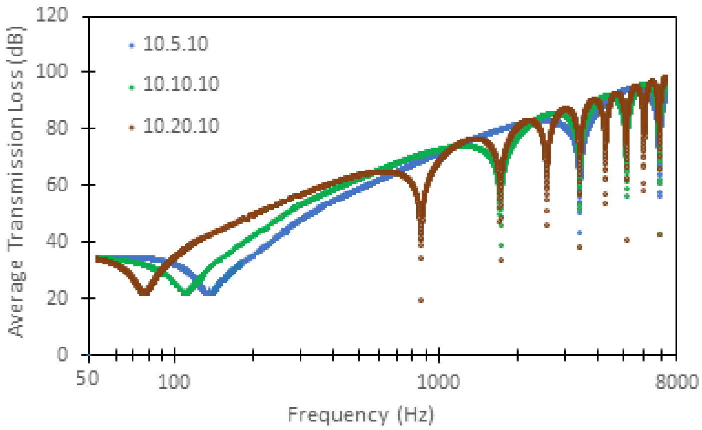

FIG 4. Influence of the air cavity's thickness on TL of a double wall structure. In the legend, the notation shows 'superficial mass of the first wall (Kg/m^2). thickness of air cavity (cm). superficial mass of the second wall (Kg/m^2)' respectively.

The influence of the air cavity's thickness is analysed in figure 4 while keeping the global mass of the wall structure constant. If the frequency of the incident sound on a double wall system is higher than the resonance frequency, the air cavity will absorb part of the sound energy, resulting a greater insulation than the one predicted in a single wall with the same global mass. Thus, by adding an air cavity, the TL will be increased specifically in higher frequencies.

As the thickness of the air cavity increases, the frequency limit moves to lower frequencies. To solve the limit frequency effects, it should be added an absorbent material which would fill the cavity partially or completely. Thus, the sound transmission dips will be smoothed.

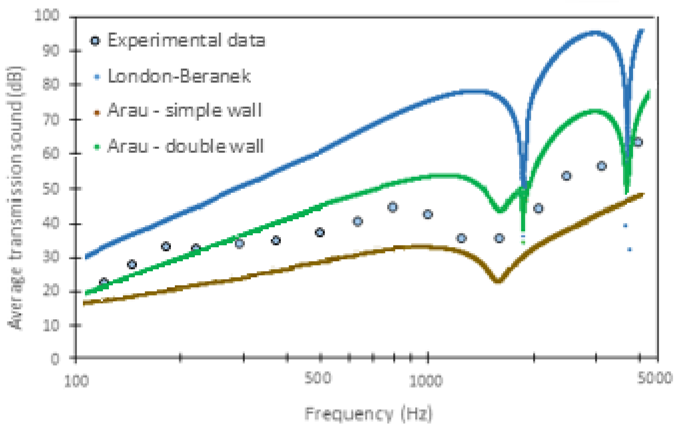

FIG 5. Simulated and measured TL of a double wall; the experimental data has been obtained from Ref [12]

The studied models are compared with experimental data in figure 5, and that is, the experimental data covering the audible range for humans. The analysed system requires the critical frequency of the simulations (shown in Tab. II) to be changed by the value $f_c = 1596\ Hz$.

Double wall models show TL dips corresponding to the limit frequencies. However, Arau's model also shows different dip due to the critical frequency. It is for this reason that in practice one should choose a critical frequency as far as possible the audible range.

Arau model of double wall was in best agreement with experimental data.

IV. CONCLUSIONS

In this paper, the TL of simple and double walls has been investigated. The simulations confirmed that the sound insulation provided by a double wall is more efficient than a single wall with the same global mass.

The influence of the air cavity and masses on the TL has been studied. In lower frequencies, thicker cavities improve TL, while in higher frequencies a thinner cavity is preferable. Keeping global system mass constant, the transmission loss is better —especially in higher frequencies— when walls have a different mass.

TL values were computed using Arau and London-Beranek models and compared with experimental data. Arau model is viable because despite being a little more complex than the London-Beranek model, it can achieve better results.

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